

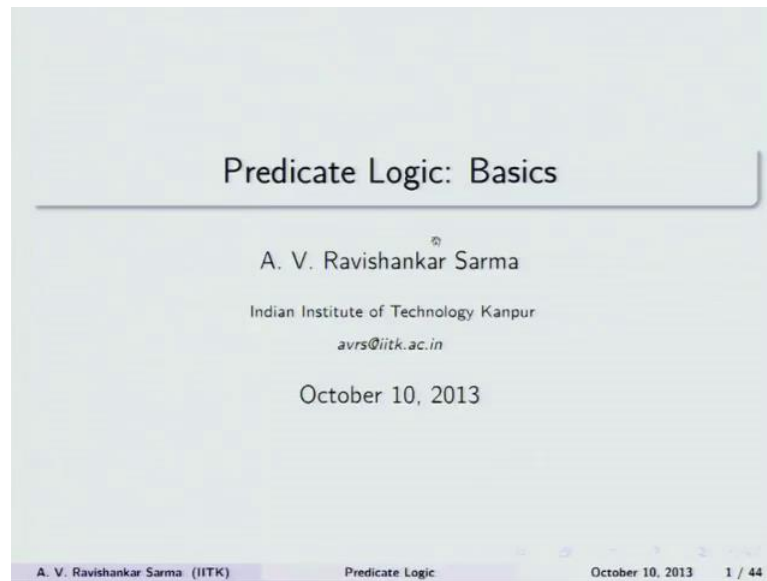
Introduction to Logic
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Lecture - 33
Outlines of Predicate Logic

Welcome back, now we entered in to the third face of this course introduction to logic. So, in this module we will be talking about some the basic concepts of the Dedicates logic. So, we were discussed we started with basic concepts of logic where we discussed about what you mean by arguments etcetera. We talked about various we kinds of arguments and were this arguments occur etcetera. Then we moved on to the logic of propositions where we where we basic units of our logical system his are the propositions they propositions contrite the sentence which is which can spoken as either true are false. So, we have cleverly choose the sentence is such away.

That we can clearly draw a line between let say a motel and non motel etcetera. So, a sentence can be spoken as either true are false all the sentence are presented by simply by means of sentence letters that is the what was the cases of propositions logic. So, propositions also considered to be the logic of the minimal logic of connectives this connectives are like this negation and n r implies if in only if. So, these are the minimal kind of the logics with which we can represent our knowledge.

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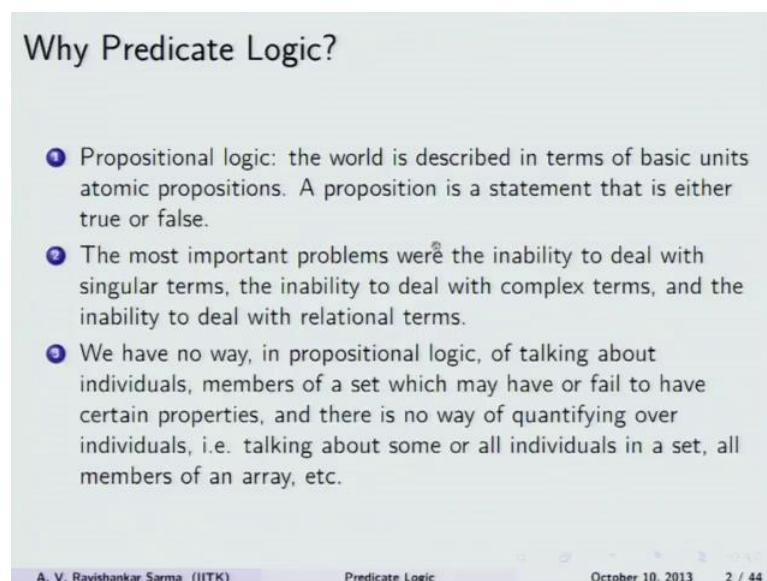
Predicate Logic: Basics

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So, now what we going to say in this class in this lecture is that we introduce propositions logic we will discussed the rational for introduce predicate logics what are the limitation of propositional logic. And then we will talk about some other importance basic contrite of predicate logic that are predicates terms etcetera.

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Why Predicate Logic?

- 1 Propositional logic: the world is described in terms of basic units atomic propositions. A proposition is a statement that is either true or false.
- 2 The most important problems were the inability to deal with singular terms, the inability to deal with complex terms, and the inability to deal with relational terms.
- 3 We have no way, in propositional logic, of talking about individuals, members of a set which may have or fail to have certain properties, and there is no way of quantifying over individuals, i.e. talking about some or all individuals in a set, all members of an array, etc.

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So, in this lecture I will be talking about these things why approval need to move to predicate logic if we have if we already have propositions logics with us. So, in the case of propositional logic the world is described in terms of basic units and this basic units consider to be atomic propositions an atomic propositions is a consider to be sentence which can clearly spoken as either true or false they all they can also be treated as declarative sentence and all.

So, if you have given any sentence is on for example, if you have sentence like it is raining if simply represented it has t r not q etcetera. So, propositions considered to be statement that is either true or false. So, the basic unit in the propositional logic are sentence and this sentence is are represented by sentential variables like p 's q 's r 's etcetera.

And we have finitely many number of such kind variable with which can ah the present our propositional proposition. So, most importance problem that are associates with propositional logic is his that they fail to deal with singular terms, and they need to complex terms they fail to deal with a complex terms and even when it comes to the relation terms proposition logics fail I will talk about ah this things with some examples little bit later, but when it comes to dealing with individual terms etcetera propositions logics fail and other things is that we are no way in propositions logic of talking about individuals members of asset which mean which may have or fail to have certain properties etcetera.

There is no way of Quantifying over the individuals that is the main reason why we will be augmenting our proposition logic with 2 more quantifies these quantifies are for all x and there exist some x . So, there is no way in which you can talk individuals in a set for all members of an array etc. So, it is all there are all represented with some simple kind of prepositions p . So, this all the limitations of proposition logic and that is the reason in order to explain in incorporate all these things we move on to predicate logics.

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Example

Predicate Logic vs. Propositional Logic:

For example consider the following statement about the natural numbers, (which asserts that there is no biggest number).

For every number x there is a number y such that $x < y$.

The above statement is written symbolically: $\forall x \exists y (x < y)$.

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So, to consider simple example let us consider a simple example such as the 1 which is in red color which appears in the slide. So, for every number x there is a number y such that x is less than y for example, you have a natural numbers 1 2 3 4 5 6 etcetera and all for any number that you take into consider let us see you take to that is always be 1 number bigger to that 1.

That number is smaller than the other number its successor 2 is always less than 3 or 3 is always less than 4 etc. So, now, these kinds of sentences which we need to invoke some kind of relations between the individual terms x y etcetera for this if you represent in terms of propositional logic it may not capture the deep structure of this particular kind of sentences. So, this simply represented as some kind of sentence p , but propositions logics are the most simplistic kind of logics that are in which usually propositions are represented by simple variables and all. So, they are they are also important in the sense that in proposition logic we have some nice features such as proposition logics are complete proposition logics are set to be consistent and even they are consistent to be sound. So, they are the wonderful logical feature that will fine in the case of propositional logics, but uh.

If you want to explain the interview mathematical reason in this propositional logic for

short of many things mathematics requires short of relations etcetera an all like in this example we need to going to the in depth of in this particular kind of sentence then we need to analyze particular then we will come to know whether are not this particular sentences true now for examples this particular kind of sentence for every number x there is number y such that x is less than y is represented as symbolically as this for all x there exists some y . So, that x is less than y . So, this is simply is not sufficient for the threw of this the particular kind of sentence, but for that we need to have some kind of domain. So, we are we talking about natural number we talking about integers a talking about rational numbers etcetera all this since its to be Stated. So, that we can talk about true of this particular kind of sentence. So, we are going in to the depth of this particular kind of sentences then we a talking about we are working a relationships etcetera an all to the predicates usually will take care of this relationships.

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Limitations of Propositional Logic:

- 1 Objects in elementary statements, their properties and relations are not explicitly represented in the propositional logic.
- 2 Statements for groups of objects require enumeration. We need $\forall_x \exists_x$.
- 3 **Predicate Logic:** The area of logic that deals with **predicates** and **quantifiers**. It is also called First-order Logic.
- 4 FOL permits quantification over variables.
- 5 Higher order logics permit quantification over functions and predicates: $\forall_P, x[P(x) \vee \neg P(x)]$

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So, these are the sum of things which are also considered to be limitation of proportional logic. So, the object object in elementary object in propositional logics are elementary statements that properties and relations are not explicitly represented in propositional logic for example, in the last sentence x x less than y a that simple sentence is represented as 1 particular kind of thing p just 1 letter p . So, that is not sufficient in enough us especially when you are talking about the relationship between 2 individual

object etcetera.

So, object in object in propositional logic are simply construct to be elementary statements and the statements are groups of objects especially like 1 which will discuss earlier requires some kind of enumeration like for example, if you want say all men are mortal we cannot simply represent this sentence as just $p \rightarrow q$ etcetera say need to quantify. So, that is for all x if x is an in x is considered to be. So, we need quantifiers. So, for all x there x exists some x are considered the 2 main quantifiers that we will using it the predicates logic. So, in at an shall predicate logic is considered to be an extension of in the proposition of logic with is 2 quantifiers there is for all x an there is exists some x . So, predicate.

Logic is also considered to be an area logic the deals with basically predicates which talks about the relationship between the objects a whether are not an object process that particular kind of property etcetera an all for example, if you say all men are mortal motility is the property which is attributed to the human beings. So, motilities considered to be predicate. So, predicates given much for important in the predicate logic. So, the other term for predicate logic is what we called as 1st order logic 1st order logic means is a predicate logic less proposition logic is already there sitting at the background. So, 1 of the advantages of 1st order logics are predicate logic is that it permits quantification a very bits like all men are mortal etcetera is that permit says that motility quantified over all the human beings.

So, and if you take in to consideration higher order logic then it permits quantification over functions and predicates if the quantification happened over only variables then it is called as 1st order logic if the quantification happen over the functions and predicates etcetera an all in this called higher order logic this is one of the important different between differences between 1st order logic and the higher order logic, but will be resisting order attention on quantification over variables.

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Valid but cannot be established in PC

- 1 All men are mortal. All IITK community are men. Therefore all members of IIT community are mortal.
- 2 All circles are figures. Therefore, whoever draws a circle draws a figure.

1 $p, q \vdash r.$

2 $p \rightarrow q.$

Validity is not merely a matter of how simple statements are related by propositional connectives, but depends on the inner structure of the simple statements.

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So, let us consider a simple example and we will see why propositional logic fails to explain the particular kinds of arguments like this. So, to note that when we introduced Aristotle in logic there is also considered to be a kind of predicate logic, but it has limitations. So, because not all the sentences can be put in the form of the categorical proposition and then we can talk about validity of an argument and then Aristotle in logics which also considered as traditional logic which has a limitation that it talks about terms groups of terms which are represented to a group there is an individual term individual beings individual things. So, some are the limitations which are talked about in Aristotle's logic. So, that is considered a simple example that is this famous example: all men are mortal, all IITK community are men.

So, all members of IITK community are considered to be mortal. So, these are the three sentences we have. Suppose if you represented with help of propositional logic when the first one 'all men are mortal' is represented by some kind of little p and 'all IITK community are men' is simply treated as an atomic proposition. So, that is represented as just simply q and then we have the final thing that is 'all members of IITK community are mortal' is simply represented as some kind of another atomic proposition. So, from $p, q \vdash r$ it follows from p, q , we can easily come up with a counterexample in which p and q can be true, but r can be false. There means the argument will be invalid if you do not

looking to if you going to the deep structure of this sentences which are expressed as premises in the same all set as a figures that for whose are transfer circles are say figures. So, suppose if you talk about this particular kind of argument again are the 1st sentence is represented as simply letter p in the 3rd 2nd sentence as a simply represented as a letter q; that means, q is reduce from p an we can easily come of the counter example.

In which p is true in q is falls at makes this argument invalidated, but actually our intuitive intuitively know that this argument is valued. So, all circles are figures in whoever draws a circle drawn a figure this is intuitively are common sense we can say that that in did follows from this, but if you take only proposition logic into consideration where each and every sentence is express as simple in propositions like all circle are figures is represent as p and whoever draws a circle draws a figure is represented also q then p these to q an. So, we know that this sentence is are valid kind of sentences an all, but how to know how to we show that this arguments are well. So, validity near is not nearly matter of how this simple statements are related by means of some kind of propositional connectives what are the propositional connectives.

And are implies if an only if negation etc. So, then many cases in which this this propositional logic works in all, but in many cases where ever you find this relationship etcetera an all this we need to going to the depth of this sentences and we need to looking to how this objectives related to each other and then then on you can talk about through of particular kind of sentence. So, validity is not simply how this sentence s are may be p s 2 there etcetera an all, but it also depends upon the in a structure of the simple statements. So, the in a structure could be in the predicate terms etcetera how these predicates are related to each other etcetera objects are related to each other all the no are in a talk about true with the respect some kind of domain etc. So, that the means same sentence can be true with respect to natural number same sentence can be false with respect to integers etcetera.

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Example:

- 1 P1: All men are Mortal (P)
- 2 P2: Socrates is a man. (Q)
- 3 C: Socrates is Mortal. (R)
- 4 Translation: $P \wedge Q \models R$.

as there is a valuation V under which $V(p) = V(q) = T$ but $V(r) = F$.

To analyze the argument as being valid, we need to break inside these propositions to capture more of the information that they convey. We need to analyze propositions into predicates and arguments, and deal explicitly with quantification.

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So, when you talk about domain 1st and when you talk about some of the building blocks which are related to the deep structure of these statements for the example in this case all men are mortal, Socrates is a man, Socrates is mortal is simply represented as 1st sentence is represented as p , 2nd sentence as q and p and q leads to r . So, we can easily come off with an assignment where evaluation p and valuation of q true and valuation of r false means you can have premises true and false conclusion that makes an argument invalid, but actually this argument is a valid argument.

So, all in mortal of it is man there is no way in which Socrates cannot be mortal, all Socrates mortal less follows from all in are mortal, Socrates a man. To analyze this argument has been valued we will break in said this proposition and to capture more of the information that they convey etcetera and we also need to analyze propositions into predicates and arguments and also deal with the quantification. What is the quantification in all they are all men, a mortal mortality is to all human beings and there is person called Socrates he considered to be human being and then there exist some explicitly Socrates that x is considered to be mortal. So, the 1st sentence is represented as a quantifier with universal quantifier in the next 2 represented by means of existential quantification. So, we need to have a part from the simple logical connective and the relationship between simple relationship between the sentences will have quantifiers.

etcetera.

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Predicate Logic

Predicate Logic extends the propositional Logic with predicate letters that are interpreted as relations on a domain.

Why Predicate Logic

- 1 The axioms and theorems of mathematics are defined on arbitrary sets such as the set of integers (Z).
- 2 We need to be able to write and manipulate logical formulas that contain relations on values from arbitrary set.
- 3 Example: Let R be n -ary relation on the domain D such that $R \subseteq D^n$, then $\text{Prime}(x) \supset N$ is set of numbers $\{2, 3, 5, 7, 11, \dots\}$.

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So, predicate logic in this sense extends the propositional logic with predicate letters p 's q 's r 's etcetera that are interpreted as relations on the particular kind of domain. This domain can be, for example, if you are talking about a domain, the domain could be natural numbers. So, if you are talking about some Indians; that means, all the people who we reside in India etcetera that can be called as some kind of domain. The same thing can be false in something which is true of natural numbers cannot be true of all in all the domains in all like domain in which we have only integers etc. So, why predicate logic the axioms and theorems of mathematics are defined on arbitrary sets such as the set of integers etcetera where always we have some kind of relationship between any 2 set of statements. So, we need to be able to write and manipulate.

Logical formulas that contain relations on values from arbitrary set let us take on simple example that is let r binary relation on the domain d there d consider to be set of it can be natural number it can be real numbers are it can be rational numbers. So, that are is subset of the domain d n were you considered that the property that your kind to in work is the prime number. So, saying that x is the prime number and which is a set of n there is a set of natural numbers if that is the case then all these things such as 2 3 5 7 11 17

etcetera 13 etcetera all these considered to be prime number for example, if I take a number such as 4 then off course that belongs to a all becomes natural numbers, but it is not considered to be does not belong to this particular kind of set. So, this is 4 is not in prime number the same kind of property that is prime of x is subset of n is inpretating particular kind of that is going to be true only when it false with in this particular kind of set if it is if does not belong to this particular kind of set then a statement prime of x does not belong ua even that belongs to n , but that sentence is going to be false.

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Singular and Complex Terms

Singular Terms
Singular terms are words or phrases that represent individual things, such as *Ravi*, *Santa*, *the Moon*, *Kanpur*, *the IITK library* and so on. Singular terms are more commonly called the names of individual things.

Complex Terms
The sentences of our everyday language usually contain complex descriptive phrases to represent groups of things. But, Traditional Logic does not distinguish between simple general terms and complex general terms. with the one small exception that affirmative terms are distinguished from negative terms

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We set in the begging of this lecture that predicate of it proportional logic fails to explain in ability which was in ability to expense single terms complex terms and also the relational terms. So, what we mean by this single terms complex terms etc. So, we will single terms are words of a trice's that we present individual things such as Ravi Santa the moon khanpur etcetera IIT k library all individual things. So, singular terms are more commonly called as names of individual things any name of anything considered to be a singular term.

So, we need to note that even classical logics traditional logics such a aristotle in logic all. So, fails to explain the singular terms it could not which not explain single terms in a proper way small the complex terms are like this sentences are over every day language

usually contains; obviously, complex descriptive phrases usually to represent group of things, but in a case of traditional logic; that means, aristotle in logic does not distinguish between simple general terms and complex general terms it is no way in condign between simple general terms and complex general terms which is the distinguish is very important for as to make. So, with with 1 small exception that it distinguishes definitely between a feel native terms and the negative terms that classical traditional logics successes make in such kind of distinction, but it fails to distinguish between singular term such as ravi santa the moon khanpur etcetera with some kind of complex phrases that we use in our day to day this course. So, there is no way in which you can discuss distinguish between singular kind complex terms.

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Example

Example

Some pink birds are long-legged. All birds are winged.
Therefore some pink things are long-legged and winged.

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And there are lots of example which 1 can give with propositional logic fail to explain particular kind of things let a say of for example, if you say some pink bird are long legged and all birds are wings therefore, we say that some pink things that is considered to be singular term are long legged and winged, so if represented in terms of simple propositional logic it will be like 2 sentences like a 1st fun is represented as p second one is represented as q and therefore, the entire statement some pink things are long legged and winged represented as r. So, that will not serve a purpose. So, we need to talk about how this sentences are related to each other that is expressed by predicates etcetera and

then we need to go to the deeper structure of these sentence and then then only we can talk about validity of sentences argument like the once which I am showing it him.

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The slide is titled "Terms, Predicates". It contains two main sections:

- Singular Terms**: All singular terms are symbolized by unique lower case letters: a, b, c, d, \dots, w (but not including the letters: x, y, z).
- Simple Predicates**: All simple general terms are symbolized by unique capital letters: A, B, C, \dots, Z . (We will also call these simple predicates.) These terms represents the properties that thing have.

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So, the single terms are usually represented by a unique letter there is a b c d etcetera an all, but x y z are usually used for variables an all. So, if you are show that the person is ravi are raju are ramesh in usually representation as a b c d etcetera an all.

Suppose if you want to represent it as x x is a human being that human being can be anything it can be rahul gandhi are it can be any 1 ravi are any 1. So, represented as variables x y z. So, now, we a trying to talking about some of the basic building blocks of the predicate logic 1st we talked about the limitations of propositional logic they certain things which propositional logic fails to explain now these things needs to be we need to be in a position to explain all this thinks with help of by the propositional logics with quantifies such as for all x in there is exits some x. So, predicate logics can in some in a certain sense it can also viewed as study of is citifies quantifies.

So, now, these are the single singular terms are usually represented by constant individual constant will be see etcetera and simple predicates such as mortality etcetera an all in all beings are mortal mortality considered to be the property which all human

beings that is considered to be the predicate we any a simple grammatical sentence we have subject and we have predicate. So, predicates takes a is a central position in predicate logic all simple general terms are symp symbolized by unique capital letter is usually we represent a predicate let us as capital letters a b c d etcetera an all. So, we also call this as simple predicates. So, these terms represents the properties that a things have for example, if you say all human beings are mortal mortality is a property which is attributed to the human beings. So, that property which is attributed to some kind of objects is called as a predicate.

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Sentences

Name-Sentence
Ravi is a painter is analyzed as having two parts: Ravi (name) and *is a painter*. The first part is a name and refers to some individual thing, and the second part is a simple predicate that identifies some characteristic.

Complex-Name sentences
 A basic rule of analysis for the new language(Predicate logic) is that complex expressions must be broken up into single, simple predications, each of which is applied to the subject. Each idea gets one sentence
 Ravi is a painter but not a Magician. $R_p \wedge \neg R_m$
 Manmohan Singh is a sneaky Prime Minister of India: $Ms \wedge Mp$

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So, there are sentences sentences are a in this then for example, if you say ravi is a painter this is analyzed in 2 parts 1st is the name that is ravi and next 1 next sentence next thing which follows that is is a painter. So, the 1st part is a name and the 1st 2 some kind of individual thing singular term that is represented by some kind of constant such as a b c etcetera and the predicate is represented by some kind of capital letter. So, that is sentence is p's has 2 parts 1st is the name sentence that is the ravi and is a painter represents a predicate the 2nd part is a simple predicate that identifies some kind of characteristic the characteristic is is a being a painter is a characteristic of that particular individual human being ravi. Suppose being a painter is take in as p capital p and ravi is considered to be a small r then it will the sentences is represented as p substitute r. So,

So, this is the way we represent sentences in the predicate logic. So, now, this sentence is represented as now, what are complex name sentence a basic rule of analysis for the new language that is the predicate logic is that complex expressions must be broken up into some single and simple predications each of which is applied to add attributed to the subject. So, each idea gets 1 kind of sentence if you have a complex sentence you break it in to some simple sentences and then each kind of property is attributed to both of the subject set that occurs here for example, lets considered a simple example if a say ravi is a painter, but not a magician.

So, now, you right it in this way 1st sentence is 1st sentence is represented ravi is painter is represented is $r p$, but not means a that we as conjunction and it is not the case that $r m$ the m stands for magician for example, if you want to say that manmohan singh is a a good prime minister of india etcetera an all. So, now, let us see the 1st sentence $s s$ stands for manmohan singh. So, now, unique a represented in this way there are 2 things which are there here 2 things which cannot to be to manmohan singh in this case sneaky is 1 property which can to manmohan singh and the prime minister of India can also be attributed to him. So, that is a reason why we wrote it as actually should be the other on $s m$ and $s p$ where p stands from prime minister and m stands from manmohan singh here. So, $s m$ and $p m$ vertical represented you can represent this particular sentence in this way. So, the idea here is is that if you have a complex sentence it which need to be broken in to simple sentences and then we need to represent this sentences in a particular kind of order.

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Syntax of Predicate Logic

Syntax

- 1 Individual Constants: $\{a, b, c, d \dots\}$.
- 2 Individual Variables: $\{x, y, z \dots\}$
- 3 Predicate Variables: P, Q, R

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So, now let us try to discuss in the predicate logics we begin with the in tax of predicate logic it tells us what kind of things set we are it in our in our language of predicate logic and then we move on to semantics of predicate logic where we discuss about what you mean by saying that a particular sentence in a predicate logic is considered to be true or false there is what discuss about it and then we will talk about some of the important decision procedure methods with which you can check the validity of a given formula that existing in a given proposition predicate logics and then in that will discuss at least 1 or 3 important methods to start with will talk about simple semantics tabacks method and then we also talk about resolution representation method and then in natural deduction in the context of predicate logic etcetera then there is another method with which you can talk about validity of a given formula that is reduce in the predicate logic formula in to its corresponding normal form say instead of talking about conjunctive isn't in to normal forms here we talked about prenex normal form.

So, if you can reduce any given formula in to prenex normal form. So, you can talked about validity of a given formula that exist in the predicate logic and then we will talked about then we move on and we talk about identity relation etcetera an know an I will talk about something talking about definite description etc. So, there are things which are there in agenda of is predicate logics to start with we need to have some kind of language

to begin with. So, the language of predicate logic consists of these things is not just sentence and the sentences are combined together with help of logical connectives and then form complex kind of sentence as is the case of propositional logic, but here many 2 point to the deeper structure of the sentences. So, in were sentences are language of you predicate logic we have individual constants whether if a to some kind of names are individual things etcetera chair table etcetera and all you referring to that particular kind of table that singular kind of things the refer to there refer by individual constants a b c d can be ravi ram raju etcetera and all set of new human beings are if a talking.

If you talking about some table which you are talking about a specific table are your talking about some kind of monkey a table are any other thing, but talking about a specific kind of monkey. So, now, they have individual variables for example, if you want say we do not know carly who that person is suppose if any properties attributed to some kind of human being let us say if you want to refer to some I i t k students are very bright. So, the brightness is a attributed to some kind of human beings. So, that is some can involve you can be many then then be at least 1 are you can n few number of people an all. So, with a we not sure who that for exactly. So, we represented as some kind of variable x x v can be anything represent this student of that is student etcetera who all come under the category of bright students and then we have predicate.

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Contd

- 1 **Quantifiers:** Universal Quantifier \forall_x and Existential Quantifier, \exists_x .
- 2 **Set of connectives:** $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \forall_x, \exists_x\}$.
- 3 **Function Symbols:** $\{f, g, h, \dots f_0, g_0, \dots\}$
- 4 (any set of them for each arity $n=1,2,3 \dots$. 0-ary symbols are simply the constants).
- 5 We dont have propositional letters (0-ary predicates)
- 6 **Punctuation:** $(), [,]$

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Let us such as p, q, r there if in general usually we represent this predicate let us by capital letters P, Q, R etc. So, in addition to that we have quantifiers. So, there are usually 2 quantifiers set as used 1 is universal quantifiers they are referring to the entire class etcetera to then you recover universal quantifiers like in the case of for all human beings are mortal mortality is attributed to the whole set of class that class of human beings and existence quantify there is represented as simply there exists some things and as usual we have this connectives which already there in the case of propositional logic. So, we need to note that we are just augmenting the propositional logic with 2 more usually with quantifies an all. So, that is in order to express this quantifies an all.

We need to have all this predicates terms etcetera an all. So, predicates logics are in away extension of enough the the propositional logics in a sense it we are this propositional logics with to quantifies let us are there for all x an there exists some x . So, these on things set that are there in our language and we have some functional symbol f, g, h etcetera an all if it is zero ary kind of thing it is which represent as zero z zero etcetera an all. So, now, here it is clear that we do not have any propositional kind of 1st let us that exists in the predicate logic because zero ary symbols are usually in this in propositional in the predicate logics at treated as constants like a, b, c etcetera an all. So, we do not have individual let us such as p, q, r etcetera an all as is the case of propositional logic because there all zero ary symbol.

But zero ary symbols are here represented as constant, but we have functional symbol with arty at least 1 2 an all. So, we do not have propositional let us; that means, zero because propositional let us are usually represented as zero ary predicates arty is zero. So, it is in that sense we do not require this propositional letters. So, we just simply use if there are zero ary kind of symbol which exists there simply treated as constants and as usual we need to have is punctuation known comma bracket square brackets etcetera to to avoid a to avoid ambiguous in the well from formalize.

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Syntax of Predicate Logic

We use individual constants to formalize **Names**, individual variables to formalize **individual variable words**, predicate symbol variables to formalize **predicate expressions**, and the two quantifiers to formalize corresponding **quantificational expressions**.

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So, syntax of predicate logic to in in to continuation that. So, we use individual constants to formalize names. Such as a b c etcetera an all an individual variables like x y z to refer to individual variable words like this man that man etcetera an all we not saying which man actually is an predicate symbol variables such as some kind of properties which the object is object posses is which are considered predicate expressions an we used to quantifiers for all x an there is exists some x the formula is the quantificational express like expressions.

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Definition of Wff in PL

Definition

- 1 If A is an n -ary predicate letter in the vocabulary of L , and each of the terms $t_1 \dots t_2 \dots t_n$ is a constant or a variable in the language of L , then $At_1 \dots t_n$ is a formula of L .
- 2 If p is wff, then $\neg p$ also.
- 3 If p, q are formulas of L , then $p \wedge q, p \vee q, p \rightarrow q, p \leftrightarrow q$
- 4 if ϕ is formula in L , and x is a variable the $\forall_x \phi$ and $\exists_x \phi$ are formulas of L .
- 5 Only that which can be generated by the above clauses in finite number of steps is a formula in L .

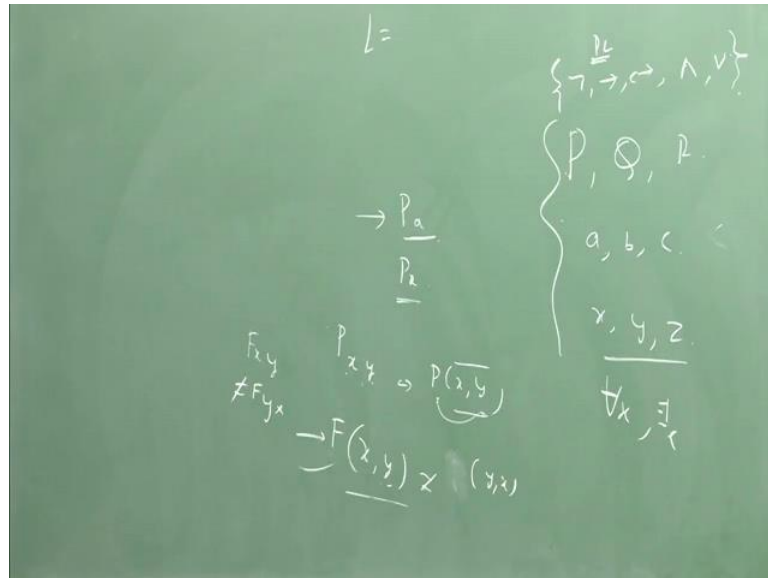
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Now just as in the case of Propositional logic we can construct some kind of well form formulas within the predicate logic p l stands for predicate logic just as in the case of propositional logic not any kind of strength we combined in form formula well form formula in the propositional logic just like that, so here also. So, formulas in combined certain way and then form some kind of. Well from formulas when it note that just like in the case of propositional logic whatever formula that you come away there is not considered to be an all are valid formula. So, they can be infinitely many strings such you can generate by using symbol that occurs in that particular kind of language with help of logical connectives, but not all are construct to be valid formula just like that even in the predicate logic also you can generates some well form formulas, but not all generated well form formulas are considered to be valid formulas valid formula is also considered to be. So, will to have some kind of definition with which you can formulate you can form is well forms formula in the predicate logic. So, these are some of the important rules at 1 employs in fining all whether a given formula is a well of form formula are. So, if a is an n array predicate.

Letter in the vocabulary of your predicate logic and each of the terms t_1 to t_2 to t_n is a constant or; that means, a basic r it can be available it it can be like $p x$ are it can be $p c$ etc. So, were p is considered to be property which is attributed to c are even to individual

variable like x then a t 1 to t 2 is considered to be a formula of L.

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So, that is means in that sense at simply says that behalf predicate P, let us say these are all p q r etcetera and then we have. So, these are individual constants are n then there are some variables which refer refers to which man that mans etc. So, now, it says that simply if you right like this that is considered to be. A well form formula p a are if a simply right this thing that is also considered to be a well form formula. So, now, what we have in our language are on this things. So, this represent predicates this represent individual constant referring to individual things and this refers to individual variables and then we have this 2 things there exists some x for all x and then as usual as in the case of propositional logic we have all this connectives. So, this implication by implication conjunction and a junction we are a set of logic connectives which are already of day.

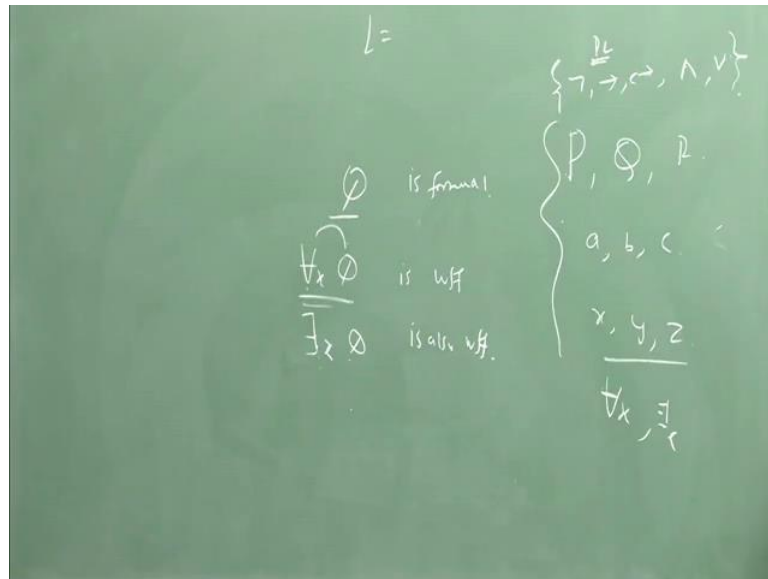
So, now, if you simply right like this p a that is considered to be well form formula are if you can any1 right p x that is what the rule number 1 tells us. So, now, the 2nd rule is is that if p is considered to be well form formula and not p is also considered to be well form formula as use the case of propositional logic the only thing which is extreme is the 1st 1. So, this is what 1 we can right it in this particular kind of way this is the kind of

predicate suppose if you 1 2 right in this way what if if you right in this way x is related to y in certain way at some property is attributed to x and y where x and y are related certain kind of way. So, these are predicates an all. So, the same thing which can be return in this particular kind of in some text books it can be it is return in this particular kind of sense the x y is in some kind of order x is related to y in a certain way and they have the property p. So, it can be like for example, there are 2 objects such as x and y for example, x is considered to be father of y then if put like this you right in this. So, this will there is some kind of order which is there in this 1 x is considered to be father of y. So, this is different from f y and x.

So, now, need to replace this with son of. So, f of x y if change order that is not equivalent to its corresponding formula. So, there is some kind of order which is they in this 1. So, here in this way formula this can be even return as f x and y order in some text books prepare to written in this x y and this is the predicate as these are individual variables is also considered to be well form formula in some other text books you will find it in this way f x followed by the y example if want to in work this particular kind of relation that is relationship between x and y is father being a father.

So, that comes 1st an followed by that is particular kind of order. So, this should be read as x is a father of y. So, x is a father of y suppose if you this is different from y and x x is father of it does not mean that y is a father of x. So, this 2 are different kind of formulas. So, as in the case of propositional logics we have suppose if any variable any things is considered to be well form formula not p's also considered to be well form formula and p and q are well form formula is than a conjunction rejection implication and by implying there are binaring connectives if you can combined with any 1 of the this individual variables with these things set as the also considered to be well form formula; that means, all the well form formula. So, proposition logic already retained in the predicate logic it is in that sense predicate logic is considered to be an extension of propositional logic. So, the forth will is is that if 5 is a well form formula in 1 an x is a variable then this is the 2 other things which we have...

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So, they are like this. So, the 2 extra things such will finite in propositional logic is this suppose if anything like phi is a formula well from formula just a constant are a simple individual variable in also considered to be well from formula an these to combined and then form another kind of well form formula. So, now, innovation to that phi is also already considered to be well form formula then this is also well form formula, but if you right like this and this is not a well form formula. So, we define it some such a way that 1st there is quantifiers followed by that there is a sentence in the same way if something phi is a.

Well, from formula even there exists from x there is at phi is also well form formula. So, these are the additional things which will find it in the case of predicate logic. So, now, finally, only which that which can be generated by using 1 of this 4 things is considered to be a well form formula we should not be in position to derive any other kind of thing apart from this particular kinds of rules. So, so these this is not we mean by well form formulas in the case of predicate logic the only thing which extra thing which we will finite here is is particular kind of thing. So, that is phi is a well form formula then for all x phi is also considered to be well form formula and phi is a well form formula they exists some next phi is also considered to be a well form formula.

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The slide is titled "Basic Building Blocks of Predicate Logic". It contains three numbered points:

- 1 Objects: (things in the world): can give these names such as Umbrella, Person, John, Earth etc.
- 2 Relations: properties, relations between objects: Wife (Ravi, Sita)
- 3 Functions: (types of relation that maps an input to a value)

At the bottom of the slide, there is a footer with the text: "A. V. Ravishankar Sarma (IITK) Predicate Logic October 10, 2013 16 / 44".

Let us set of something about. So, the basic building blocks of predicate logic and we this and this particular kind of lecture. So, just like in a case of positional logic we have the basic building blocks for the propositional logics arte the proposition the propositions are the sentences which can be clearly true or false, but in a case of predicate logic there are the objects; that means, things in the world such as duster table chair etcetera an all individual things an you can gave this names such as umbrella person john ravi etcetera an all individual things. So, their considered to be the building blocks of predicate logic and then in addition to the we have religions such as properties relation between objects such as example ravi is related to sita and such away at sita is considered to be wife of ravi right to here and we also have functions. Such as functions talks about types of relation that maps an in input in to some kind of value.

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Building Blocks: Predicates

- 1 **Predicates:** We use capital letters, (A, B, C, etc.) to represent predicates. A predicate letter will usually be associated with a list of at least one variable. For example, $A(x)$, $B(x, y, z)$ etc.
- 2 A predicate is used to represent a property of its variable(s) or a relationship between its variables.
- 3 The connectives are $\{\wedge, \vee, \rightarrow, \leftrightarrow, \neg\}$. These are the same connectives we used in propositional calculus, and they mean exactly the same thing.
- 4 L_x , L_{xx} , are one place predicates; L_{xy} is a two place predicate. G_{xyz} is three place predicate.

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So, now these are some of the building blocks we just now we talked about the particular kind of thing to start with. So, is the predicate logic is all about study of predicates. So, predicates are a occupies central position here. So, with this I will end this lecture. So, I talk some I will talk something about the predicates. So, we usually we discuss we it present this predicates with capital letters and the predicate letter will usually be associated with a list of at least 1 particular kind of variable for example, if there is only 1 variable here it is called as a unit predicate an if you a 2 letters x and y it is a binary predicate if you a t v letters such as x y z for example, a will talked about the example in a y from long for example, if you say ravi is a bright student. So, you simply right it as b r. So, were b is considered to be predicate that is being bright is considered to be predicate and then the constant that is here b are different to r.

So, that is we will to the represented as b r that is considered to be for example, if you want to in work relation between x and y on like x is a wife of y then you right w x y w is w stands for being wife of some on any need to follow some kind of order. So, that is a binary predicate suppose if you a are trying to talk about the h8s of 3 people an all x y z suppose x is less than y less h8 x x h8 in less than y are y's h8 is less than z etcetera an all. So, need to required relationship between 3 people. So, that is considered to be twister predicate an all. So, like this based on the number of variables we have laniary

binary and a kind of for may be n nary kind of predicate. So, predicates is usually used to represent property of is variable that is for example, if you say all in a mortal mortality is a predi predicate which attributed to the variables that is all human beings r a relationship between x variables like for example, x is wife of y etc. So, now, we have the connectives and are implies if only an if etcetera and these are same connectives we used in propositional calculus any addition to the that we have \neg \wedge \vee etcetera there all \neg \wedge \vee is considered to be 1 place predicates, because x is related to x \neg here \neg \wedge \vee is considered to be a 2 plus predicate an if you want to...

In work the relationship between 3 in 3 people for example, then in equal 3 place predicates an all. So, in this lecture we just introduced some of the limitation of propositional logic. So, we discuss that not all things we can be represented terms of simple relationship between the sentences that is what was d1 in the case of propositional logic although propositional logics were considered to be sound complete in constant etcetera wonderful features are there in that 1, but that is not sufficient enough to capture many parts of your mathematical reasons. So, the basically our goal was to capture the mathematical reasoning with the help of this 1st order logic 1st order logic I mean it is it is combination of propositional logic and the predicate logic in this class we we discussed about some of the important building blocks of the pre the predicate logics.

So, in the next class I will talking about what exactly will mean by is predicates terms objects the function etcetera an all. So, when I talk in greeted it tell about is the syntax of pre pre predicate logics I will deal with some of this important concepts are the basic building blocks of predicate logic. Then we will move on to when do we say that we given sentence is true are given sentences is false etcetera in than we will continue with this, I think the next class.