

Introduction to Logic
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Lecture - 27
Resolution and refutation method: Examples

The last lecture, where we discussed in extensive detail about the resolution refutation method resolution refutation method is also considered to be one of the important decision procedure methods. With which 1 will be able to know whether a given well-formed formula is tautology or valid or when 2 or 3 groups of statements are consistent to each other or certifiable to each other etcetera. All these things 1 can come to know.

So, resolution refutation method is also like semantic tableaux method where we looked for a counter example. So, here what we will do is given some set of clauses which are usually ah in disjunctions. So, given these clauses; so, we will try to prove an empty clause you know.

So, after resolving these clauses and all, so 1 will come up with an empty clause then that proof is called as resolution refutation method. So, that mean you have used the resolution method and then you are refuting it in the sense that you are come up with an empty clause that is the contradiction.

So, in this lecture what we will be doing is we will be studying some more examples based on resolution refutation method and then second in the second part of this lecture. We will be talking about comparative evolution of various methods that we have learnt so far.

So, far we have learnt truth table method to start with this is considered to be the most simplest method and then truth table method we have used in direct truth table method. So, 1 need not have to construct all the rows and all it is like a constructive method say especially when the number of rows increases number of variables increases and the number of rows increases then truth table method may not be an effective kind of decision procedure method may be partly a computer can compute this things in a better way.

Then especially when the number of propositional variables increases to 4 or 5 more than 5 things will become unmanageable and all for us because for checking the validity you need to check all the rows in which your 2 premises in a false conclusion that establishes that a given argument is invalid.

So, then we talked about semantic tableaux method and then we tried to solve 1 single problem with the help of all these methods 1 is truth table method the second is semantic tableaux method, and the third 1 is reducing it in to c n f and d n f and the 4 th method is the semantic sorry the resolution method and we also used some kind of syntactic methods. So, that is the natural detection method where we used conditional proof and the add kind of proof.

So, what I will be doing in the second part of this lecture is that I will take up 1 example then I will I will try to solve that problem using all these methods and then when the time arises I will talk about the importance the merits and demerits of these particular kinds of decision procedures that are available to us to start with in continuation to the last lecture here, let us take some examples some more examples. So, that we can understand this resolution refutation method in a better way basically we it is a decision procedure method with which you can establish whether or not a given formula is satisfiable or unsatisfiable or 1 can even come to know whether or not a given well-formed formula is a valid or in the same way when you are talking about an argument you will come to know whether argument is valid or invalid.

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Problem Solving

Find Inconsistent clauses[®]

- $\{(p1, p2, p3), (p1, \neg p3), (\neg p1, \neg p2)\}$.
- $\{(\neg p1, \neg p2, p3), (p1, \neg p3), (\neg p1, p2)\}$.

Validity

- $\{(A \rightarrow B), (\neg A \vee C \vee D), \neg C \vee (D \wedge A), (C \wedge \neg D) \rightarrow \neg E\} \models \neg D \rightarrow B$
- $\{\neg A \wedge (\neg B \vee C), (B \wedge C), (C \rightarrow D), (D \vee \neg A)\} \models \neg(\neg D \wedge A)$.

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So, let us consider 1 simple example where we are trying to find out the inconsistent clauses inconsistent clauses in a sense that all these clauses that is $p_1 p_2 p_3$ in the first example and the second clause is p_1 and not p_3 , and the third clause is naught p_1 and naught p_2 whether all these clauses after applying the resolution principle whether it lead to an empty clause or not is a 1 which we tried to look further if you come across an empty clause that is considered to be these clauses are considered to be inconsistent otherwise it is consider to be consistent.

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Handwritten notes on a green chalkboard showing logical clauses and their resolution:

- $C_1: \{p_1, p_2, p_3\}$
- $C_2: \{p_1, \neg p_3\}$
- $C_3: \{\neg p_1, \neg p_2\}$
- Resolution of C_1 and C_2 yields $C_4: \{p_1, p_2, \perp\} = \{p_1, p_2\}$
- Resolution of C_1 and C_3 yields $C_5: \{p_2, \neg p_2\}$

So, now we have these clauses $p_1 \vee p_2 \vee p_3$ usually this is expressed in this particular kind of format p_3 and the second clause is $p_1 \vee \neg p_3$ usually we write it in this way it is a set which consists of all these clauses and all and the third $\neg p_1 \vee \neg p_2$.

So, basically what we are trying to do is that. So, these are the clauses c_1 , c_2 and c_3 . So, now, what we will be doing is we will apply we will be applying the principle of resolution to it and we will find out the resolvent of these clauses and all ultimately if we can reduce this empty clause this contradiction then it is considered to be inconsistent all these clauses are considered to be inconsistent.

So, now apply the resolution principle for the first 2 things like this and this because we have a literal p_3 and we have a literal $\neg p_3$ applying resolution on a literal p_3 taking into consideration the other things as formulas $p_1 \vee p_2$ and $\neg p_1$ etcetera.

The what you will get is this $p_1 \vee p_2$ or and this will go away $\neg p_3$ will go away then you have another p_1 . So, now, after applying the resolution principle we have to ensure that there is no redundancy and all for example, in this case $p_1 \vee p_1$ twice you know. So, this is as good as same as p_1 and p_2 .

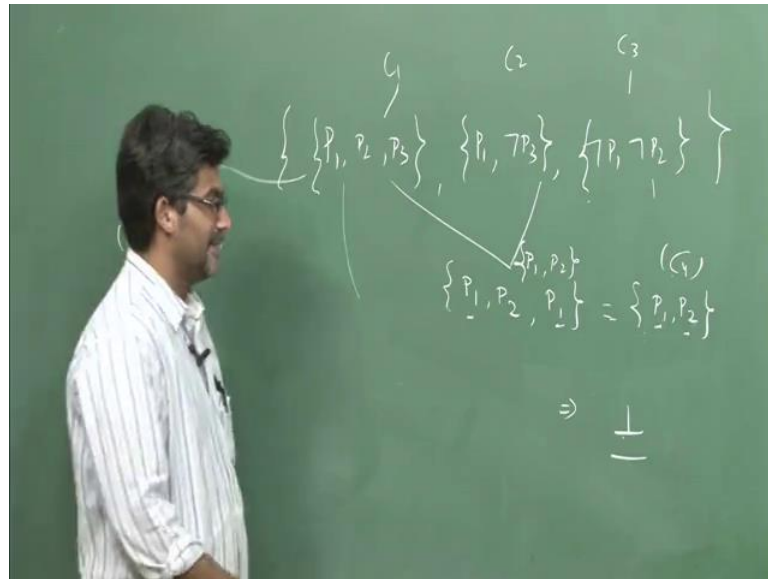
So, now what we got is p_1 and p_2 . So, now, once again you apply the resolution on these 2 things example apply resolution on these $p_1 \vee p_2$ and this $\neg p_1$. So, now, we would write it write down here now we have $\neg p_1 \vee \neg p_2$. So, that is the third clause and all.

So, now applying simultaneously for example, applying resolution principle on p_1 that leads to $p_2 \vee \neg p_2$. So, now, now once again applying this particular kind of thing resolution principle on these 2 clauses c_1 and c_2 . So, now, what we have here we have $c_1 \vee c_2 \vee c_3$ and this is c_4 for example, and then c_5 now this is this is as it is c_3 only this is c_5 .

So, now once again applying on this $\neg p_1$ you will get these 2 on resolution you will get $p_1 \vee p_2$ will go away because you have p_2 and you have $\neg p_2$. So, that will go away and then. So, what is left is p_3 .

So, now here in this case. So, once you have this particular kind of thing.

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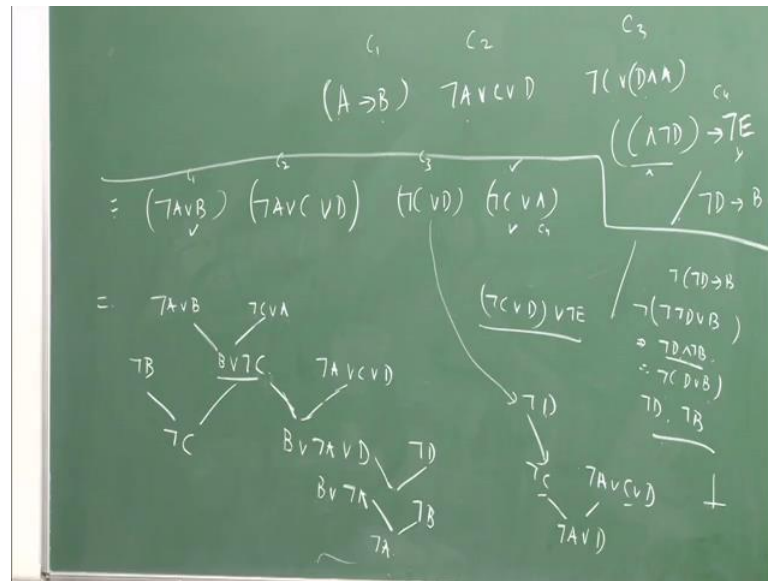


So, now, you have P 1 here and you have p 2 here and you have not p 1 and not p 2. So, now, applying resolution on both p 1 and p 2 simultaneously, then you will get this particular kind of clause since you have a literal here and you have ah negation of this literal here and this will be not p 2 here and p 2 here.

So, applying simultaneously the resolution principle on both the literals p 1 and p 2 you will left with only an empty clause, so that means, that given these 3 clauses are said to be in consistent to each other, because when it got resolved by using resolution principle it lead to some kind of empty clause that is a contradiction.

So, empty clause is always false; that means, there is no interpretation which makes this particular kind of formula true; that means, the formula is considered to be unsatisfied. So, now, let us assume that another example where we are trying to check for the validity of a given argument. So, in the second example that you are seeing there in the slide. So, let us try to solve this particular kind of problem by using resolution refutation method.

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So, these are the clauses that we have a implies b the first 1 second 1 is not a or c or t not a or c or d and the third one is not c or not c or d at a not c or d and a and c and not d c and not d implies not e. So, these are the clauses c 1 c 2 c 3 c 4 etcetera actually these are not in the actual conjunctive normal formula you should note that resolution refutation method applies only to those formulas which are in the conjunctive normal formula.

Suppose, if the formula is not in the conjunctive normal form then we should ensure that we convert it into the appropriate conjunctive normal formula. So, each and every formula that occurs in the preposition logic will have its own corresponding conjunctive normal form. So, converting like any given formula into conjunctive normal form may not be that difficult. So, now, this leads to a conclusion which is not d implies b.

So, now in the resolution refutation method what you will do here this is that given these clauses c 1 c 2 c 3 and c 4 and then in addition to that you take the negation of this particular kind of formula. So, now, let us write the converted into appropriate form and all.

So, the first clause can be written it this way because a implies b is nothing, but not a or b now the second clause can be written as say as it is which is already in the disjunctive form. So, we need not have to do much. So, now, the third clause is not c or d. So, now, you need to apply distributive law then it will become not c or d and not c or a. So, this is the third clause.

So, now the 4th line can be written in this way. So, now, this whole thing is taken as x and this as y . So, now, this is not of c and not d or not e . So, these are the clauses that we have c_1, c_2, c_3 and c_4 first we will write it in this particular kind of form and the third line is that. So, now, what we are trying to do is that we have listed out all the premises and all and now we take the negation of the conclusion.

So, now negation of the conclusion t is not d implies d . So, first of all not d implies b is same as not not d or b . So, now, if you apply negation to this line. So, this will become not d and not b because not not d is d only negation of d is negation of d negation of disjunction will become conjunction and the negation of b . So, this is the line which we have.

So, now this can be written as the same as this now. So, now, this is the final clause and all not of d or b . So, now, this particular kind of formula we further converted into an appropriate form then it will become same thing will become not c or not not d is d or not b . So, this formula is same as this line.

So now, what is that we are trying to do is that we now we need to apply resolution principle on these clauses and then in addition to that if we take the negation of that line ultimately you need to generate this particular kind of thing the contradiction that is the empty clause if we can generate empty clause and that means, you have with using resolution refutation process you have showed that all these things x_1, x_2, c_1, c_2, c_3 etcetera and the negation of this particular kind of formula is unsatisfiable.

So, now what you need to do here is this thing. So, now, you start applying the resolution refutation method on this particular kind of thing. So, so there are 2 ways to prove this particular kind of thing either you resolve this consequence sorry the terms c_2, c_3 etcetera and ultimately you will generate this particular kind of formula or you take the negation of the conclusion and show that it leads to contradiction.

So, these 2 for example, this will not go to anything not because you are not here now taking into consideration any one of these things. So, not c or d and not c or a so that means, both the things are there. So, you now have to write anything and all. So, you can skip this particular kind of thing because all these are premises only all the premises will be in the form of conjunctive only c_1, c_2, c_3 and ultimately relates to some conclusion x .

So, where we can apply this particular kind of thing. So, you have a here and you have not a here. So, now, what you will do is not a or b and not c or a. So, now, you have listed out c 1 and c 4 for example. So, now, these 2 resolution you will get b resolution applying on this particular kind of literal a leads to b or b or not c these 2 you will get this 1 this 2.

So, now you apply resolution on these 2 things not c or d and b or not sorry not this 1 somewhere else we have literal and its negation is the 1 which we need to take into consideration. So, now, b or. So, now, this will become b or not c now here you have seen here and you have not seen here. So, now, you write it like this not a or c or d.

So, now these 2 applying resolution principle it is applying on c then this c will vanish. So, now, what you have is b or not a or d, because c vanish vanishes here because you are not c here and c here. So, applying resolution principle on the literal c you will generate this thing b or not a and d.

So, 1 can apply resolution principle many times and all ultimately in the process you should be in a position to generate whenever you come across an empty clause you need to stop the proof procedure and then that is that will serve as proof for this particular kind of thing of the conclusion leads to contradiction.

So, now, observe these 2 things. So, here we have not d and anyhow not b and all. So, that is what we have, so not of d or b means not d not b. So, now, applying the further first thing for the not d you will get you are applying resolution principle on d you will get b or not a. So, this is what you get.

Now, once again applying resolution principle on not b which is there here already then this b and not b goes away and then you will get not a. So, now, we need to see whether you will generate a from some other things and all like. So, now we have not c or a and then we have not a. So, this leads to a particular kind of things. So, b are not c not a or c or d . So, now, we need to apply on this thing. So, not a or c or d not c or d a and the n somehow we need to take this clauses in such a way that you generate this empty clause.

So, now instead of taking this into consideration now you take this b or not c and not b. So, this 2 you will generate not c. So, this is what you have generated from not b b or not

c. So, now, apply this particular kind of thing now not a and not c or a these 2 under resolution you will get not c not c.

Not c or a 1 second this is not serving our purpose. So, somehow we need to get this particular kind of c or a b or not a not b we will get not a. So, how to get this 1 this thing which we are trying to look for not a or b not a or c or d. So, not c or d not c or a etcetera somehow we need not still generate this particular kind of contradiction and all you take into consideration not a this is not not d or b not d and not b and not of d or p not c or a. So, this and for example, in this case not c or d and then not d this will you will generate not c and this 1 not c and not a or c or d you will get these 2 on resolution you will get this particular kind of applying resolution on c you will get not a or d.

So, ultimately what is happening here is that we have operate resolution n number of times and all somehow we are not able to generate contradiction c and not d or e ot of c and d or not a. So, if we can generate contradiction out of these 4 clauses and all then you have you have proved not d implies b negation of not d implies b leads to unsatisfiability and hence you are you are set to shown that 2 premises in a false conclusion that is invalid kind of argument and all.

So, somehow this needs to be work out in greater detail I will come back to this problem in a well form now. So, what we will be doing is you will take up 1 simple problem and then we try to see how this problem can be solved in 3 different ways around that is using different decision procedure methods that we have leant here.

So, let us consider some 1 more example and which we will come to know how to solve this particular kind of problem by using various kinds of method. So, the 1 which is explained on the board in the first problem. So, ultimately what 1 needs to solve here is is that. So, you apply resolution on in n number of steps and all and then resolve those things and ultimately. So, that has to be contradictory with this particular kind of thing.

So, then you are said to have achieve you are particular kind of task and all. So, here what we have done simply is that we started applying resolution principle on different kinds of conjuncts and all ultimately we generated some kind of formulas. So, somehow we are not able to get to our conclusion that is the empty clause.

Suppose if you do not get this particular kind of empty clause of tripling resolution exhaustively this resolution on all these steps and all then the argument is considered to be invalid and all ok

So, now let us consider another example. So, we will see how we can apply this resolution principle in solving this particular kind of problem not a or b yeah this is what is creating problem here. So, I made a mistake here. So, that is the reason why I am looking for the solution here actually this is not of a implies b.

So, now this will become a or not not a means a a or b. So, now, as you can clearly see here. So, now, I need to apply again and all. So, this problem changes now. So, of if it is simply a implies b and not a or c or d then there is no way in which you can generate a contradiction here; that means, the argument is going to be invalid; that means, not d implies b does not follow from this 1. So, the actual problem which I could have taken for proving the validity of this given argument is in this sense.

So, this negation is missing here if I take the negation sign here then by applying resolution principle then may be 1 can come up with a kind of reputation. So, let us try to see let us try to solve same kind of problem by taking into consideration this negation sign and then we try to solve the same problem with the help of other methods as well

So, actual problem is this 1 negation of not not a implies b and not a or c and then not c or e and f not c or e and f this problem is different and all. So, forget about this thing. So, now, let us come back to a different kind of problem this this problem will try to solve it by using different kind of methods.

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Example

There was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

- 1 Nobody else could have been involved other than A, B and C. $(A \vee B \vee C)$.
- 2 C never commits a crime without A's participation. $(C \rightarrow A)$
- 3 B does not know how to drive $(B \rightarrow [(B \wedge A) \vee (B \wedge C)])$.

Is A innocent or guilty?

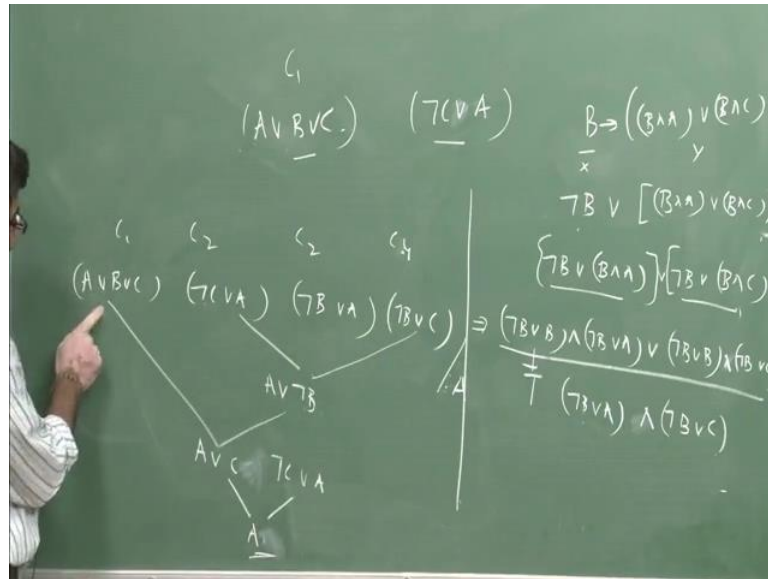
A is Guilty

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So, let us consider a simple example. So, this problem is the 1 which we have already seen. So, this is like this here is a puzzle in which there was a robbery in which lots of goods are stolen and the robbers left in a truck and it is known that nobody else could have been involved rather than a b c; that means, it is simply represented as a or b or c and c never commits a crime without as participation; that means, c implies a and b does not know how to drive; that means, b requires the company of a and c. So, this is represented as d implies b and a or b and c .

So, now we need to find out whether a is innocent or guilty now this problem we try to solve it by using various kinds of methods first we will start with the simple dissolution refutation method. So, in that method what we will be doing is you will be listing out all the all the conjunction all and we will start applying the resolution refutation method on this particular kind of thing. So, then if we can generate an empty clause from these things and; that means, we are said to have achieved our particular kind of thing. So, so what is that we are trying to do here in this example is like this. So, here are these conjunction and all a or b or c.

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So, now what I will be doing here is that I will be trying to solve this problem by using 1 of the some of the methods that we have already discussed. So, far. So, the first c 1 clause is like this and second clause is c and a sorry c or a and the third 1 is b implies b and a or b and c. So, now, this is already in the design tube form this is also in a this consists of disjunctions and now we need to convert this thing into appropriately into corresponding disjunctive form.

So, now this whole thing is taken as x and this as y. So, now, this will become not b or the whole thing b and a or b and c. So, now, we need to apply associative law here. So, now, this will become not b or b and a that is the first 1 and the second 1 is not b or this is or not b or b and c.

So, now these are the things which we have and then we can further expand it in this way using distributive law here. So, now, this will become not b or b and not b or a. So, this is the first 1 if you apply distributive law this is the thing now or now the other 1. So, that is not b or b and then or not b. So, this is not b or b and not b or c. So, this is the 1 which we have

So, what are the clauses now for us. So, these are the clauses that we have first 1 is a or b or c and not c or a and the whole thing. So, now, in this case not b or b is; obviously, is going to be true only. So, you need not have to worry much about it which is written as

top always true and all. So, now, whatever is left here is these things not b or a or again here not b or b is always true and all. So, we need not to worry much about it.

So, and then not b or c so; that means, not b or a and then the other clause is this not b or c. So, now, 1 can do it in various ways and all or playing resolution principle and this 1 whether you will get a as an out come or that is the problem already it is the question that we are trying to answer here.

So, each resolvent is considered to be a logical consequence of the above 2 clauses. So, these are the clauses that we have c_1 , c_2 and c_3 . So, now, the conclusion here for us is that now we are trying to find out whether or not a is. So, assuming that a is guilty suppose if you take into consideration that naught a whether that is inconsistent with this thing or not is the 1 which we are trying to see.

So, now a or b or c and this 1. So, we need to apply the resolution principle on this 1. So, now, these 2 applying dissolution on literal c you will get not b a or not b. So, because not c will go away. So, what is left this a or not b. So, we have to add add this things this will become a or not b.

So, now applying resolution on these 2 things you will get applying resolution on the literal d you will get a a or c or a. So, now in order to avoid redundancy here. So, you will simply write a occurs twice here. So, you will just simply write a or c. So, now this is what has come as an outcome of this resolution . So, now, you apply resolution on these things not c or a applying resolution on the clauses a or c and the clauses that are that is there here not c or a. So, now, this c and not c will vanish then you will get a or a.

So, now this a or a is in order to avoid redundancy and all we simply write a. So, what is that we got we listed out all this the information that is there here this is the first clause a or b or c and not c or a and then the third clause is this 1. So, we simplified this thing into this particular kind of format by using distribution and associative law and here is what we got.

So, we got it because of this particular kind of thing not b or b is always true. So, we need not have to worry much about it. So, so what is left here is not b or a and not b or c that is what we have written here this is the 2 clauses and applying dissolution on c_2 and

c 4; that means, this is a logical consequence of this 2 clauses c 2 and c 4 we got a or not b and again applying c 1 and let us say this is c 5 and all.

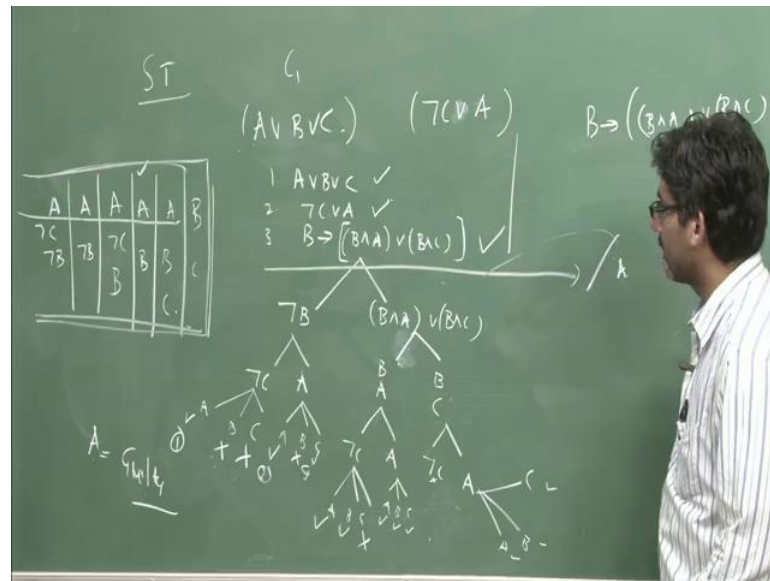
Resolution on c 1 and c 5 we got a or a or c and we already have this particular kind of thing not c or a. So, there is what is already there and now again once again you apply resolution principle on this particular kind of thing comes end with the literal c whenever you have a literal c and negation is there it cancels and he lets then what is left here is a or a.

So, a or a in order to avoid and see we got a and. So, a is the 1 which we got it as a outcome of all these proposition; that means, a is in the original interpretation a means a is guilty if it is not a if you somehow you generated not a as a result of applying the resolution here then not a means a is innocent. So, now, the same particular kind of thing which can be which can be solved by using the semantic tableaux method as well ah we can show that whether or not is guilty or not.

So, now in the semantic tableaux method what we need to do is this thing. So, these are the 3 propositions that we have. So, now, we are trying to say that a is assuming that a is guilty. So, now, we are assuming that suppose let us say a is the conclusion from this 1 and there are 2 ways to solve this particular kind of problem using the semantic tableaux method.

Semantic tableaux method always looks we look for the counter example; that means, if you assume that a is guilty in the beginning then what you need to do is you have to consider not a and then you consider semantic tableauxity, and then you need to generate a contradiction.

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So, now you list out all the premises not c or a and b implies b and a or b and c. So, comparatively evaluating checking this this particular kind of problem with various methods. So, that you will come to know the merits of this methods so. So, now, right now I am trying to solve this same problem by using semantic tableaux method.

So, now you list out all the premises and all. So, now, the problem tells us that these are satisfiable and all; that means, all these statements are consistent to each other at least under some interpretation this formula is going to be true x 1 x 2 x 3.

So, now you expand you rules and all to expand this particular kind of thing. So, now,. So, you apply this 1 x implies y then it will become not x and y. So, this will become not b and this is a b and a or b and c. So, this is o now this can be further simplified into this thing.

So, either it is b and a or it should be b and c. So, now, this we checked it and all. So, now, we need to look into this particular kind of thing. So, now, wherever the open branch is there we need to write this particular kind of information. So, this is the branch it will be like this not c or a and here also if you write the same information and every branch which is open you need to write this particular piece of information.

So, now each time you finish with the checking with this formula you need to see whether literal its negation exist in a in a branch. So, this is considered to be 1 branch

this is considered to be 1 branch like this. So, now, in this case we do not have we do not still have a literal and its negation all the branches are open.

So, now apply rule for this particular kind of thing a or b or c. So, then this particular kind of information needs to be written on every branch and all like this thing a b and c. So, this information a or b or c we have written it under whatever branch which is open. So, now,. So, this is a or b or c a b or c a b and c like this a b and c.

So, now we need to inspect all this branches and all wherever a literal negation occurs the branch closes. So, now, this branch goes like this a not c not b. So, that branch is open. So, now, you have b here and now b here which closes here and c not c this closes here itself now this branch is open you have b here and not b here this branch closes and this is also open.

So, now we have a not c and a b and there is no c here; that means, this branch is also open and this is also open, but you have here you have c and not c this closes. So, big branch and all. So, sometimes some resolution refutation method might be better than the semantic tableaux method because the number of branches are more here.

So, in the resolution refutation method we have seen in the in the last slide that that will simplify our task. So, now, here this is also open this is open and all the open branches such as suggests us that that makes this formula true; that means, if you observe this particular kind of open branch says that under the interpretation which a is true c is false sorry.

This branch closes here itself. So, need not have to worry about because you have b c here and then not c here this closes here itself. So, you need not have to worry much about whatever fall of. So, now, in this case all the branches are open. So, now, open branches tells us the answer and all. So, that satisfies this particular kind of formula.

So, now any open branch which you taken take into consideration the first 1 is like this a c and not b that is the first thing and the other open branch you will find it here a not b and we do not know whether about c in this particular kind of branch this is the first 1 second 1 and third 1 and the third 1 is a not c and b this also another kind of solution and then the other 1 is a b. So, this is the 1 which we have and other things are a b again the same thing which need not to write it again another thing which satisfy this formula is a c

and $b \wedge a$ and c and the other branches are like this $a \wedge b \wedge c$ and $b \wedge c$, we do not know about a here.

So, now the last 1 is this thing $a \wedge b \wedge c$ which is already we have listed. So, all these things are considered to be the interpretations which are gone to satisfy this particular kind of formula; that means, when a is true c is false b is false; that means, these 3 sentences true and in the same way when a is true b is false that also satisfies this particular kind of.

So, now in all these cases you will observe that you have turned out to be the case $a \wedge e$ that we got only a so; that means, that a is considered to be guilty. So, our interpretation is that when I write simply a ; that means, a is guilty when I write simply not a ; that means, a is considered to be innocent.

So, now the same kind of thing can be solved by using truth table method in the, but in the truth table method the problem is that you have 3 variables; that means, number of entries would be $2^3 = 8$ rows will occur in the truth table.

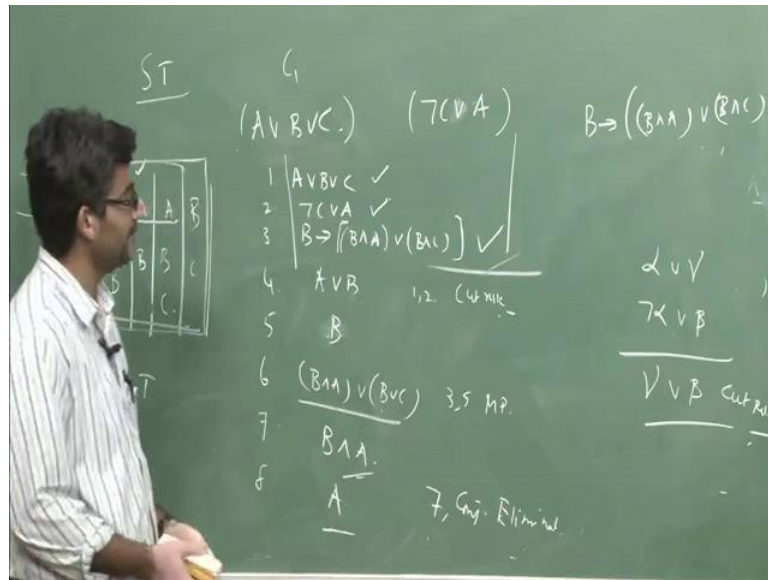
So, now in the truth table what we need to look for is a row in which all these statements are considered to be true; that means, conjunction of $c_1 \wedge c_2 \wedge c_3$ whenever that is true and all. So, that row corresponds to our particular kind of answer. So, same thing which you might get it. So, a particular kind of example can be done in n number of.

So, So, there in natural deduction what you will do here is that from $a \vee b \vee c$ and not c are a and all these things. So, what essentially do here is this that we list out all the premises and all. So, now, we need to assume something and all here suppose if you start with a is guilty and all. So, that is the original conclusion. So, you need to assume something as a then you take the negation of this thing into consideration and then see whether that leads to contradiction or not.

So, for example, in this case you list out all these things in 1 $a \vee b \vee c$ is c or a etcetera and all. So, what essentially 1 needs to do is by taking all these premises and all ultimately you should be in a position to reduce in that case a follows from these things with using resolution refutation method we showed that a will come as a logical consequence by applying resolution principle.

So, in the natural deduction method what you will be doing is you will be doing the principles of logic etcetera and all and then you will generate something like a and all from this things. So, let us see whether we can reduce a from this particular kind of premises.

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So, you have a or b or not c and you have not c or a. So, from this 2 things you will get of course, a or b or c and not c or a. So, for this you will get a or b or a. So, in order to avoid redundancy you will write it only once. So, this will be a or b simply a or b.

So, this rule is which we have discussed here this rule is called as cut rule. So, that says that alpha or gamma not alpha or beta suppose if you have a formula like this then. So, this will become gamma or beta this is called as cut rule which is also the rule responsible for the resolution principle same rule which we apply here.

Then you will get a or b. So, 1 end to this is a kind of mode rule this is special kind of rule you can use any rule which preserves that truth that you can here. So, now, . So, now, this means something is a or b is true means definitely some 1 of these things should be true and all. So, assuming that b is true here irrespective of whether a is false or a is true let us assume that b is true or not.

So, now these 2 3 and 5 you will get b and a or b and c. So, how did you get this 1 3 and 5 you will get this. So, now, somehow we have to deduce a as an outcome of this

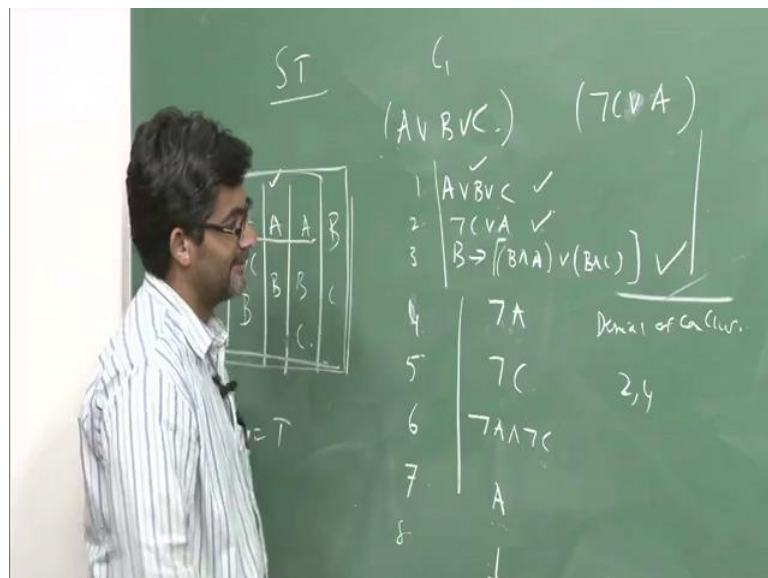
particular kind of. So, now, from this since this is true and all b and a a and b or c is already true.

So; that means, even b and a is also true because all this particular kind of thing x and x or y is going to be true then both x and y are true even in the case in which 1 of the disjunct is all then that also it leads to this thing. So, now, since b and a is that case.

So, now from this you can eliminate this conjunction conjunction elimination role which we use and then from this you will get this 1 conjunction elimination group you will get a; that means, we have reduced a from this 3 premises using a kind of proof method which is called as natural detection method. So, now, you have to note that in this particular kind of method each step is considered to be assumptions are; obviously, considered to be true and then if that has to be true this also has to be true each step is considered to true and hence the final last step of your proof which is considered to be. So, that also turn out to be true.

So, this is another way of showing that in this particular kind of problem a is said to be guilt and all in this particular kind of way. So, there is another way of doing this particular kind of thing within the natural detection method. So, now, what you do here is this particular kind of thing.

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So, now assuming that let us assume that a is guilty in 1. So, now, for the sake of argument you take into consideration not a so; that means, you are assuming that a is not guilty. So, now, we are trying to come up with some kind of contradiction.

So, in the 4th step since not c are a and you have a here. So, this option is ruled out what you get is not c. So, now, in the second principle should. So, how did you get this 1 not not a is of conclusion. So, now, you got from 2 2 end 4 we got this not.

Ah. So, now, since not a and not c are true we have not a and not c is also going to be true. So, now, 7th step 1 second not and a not c. So, now, these 2 1 second. So, from this particular kind of thing not c or a is already true; that means, a is also considered to be true from this particular kind of thing we can take into consideration assumption that a is also true. So, now, we have you have come across a situation where you're not a is true and a is also true.

So; that means, this leads to a contradiction; that means, the negation of this particular kind of formula a assuming that a is considered to be not guilty leads to this particular kind of contradiction. So, in this lecture what we have done is simply this that we started with some examples of resolution refutation method in that method what we essentially show is if you want to show that a given argument is valid and then what you need to do is given this clauses the premises you add the conclusion to it the denial of the conclusion and then you will be deriving an empty clause if you can derive empty clause from that 1; that means, denial of this particular kind of formula denial of the conclusions is unsatisfiable; that means, the original conclusion holds.

So, that is 1 way of showing that given argument valid in case using resolution refutation method. So, then in the second part what we have seen is we have we have taken up 1 simple problem that adjust in the propositional logic and then we showed with various methods the methods that with which we have shown are semantic tableaux method and we also shown the same thing by using a syntactic method that is a natural detection method with which we showed that. In fact, a is considered to be guilty that is what we have deduced from a given set of premises and all that is the natural detection method.

The same kind of thing which we did not shown in this class is is that we can show the same thing with the help of truth table method; that means, you list out all the premises

and then you assume that not a and then you will come across a row in the truth table in which you are 2 premises and a false conclusion.

So, in that case you can show that this argument is invalid; that means, not a leads to contradiction; that means, a has to be true. So, in this sense 1 can solve same problem by using any 1 of this decision procedure methods it is all up to our convenience which method to employ.

So, in some cases natural deduction method might be simpler are in other case semantic tableaux method may be better. Or in some simple cases to begin with truth table method better than other methods are in some other cases resolution refutation method especially when you have conjunctive normal forms. Then we can straight away apply the resolution principle and simply get your answer because each resolvent is considered to be a logical consequence of the conjunctions. That you are taking it to consideration that conjunction is nothing, but each conjunction will have its own corresponding disjunctions.

So, that is what is conjunctive normal form, so in the next class what you will be doing is that we will be taking up the axiomatic propositional calculus. In that what you will be doing here is that we will be taking we will be taking into consideration the primitive axioms. And then we will be using as many little as many minimal kind of principles and all like only is the 1 which we use. And we use transformation rules and then we will reduce all the validity formulas within your logical system.

So, in the next class we will be taking up the axiomatic propositional calculus there we will study 2 axiomatic systems 1 is due to Russell and Whitehead that is in the Principia Mathematica. And the second 1 is due to Hilbert and Ackermann Hilbert Ackermann axiomatic system.