Introduction to logic
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Lecture - 34
Building blocks of predicate logic

Welcome back in the last lecture we presented concept of predicate logic were we introduce syntax of predicate logic. So, in this class I would be in continuation with the last class will be talking about some other basic building blocks of predicate logic. The Basic Building Blocks of Predicate Logic is, this that predicate first thing to start with and you have objects and we have terms etcetera and all, so will define of all these things 1 by 1 detail which some example. So, this class is dedicated to syntax of predicate logic. So, what will be doing is when do you say that, a given well form formula is given formula is considered to be a formula well form formula or when do we say that a particular formula is considered to be a kind of formula in predicate logic etcetera and all.

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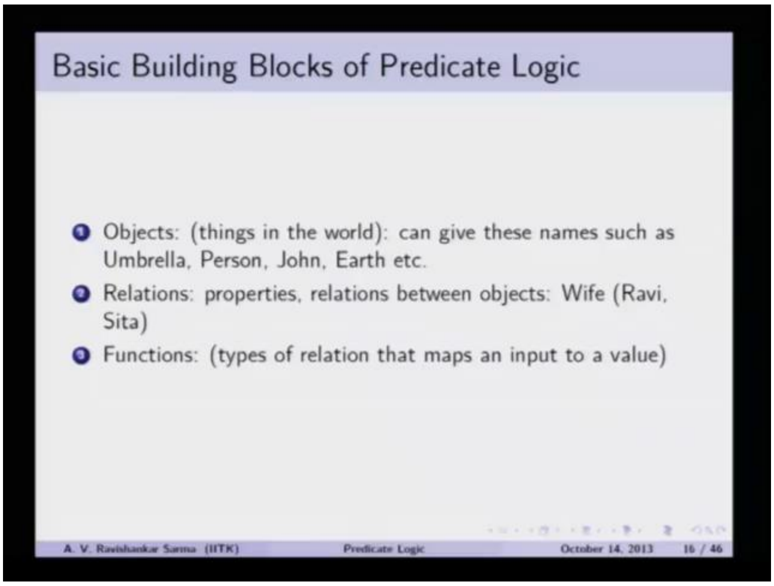

So, to start with these are some of the building blocks of predicate logic, to start with we need to have some kind of objects. There the things which exist in the world, you can be cats, dogs are anything; we can give this name such as umbrella, person, John, earth etcetera and all. These things are referring some kind of objects and we have also relations and properties relation between these objects such as, the relationship between Ravi and Sita is wife or siblings and etcetera and all. So, we need to have relations and we need to have functions, such as a type of relations that maps and input to some kind of value there are considered to be functions.

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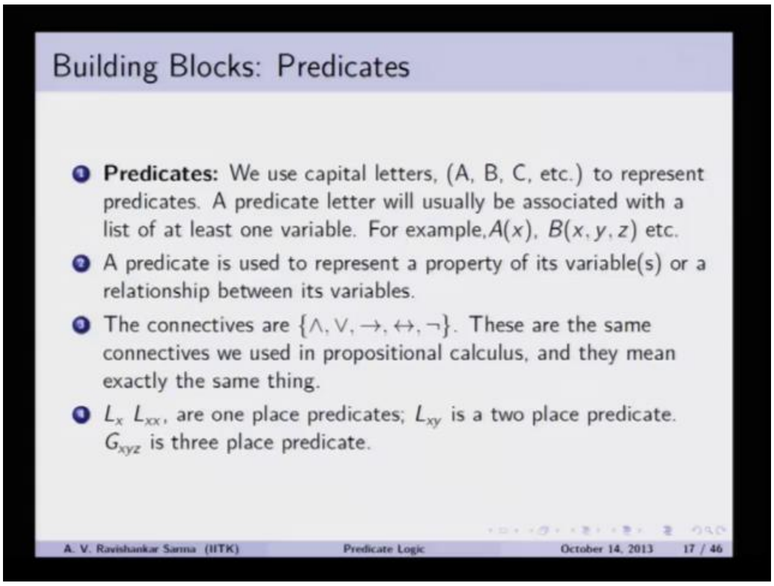

To start with usually we represent predicates with capital letters, actually what we 1 what we need to know note before this is that, in the context of pretty propositional we represented all the sentences as some kind of propositional variable. For example, if I represent x; that means, stands for may be like is raining etcetera. So, the compound sentences are form by combining these atomic sentences, which are represented by some kind of propositional variables.

So, our logical connect is and, or implies etcetera we will take care of in combining this simple sentences into a compound kind of sentences. So, in the predicate logic the story is little bit different. So, we need to go in to the deep structure of sentences. So, were what is essential for us is the predicate and the relations, objects etcetera. All these things are very important for us. So, all these things are important especially when you are time to interpret a given sentence.

So, sentences in predicate logic are not so simple like, the once which you have already seen the case of propositional logic. So, if you we're ask you say if you're ask to find out the truth value of it is raining, it is not raining it is so simple whatever, value that it is raining and not raining takes it is p and not p. Obviously, the truth value of the particular kind of sentence is false because, it is a contradiction.

But it is not, so simple in the case of predicate logic, predicate logic we need to going to the details deep structure of this particular kinds of sentences; were we need to interpret especially predicate, relations, objects etcetera and all. So, the semantics of predicate we will talk about in the next class, but we will focus our attention on our basic building blocks.

To start with predicates, the predicate later will usually be associated with list of at least 1 variable that is the suppose, if you have ax than it is considered to be a unary predicate. And if we have let us say, if you represent x and y x and y are considered to be variables and then, a is considered to be a predicate. So, that is considered to be binary predicate a can be is some kind of relation etcetera and all.

Suppose, if we want to say in between x, y, z etcetera or in between x and y we need to have a kind of predicates B x, y, z. So, usually a predicate is use to represent a property of its variable are relationship between its variables. When you have a unary predicate, it is talking about the property of a particular human being. For example, if you say is mortal then mortality is a property which is attributed to the variable that is Socrates.

Here in this cases x, so you represented it h, n, x for example, if you want represent all minor motel than mortality is attributed to all human beings that is h, x etcetera. So, a predicate is use to represent a kind of property of its variable or a kind of relationship between its variables in the case of father x y in a particular kind of order x is consider to be father y.

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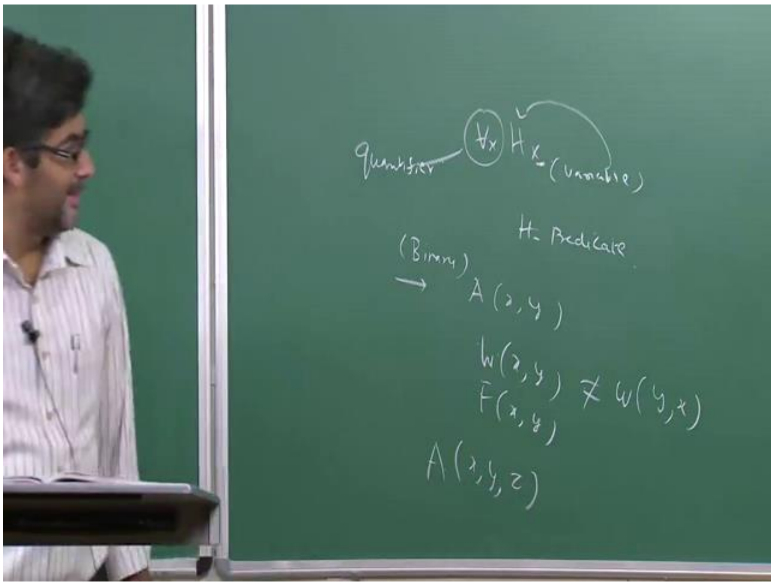

Suppose, if you represent like this in the first case it is like this. So, now in this 1 H x is a variable this is consider to be a variable and some kind of property is attributed to this 1 H is consider to be a predicate and then, this is consider to be A quantifier. So, this is the structure of this particular kind of thing. So, a predicate can also be represented in this particular kind of way.

So, now in the second case it is talking about the relationship between x and y. So, it can be a less than or greater than y if you talking about natural numbers. Suppose, if you are talking about human beings you can say that let us, if I write like this there is a particular kind of order which is followed here. So this means, x is a wife of y or if or if write like this x is father of y.

So, this is different from wife y and x if you change the order here, order of variables terms that is what you call it as terms representing some kind of variables if you replace this 1 like this these 2 or not the same. So, this is kind of binary kind of predicate and then, If you want to represent more than 2 than you represent in this way x y z.

For example, someone is sitting in between x, y and z and in between ness for example, requires let us say more than 3 variable and all. So, like this you can go on and on you can talk about n binary kind of predates and all.

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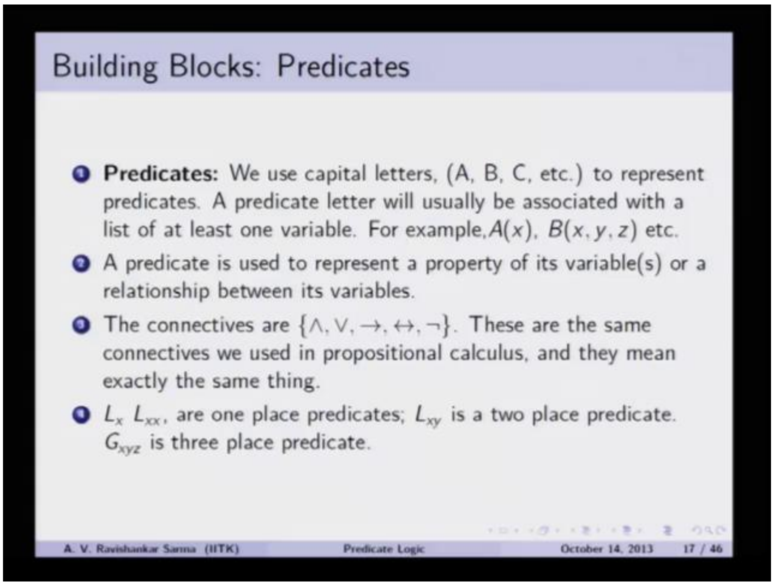

So, as usual we have the connectives and or imply, double implies, negation etcetera. This is the same as, in the case of propositional logic and they mean exactly the same that is n stands for all, n stands this upside v stands for end and v stands for r and implies for implies and if and only if is stands for this thing a b y directional kind of connective and as it is. So we have unary, binary, ternary, kind of predicates and depend text books it represented in a different way.

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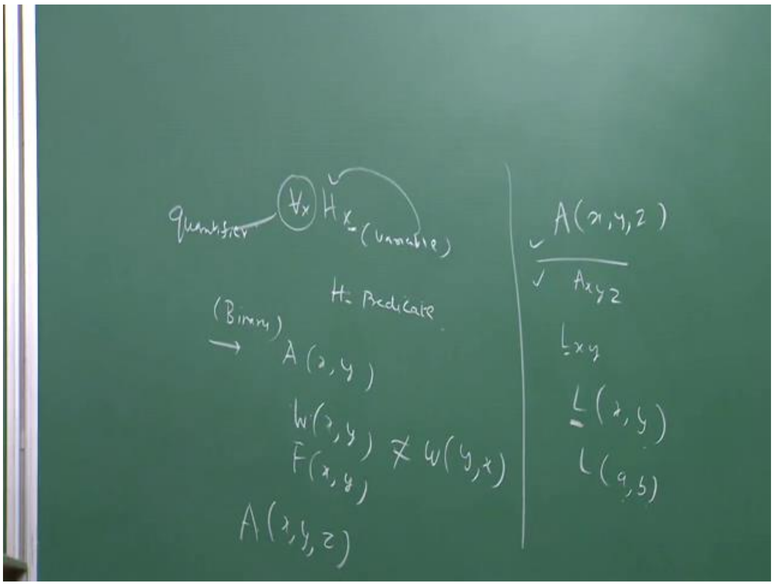

So, sometimes you can write like this A x, y and z or in some text books it is written in this way. So, they are morals the same on in a same way for example, L x, y some kind of property L let us say L x loves y and all. So, in text book it is written as love is considered to be the predicate which comes first we give lot of emphases to the predicates and these are the objects. So, they represent some kind of individual variables and when you represent replace this the x and y with some individual variables. Then for example, it takes value a and b, a and b are referring to some specific kind of individuals like John, Ravi, Sita etcetera.

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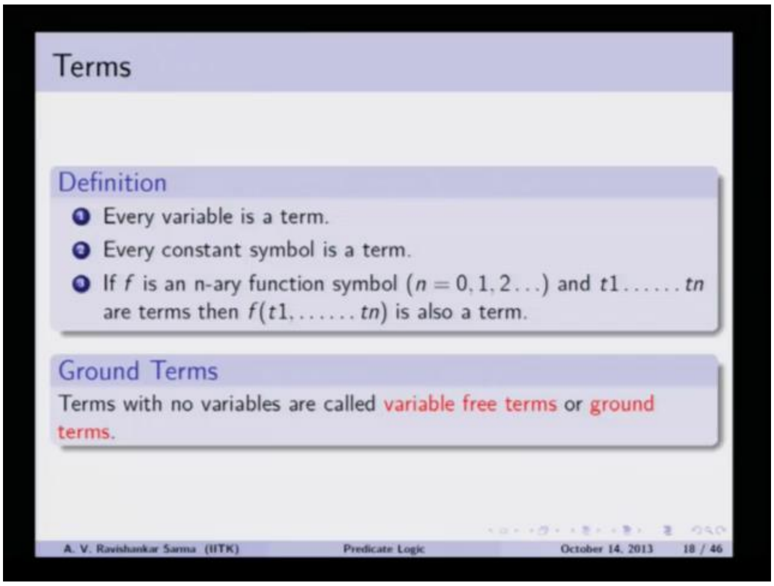

So, these are the basic building blocks to start with after the predicates predicate consist of terms and all terms in particular and what you mean by a term in the predicate logic? So, the definition is like this so we will be following some kind of definition, some of the definitions sound technical. But I will try to explain or I will try to make it as simpler as possible. So, now to start with what you mean by a term. So, every variable is considered to be a term, so any variable that you taking to consideration. So, that is going to be a term.

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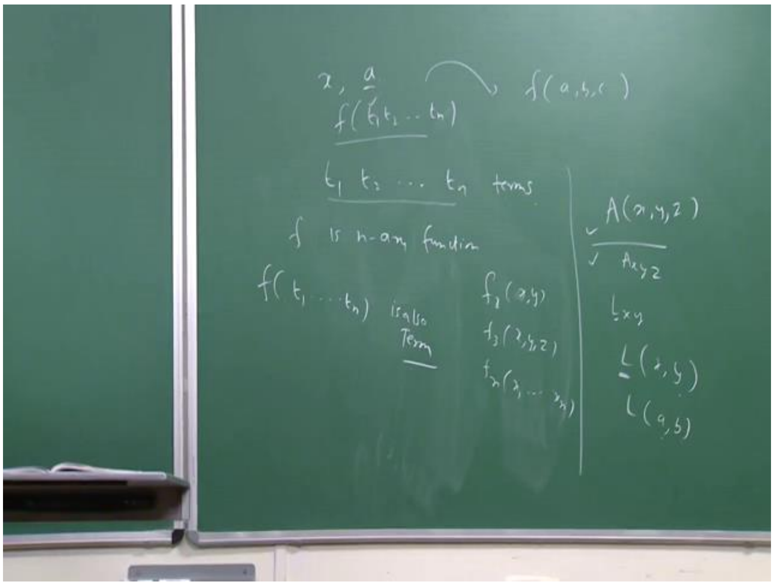

So, suppose if you represent this thing so, this is considered to be a term is an individual constant like this can also be treated as a term. So, every constants symbol is also consider to be a term like a, b, c's etcetera so that is thing. So, these are the primary things need to not either if you write individual variables like X stands for anything like: Sita, Geeta, Ram, etcetera and all.

And individual constant to a that also considered to a term usually predicate consist to be a terms. So, if f is a kind of n-ary function symbol; that means, n= 0, 1, 2 n and then, t1 to tn are the terms then f t1 to tn is also considered to be a term. So, it is like n-ary functions means for example, suppose if we are representing only 2 variable is binary function and all it is a relationship between x and y.

So, these are considered to be terms in all so, now we have t1, t2, tn are terms. So, these represent either individual variables x, y, z etcetera and all. This xyz can be replaced later or it can be individual constant like: a, b, c etcetera and all this specifically referring to individual object like: duster, chock pieces etcetera. So, now these are considered to be terms and all and if f is n-ary function that means, if it is binary function.

We have 2 variables x and y like this 3 means x y is z etcetera, fn means x2. I can take in this way x1 2 xn if it n-ary function. Then, these are also these are already consider to be terms than f of t1, tn is also consider to be at term. So, that is what it says so, at term has to be depend at least these 3 ways anything which is not defined in this kind of thing is not considered to be a term.

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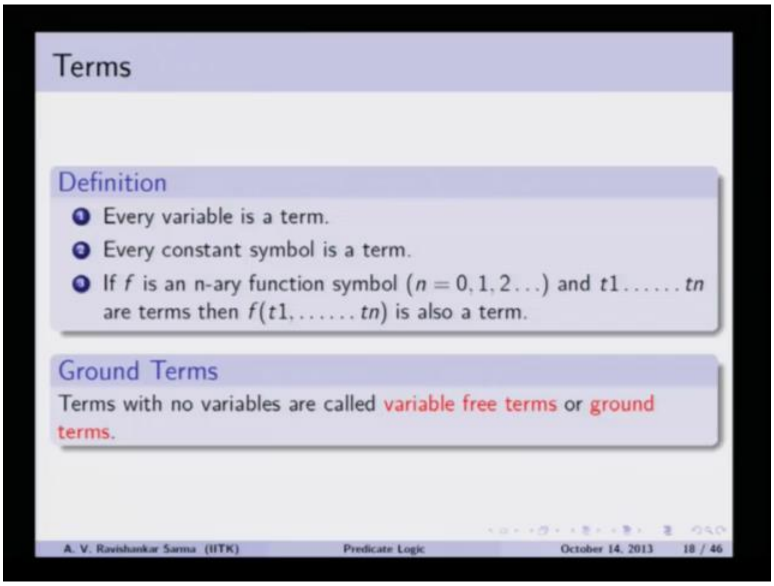

So, out of these terms there are certain specific kinds of terms which are considered to be ground terms. So, what are this ground terms? These the terms with no variables these are consider to be ground kind of terms. So, in the sense that you're already exhaust replaced is individual variables with these variables with some kind of individual objects. Like: x, y, z are replace by some constant like: a, b, c there referring to some specific individual objects.

In that case, these are considered to be variables free terms are they can also be considered as ground terms. So, in these case for example, f t1, t2 etcetera tn. So, now you replace this terms with some kind of individual variable, individual constant like: a, b, c etcetera. So, then this is considered to be these terms are considered to be the terms which exist in that particular kind of n-ary function f are considered to be ground terms.

So, sometime you will have only 1 variables, sometimes you can have more than 1 variables with 1 free variables etcetera and all.

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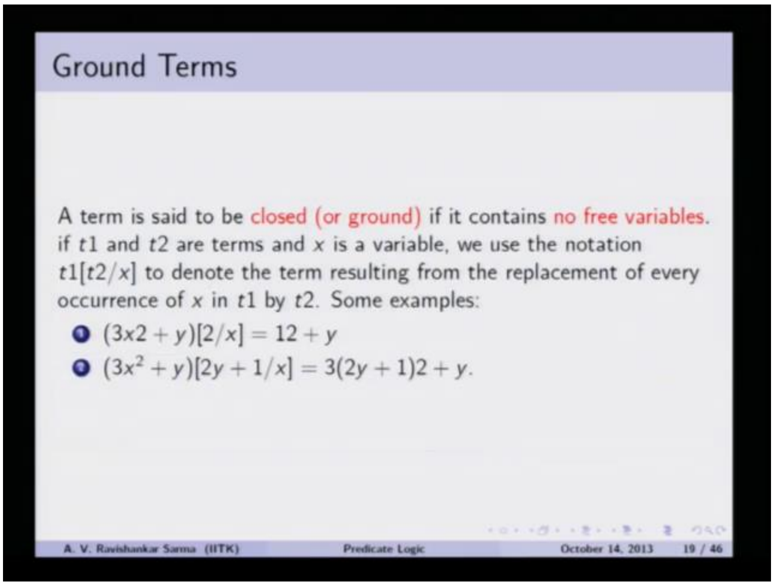

So, now to make it more express it a term is set to be closed are a ground term if it contains know free variables. For example if t1 and t2 are considered to be term and x is considered to be variable, we can use different kind of notation. So, a term t1 is such that is such that t1 is like this a t2 replace x is replace by t2 that means, t1, t2 obligates. It denotes a term resulting from the replacement of every occurrence of x in t1 by t2. So, let us some example which we taking to consideration and we will try to make it clear.

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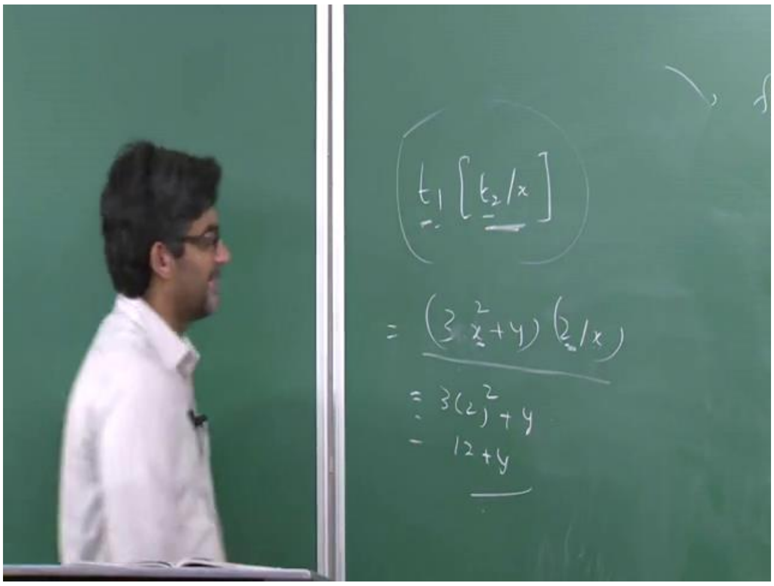

So, now let us say this is the notation that we are following t2 and x or you can... So, these are the terms in all t1, t2 and x is considered to be a variable. So, now so in this term t1 so t1 is resulted especially when wherever you have x that is replace by another term t2. So, then that is what is resulting by this a particular kind of formula. So, this results in by replacement of every occurrence of x in this term t1 with t2 some examples which you can take to consideration.

For example, if we have this particular kind of thing 3 into 2 plus by 2 and x, 3x plus y and 2x that means, stands for this thing that wherever you have x you're replaced it with 2; this is the sum of notation that we can use and all. So, now this is considered to be a formula in that you have a term x stands for individual variable. So, now this x is replaced by 2.

So, now this will become y let us for a time being we have a formula like this 3x plus y. So, now you replace x with 2 so, now this will become 2 square plus y so, now this will become 12 plus y. So this means, where ever you have x you replace with 2 that is what it 6. So, what is a 2 in the a kind of individual constant which are replacing x with 2 so, then the formula will become 12 plus y. In the same way, in this formula 3x square plus y you replace x with 2y plus 1.

So, then this formula will become like this. So, wherever x is there you replaced with 2y plus 1 power of 2 plus y. So, whatever formula which results in is considered to be the resultant term; so a term is set to be closed if it contains no free variable. So, this tells us how to replace the individual variables with some kind of constants.

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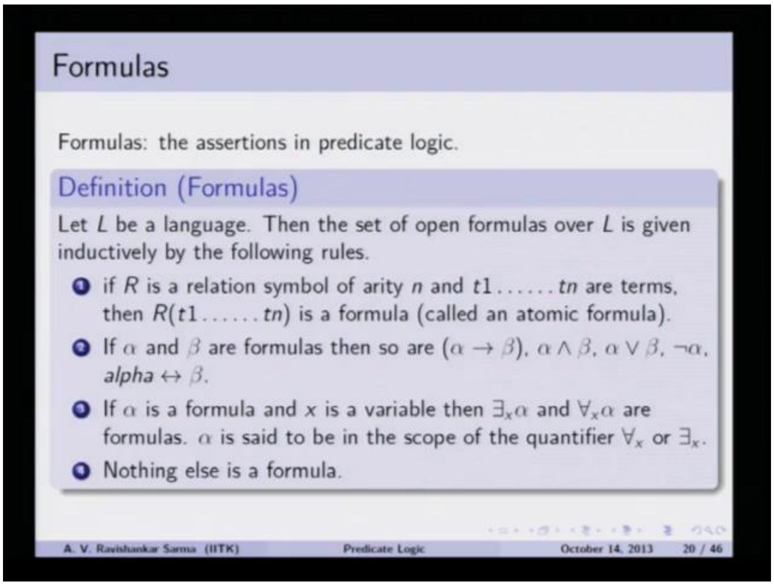

So, now we spoke about what we mean by terms then, when individual some variable can be replaced with some kind of constant in a term or this is which we spoke about it in a some kind of details manner. So, now let us talk about what we mean by a formula in predicate logic. So, predicate logic is a also consider to be term logic just like in now in the case of Aristotle logic; in the case of classical Aristotle logic it also talk about terms.

But it fails to explain those arguments in which consist of complex terms are simple singular terms and complex terms and the terms which includes some kind of relations and all. So, predicate logic the Aristotle logics in a waves all shot of explaining this complex terms and singular terms in a satisfactory manner. So, in a sense predicate logic is also considered to be kind of term logic or predicate logic that may be even called as first order logic.

So, we will talk about what we mean by first order logic little bit later. So, let us consider what we mean by saying that a given string of symbols are considered to a formula. So, these are also considered to be as ascension in the predicate logic the definition goes like this. So, we need to have some particular kind of language that is, the language of predicate logic which consist of all the individual variables, constant, terms, predicates, etcetera and the functions etcetera.

The set of open formulas over l is given inductively by the following rules. So, let us consider R is a relation symbol of arty n; that means, unary predicate, binary predicate etcetera. And t1 to tn are consider to be terms then R of t1 tn is also considered to be the formula. So, it is like p t1, t2, tn is considered to be a formula will also considered to be the atomic formula in the predicate logic.

So, you have terms and you have a relation that is what in a for example, if you can represent R t1 to tn as simply w, x, y. And all were w stands for wife and x stands for Ravi and y stands for Sita for example, Sita is wife of Ravi so w, x, y in that particular kind of order. So, we have t1, t2, tn here so anything if R is a relation symbol which relates some kind of objects in your language. And then R t1, t2, tn is also consider to a formula.

So, as usual in the case of propositional logic if alpha and beta are 2 formulas alpha and plus beta, alpha and beta, alpha or beta all these things are also considered to be terms. And the third thing is that, if alpha is a consider a formula and x is considered to be a variable then, the only thing which is different in the case of predicate logic are these things. So, there exist some x alpha is also considered to be formula these are the 2 things 2 quantifies that we will be using in the predicate logic.

So, it is in that sense it is an extension of propositional logic. Anything which you extend it with 2 more extend it; extend the propositional logic with 2 more quantifies like for all x exist some it will become predicate logic. So, then there exist some x alpha are for all x alpha is also considered to be a formula, where will the additional things which you find it you in in the predicate logic.

An alpha is usually considered to be within the scope of your quantify example if you have... So, this essentially says that then you say that a given kind of sting of let us that am going to write on the board is considered to be a formula.

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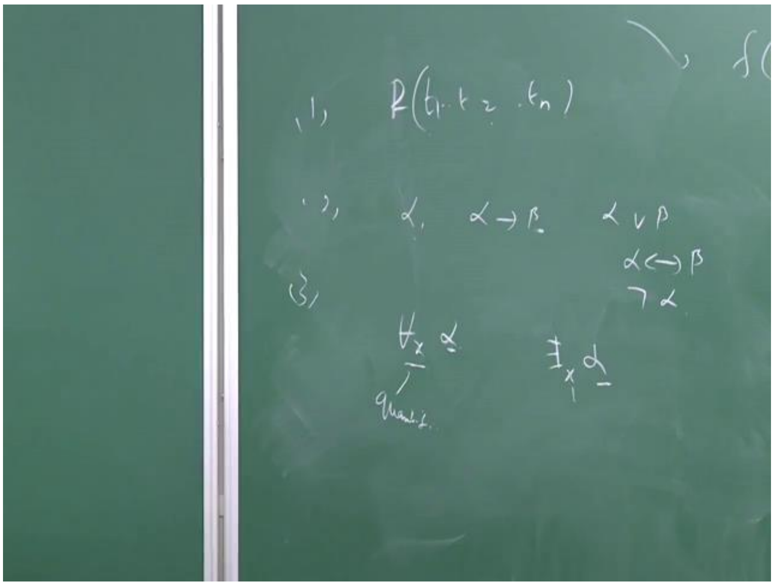

So, the first thing is this that t1, t2 individual terms are also obviously, considered to be formulas and all. And anything which you write like this tn if t1, t2, tn of formulas anything which you write it like this if t1, t2 is also considered to be a term which obviously, is considered to be formula. Then, this usually we request it with R sometimes you can view and write it as p.

So, now the second thing is this that as usual in the case of propositional logic if alpha and beta formulas and only syncs are also considered to be formulas and negation of alpha etcetera. This is the second thing and the third thing is this that, for all x... so, the other thing is this that there are 2 quantifies which you can use there exist some x alpha. So, this is considered to be quantify which talks about for all x alpha is alpha goals for all the property all the objects x.

So, now alpha holds per some x so, here this x stands for variable individual variable and alpha is within the scope of this particular kind of quantify let’s what it says. So, here alpha is set to be within the scope of the quantify; the quantify is in the case the first 1 is for all alpha and the second case it is there exist some kind of alpha there is some x alpha. So, anything which is define in following 3 ways is considered to be a formula.

But anything which is define not in this particular kind p f sense is consider to be not a formula. So, this is just like in the case of propositional logic we talked about some kind of formation role any sting of formula cannot be considered as a well form formula. So, like this we have in addition to these terms etcetera which are known here and we have 2 quantifies for all x and there is exist some x. So, we need to talk about what we what exactly these quantifies are all about.

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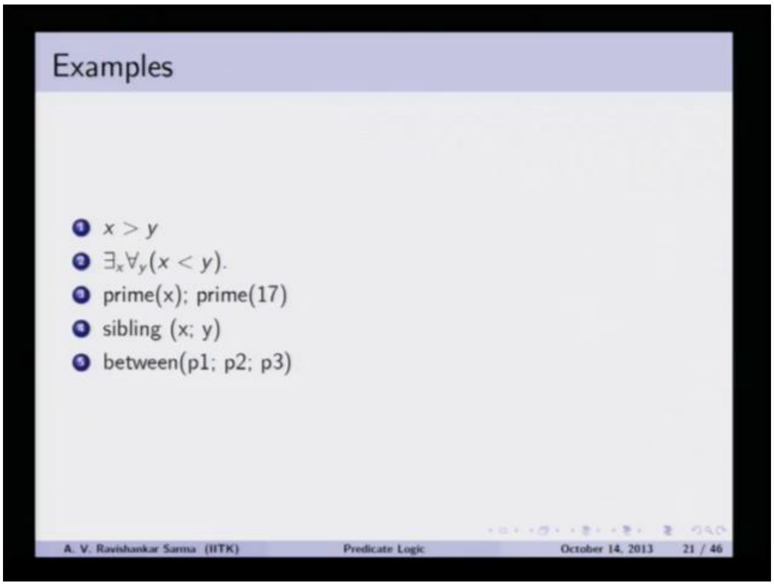

So, some examples of formulas are like this, so x is greater than y because x greater than is property which is relating x and y. So, we can written as R x y simply it shows that greater than is a sign which represented by relation are the property or we can say for all there exist some x for all x is less than y or you can simply write x has a particular kind of property that the prime number.

So, in that sense 17 is considered to be prime number so, prime 17. You can write sibling can be sister, brother anything x and y x is the brother y; y is a sister of x etcetera. So, want to be present anything between p1; p2, p3 that is b p1; p2, p3. So, 1 can write it in the linear order or sometimes you can even write in this particular kind of thing. Suppose, some text books it is written like in this p x, y, z and if I want to avoid is particular kind of notation I can write this way also p x, y, z it means, the same thing and all. Some text book maintains this particular kind of notation; in some other text book will find the other notation.

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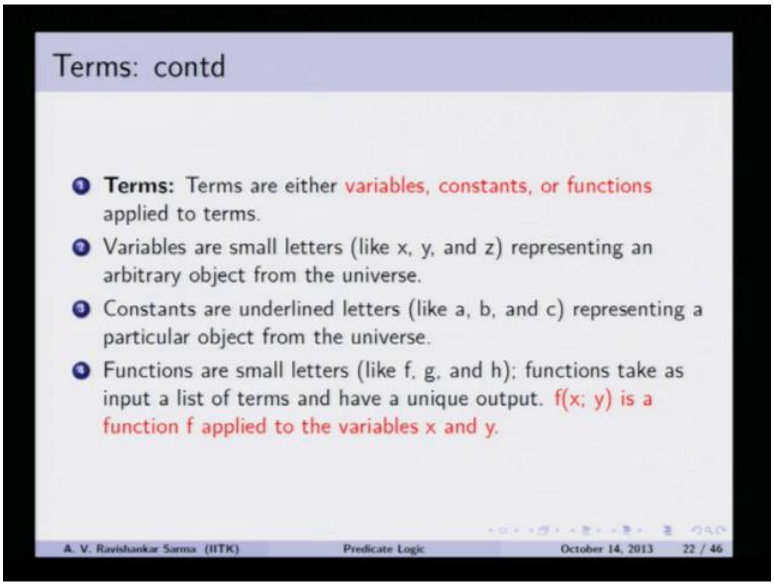

So, to continue with what we are discuss so far the terms: terms are either considered to be variable, constants and functions which are functions applied to terms there is f t1 t2 is considered to be a term. Just a summary of what we are discuss so far and variables are also considered variables are usually represented to xyz. Usually, these are replace by terms t1, t2, a, b, c etcetera and all.

Constants representing some kind of arbitrary objects from the universe like: some people, some man, or some bright students of like that. So, we are referring to arbitrary objects within some kind of universe. So, we have to note that in predicate logic whatever we are trying to talk about make sense only when if refers to some kind of domain or universe of this course.

It does not make any sense if you do not have some kind of universe of this course. So for example, something which holds for natural numbers may not hold for the real numbers etcetera are integers etcetera. So, we need to specifically talk about the domino that you're trying to talk about in the beginning of analyzing this sentence in predicate logic.

So, constants are underlined letter such as a a, b, c etcetera representing particular object from the universe. You want to refer to this particular kind of duster it is this thing or this particular kind of practice etcetera there all the referred by constants this what we have discuss so far. And functions are usually, represented as again small letters except the predicates we will see all the things which you'll find it in your vocabulary will have f x, y means, f is a function which supply to variables x and y.

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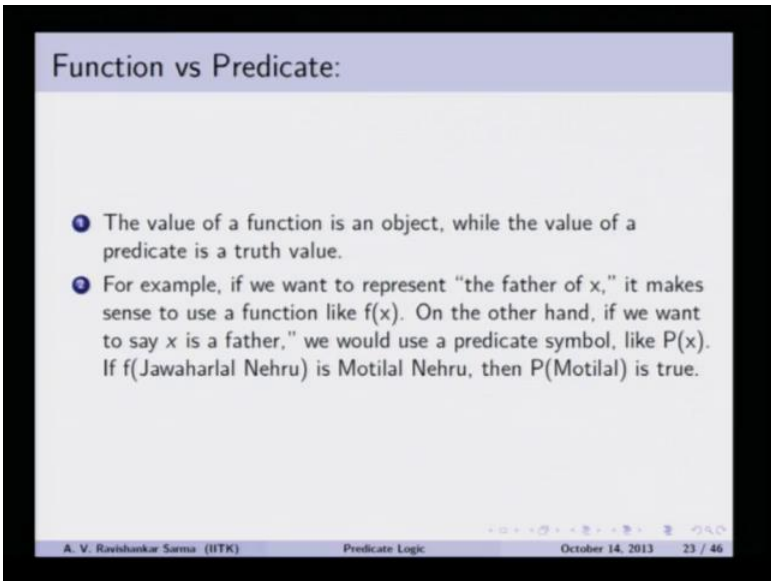

So, now is in sometimes there might be lot of confusion between function and predicate the usage of function and a predicate. So, here is the distinction between a function and the predicate. So, the usually the value of the function is an object while the value of predicate is considered to be a truth value. So, let us consider some example so that this distinction will become clear.

So, this one of the important distinction that is value of function is consider to be an object whereas, the value of predicate is consider to the a truth value. So, that means, the for example, if you say Ravi is husband of Sita for example, so this is usually represented as this thing.

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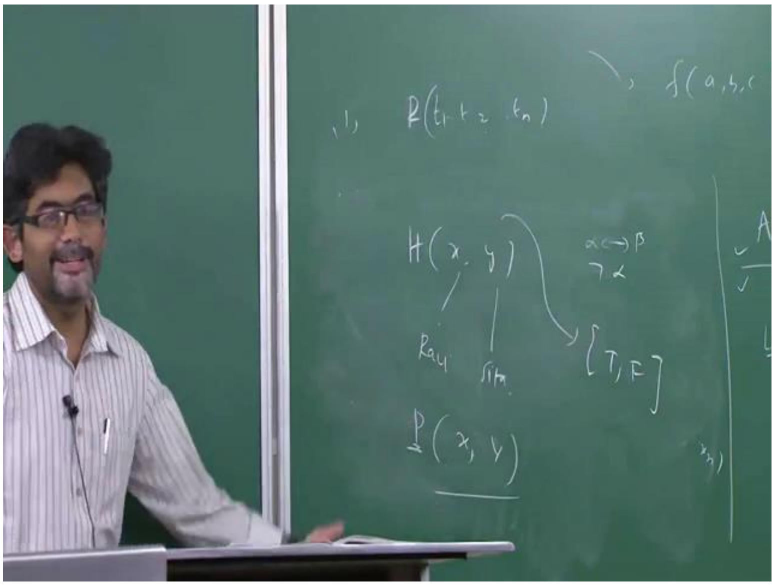

So, this is the predicate so, being husband is consider to be a predicate and then let us say, you write it in this particular kind of thing this says that x is husband of y were x is considered to be Ravi we replace it with individual terms. Then, it will become this thing Sita Ravi is husband of Sita, so this is in a particular kind of order. Suppose, if x stands for Ravi and y stand for Sita and an indeed theses are considerable states of a effects.

Then obviously, takes a value either T or F so that, is what the predicate does and all. And the value of the predicate is always considered to be some kind of truth value either it has to be 0 or it has to be 1. For example, if you say Manmohan Singh is Prime Minister of India and all. So, suppose if you say that particular kind of thing Prime Minister is to consider to be the predicate.

Then, x is to b considered to be the Prime Minister of India x is referring to Manmohan Singh and y is referring to India. Then, this is the property that is being the Prime Minister is consider to be the property. Suppose, if you say some other name and then replace it with this 1 it takes the value F that means, the predicate the value of predicate is will take the value F.

So, the value of the predicate always takes some kind of truth value either it is true or false. Suppose, if you want to the represent the father of x particular kind of function f x father of the nation etcetera. It refers to some kind of object such as, Mahatma Gandhi etcetera a Prime Minister of India that is referring to Manmohan Singh some kind of object we can refer to f of x.

So, that is referring to some kind of object it is mapping to some kind of object and all. On the other hand, if we want to say that x is a father then now it should take some kind of truth value. So, now you use P x that is let us see if you want represent this thing father of Jawaharlal Nehru. f of Jawaharlal Nehru is referring to some kind of object I mean another object which is exist within your domino and it considered to be Motilal Nehru we know the fact.

So, that is why now predicate Motilal for example, that is P stands for is a father of Motilal that can take only value at the true or false. If it refers to states of refers that indeed Motilal Nehru is considered to be father of Jawaharlal Nehru an off course, this statement is true which occurs in the law it is true otherwise, this going to be false. So, this is the final distinction between the function and predicate we often make mistakes in using these things in our language that is function was as predicates are like this.

So, the fine the menudo distinction that we need to note here this is that, the value of a function is always consider to be an object it maps to some kind of objects whereas, in the case of predicate. The value of a predicate is always going to be some kind of truth value either it has to be true or false.

So, this f of Jawaharlal Nehru is referring to some kind of object that is Motilal Nehru whereas, P being a father of Motilal rather or not he is father of a father then the statement is going to be true, otherwise it is going to be false. The value of that 1 is either to or false that is the minor distinction between the function in predicate.

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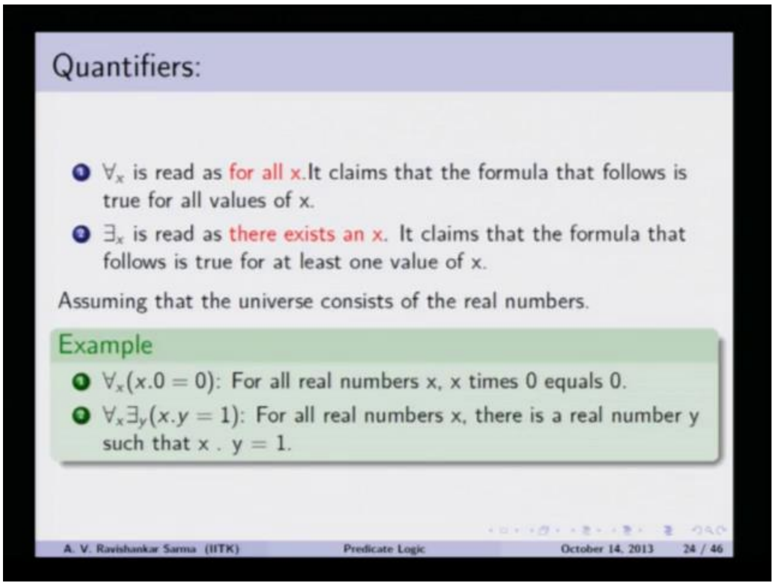

So, so far we discussed about some of the basic billing blocks of predicate logic to start with we started with the terms and then, predicates. And now, 1 of the important things at you need to discuss is the quantifies I mean or the quantifies. So, the 2 quantifies that you'll see in the predicate logic 1 is, if you want to quantify over the entire universe of discuss you use universal quantify for all x.

If you are trying to refer to some kind of individuals within your universe of discuss, you refer to existential quantify. So, for all x is for that is the name for this for all x which is in the red see it claims a formula that follows is true for all kinds of values x. For example, if you wants to say all human being are motel mortality is attributed to not a single human being. But it referring to the whole group of individual that exist in the given domain.

The domain is here is the domino of universe of this course which consists of all human beings. Suppose it refers to animal and that thing is going to be may be that made also true, but if you re referring to some other kinds of object domain, maybe might be false. So, there exist some x is really read as there exist an x it claims that that the formula that follows anything after the quantify is going to be true for at least 1 value of x.

So for example, if you say that all some IIT case students are intelligent then, it refers to only some IIT students are intelligent means, at least 1 is intelligent that itself shows that some or consisted to be intelligent. So, that is satisfy is this particular kind of sentence; if you have more it is good enough, but definitely you want to make a distinction between all IIT students are intelligent and some students are constant to be intelligent.

So, that is a difference between existential quantify and the universal quantify. There is some kind of important we need to maintain this particular kind of distinction. And there are some efforts to say that, whether for all x ax is to implies that they exist some x ax or not. So, these are the kind of inferences that a troublesome kind of inference and all.

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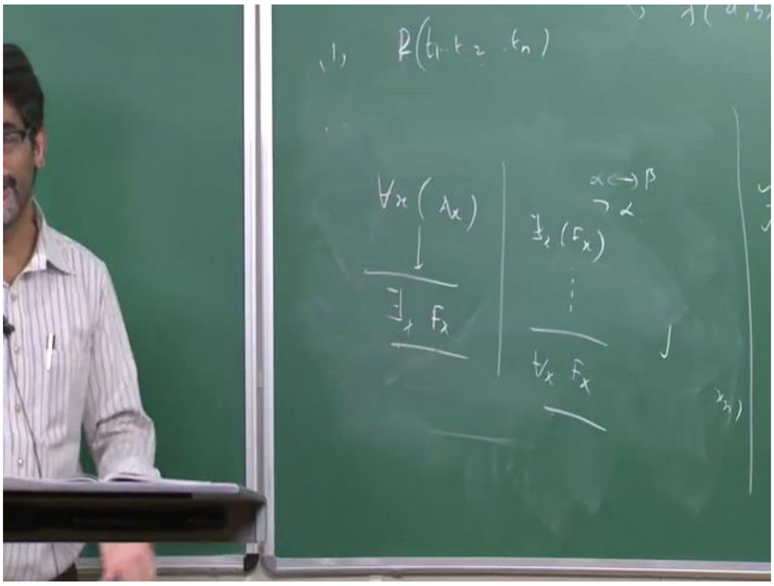

So, usually or temptation is this that for example, if you say for all x ax and from this can be infer there exist some x Fx. Or the other way, around if we have something like there exist some x Fx from this can be generalize we can say that it is for all x Fx whether these can be considered to be rules of inference and all. These the things which we will talk about it later, but lot of issues which are surrounding this particular kinds of inferences Aristotle was also talking about this particular kind of inferences.

Suppose, if you again something like for all unicorns intelligent and this stands for some unicorns are intelligent from that you obtain in this particular kind of thing. Since the intelligence is attribute all human beings may be some are also consider to be intelligent. But once you infer this particular kind of thing, there exist some x here the problem here is that your importing existence in to the conclusion which is not there in the premature.

In the premature, the there is no commitment for the existence of the unicorns here, but you can still use an obstruct terms all unicorns are intelligent etcetera and all. And the premature, but still in at out to have unicorns to exist in the world. But here I once you say that, there exist some x Fx and there is some kind of we imported existence which is not there in the premises to the conclusion.

So, that kind of inferences is consider to be a kind of which Aristotle talks about it as the existential in the more modern terms , Boolean terms it is considered to be an existential velecy. So, how they resolve this is the existential velecy you know and all we will talk about later, but... so, this is the manure distinction between for all x and there exist some x. Let us consider some examples so that will understand this concept in a better way.

So, for all x x into 0 is 0 that means, any term multiply by 0 is going to be 0 it happens for any number that you going take in to consideration whether it is real number or complex number or any number that you take into consideration. So, now for this whether or not the statement is true etcetera and not we require and domain, either we need to talk about natural numbers, real numbers, integers, rational numbers etcetera and all.

Within the domino we takes x has some kind of number, if I talking about natural number you take 1 2 3 etcetera and all. Then, for all x any x you taking to consideration if this property x into 0 is 0 holds for all the let us that your exhaust fully listed out in that case it happens it holds for all the numbers, all the members of x; x times 0 equals to 0. So, in that case it is going to be true we will talk about truth and false it little bit later.

But right now, we are trying to talk about focusing attention on how we are using the quantifies. So, we are we did not entering to the details of what we mean by this quantifies. It semantic will take care of what exactly we use this quantifies and all how we use this quantifies etcetera will talk about it when we discuss about the quantifies. Second sentence for all like were exist some y x multiplied by y is equivalent to 1. So, now your domain is real numbers here and there is a 2 real numbers x and y and if you multiply this 2 real number x and y it so happen the multiplication these to 1.

And this holes for all x at least for all x means you take 1 2 3 etcetera and all. For all these elements and all there exist at least some y, that y is also another real number for all these say there is some y and all. For that, if you multiply some of things which are there here, it some of the element here, then you will generate a number 1 whether it is true or not we will talk about little bit later.

But this is what it conveys the information that it conveys is this thing for all exist some y. Any number x is multiplied by y whatever number randomly you take into consideration. And then, they you choose another y from y which happens to be another real number the multiplied x multiplication these to 1 some example of quantifies

So, now 1 example could be like this we need to talk about the domino of this course, usually in the case of natural number, in the case of numbers you have to define it properly. It can real numbers, it can be relational numbers, complex number etcetera. So, now let us consider natural numbers as universe of this course. So, let 5xy denudes a kind of relation and all between x and y x is less that y.

So, now we have a function x which relates x and y were x is considered to be domino, y is range which is represent in terms of order x y. That is considered to be binary function, which talks about this things x plus y and a b c are considered to be constants naming some kind of numbers. Suppose, if you are referring individual number on that is 0 stands for a, 1 stands for b, 2 stands for c etcetera and all the like that you go on and on.

If exhaust this constant and we can use even a1, a2, a3 etcetera, so now in that context in given the universal of discuss, natural numbers and then we have relation x less than y and we have function that is x plus y etcetera. So, here x is less than y less than is credited that is property which is on this 2 objects. So, these 2 objects are related by predicate being less than.

So, now there is exist some x 5xy it is considered to be an unary predicate which says that of y that there is at least 1 natural number less than it, where y is not equivalent to 0. So, you take any number for example, 1 2 3 to n and all you take any number other than 0 let us say, take 1 that is obviously, less than any other element that is 2 etcetera. In the same way, 2 is less than 3; 3 is less than 4 etcetera.

So, now in the second context for all x there is a exist y phi of xy starting that for any natural number x there is a natural number y; which is greater than x it is in the context of first 1 x is less than y. So, same thing is represented in terms of this suppose, if you want to represent that for any natural number that you taking to consideration there is always a natural number y which is greater than x.

So, that is represented by this thing for all x at least there is some y such that phi; phi is stands for less than y and all phi is stands for property been less than something. X is less than y exist some y x is less than y is a presented by phi xy. So, this is a way we represent in this thing in this 1, for all x there is some y are considered to be quantify. And phi stands for predicate and xy stands for individual variables when it refers to individual constants like that numbers like 1, 2, 3, 4 etcetera and all.

Then, it takes some kind of value. So, for we discussed about terms, predicate etcetera and the quantifies. Then, given any formula in particular we can talk about to what extent this quantifier operates and all. So, that is considered to be the scope of the quantifier. So, depending on the scope of the quantifier we can say that the variables existing formula are considered to be either free or bounded this is what we mean by foundation freedom.

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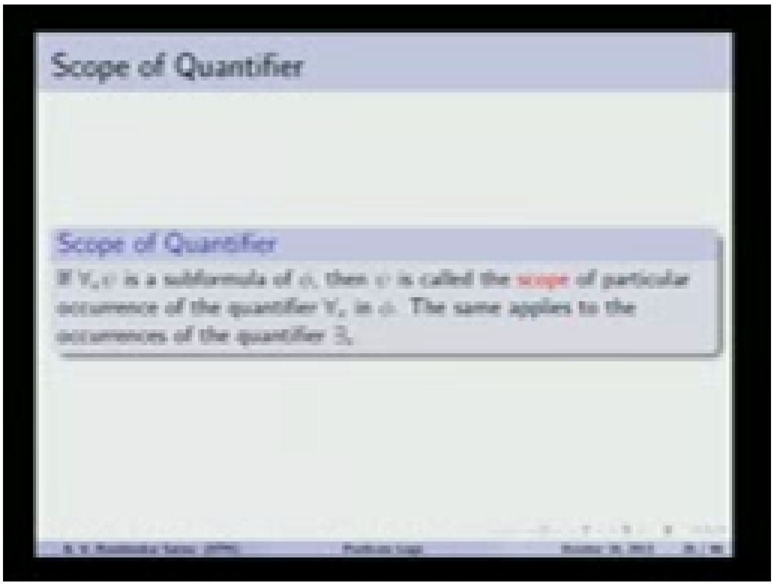

For example, we can have for all x phi is a sub formula of another formula phi; phi is big formula like a x are y are z etcetera and all. In that you taken to consideration, the part of it and all that is some consider to be it is a formula of phi. Then, phi is called as the scope particular occurrence of a quantity for all x and phi. So, that is consider scope of it the same applies to the occurrences of when you're taking about there exist some x.

So, you have a formula in that formula only the quantifier operates over some kind of sub formulas, not over entire formula and all some kind of sub formula. So, its scope is still that extent only. So, beyond that there is no the other variable is that exist there are going to be free variables. So, we will talk about the scope of the quantity of with some examples and we will talk about what about what we... what is the significant of this 1.

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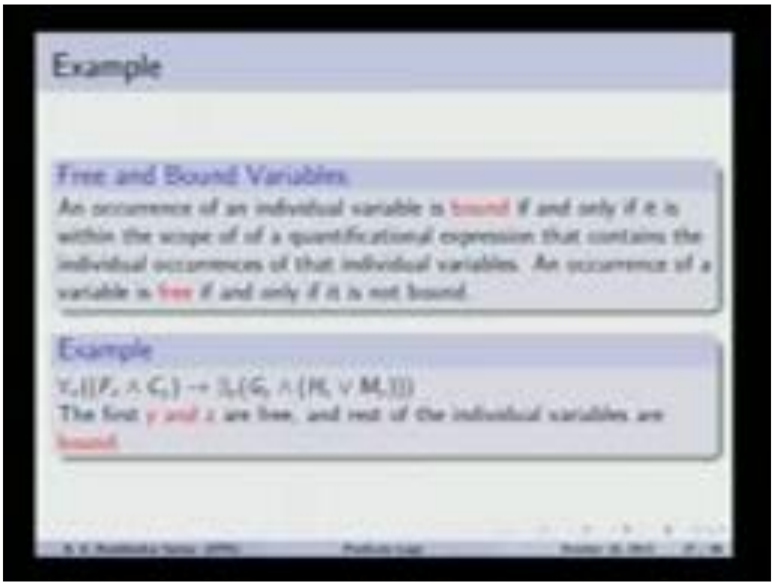

So, some examples which we can talk about so, what we mean by first of all what we mean by free and bound variable? An occurrence individual variable; individual variable means, xyz etcetera and all. There considered to be bound within the scope of quantify if an only if it is within the scope of quantificational expression. That contains the individual occurrences of that kind of individual variables. An occurrence of variable is considered to be free if an only if it is not considered to be bound. Let us consider one example, so that we will understand what you mean by scope of a given quantify.

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So, the example that is there here is like this for all x Fx and c by some terms which are presented in this way. So, other is 1 bracket here implies So, I am just listing out 1 formula and then we are trying to see what the scope of the quantifier is the quantify for all x and there exist some y Gy there exist some y Gy and Hx some strange randomly also we can take into consideration Mz.

So, this is closed by this thing and close by this thing. So, these brackets are very important. So, this quantify is operating till this extent. So, this is a till here and then we are closing this 1 which this and this is closed by this thing that is why there are 3 brackets here. So, now let us assume that this is the formula and all now we are trying to talk about the scope of the quantifier.

So, now what are the variables that exist here x, y and z these are the individual variable that you'll see in this particular kind of formula. So, these are considered to be quantifies so, now in this formula so this is over the whole thing. So, the occurrence the now we need to talk about sub formulas. So, what are the sub formulas this is 1 and this can more and whole thing can be set be another kind of sub formula; that means, Fx and Cy can be 1 sub formula and there exist some y.

All these things this can be 1 sub formula the main formula or it can be only simply this particular kind of thing. So, now with respect to the universal quantify for all x, y and z are considered to be free. Because, this is bracket is still is extent so, now with respect to this particular kind of quantify so, this is considered to be bounded. Because, this is within the scope of this particular kind of quantify and even this is also in the second sub formula also this is bounded by this particular kind of quantify.

So, now what are considered to be pre with respect to this universal quantify? These are the variables that are consider to be free with respect to this thing. Even then, the second sub formula also so, these z is also considered to be free with respect to this particular kind of quantify. The first y and z are considered to be free whereas, the rest of individual variables are considered to be bounded with respect to both the quantifies that is for all x are they exist some y.

So, now with respect to x the first occurrence this 1 this and z are considered to be free. So, now Fx is bounded by for all x here, y is free here and then with respect to this particular kind of quantify and y is bounded by this thing. And with respect to this individual existential quantify x and z are considered to be free. So, now we can list out this thing so, it is like this. So, what are considered to be free here are things; in the first occurrence in this particular kind of formula x is considered to be bound.

In the second sub formula this second sub formula so, this is considered to be bound and all. But in this case y in the first occurrence y is free first occurrence means, first sub formula y is considered to be free. In this case, y is consider to be bounded, but in the second occurrence of this quantify there exist some y x and z is also considered to be free.

So, in this sense 1 can find out when we can say that a given quantify bines the individual variable. So, in this example the first y and z are considered to be free; first y z means, the moment you start the formula from the left y and z are considered to be free. In the second case, in the seconds sub formula y is bounded whereas, x and z are considered to be free and this are the individual considered to be bound.

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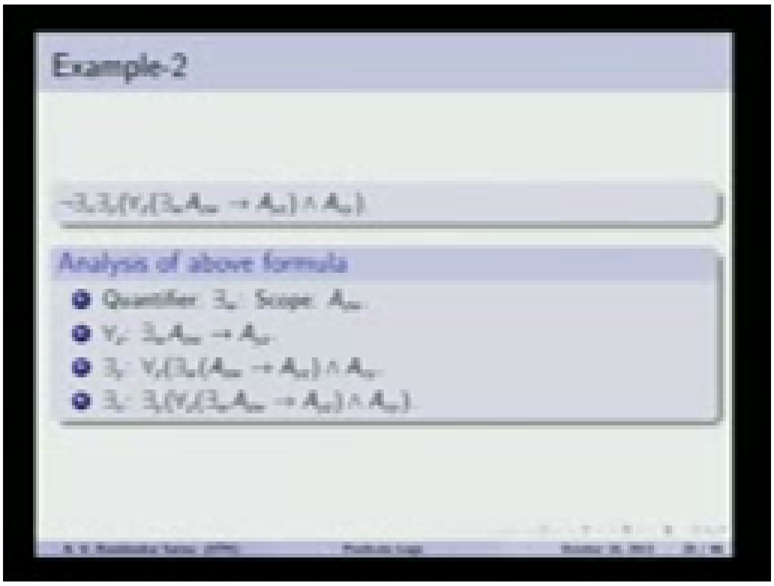

So, in the same way 1 can talk about this particular kind of thing we can talk about some more example like the quantify. For example, in this case are the 3 quantifies it does not exist that some x they exist some by for all z is a complex kind of thing. And all the exist some w Azw imply Cyz and Axy. This is a let us say, this is consider to be a complex kind of formula with this kind of thing I will end this lecture.

So, now here what are the quantify there exist some x, there exist some y and the exists some w are considered to be a quantify. To start with a s w is the 1 which you're seeing this formula is bounded by the quantify there exist sum w. So, now the scope of that 1 is is Azw. So, with respect to for all z entire thing is considered to be bounded that is there exist sum w Azw thus ay z hat is considered to be bounded.

So, with respect to the quantify there exist some y and whatever follows after that 1 in the brackets that is for all z there exist sum w Azw plus Ayz it end Axy that is considered to be within the scope of there exist of sum y. Now, with respect to there exist of sum x and whatever follows after that 1 is consider to be within the scope that particular kind of thing. So, this is the way in which are the quantify operates over it given formula.

So, later we will see the significance of why we are talking about why we need to know at a given formula is bound with respect to a formula are when, why we need to know that a given formula is free. So, in this class discussed about some other basic building blocks of predicate logic and the started with predicates which consist of some terms etcetera.

So, we defined what we means by term and then I introduce 2 quantifies and I do not discuss about the relationship between in this 2 quantifies. These 1 quantifies can be define with respect on other 1 can reduced to the other 1. So, the other thing which I discussed is you say that given variable is considered to be bond it's considered to be free.

So, we need this information especially in knowing the validity of a given formula are given formula is considered to be a well form formula. Especially, are a sentence in the predicate logic especially when it is bounded etcetera. So, we need to know some information about whether or not a given variable is bounded or not to have some kind of sentence in the predicate logic.

So, we will discuss about we will continue with the syntax in the next class and we will also talk about the semantic of predicate logic what we mean by saying that for all x a x y is equal to be true etcetera. So, these are the thing which we are going to discuss in the next class.