Introduction to Logic
Department of Humanities and Social Sciences
Prof. A.V. Ravishankar Sarma
Indian Institute of Technology, Kanpur

Lecture - 26
Resolution and Refutation Method

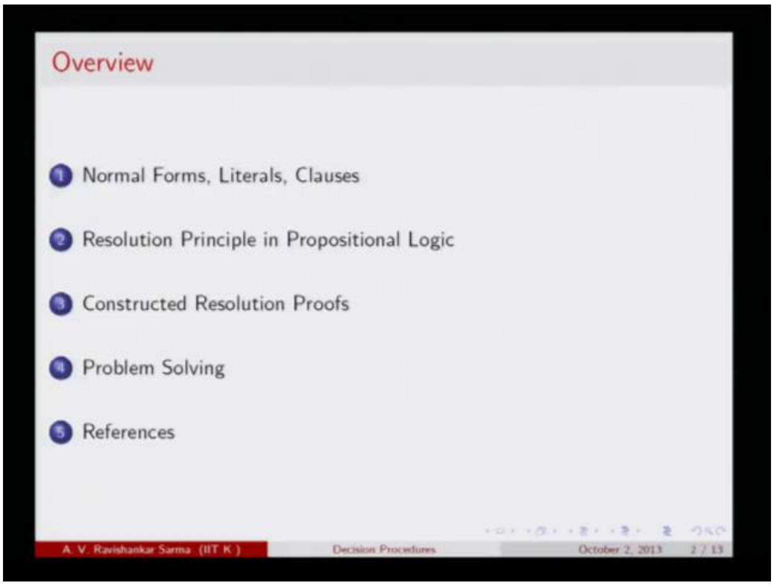
Welcome back, in the last few lectures we discussed various decision procedure methods, methods in the context of propositional logic for instance. We first began with, the most simplistic kind of method. So, that is considered to be the truth table method, and then we saw that when, the number variables increases from 2 to 3 a maybe 5 etcetera and all. So, number of entries will also increase, the number of rows will also increase then it will very difficult for us to manage and in that context we discussed about, another method his directly relevant to the truth table method. So, that is, the indirect truth table method instead of checking all the rows for the validity of a given argument we checked only a few rows, where you have 2 premises and a false conclusion.

If a come across that particular kind of a think we said that it is invalid argument and in the second method. So, that is due to semantic tableaux method. So, this depends upon constructing a counter example; that means, so if we want to show that the given argument is valid, then what you need to do is, you have to deny the conclusion and then you need to construct a tree, based on some kind of a tree rules. And then when you, come across a situation where, there is some contradiction in the branch, then the branch closes means, we have established that not x is unsatifiable; that means, x has to be valid, x has to be true. That is the second method, which is discuss and the third method is synthetic kind of method. So, that is due to natural deduction method.

So, where we employed some kind of basic principles of a logic, such as Modus ponens, Modus tollens, constructive dilemma etcetera and all. And then we proved lots of theorems and all. And then we will also discussed something about, reducing the given preposition logical formula into its corresponding conjunctive and decentive normal form. Any proposition given, propositional logical formula can be reduced to it is corresponding conjunctive normal form that is, conjunctions of disjunction or disjunctive normal form that is digestion of conjunctions.

So, once you reduce the given formula into conjunctive and disjunctive normal forms. We can talk about, the satistability, and then once we established unsatisfiable; that means, not x is unsatisfiable then; obviously, x has to be valid. So, today we will discussing another important and interesting method, which is widely you use in the context of Automatic theorem improving in the computer science, that is the name of this method is called as Resolution refutation method.

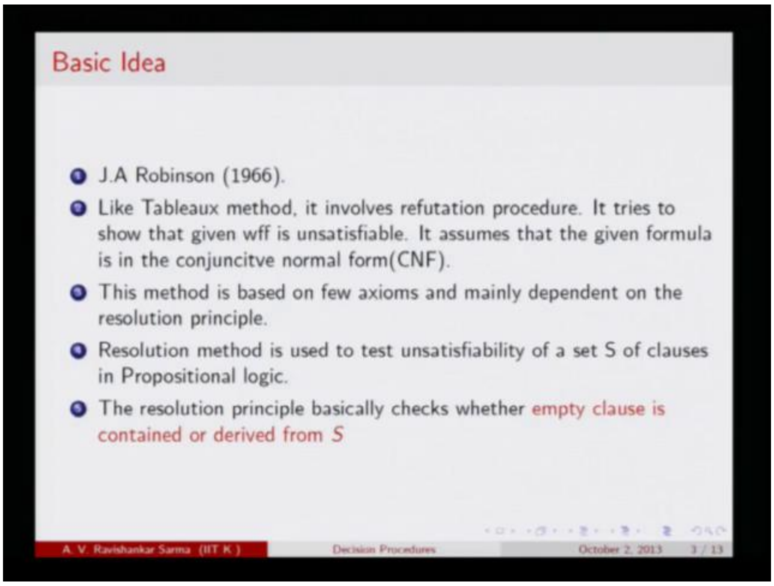
(Refer Slide Time: 03:00).



So, what we will be, basically doing is like this. So, this is a method which works for only those formulas, which are conjunctive Normal forms. So, if say it can also be called as a special case of a formulating kind of conjunctive we normal forms from a given formula. So, first we will talk about what you mean by the normal forms, we discuss in greater details in normal forms in the last few classes. That is conjunctive and disjunctive normal forms are considered to be normal forms. And then we will introduce some of the definition; such as littler clauses etcetera, and then we talk about, what occupies central position of this resolution refutation method. That is, the resolution principle, and then once you state this Resolution principle then we will talk about construction some proofs based on the resolution reputation method. Then will do some kind of bit of problem solving to no let it with this particular kind of methods.

So, this is also consider to be one of the important decision procedure method. As we, the case other decision procedure methods we can talk about, whether or not a given well form formula is a Tautology that is the valid whether or not the formula is valid or you can you can talk about when 2 groups of statement that consistent to each other there is a satisfied to each other. All these things 1 can come to know with the help of this particular kind of decision procedure method. As we case the other methods Resolution Refutation method is also consider to be sound consistence and even sound and system.

(Refer Slide Time: 04:55).



So, this based idea is like this, it was introduced by John Abraham Robinson in the context of automated theorems proving. So, when we talk about predicate logic in Resolution Refutation method in the context of first order logic. That is, the predicate logic we will a more about Abraham Robinson direct what on Resolution Refutation method. So, this method has extended has reduced in the Computers Science. So, like a semantic Tableaux of methods, it involves refutation procedure; that means, time to conjunctive counter example. So, in this method we try to show that a given a formula is unsatisfied.

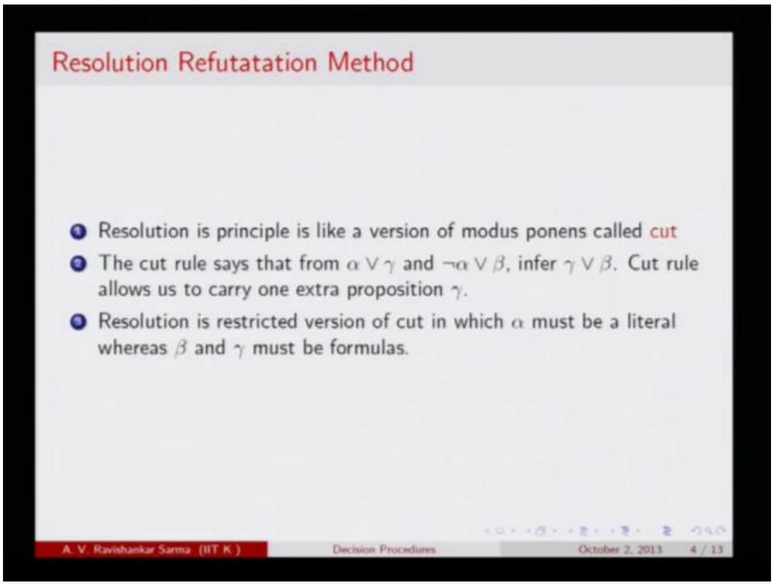
So, that the semantic Tableaux of methods what we did, we have denied the conclusion, and then we come of the branch closer; that means, we have the un satisfability of a given well form formula so; that means, the original formula has to be invalid formula. In the case of a premises and a conclusion in the format, we denied the conclusion then the conclusion; that means, x is has to be truth. So, this method assumes that, given formula is in the conjunctive the normal form. So, all of us no that any given formula can be either express in the CNF that is the disjunctive normal form, are it can be also expressed in terms of conjunctive normal form.

So, conjunctive normal form is a normal form, in which a only a negation and disjunction and conjunctive off course, this is no implication and double implication which exist the conjunctive normal form. So, this conjunctive normal form is conjunction of disjunction, if each if any cross that is C1 C2 and C3 etcetera. Were each even is a combination of for some dissection on all. So, in the dissection for formula contains littler next negation then; obviously, the dissection is do not be truth. Then if, all the dissections are truth then; obviously, conjunctive normal form when hence it is valid, but here, what we will be do is is that, given in formula in the conjunctive normal form.

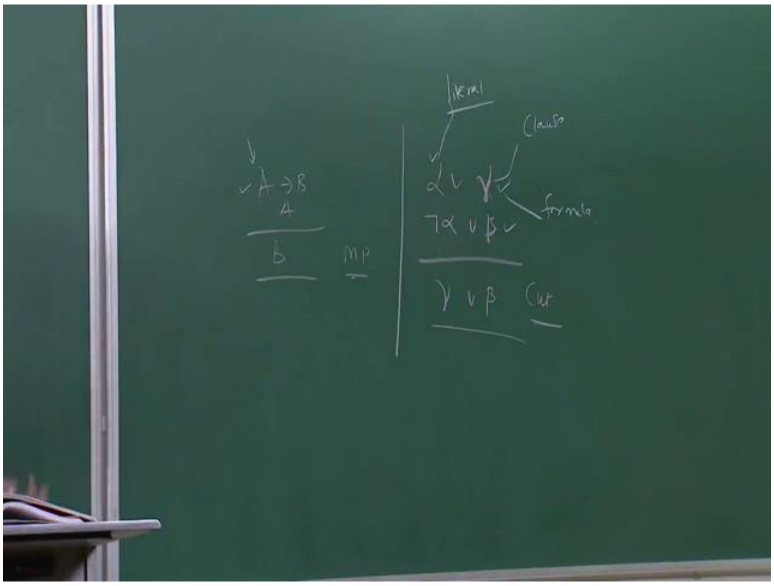
So, we will be will be applying Resolution principle, which will be taking about a while from. And then will derive some kind of contradiction in all that is empty set empty clause. So, this we will talk in detail avail from now. So, now, this method is based on very few axioms as many axioms as in the case of axiomatic method, which will be talking about it next few classes, but it most few absence and its mainly dependent on Resolution principle.

So, resolution method is used to test as in the case of Semantic Tableaux method, it used to set test unsatisfiability of a set of clauses in the propositional logic. So, there some clauses, C1 C2 C3 etcetera and all; all these things combined together, will lead to a some kind of contradiction. So, the resolution principle basically checks, whether empty clause is contained or it is derived it in thing the language S. So, that is what it checks in all. So, the Resolution method is based on the resolution principle

(Refer Slide Time: 08:50).



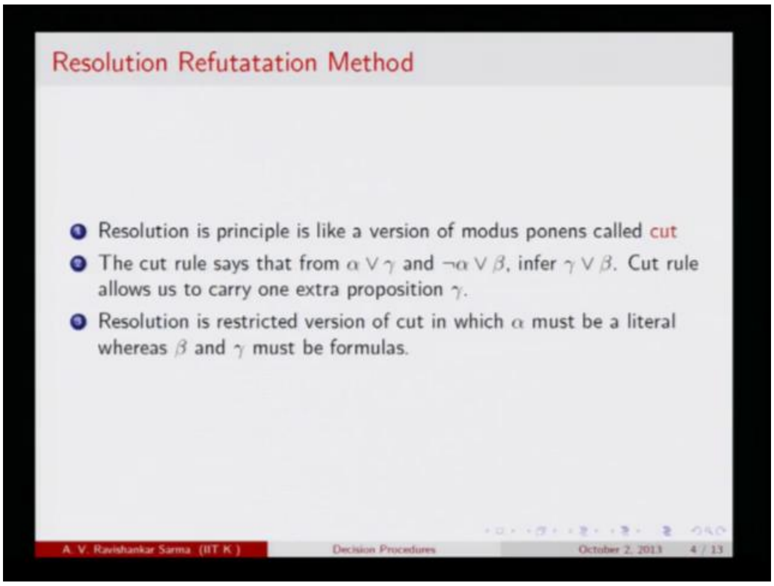
(Refer Slide Time: 08:56).



So, Resolution principle is like, a kind of Modus ponens, this is also called as cut rule. So, what is modus ponens we have A implies B and you have A and then A gets detach. Now what follows is B. So, this is what Modus ponens rule. There is another rule which is nothing but, instant of Modus ponens, which is called as a cut rule. So, for example, if you have alpha r gamma and not alpha r beta. So, from this you can infer gamma r beta. So, happen here is it that, you have literal alpha and it is negation in the second formula. This 2 cancel each other, and then what remains is only this things gamma and beta.

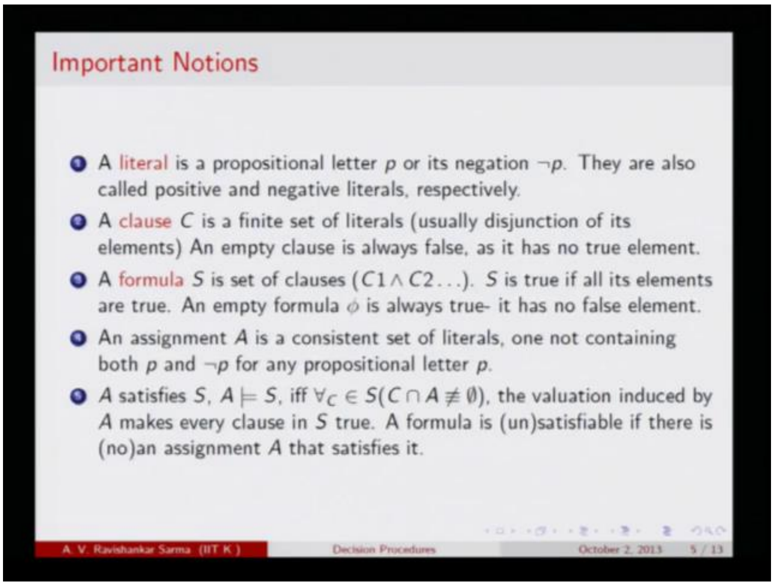
So, this is another kind insistent of Modus ponens rule, is called has cut rule. It is cuts alpha and not alpha and not alpha loses, and then it nor remain in the final constitution all; what remains in gamma and beta. This rule is allows us to carry instated of, in the case of Modus ponens, it will not a loved this particular kind of thing A because it get attached in the Modes pone's in the rule, but 1 of the advantage of the this cut rule is Regulation principal, it is that 2 this 1, we can add 1 more kind of preposition here its comes the with the extra composition extra gamma, that is the 1 advantage of this rule.

(Refer Slide Time: 11:32).



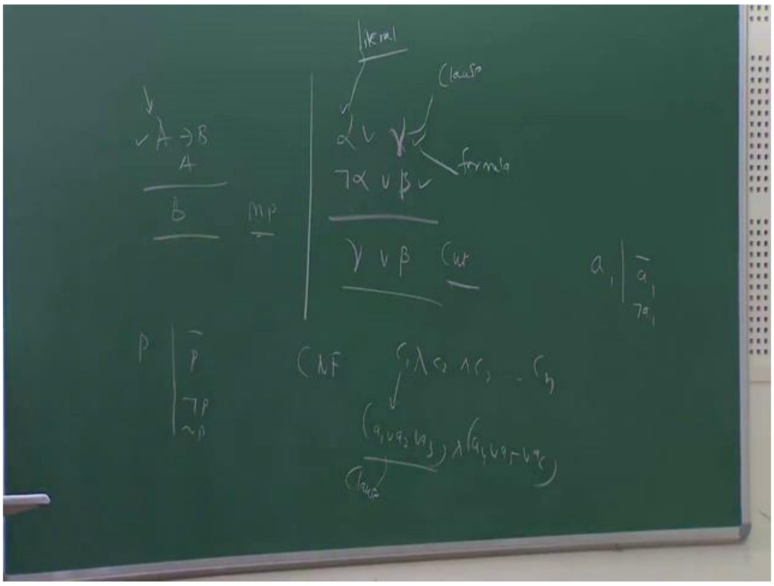
So, now, the regulation is verification of particular kind of cut rule in which alpha must be a literal whereas, b and must be formulas. So, this can be any formulas an all, but alpha that exist here, as to be a literal. Now, we need talk about what need by literal, what been mean by close and what mean by a formula extra and when, you say that a given well formula. Is in the conclusion from each note that is method works only, in case of the formula which already in the normal formulas.

(Refer Slide Time: 11:45).



So, now, here has some of the important notes and before going in to the details of the regulation methods of refutation method. So, we need this important notations it is go in the details of 1 by 1; first important things is that I need a talk about, a literal is a profession letter simply, when you positive a and the negative a the navigation of the a not p usually in the language of logic what we right it is.

(Refer Slide Time: 12:16).



This it is thing p and it complement and this is same as not p in some test books it is writer's as till p. So, am using this particular kind of symbol. So, it is negation of p you can also write it or p bar. So, now, this is consistent set of literal; literal like this a proposition verbal proposition letter p q r and extra, and complement r not p r p bar. So, please consider possible literal and p bar not consider to be a negative letter. So, now, this is what we mean by literal, it what use by literal this is what by use in the re valuation method. So, this is the section instead of cut rule of p and the last light. Now, the second thing is which important and the of the regulation recitation method, is a close, see is consider to be final set of literals is the combination of has can be p or q or r, we can be p bar or q bar r an. If can be of course, an we can expression same thing as not pr not q r r.

So, literals combine together and form a close. So, usually in the case of congestion normal forms, this process will be this is the action. So, what is a conjunctive normal form; conjunctive normal form is like this C1 C2 C3 extra Cn; where each C1, each consider to be a dissection a3 and a4 and a 5 just for take up understand we are write in like this. So, this whole thing is consider to be close. If it take 1 this individual letters in to consideration their consider to be literals. So, if and positive thing and negative literals stand for this 1 are it can even, written has this 1 not a1 that is the exact opposite 1 the letters a1.

So, this is what we mean by clause in the case of CNF obvious that, cone C3 extra, all consist of disjunction it consistent. Now, it is the important things of which we note and empty clause where is the any element very literals that is the always consider false, has it has true element. So, that is important things which 1 we needs to no, empty clause always good be for clause because it do sent consider true elements. So, now, a formula S is consider to be set of clauses now, we need to talk about what a mean by a formula. A formula S is a set of clauses C1 C2 Cn extra, the whole thing the conjunction of these clause will become a formula.

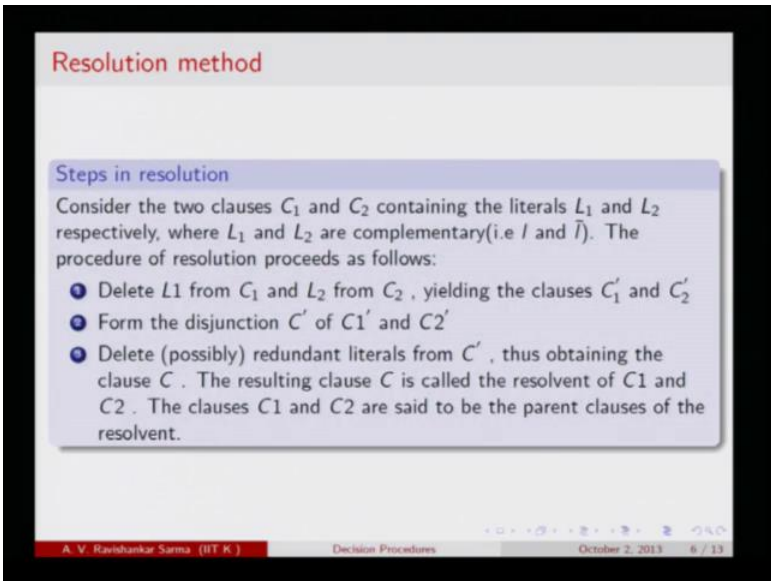
So, that is going to be true as we all as a convention is going to be only true; that means, all the are going to be true in all; that means, C1 C 2 and Cn is true if all of elements are true. And an empty formula, which is written as empty set as does not as any clauses is always true it as know falls element all. Some other is a mind would difference between empty clause and empty formula does not consists of anything a not does always going to be true consists of anything we can all going to be true we can empty set is a subset of all sets at all. So, since it has no false. So, these are the few things which need to be do not.

Now the third think is that an assignment A is consists set of letters 1 not containing both p and not p for any proposition letter p. So, suppose if are saying value to a given formula. So, that has to be we can r f both t and both f. So, that is what is says it has to be c consistence set of later. So, know the rotation that we a make use of it, in that relation deputy method.

So, when a formula is A satisfies particular kind of a S set up formulas especially, details written as the A double turns. If an only a all see for all C is belong to your language Sand C and A the intersection of C and A is not going to be empty. That is empty, that is going to that is false and all the as is the case of, the second 1 empty clause is always going to be false. So, he has to ensure that C and intersection of C and A non empty, that is 1 valuation in which the formula is going to be true. That is want to be need to t he satisfied the valuation use by a makes every clause in S true. The true whether only the same formula is going to be true, because is in the conjunctive normal form which C1 C2 C3 all these things have to the true.

So, that in all your conjunctive normal form has to be true. So, now, a formula is going to be unsatisfable, if there is no assignment A is that the formula is going to be true. So, these things, which we all ready no. So, 1 has to a ensure that, there is no empty clause, empty clause is a is going to be the formula is going to be false, but empty formula is always is going to be true. So, now, this is a resolution method.

(Refer Slide Time: 18:25).



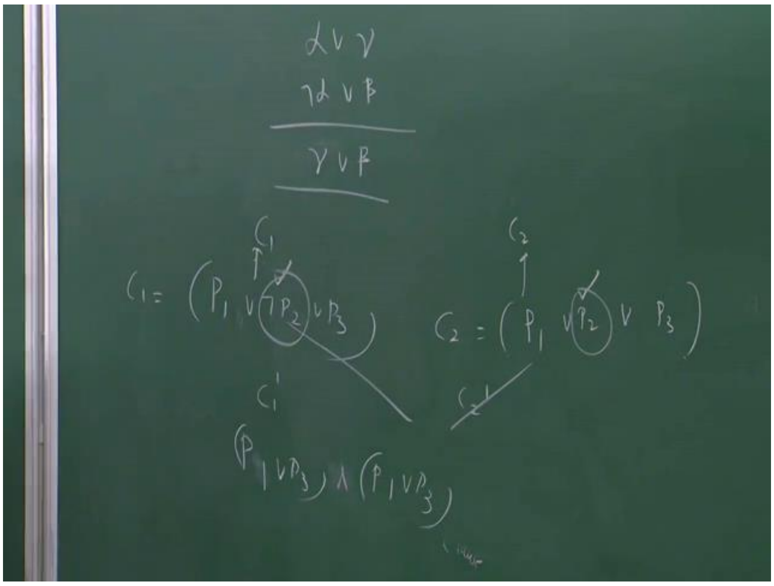
So, what are the various steps that are they other things which be employee in the resolution method. What is that we are trying to resolve. So, now, considering the 2 clauses C1 and C2; let us say, these are the 2 clauses and first up all need to the note that these are place to only conjunctive normal form. It is only, in the conjunctive normal form we can talk about resolution refutation method. Now, consider the 2 clauses even L1 and L2 respectively and L 1 and l 2 are complimentary to each other, 1 letter is L and other letter is literacy L bar it is not L; there are the 2 literals we have.

So, we have 2 clauses C1 and C2 and the literals the exist in this C1 C2 all, like this. A literacy there in the given formula. So, now, the resolution procedure is as follows we will be solving some more problems of that will understand this particular kind of method. So, now, the first step that is involved here is that delete L1 from C1 and L2 from C2 yielding clauses C1 prime r C2 prime. So, then there is a first think we need to do, first we need to who elements the literals. So, where C1 consists of a literal L and C2 consists of literal not

L. So, that why L and not L leads to f and all that exhilaration kind of a. So, know to form the distinction of C prime of C1 prime and C2 prime.

So, now, then what will do is delete if there is any classes like p r p r p etcetera in all. That reduces to only p; thus obtaining a some kind of a final clause C, the resulting clause C is called as the re solvent of C1 C2. So, that C is called re solvent of C1 and C2 set to be parent clauses of the re solvent. So, let us consider some examples of that you are understand the is method in a better.

(Refer Slide Time: 21:01).

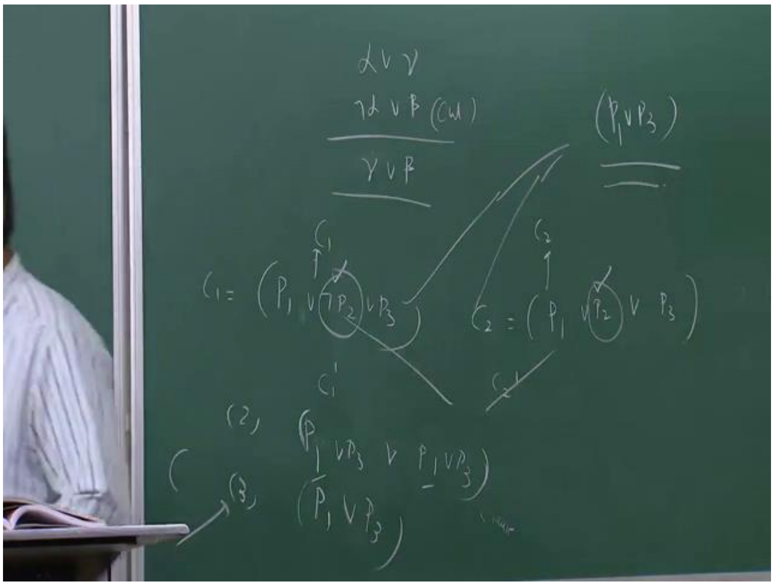


So, let us consider some examples of that, you are understand the is method in a better. So, the idea is the it is based on the rule alpha r gamma. Not alpha or beta and then u get alpha gamma r beta. So, what is done here, so let us consider a s some examples P1 or P2 or P3 this is called P3 and then C2 another clause like P1 or P2 or P3. So, now, these 2 gets resolved and all the first step that we need to do is to find out a literal and its negation now. So, in this case we have now, P2 here, and then you have P2 here. So, that vanishes at all.

So, now, there is a first step that we need to make use of the first steps tells us that delete L1 from C1. The literal here is, what is need to delete is not P2 here this later and then in C2 need to delete its corresponding positive letter that is p2. So, now, we have to deleted that particular kind of think; now, in the second step form the digestion of these 2 things. So, this is consists now, it is a change the formula C prime C1 prime are C2 prime. So, let us consider this is C1 and this is consider to be C2. So, now, the literals got deleted in the all, now it becomes C1 prime this formula has change to C2 prime we deleted the literals.

So, now, as from disjunctions of whatever remains here. So, that is the P1 are P3 are P1 are P3. So, that is what happens, soft training the kind of the clause C as a re solvent as C1 and C2. So, now, so this is consider as this is 1 clause and this another clause. So, this goes away and then form disjunction whatever remains here is considered to be is disjunction and then remains here P1 are P3 is going to be another disjunction.

(Refer Slide Time: 24:05).

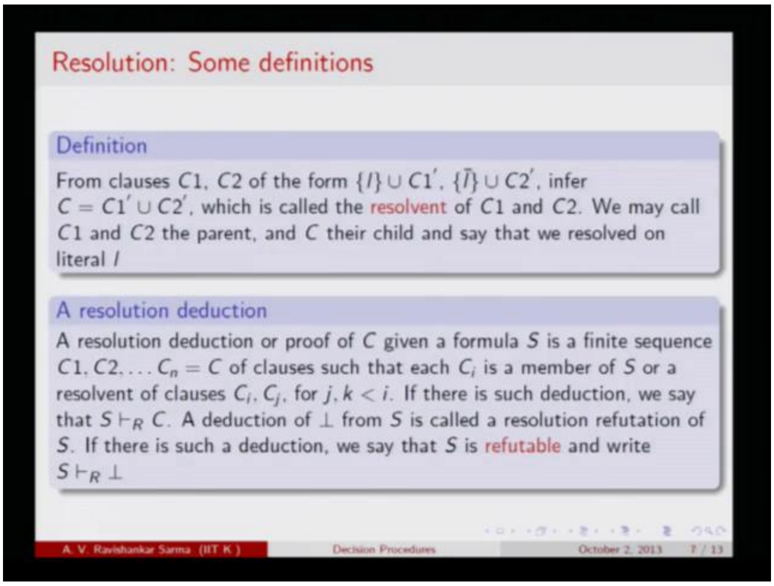


So, now, what he need to for this in the second step. So, that is the removing the literacy negation, you form the disjunction of all these things and all. So, that is P1 are P3 are P1 and are P3. So, now, in the third step, since we have P1 exists trice in all here. So, that is what is the third step, delete the redundant literals from C prime. So, what are the redundant literals here P1 is twice even if u 1000 s of times and all, even are P1 P2 P3 P1 are P1 are p1 is always same as P1. So, now, this reduces to P1 are P3 is again use twice and all, it is simply P3. So, now, this is called as re solvent of C1 and C2.

So, these to gets resolved and all and then we will get, P1 r P3. So, this is what we need by a the re solvent 2 clauses. So, this makes use of for this particular kind of rule, which is called as cut rule. So, now, let us consider some more examples. So that, we will understand this idea in a better way. So, as the method is very clear in 1 clause as a literal L and another clause we have another literal with a negative sign; that means, it has a P here, if have not P in the other clauses. So, what he need to do is he need to delay the literals, which are positive and negative and all. Then these are the things the group together with the help of disjunction that is the second step.

So, after a grouping a in the form of a disjunction, then what he need to look for is whether this formula consists of any redundant literals and all p r p r p etcetera. And all if u the same letter as a twice a trice a it is a same as P1. So, that reduces he 2 just simply literal P1 P1 are P1 are same as P1. So, now, the clauses C1 and C2 that resultant clause after formulating is way is called as a re solvent of these 2 clauses. So, that is what is consider to be in the case, the same kind of first thing can be applied in the simplifying a digital searching circuses then all provided they are in conjunctive normal form.

(Refer Slide Time: 26:43).



So, there some definitions 1 needs to follow before going in to it is the examples and all let us talk about some more definitions. So, from clauses C1 C2 what are the C1 and C2; C1 consists of some kind of disjunction. Now, P1 are P2 P3 etcetera. So, they are of this particular kind of form, C1 consists of a literal L union some kind of form C1 prime. So, there is a first thing and C2 consists of its negative literal L bar union C2 prime. So, now, from these 2 u can in to C1 prime union C2 prime, which is what we called it as a re solvent of C1 and C2. So, we may call C1 and C2 as parent and C as their say that we need on literal L, anyway because L and not L needs to f. So, that goes way. So, we resolved on that particular literal L.

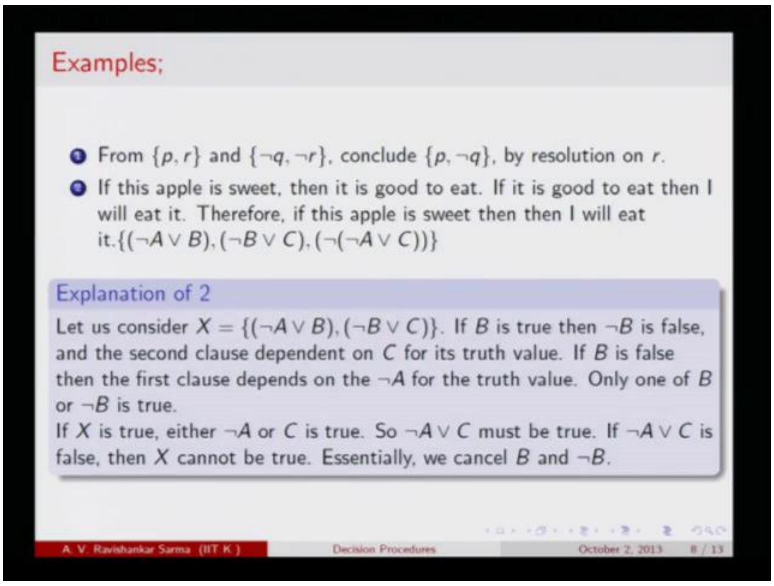
So, this is what you mean by re solvent. So, now, a resolution deduction process is like this.

So, is a resolution deduction these also be considered as a proof, as a natural deduction here what we are reducing is the re solvent and all. A resolution deduction as a proof of see given a formula S; that means, we got a see from S a nor there is C from S it is a finest sequence like C1 C2 Cn and ultimately leads to see C of clauses such that, each C I; that means, each step your proof, is a member of S r are solvent of clauses Ci Cj and Cg. So, there are 2 re solvent then all then either it should be belong to a member of it has to be a member of S it has to be a or some theorems already it proved in S R it has to be a re solvent by using this resolution principle is a insects of cut of rule.

So, only in this case u called it has; that means, it has to obtain all the steps of your proof, as a result of applying the resolution principle are it has to be already member of S. I mean it has to be theorem some other axiom or something other. So, resolution method involve very few axioms, and then mostly depends upon there is a resolution principle, each time will be applying resolution principle you will getting the corresponding. The re solvent and all that adds to that clause other clause will lead to another re solvent etcetera. In that since this proof is also considered to be a effective proof. So, because an effective proof is a proof, which ends in finest step and finest interval time.

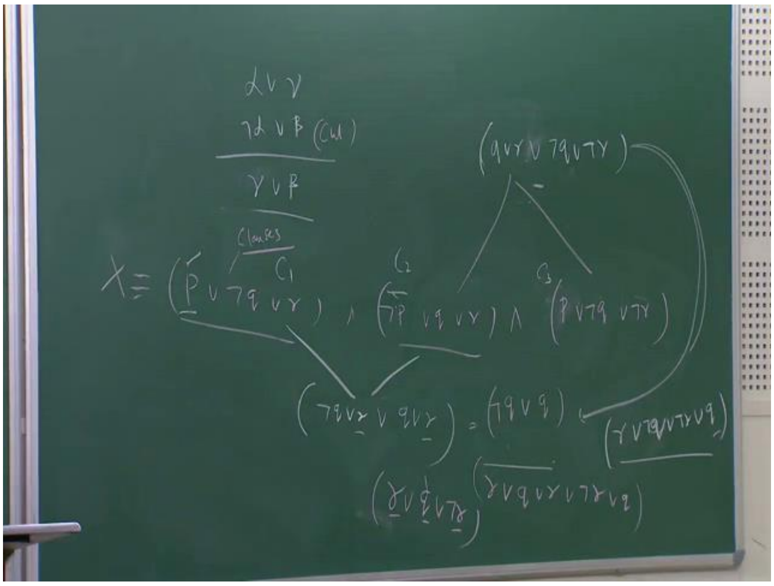
So now, a deduction of empty set empty clause from S is called as resolution refutation of S. Suppose, if u have a clauses even C1 and C2, and then you got an empty set then what you did an empty set if there is such kind of deduction we say that S is consists to be refutable. And we simply write it has a contradiction are clause derived from S; that is S single terms, styles are with respect to resolution refutation we got the contradiction. So, the empty clause, empty clause is also going to be a false.

(Refer Slide Time: 30:45).



So, now some examples we let talk about some examples. So, that is we can understand this method in a better way. So, we have 2 formulas less p r and not q not r. So, we can compute p not q in this particular kind enough way.

(Refer Slide Time: 31:10).



So, let us talk about some formulas and all not q r r C1 is a conjunctive normal form and all now let us say, p r q r r and another 1 is just for a sake of as anything p r not q r not r.So now, these are the 3 clauses and all. So, now, we will be applying resolution methods of this particular kind of formula. So, this is the considered to be a formula. So, now, each 1 is considered to be a clause, there are considered to be clauses mostly there are combination of disjunction and all these a conjunction and all a conjunction normal form is a conjunction sub disjunction each 1 is a dissent.

So, now, each and the dissolution a method of only to the conjunctive normal forms; that means, 1 has to convert given to formula, in preposition logic into the conjunctive normal forms, then we will talk about the resolution refutation method. So, now, a 1 way use this resolution principle tries a even tries also depending upon. So, now, these 2 what are the clauses that p and he have not p. So, these gets resolved and all not p. So, what remains here these 2 trying resolution principles in this way, needs to not q r r because we are resolving on p, then q not r ; is a disjunction of this things whatever is left these things particular kind of formula.

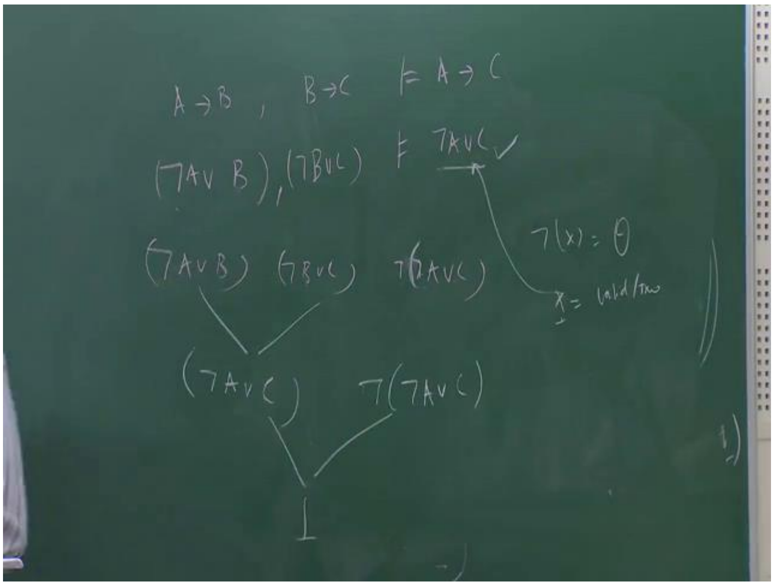
So, this is the 1 which we have. So, now, since r occurs twice in all we need to get away from the at all. So, now, this formula will become not q r q; now, for a examples, we trying to resolve this formula and all. Now it is so now, suppose if he have trying to resolve this particular kind of think then we are resolving again p only here. So, then u will get this formula q rr not q r r not q r not r. So, now, original formulas are plus r C1 C2 C3 etcetera and all. And now, we are getting is corresponding formulas and all some other letters which can be use, we can say particular kind of. So, now, we can apply again a resolution principles on these 2, and then we can say that since you have q here and not q here away and then in the same way, not q here and q here. So, first time when u applying the resolution principles to this 1, it will become r are since u have not q also here this also goes away and then what remains here is this think r q r r not q and this becomes, these to will resolved in to this particular kind of think r r not q; again, u write the disjunction of these thinks and all whatever remains here, r r not q r not r disjunction because q not q is gone and all.

So, whatever is remained is q. So, this is what we have r r not q r not r r q. So, now, what is that we are trying to say is, applying this resolution method n number of times till u obtain a contradiction and all. We do not get a contradiction of a given formula is going to be true. So, now it is happed here. So, this is not q and q again if u are play resolution principle on this 1, you will get is not q is a here and q is here is goes away and u will get r r q r r r not r r q. Now u can remove the reference at all q occurs twice here and also occurs twice here. So, now, this will be become r r q r r q r not r. So, in this digestion since you have, a littler and its negation is already they here. So, then; obviously, whether or not whether will q is true or false, his resolve is always going to be true.

So, that means the q has to be even if is true or false it does not matter, because you already have littler and legation her. So, the descent always going to be true, the clause is always going to be a true. So, in this way 1 can find out ah resolvents and all. So, let us consider another simple example, how we can apply particular kind of resolution principle and all. So, here is an instance it is like this, a natural language sentence is a given to a us. So, that is, if this apple is sweet then it is go to eat; obviously, sweet apple; obviously, will like to eat and all. Now, the second statement is like this, if it is good to eat then I will eat it therefore, this apple is sweet then i will eat it.

So, these are the 3 sentences corresponding to 3 different formulas and all the clauses. So, that is a combination of C1 and C2 and C3. So, in an argument all the premises P1 and P2 and P3; these to some kind of conclusion form. So, here the first 2 consider to be premises and whatever follows after therefore, consider to be the conclusion. So, now, we would like to no whether, this is going to be true or false. So, in that case what we will do, here is a z he employee resolution refutation method. So, what is resolution refutation method? What in that method, what we will do is we take the premises as it is and we derive the conclusion. We take the negation of the conclusion and if we can derive empty clause then the original conclusion; that means, a negation of the conclusion is an unsatisfable.Then; obviously, the original conclusion and underivable form is going to be valid.

(Refer Slide Time: 39:32).



So, let us consider these 2 thinks not A r B. So, that is the first think, if the apple is sweet then it is good actually that is in this particular kind of format. So, this is nothing but, not A or B by definition it will be A are B and the second statement is also like this B implies C. Now, the conclusion then is like this, if this apple is sweet then I will eat it so; that means, A if this apple is sweet then I will eat this. So, now, the corresponding definition you can write like this.

This is each 1 is consider to be an conjunction C1 and C2 and a; obviously, something eats too. So, it is not B are C and then we have not A r C. So, now, in the resolution refutation method what we will do is. So, we will take the combination of this think A r B and not be r c and u take the negation of particular kind of think and see whether leads to empty clause are not A r C. You have to derive is conclusion and if this leads to an empty clause, then taking the negation of conclusion leads to un satistability; that means, the conclusion taking the negation of conclusion leads to contradiction; that means, negation of x is unsatisfable; that means, if negation of x is leads to some kind of empty clause then; obviously, x has to be valid, are x has to be true.

So, now so given this particular kind of problem, we translated in to its corresponding clauses first 1 is translated is not A r B. The second 1 B implies C translated this 1 as a third 1 is this. So, now, as a combination of all these things, it is to lead to unsatisfability and all because he has taken negation of the conclusions. Now, we to apply resolution principle on this particular kind of thinking. So, now, so little bit change it all. So, this is nothing but, A not A is A. A negation of disjunction is a conjunction and this will become not C negation of C means negation of Cm.

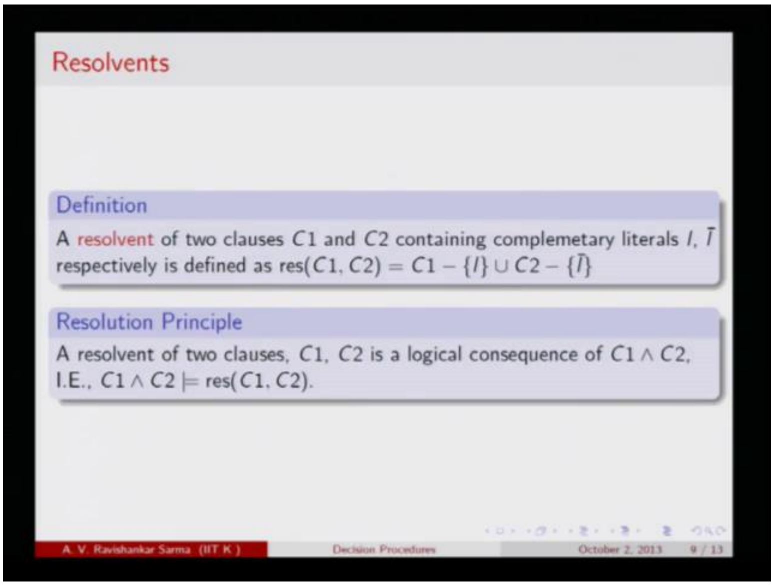
So, now, this is not in a proper disjunctive disjunction all. So, some of u need to use demerges law and we need to convert into the corresponding think. So, this is negation of negation of A r C off course, this is same as this particular kind of think. So, now, what will be here is this think, the first you to resolve these 2 thinks and B. So, what on the literals that the exists here, if have B here and u have not B here. So, these to vanishes and all and then what we not do is, we need to take the digestion of whatever means here. The literal is negation goes to the way and they whatever is there is this 1.

So, these 2 after applying the resolution principle you will get not A r C. So, now, you have not of not A r C. So, now, this is the exactly opposite to this 1 it is x and it is not a and all. So, this lead to empty clause. So, what is that we have derive we derive empty clause taking the negation of the conclusion; that means, this is consider to we are resolution refutation method procedure for finding out that denied of the conclusion leads to an empty clause. And all if it do not denied this conclusion it would have let this empty clause. So, it is in that sense, in the process of consisting counter example we have come of with an empty clause. So, that why, the original conclusion here.

So, that is negation of this 1 original conclusion these to unsatisfability it is empty clause. So, that the x has to be valid are x has to be true. What is x here, x is the particular kind of a; that means; this conclusion follows from the premises and all. The expanses of 1 is like this, we consider not A r B and B is true then; obviously, B has to be false, B is to substituence not be we become false. And the second clause, in a particular that is not B r C because not B is already for false, in order for this statement to be true it depends upon the value of C. If the C is always false and all obliviously, the not B r C is going to be fall so; that means, now that is a case and all C has to be true in that case.

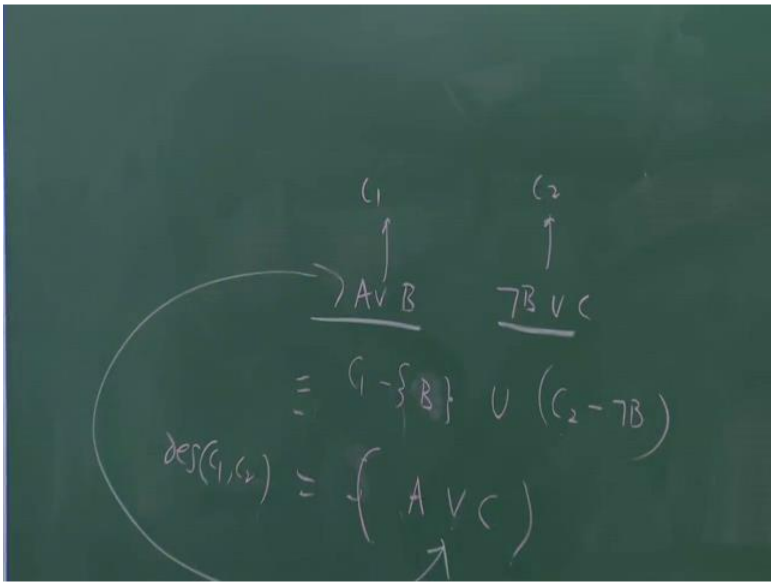
So, now, if it take be has false and all false then obliviously not B is going to be true, then in the first clause that is not A are B; B is already for false then the truth value of not A are B depends upon, what value not a take. If not a takes a value F and whole design to be false and is not has a true so; that means, only 1 of the thinks have to be true, that is either B has to be true and not B has to true. So, now, the explanation for the above is it is, if the x is true either not A are C true; obviously, then not A are C must b true. If not A are C is false then x cannot be true. So, essentially here in this case we can see B and not B.

(Refer Slide Time: 45:45).



So, now these are the examples of, resolvent of 2 clauses. Let us C1 and C2 is consists of conjunctions, usually C1 and C2 consists of at least 1 literal L, and then the other clause will have exactly the negation of the 1; is a literal L bar is there in the other clause, then re solvents is defined in this sense of resolvent of C1 C 2 is nothing but C1 minus n union C2 minus that nega negative literal. So, it is like in this case example. So, this let us say, not A are B, A not B r C not B r C.

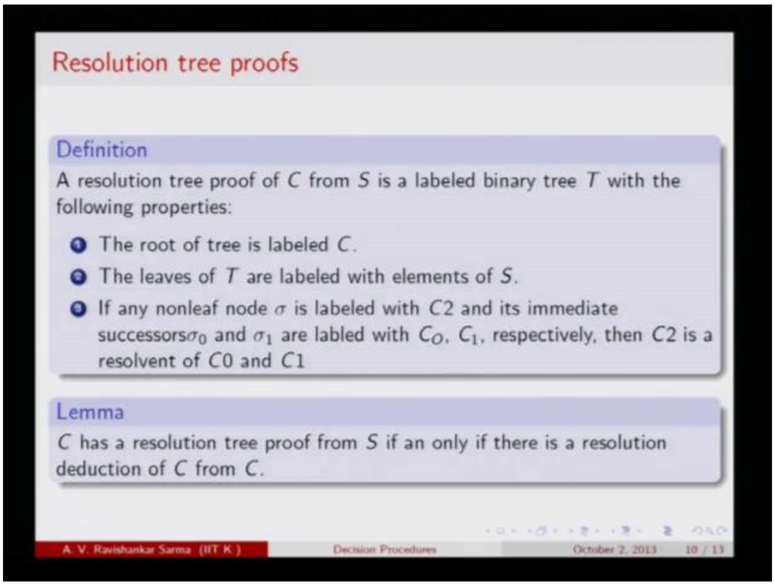
(Refer Slide Time: 46:35).



Now, here it is C1 and this C2. So, what is the re solvent of for C1 and C2, they are like this. So, now, it is C1 minus this literal whatever he say here negative literal not A union C2 this is C2 minus is not a B C2 B. So, now, in the sense it will become the simple formula we remove this B, if have A here C1 minus B is a union means here it use it has disjunction, union is same as disjunction there as same as conjunction not to is removed from here, then it is C. Now, this considered to be resolvent of C1 C2. So, what is essentially says now, the resolution principles, which is which occupies the enter position we have resolution method as a resolvent of 2 clauses C1 and C2 is considered to be a logical consequences of C1 and C2.

So; that means, each resolvent of any 2 clauses is automatically consists of to be a logical consequences of this. For example, if we take this 2 into consideration the logical consequences of the this 1 is its corresponding resolvent. It is in that sense, in your proof each time many of applying in this resolution principle you are coming of this resolvent and that particular kind of resolvent is consider to be an logical consequence of is these formula C1 and C2.

(Refer Slide Time: 48:37).

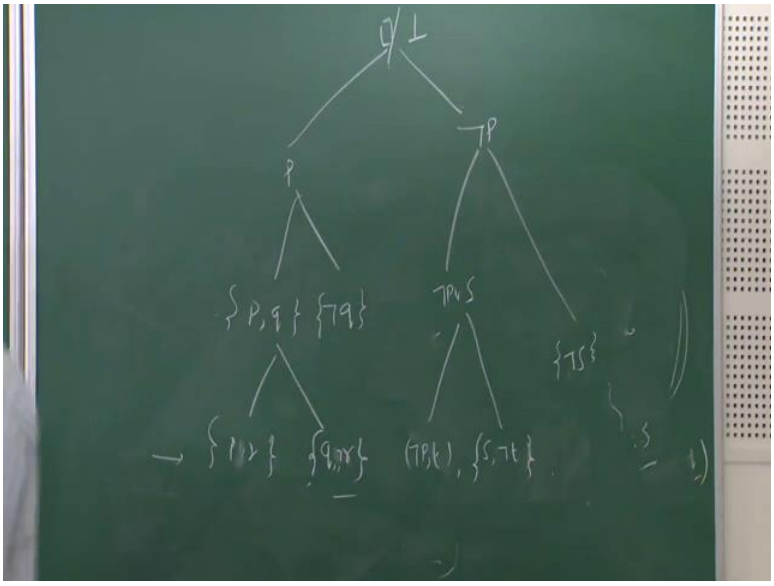


So, now, 1 can the better proof with the the help of resolution tree proof and all. So, this resolution tree proofs looks like this the definition of this 1 is like this. So, the same think it will be what be trying to do is is put in a and structural format. So, then it will it will look like ah the semantic tableaux method, but it is 1, but looks like a a tree. And then ultimately what is generately it is the tree what if we get its contradiction that is a empty clause, then the original conclusion is un satisfable i mean negation of the conclusion unsatisfable; that means, the actual conclusion holes. So, now, the resolution tree proof are like this, a resolution tree proof C from S in a labeled binary tree t is have a particular kind of a properties. This is the root of the properties tree is labeled as C.

So, that is what is, what is that we are trying to reduce the and all proof and ur branches and all.In the semantic tableaux method what we have is the given formula occupies the root, but it we are considered the upset down from the tree, were the root is there in the upstairs the occupies the set position were as the other formula. We come across are there in the notes in all. So, now, the root of the tree is going to be your clauses and all, whatever clauses that we taking a consideration and the resolvent is considered the logical consequences corresponding notes etcetera is like a branches.

So, the leaf of the tree are labeled which elements of S and if any non leaf node sigma is labeled with C2 and its immediates successes are labeled as any other letter other than sigma 0 and sigma 1 are labeled with 0 and etcetera. Then C2 is considered to be a resolvent of C0 and C1. So, C has a resolution tree proof from S; there is resolution deduction from reduction deduction enough C from S. So, let us consider some kind of resolution tree proof and all. So, now, here is the case and all; so just I will draw 1 simple I will proof some, simple proof resolution proof for given the problem.

(Refer Slide Time: 51:23).



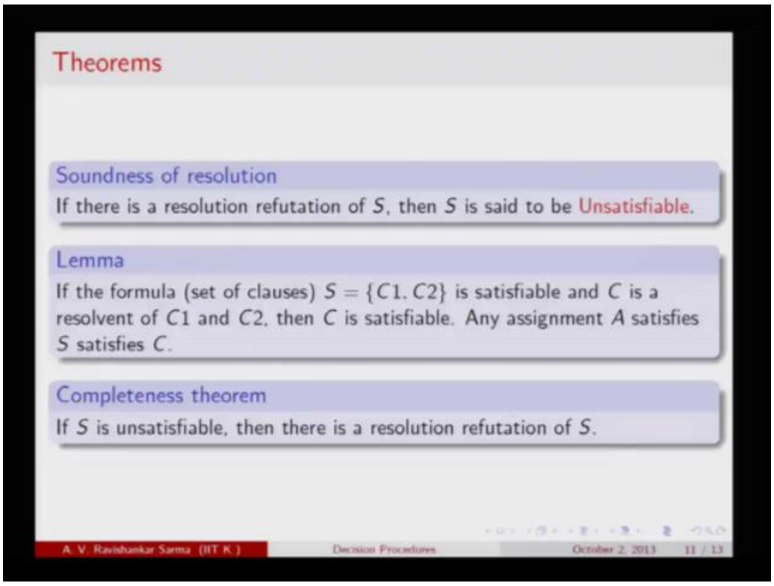
So, that is you have this formula that is p r q r these are the clauses at all p, r is p and r and a right, q come r that is q and r then not p t an then if had S not p. So, now, we have another kind of formula which is not S and not S is here. So, now, so these 2. So, what occupies that root kind of what occupies the clauses here, difference types of clauses C1 C2 C3 C4 etcetera. So, now, these 2; I think should be 10 in the p r q r something like we with like this. So, now, these 2 resolution you will get p r and q not r for example. So, now, these to after resolution get clause p q and you already have a claused q. Off course, there are all the which are already there here formula is p r r and second q r not r not p s r not p not S any 1 not q also there and these to, resolution you will get q and not q here. So, what will get is p.

So, now, observe these think here not p t here S n not t here. So, these to resolution you will get not p r C not p r S. So, now, we have not takes here, not p here these 2 after resolution you will get not p. So, now, this p and not p these 2; u can called it has box are sometimes you write it has this particular kind of symbol. So, this is called as contradiction. So, this will serve as a some kind of resolution refutation proof for, a given formula. Now, for example, if we take this 1 2 3 4 may be 5 and any taking to the configuration not as your conclusion. Now, actual conclusion s and all S, but we have taking into consideration not S.

So, now, the applying resolution principle 2 3 twice etcetera and all. And then if we take this not assign into consideration that the denied of the conclusion and that leads to contradistinction of so; that means, in this case may be s is going to be a conclusion. So, the derive of the conclusion leads to some kind of m t clause. So, in this way, 1 can find out proof for a given kind of formula for example, last show that a given formula is, argument is valid etcetera

What do is another conclusion and you constrict using the resolution principles. And you will formulate, you will come with a contradiction that is a empty clause; in that case denied of the conclusion is unsatisfability means, the actual conclusion. So, in this we discussed about resolution refutation method. So, were it applies to only a conjunctive normal forms whenever you have 2 conjuncts C1 C2 and these 2 gets a resolved in 2 another kind of conjuncts. Especially, when you have a literal and negation is there in the other clause and all.

(Refer Slide Time: 55:52).



So, now, it has a his own this method has his own advantages, this resolution method is consists to be sound. It means that is, the resolution refutation of S then S is a consists to be we are saying effectively that a given s is considered to be unsatisfable at all and is corresponding is the formula s is a combination of C1 and C2 satisfable and C is a resolvent of this 2 clauses then; obviously, C is also considered to be satisfable. So, and this resolution refutation method is also considered to be complete. It is, if you can sum more S is to unsatisfable then; obviously, there is resolution refutation of S like, any other deduction dissolution refutation method is also considered to a we consistence sound etcetera. So, now, a we will be talking more about, this particular kind of resolution refutation method in the context of dedicates logic. There we will talk about this particular kind of method in greater details.