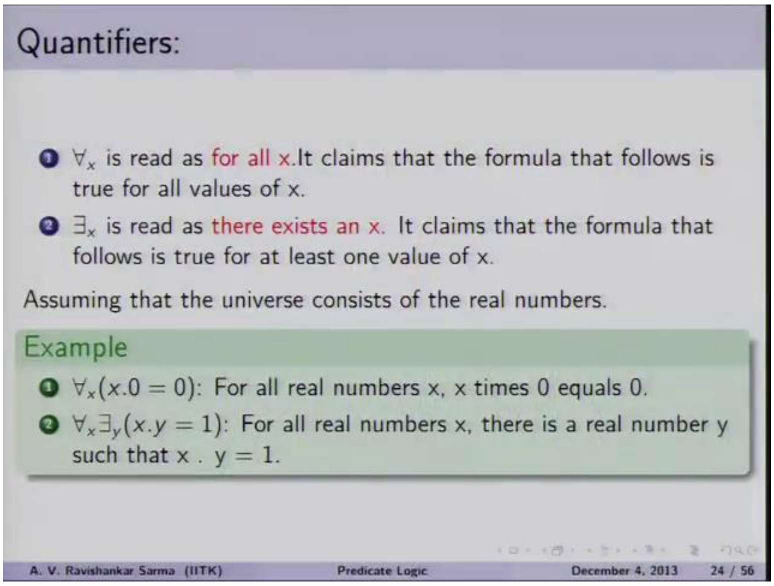
Introduction to Logic
Prof .A. V. Ravishankar Sarma
Department Name Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 35
Quantifiers, Freedom, Bondage

Welcome back, in the last lecture we discussed something about the basic building blocks of predicate logic were to be introducing various thinks such as what you mean by predicate, what to mean by term, what do you mean by functional symbol etcetera and all. So, all the since we discussed in the last class. So in this class, another major building block of predicate logic apart from predicate etcetera a terms, relation symbol etcetera and all the quantify.

So, infarct predicate logic are also called as quantificational logic are it is also called as another name for this one is the first order logics. So, in this class what will doing is will be discussing something about, what you mean by quantify, why we need this quantify.

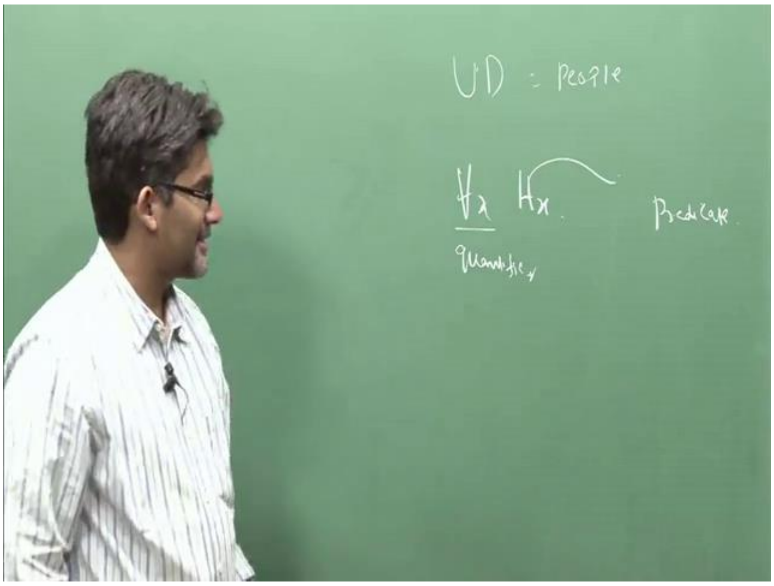
(Refer Slide Time: 01:05)



And then we talk about so the important loss of this quantifies. And then we will discussed some other important things which other come under the category of syntax; the syntax of predicate logic. So, basically we are in the syntax of predicate logic basically we are discussing about some other building blocks of predicate logic. So, let us start with a what to mean by a quantifier. So, there are 2 quantifier set use in the predicate logic: the first 1 is called as universal quantifier it is represented as for all x; the symbol for this 1 as returning this way for all x it is read as for all x it claims that the formula that follows is true for value of x.

For example, if is say all men are mortal; a mortal it is attributed to all the human beings, so the mortality to all the human beings. So, whatever follows the after the quantify is that formula is going to claim that, again claim to formula that follows what follows after the quantifier is true for all values of H. For example, if is a for all human beings are happy this is simply represented as this for all Hx.

(Refer Slide Time: 02:28)



For H stands for the human being and x stands for and any individual human being, and then we have some kind of domain of so that is universe of discourse usually called it as UD means Universe of Discourse, which use of consist of human beings of people, and then this stands for a quantified and this stands for usually predicate. So, being mortal or being happy is considered to be the predicate that is the attributed to a 1 single variable x

If that happens for all the human beings then we represented in this way for all x Hx. So, another quantify that we will be using ha every very often frequently is the exist essential quantify. So, it is represented if that there an is x, it is claims that the formula that follows is after this quantify is true for at least 1 value of x. So that means, suppose if say that at least 1 swan is considered to be usually stands are whiting color.

If you find if you figure out such the figure it out in any way and swan that it look that is considered to be black. So, we want to say represent that particular kind of thing you usually, represent in terms of there exist some x such as, that the particular x consider is swan which is consider black. So now assuming that, the universe consists of real numbers, so depending upon what you taking to consideration.

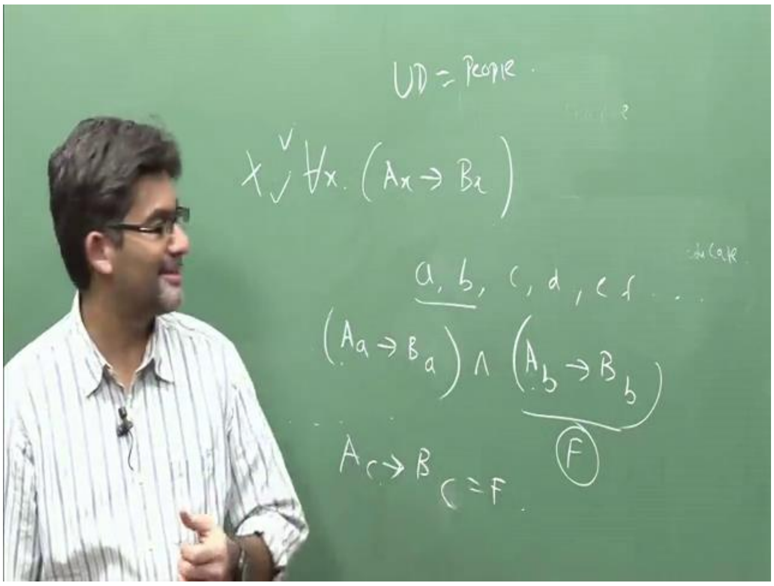
The formulas represented by, quantify changes an all formulas is going to be true sometimes, same formulas going to be false some other occasion. For example, if you consider the universe of discourse as real numbers are then suppose if you want you represented this particular kind of sentence for all x x multiplied by 0 is equal to 0 any number; any real number x which is multiplied by 0 which is obviously, will give you 0.

So, x in to 0 is equal into to 0 happens for all x whatever, the x that going take into consideration which falls within the domain of real numbers; the proper that x into 0 is equal to 0 holds for all x and all. That is a listen by we wrote it and this way for all x x multiplied by 0 is equivalent to 0. So, this is 1 way of representing this particular kind of thing. Again, if you considered same universe of discourse has real numbers and the other thing which represented in this 1 is, for all x the exist some y for at least 1 y such that x multiplied y is equivalent to 1.

So, here we have used to 2 quantifies the first 1 is considered to be universe of quantifies. As second 1 is, the existential quantifies and just stating that you know these are the some of the things which you commonly come cross mean… Later will be talking about translation part; where will be taking about, how to translate the sentence appropriately in to the language of predicate logic little bit later.

So, write now there are 2 quantifies that we need to study detail and because, predicate logics are called as quantificational logics. So, usually we represented in this particular kind of thing suppose if you want you represent universal quantify it is considered to be like this.

(Refer Slide Time: 06:18)



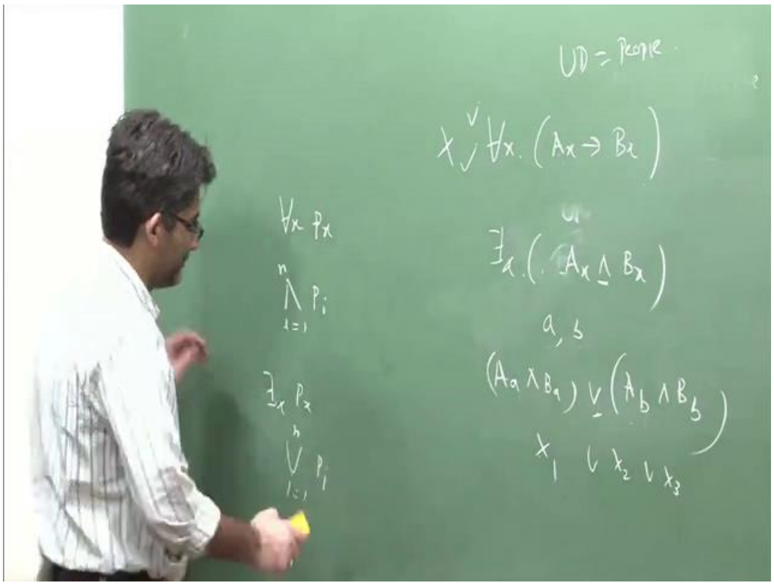
For example, if you have this particular kind of thing Ax in place Bx it is an all human being are mortal. Where A is represented as human beings and B is represented as mortal. So, now latest considered that there only 2 individual human beings it makes sense to talk about simple formulas in this way. For example, this can be return as Aa implies Ba were x is replace by A and Ab implies Bb.

So, in there are only 2 in this universe of discourse is considering to be people all people are human beings. In that way, we have taken in to consideration and all the 2 human beings A and B let us say Aristotle and Socrates. So, if we want to be represented this particular kind of things because, there only 2 people in this universe a mean then domain. So, we can represented this this formula simply as this 1, but when the number increase then all a, b, c, d, e, f, x an all it is no way in which we can represented in this particular kind of form.

Because, is string will this string will go on and on all. So, in order to represent this particular kind of thing this property is going to true for all x then we require to in this particular kind of universal quantifying. So that means, here another thing which we need note is that if any 1 of this thing is false then the whole this particular kind of formula is going to automatically false. That is as could as saying particular kind of universal suppose if want to say the all close are black and you represented like this.

Found they crow white as black crow A, if that is a crow then a as to be black. The same way if found another crow your naming it as Bb is considered to be a crow b as to be black; like that we goes that we on and on. Suppose if you find and instance were for example, A is false and all. So, third instance found a white crow then we cannot say that for all x if x is a then x as to be black and all, because that particular kind of thing is false even if in is 1 instance is false, we cannot represented in this particular kind of way. Usually, mainly last statement in science is usually represented as universal generalization. But in it note that all universal generalization haves on has is on exceptions.

(Refer Slide Time: 09:18)

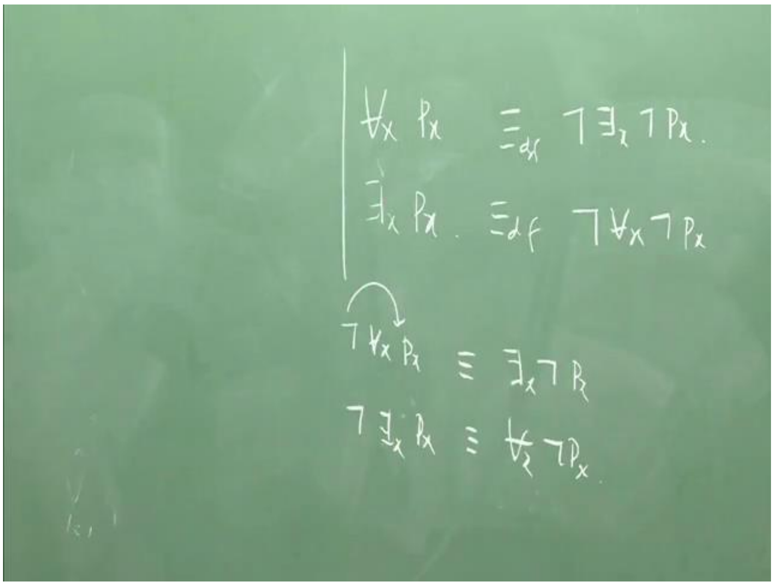


So, this the way in which you represent universal quantifies were as existential quantifies same things Ax and Bx. So, this is for example, for if you have again 2 you are taking in to consideration 2 people from this universe of discourse. There is n number of people of there; out of that we are selected on the 2. At assumed that, there only 2 human beings existing in this world for example, then you represent this formula as represented formula in this way Aa and Ba or Aa Ab and Bb.

So, here it is a conjunction here it is a disjunction that means, at least at least on of this things, so it's satisfied particular kind of things then this formula going to be true. So, later when talk about semantics of predicate logic we will be discussed in the detail, how to interpret this particular kind of formulas enough.

At this moment it is like this a for example want to represent for all x some px then it is a usually conjunction of all this things i goes to 1 2 n etcetera pi. So that means, p1 to p2 p3 etcetera and all; each p consider to a formula if you want to you represent this things there is exist some x p s it is considered to be disjunction of all i 1 goes to and i infinity also and p i etcetera. So, the each an every formula will be like this. This is some x1 x2 x3 etcetera see even if at least one of this formula is true that is going to universal existing quantify is going to hold and all; that means, going to be true. So, these are to be quantifying that to be come across and these to be quantifiers interrelated to each other in this particular kind of way.

(Refer Slide Time: 11:29)



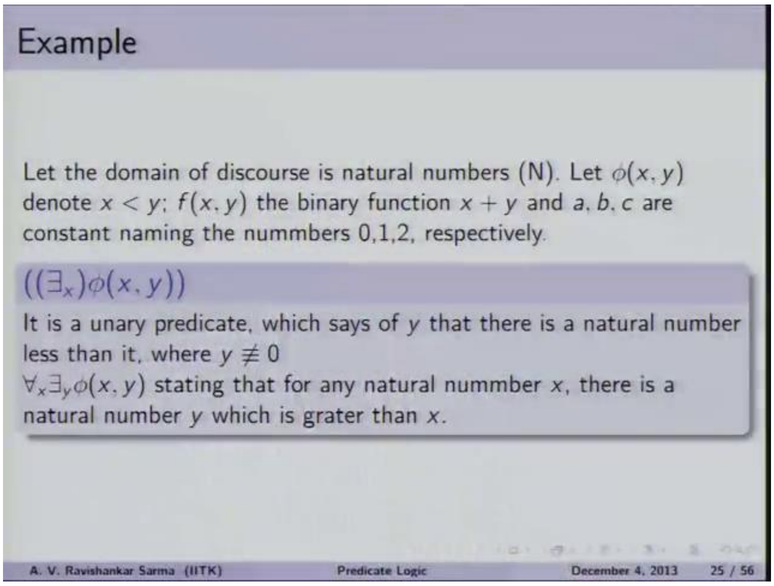
So, these are considered to the dual to the duals. So, for examples for all x p x can be representing these particular kind of by definition it can be written in this sense. So, universal quantify can be defined in terms of existential quantify these particular kind of way. So, it is says that all x px means these does not exist some x does that it is not px. So, that is what it says then the same way the exist x px by definition it is same as not for all x not px.

So, this is what we have and then suppose if you negate these particular kind of in universal quantifies it is the all the case of all x px means, you need to push this negation inside. And the negation of universal quantify in become existential quantify and we need to push this negation inside. So that means, it becomes not the x, and then negation of x px; that means, if the negate the existential quantify the examples these stand for x is happy or something like that.

So, the exists some x or x is happy or there exists some a sawn, which is considered to black ink color; if the negate that 1 these is going to be a universal quantifier for all x and these is push it inside and then it will become px. So, these says that for all x for all birds etcetera and all there exists at least 1 swan which is not considered to be white; that means, it as be black in color. So, it not in says say.

So, these is the relation between universal quantified and the existential quantifiers it always existing duals. So, existential quantify can be defined in terms of a universal quantifiers in this way. In the same way, suppose if the negate these universal quantify we can talk about these thing in terms of existential quantifier; negation of existential quantifier we have universal quantifier.

(Refer Slide Time: 13:49)



So, let's talk about something in detail for example, if take the domain of discourse as natural number. So, in the we have note in the predicate logic everything make sense only with respect to some kind of domain. If the domain is not there is does not make any sense to talk about any formula at all. Because, something happens something which is true in some kind of domain that we have taking into consideration may be false in some other kind of thing.

Suppose, if we take if we take into consideration real numbers, and then we talk about some kind of formula which might hold. And the same kind of formula, by the false of in terms of in case of some other numbers such as complex number etcetera. So, domain is considered to be the most important thing which we will talk about it when we discuss, the semantic of the predicate logic in greater detail in the next few classes.

So, now we considered the domain of discourse is considered to be natural numbers like, 1 2 3 4 etcetera and all 0 is not there. If we add 0 it will become whole numbers. So, another we have predicate phi, which relates x and y a some kind of relationship between x and y. And that relation is, defined as and defined in these way x is less than y; so then we taking into consideration natural numbers.

Then we have a function x y which as a considered to be binary function, which is defined in these way x plus y; plus is considered to be a binary function. Because, it connects x and y multiplication plus divided by etcetera all binary functions in are the meeting. And a b c is considered to be constants, which stand for some kind of numbers 0 1 2 etcetera.

So, now we can talk about 1 particular kind of formula there exists some x x phi in constants of natural numbers. So, these is considered to be a unary predicate which essentially says that, the exists some kind of at a should be exists some x phi x phi which says that of y that there is a natural number less than that particular kind of thing x; where usually we considered it, since we considering natural numbers definitely y is equal in to 0.

So, it might hold as long as we do not take into consideration y is equals to 0. So, there is a always a number which is less than suppose if we take x is 1, y is 2 in we have situation where it satisfied is particular kind of formula. There exists some x phi x phi holds in this particular kind of domain of natural numbers. Provided y is not equal into 0 and second 1 lets considered another examples where for all x exists some y and predicate x y which is stating that.

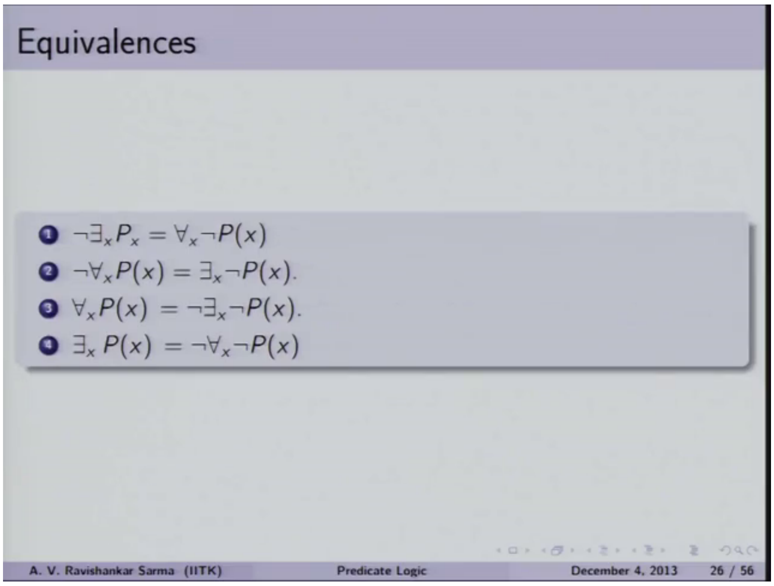
For any natural number x; that means, for all x means for any natural numbers whatever number that we have taking into consideration. So, that numbers have be natural number x there is always there is exists some y it is not saying that for all y. There exists some kind of y there is at list 1 particular kind of y, there is also a natural number y which is greater than x.

So, we take 2 numbers in a domain in all pick up to domain a true numbers from a domain. So, then a we take any such kind of natural number for all x there is always kind of natural number y which is greater than x for example; which if we take though into consideration there is always another number; which is greater than 2 that is 3 might be 3 is greater than 2.

In the same way, fixed 3 and there always another number which is greater than that 1.

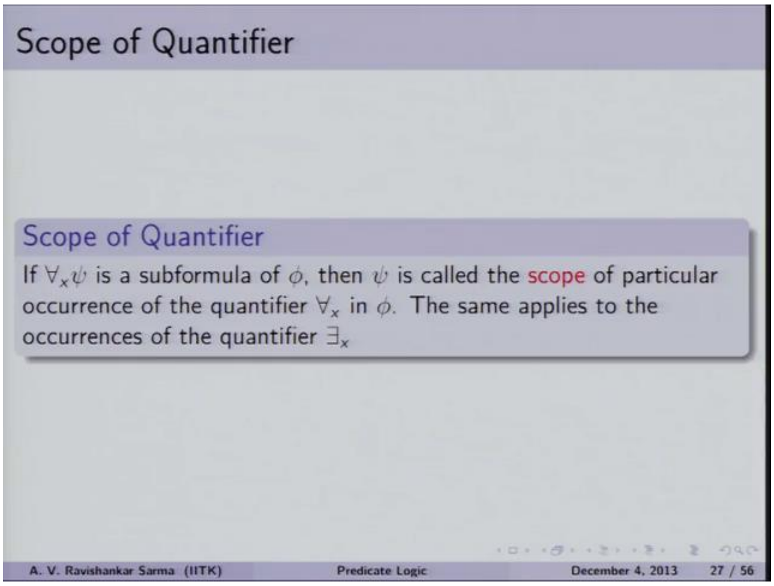
So, in these holds for natural numbers, but if we take into consideration the real numbers; that means, all the irrational, rational all these numbers in all than it may not hold. The same kind of formula we working some kind of domain might be false in another domain.

(Refer Slide Time: 17:59)



So, this is some of the equivalences between this quantify, some of these thinks which we have explain on the board an all which connect these equivalence relations connects universal quantify with the existential quantifies.

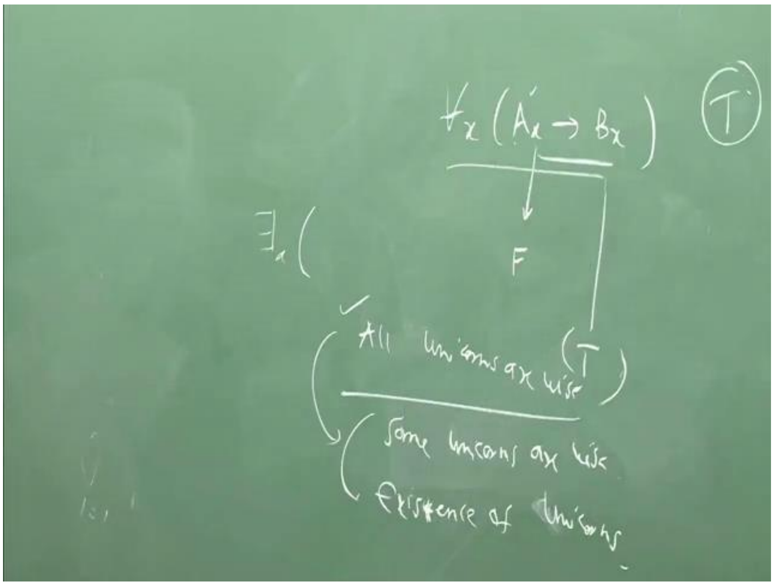
(Refer Slide Time: 18:15)



So, now so for we are discussing in a some kind of detail about the quantifies if we do not have this quantifies, there is no way in which we can express particular kind of he will be reclusively writing all the formula without any end. For example, if the want for represent all course of black. Then, we will be writing there are supposing the domain there are 10000 crores and all; which represented by a b c etcetera and all a1 b1 b2 etcetera and all.

Then, if we start writing about an what if start representing that particular kind of formula then suppose there are 10000 words that you have taking into consideration then your string we were welfare formula will have10000 sub formula in all which is very difficult for as to manage. So, for that reason we require these particular kind quantifies. Another important think which we need to note is that in the case of universal quantifier, there is a different between these particular kinds of thing.

(Refer Slide Time: 19:21)



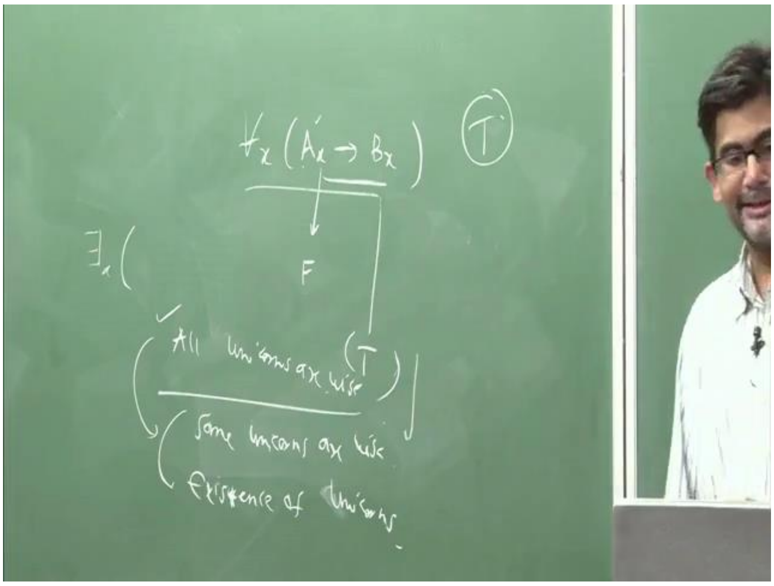
For example, if represent this particular thing Ax implies Bx for example, in this 1 Ax is considered to be empty; that means, this is false are empty are it is false and all. For example, we can talk about these particular kind of thing, that all unique earns are wise. So, these particular kind of that we can expressive in terms of quantifies that might holds in might be hold in might be true or might be false also. So, depending upon what values how we interpret is thing.

So, suppose these particular kind of thing Ax is false then irrespective of your consequence Bx is whole formula going to be true another is formula wholes in of for all. This formula can be true even without an existence of the unicorns in the actual word. So, we can still talk about a universal quantificational formula without talking about, whether or not exist in the word Ax can be false Ax.

If Ax can false than the is whole formula is automatically going to be true and all. But in the case of existential quantify suppose if we say that unicorns… for example, if a say that all unicorns are wise in the unicorns does not exists and all. So, form these suppose if you in for that some unicorns are wise. So, these as no problem in all as such because, this is the statement can still be true provided Bx Ax is false for if antecedent is false.

The condition to be automatically true, even if it does not exist also does not make big difference and all. But once is say that some unicorns are wise in all, these prepositions the existence of unicorns are in the actual birds. So, these does not require we do not have any commitment that we now, unicorns actually existence word we know. But we can talk about is particular kind of formula, but once we talk about these particular kind of formula, these presuppose some kind of existence.

(Refer Slide Time: 22:27)



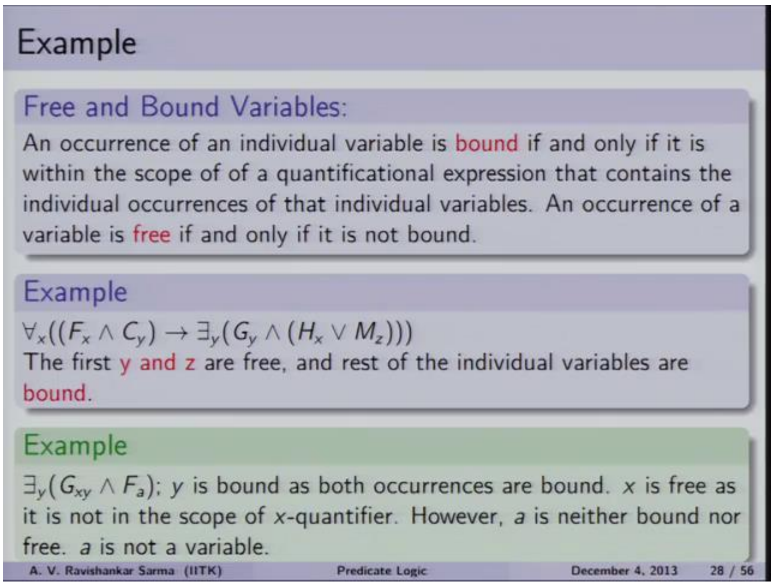
So that means, unicorns we have to actually existing in the word an all. So, this is another interesting issue which is which we talk about it, the next few classes these again is problem is a raised by Aristotle the problem is called as a existential kind a parasites. Suppose, this is considered to be existential paillasse in the model logic, but Aristotle taken into consideration that from a proposition; this is considered to be a preposition that is all unicorns are wise.

So, From that we can still in for whether have are not in for some unicorns are wise or not. So, this is if we inform in these way it is called as a existential paillasse; what is problem here is that, the problem is here is that we are importing existential conclusion which is not there in the premises. For example, if we say all unicorns are wise. That does not pre suppose any existence of unicorns wise, but once we talk about some unicorns are intelligent wise, presupposes that unicorns actually exist in the word. So, we will talk about these problems of existential import the end as a limitation of the first order or this quantification logic.

So, let us talk about what we mean by the scope of the quantify, so we have just times to talk about the basics of predicate logic still we are in the in the part of syntax itself. So, what we mean by scope of quantify? Suppose if we say for all x phi is considered to be sub formula of phi; that means, the psi consists of some kind of sub formula phi.

Then, psi is called the scope of particular occurrence of a quantify that is for all x and the particular kind of sub formula phi. The same apply to the a occurrence of the quantifies the exists form x etcetera. So, whatever is within the scope of the quantify is usually considered as the scope of the quantifies whatever, false of outside the scope is not considered to be bound.

(Refer Slide Time: 24:21)



So, now we talk about depending upon the scope of the quantify to what existence these quantified operates in the given formula. We can talk about, whether a given variable is free are given variable is considered to be bound. So, there only 2 thinks which exists here. So, an occurrence of an individual variable I considered to be bound if and only if, it is within the scope of the quantificational expression that contains the individual occurrences of that individual variable.

So, where as an occurrence of a variable considered to be free if and only if, it is not considered to be bound. Which is not in the scope of the quantifies is considered to be a not bound; that means, a free variable. So, it is like something which is in the room is considered to be bound for example, professor is teaching a class whatever whose over in the class are bounded by that particular kind of instruction, teaching etcetera and all.

But we those whichever is walking a outside etcetera and all there not there have to be there follow instruction of teacher and all. So, there is not bound by the particular kind of instructor, who is teaching in the particular kind of class room. So, let us considered some examples if we can say which variable is considered to be bound and which variable is considered to be free.

Suppose in the formula that is shown in the slide Fx and Cy in price there exists some y, Cy, Hx and Mz. In this particular kind of formula, now there are various occurrences of these variables what are the variables? x y z are considered to be the variables. That means few can the replace with the any constants etcetera and all. It will we can the place this variables some kind of constants; these constants are considered to be some other things which are some kind of objects in the domain.

It can be people; it can be any 1, it can be cross, it can be anything, so now in this particular kind of formula... So, x y and z and these are the variables that are exist in this particular kind of formula. And it occurs in various places and all. So, know the first y and z are considered to be free for example, to taking to consideration if you read it from left to right and all in this particular kind of formula.

So, this whole formula is within the scope of for all x so that means, Fx is consider the formula Fx is are; obviously, bounded by is quantified x. But were as Cy in the formula Cy; y is considered to be variable which is not bounded in; only x is bounded in that particular kind of formula in the first sub formula. So, now wherever x cercus here there obviously, it will be bounded and all.

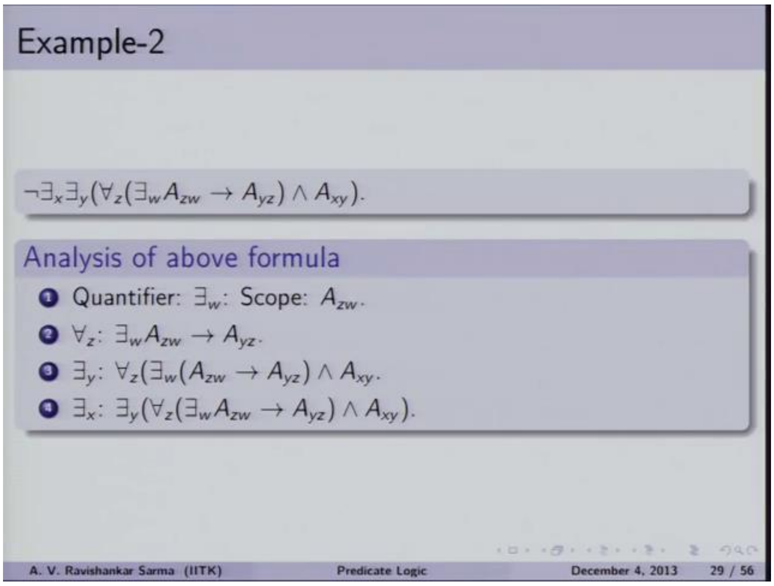
For example, in the second formula there exist some y and Cy and Hx and are Mz. The first occurrences that means, y in the first occurrences means it occurrences in Fx in Cy. That is consider to be free whereas, same occurrence of y in the seconds of formula that is the exits some y Cy and Hx are Mz. So, cy in the formula Cy it is bounded by the quantify there exist some y.

So, in the first occurrence y and z is considered to be free and if and of is rest of the individual are variables are considered to be bound. So, whatever is within the scope of the quantify, is the 1 which we are trying to talk about. Basic another example there exist some y G x y and Fa in this particular kind of formula, y is considered to be bound as both occurrences are considered to be bound.

So x is free because, it is not in the scope of the x quantifier; x quantifier is for all x we are only the exits some y. However that term a which is a considered to be a constant which is neither bound nor free; such kind of terms are called as is its not considered to be variable and all. So, depending upon the scope of this particular kind of quantify, we can talk about whether are not given variable xyz etcetera are considered to be free.

Or sometime, it is may be considered to be bound. It sometimes of occurrences of that variable in that particular formula can be bound and the same occurrence of that particular kind of formula y can be bound as well.

(Refer Slide Time: 29:44)



So, now let us considered another example so that, we can understand this scope of this quantify to what extent this particular kind of quantify operates in the particular kind of formula. The formula is a red like this in this case it does not exist x and there is exist some y. This formula is going to what is the formula; for all z there exist some w z Az z and w are related some were implies Ayz and yz and also related in to somewhere.

And Axy were not talking about what how this x and y are related to each other. It may be greater than, it may be less than etcetera or x is the father of y or x is the brother of y and anything; some kind of predicate A. So, now in this 1 suppose if a looking to inner most quantify. So, that is the exist some w and the scope of that 1, is next immediate kind of formula that exist after that that is Azw.

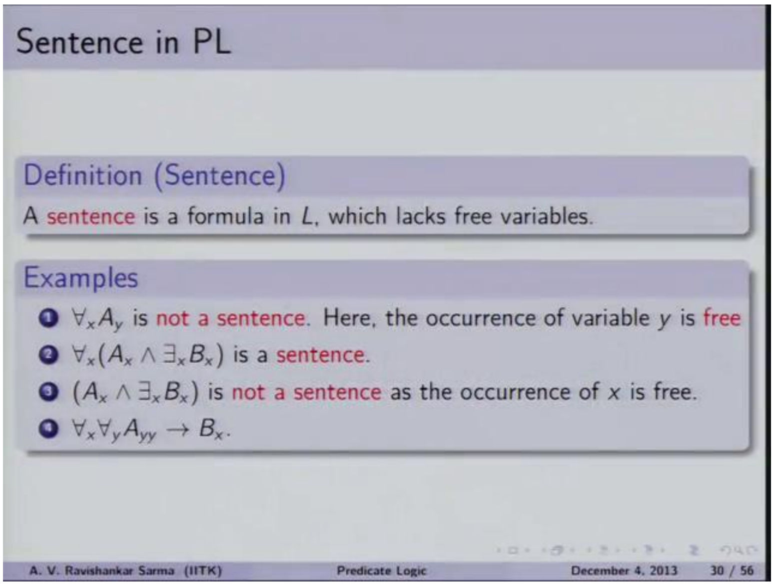
So, that is scope of that particular kind of formula, beyond that it would not operator and all. So, there is another term which follows after that 1 is that is Ayz it would not operator that particular kind of thing. Now if it consider the inner most quantify for all z and that is going to operator on the scope of that 1 is whatever is the brackets that is exist some of w Azw implies Ayz.

So, we are not try in to talk about which variable is consider to be bound, which variable is considered to be free and all. So, we want talk about that particular kind of thing it respect to this universal quantify for all z there exist some w Azw implies Ayz. In that w is z is considered to be... now is everything is considered be bounded and all.

Because, is not of which is considered to be free. So, now with respect to the next quantify that is there exist some y, that entire thing to be considered within the scope of that particular kind of formula. Whatever, follows after that particular kind of within the scope of that quantify. And then the respect to there exist of some x the entire formula is going to be within the scope of that particular kind of formula.

So, we have what we have done so far this is a understood what mean by scope, to what extent quantify operates and based on that once it on operates in all to... which variable is considered to be free which variable is considered to be bound is the 1 which we have seen. It was on significance as specially, when we are trying to talk about some of the important inferences is in predicate logic. Some other important decision procedure methods that will be using, where we will need information about which formula is considered to be free etcetera which formula is considered bounded and all.

(Refer Slide Time: 32:44)



So will make use of it in those decision procedure methods; where we can check, validity of a given formula are we can talk about when 2 sentences are consider to be satisfy able etcetera. So now, so for we talked about what you mean by quantifier and then what is the scope of a quantifier and then what you a mean by bound and free variable etcetera. So, now it is talk about what do you mean by proper sentence in predicate logic.

In proposition logic we said that, a sentence is considered to be usually sentences means, is declarative sentences; anything which can be spoken as true or false usually considered as, declarative sentences. So for example, if you referring to sentences like shut the door or open the door etcetera and all; the sentences can neither be true are nor false and all.

So, in the same way you are talking about some questions etcetera what is a your name etcetera and all. Sense can be neither true are nor false and all. In the same way, what do you mean by sentence a complete sentence in predicate logic? Is a 1 which we should be interested to know. A sentence is a formula in your language the language of predicate logic which lacks free variables.

So, your formula so considered to such a way that is no free variable in that particular kind of formula, is considered to be proper sentence in predicate logic. For example, if we take into consideration for all x Ay this is not considered to sentences because, you have a free variable y. Suppose if you have a said that for all x Ax then it is considered in a put been considered as sentence.

But here, the existence of free variable will make it not a proper sentence in predicate logic. So, why we are talking about whether not is particular sentence etcetera and all. Just like on a in the case of preposition logic, only sentence can be statement can be true or false an all we can talk about true are false it is false of particular kind of sentence. In the same way for example, if a taking consider to for all x Ax and there is exist some x Bx is considered to be sentence.

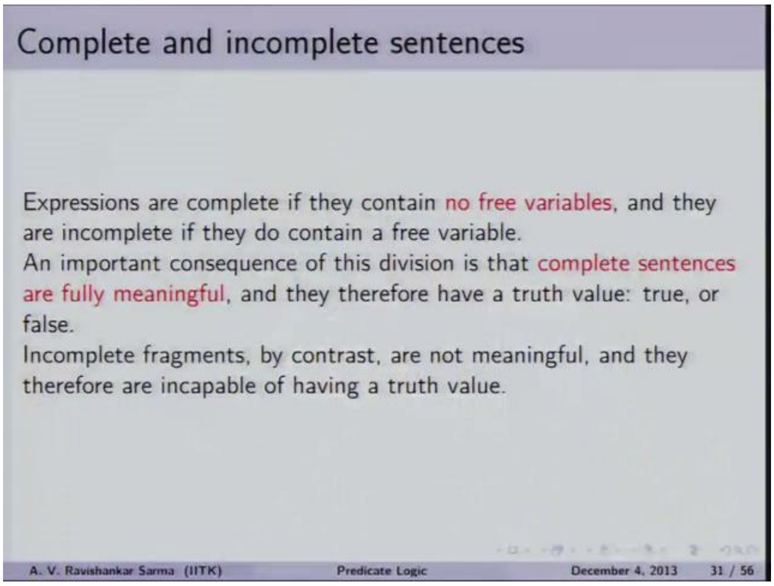
Because, is no free variable that exist in this 1, because Ax is bounded by this is quantify for all x and Bx is already bounded by this existential quantify there exist some x. Then, entire thing is within the scope of the universal quantify. So, there is no free variable which exist in this second formula. So, that is way it is also called as a sentence. So, let us considered a third 1 Ax and there exist some x Bx; Bx is a bounded by is particular kind of existential quantify there exist some x.

So, x is not free the occurrence of x in that particular kind of formula is not free x is bound in that particular kind of occurrence. And x occurs an x also occurs in the first term that is Ax. So, in that occurrence of x in that occurrence Ax; x is considered to be free. So, whenever you have a free variable that is not considered to be sentence. As occurrence of that particular kind of formula is considered to be free in the same way for all x, for all y, Ayy where y is considered to be free.

So, it is not considered proper sentences and off course, Bx is any way in that particular kind of formula Bx is obviously, considered to be free. I mean variable x is second term is considered to be free, so now what do you mean by complete and incomplete sentences in the predicate logic and what it is signifies particular.

So, just like in the case of prepositional logic, only statements can be only declared sentences are the once which were the going to take into consideration. All the other sentences which, were we cannot draw clear line between let say mortal and al mortal etcetera and all we do not take those sentences in to consideration. So, it's sets some kind of a limitation which we talk about the end of this course.

(Refer Slide Time: 37:07)



So, now what do mean by the complete and incomplete sentences in predicate logic? The expressions that is the formulas such as, a represent of a in terms of a formulas are search be complete if the contain no free variables. That means, everything is bounded by within the scope of the variables and they are quantifies there are variable such exist in the scope of the quantifies also considered to be bounded.

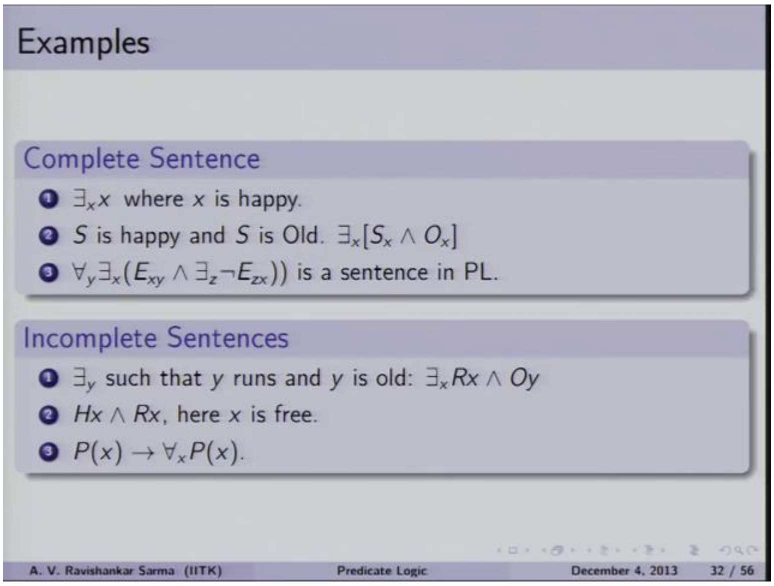
And we do not any free variables then that particular kind of sentence is considered to be a complete sentence. And there incomplete if they do not contain, if they do contain some kind of free variables and all. So, we have seen some of examples earlier. So, now one of the important of consequences of this particular kind of division that complete sentences and incomplete sentences is that.

Complete sentences are considered to be fully meaningful and they we can talk about whether are not there tautologies. And therefore, have some kind of truth value that is true are false. Incomplete fragments by contrast are not meaningful we can only talk about satisfaction. And the some interpretation that formula is going to be true and some interpretations formula going to be false.

Therefore, incapable of having a truth value we cannot clearly say that tautology or we cannot say contradiction and all; it’s just likes some kind of contingent kind of statement. It may be true or it may be false, so this is 1 of the important of significance of remarketing between complete and incomplete sentences in predicate logic. A formula which consist of no free variable its considered to be a complete sentences in the sense of predicate logic, do not taking in to consideration in the in terms of English language.

But deals context of taking this in the context of predicate logic. So that means, here important messages is that formulas is any formula to given going to consideration that construed is a complete sentence provided if it has no variables. If it has variable free variables, then it is not considered to be a complete sentences it is a usually called as incomplete sentence.

(Refer Slide Time: 39:40)



Let us consider, some example in which we can understands idea in the better way. Then, there exist some x x were x is considered to be happy for example, if you write there exist some x Hx that particular kind of sentences is considered to be a complete sentence. Because is no free variable here, in same way there some exist is x such that x is happy and x is... both that is what is both x the occurrence of x this particular kind of formula is bounded.

So, there are no free variable, that is a reason y it is a called as a complete sentences. In the same way, third sentence for all way some exist x Exy; E is considered to be some kind of predicate and we can talk about any such kind of a predicate in within contest of a domain. And this is formula is read in this way there exist some z not Ezx.

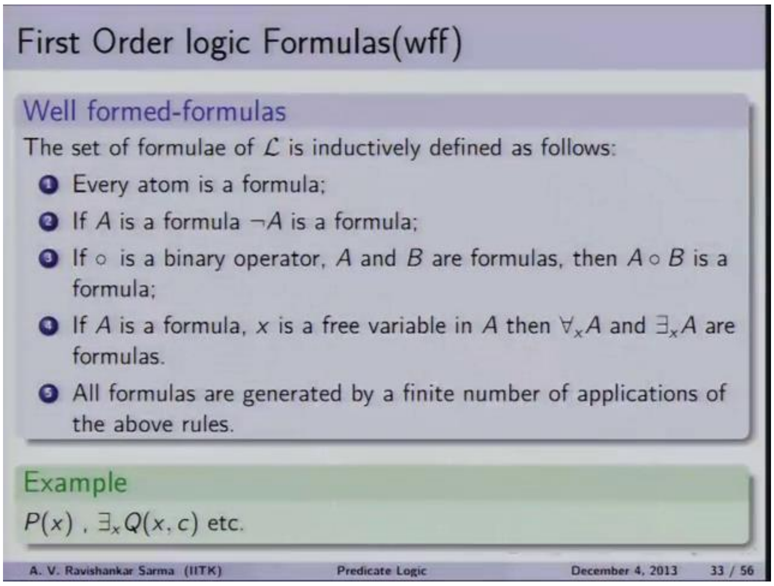
So, all the variable z exist in this particular kind of formula are bounded by either universal quantify or the existential quantify. So, that is way no free variables in this particular kind of things. So, that is way it is considered as a sentence whereas, incomplete sentence are like this. The exits some Yzy runs and y is older suppose if is represented is this way, there exist some x and Rx and Oy.

Second occurrence of variable y only once it occurs and all y in the second in the second term there is Oy is free. So, wherever you find a free variable it is not called as a complete sentence in context of predicate logic. So, Hx and Rx both as occurrence of x are considered to be free because, not bounded by any quantify and all. So, since it is the occurrence of x in this particular kind of formula going to be free.

So, it is considered as a incomplete sentence. In the same way Px implies for all x Px the first occurrence of x is free is not bounded by any quantify etcetera and all. So, that is y it is called as a incomplete sentence. So, incomplete sentences we cannot only talk about satisfiability and all whereas complete sentences.

We can talk about tautology or again, even defiantly say that its false it’s definitely we can say that it is true. That is what we are interested in either we are interested in knowing that particular kind of formula its true and raw interpretation that is a tautology or it is it is false in all interpretation and all that is a contradictions.

(Refer Slide Time: 42:17)



So, so for we have discussed about what do you mean by quantify and then scope of quantifier. And when we also said that, when a given formula is a free; when a given formula is bound etcetera, and then we also talked about what do mean by a complete sentence and incomplete sentence in the context of predicate logic. Just we have building up things.

So, now just like in the case of prepositional logic where we have we discussed in the greater detail that at whatever, will combine will not considered well form formula form and all. We need to have some kind of rules for judging whether or not given formula it is considered to be a well form formula. So, in predicate logic which is usually considered as a extensional of pre proportional logic.

So, most of the rules of proportional logic apply here also, except that there in has also another additional rule that is the rule with respect to the quantifies. So, now what do you mean by saying that given well form given formula it's considered to be well form formula and in the first order logic or the predicate logic. So, now every atomic formula that is p q r etcetera and all considered to be a well form formula we just right line.

Then it is considered to be well form formula it is some x is considered to be well form formula and not x is also considered to be well form formula. If circular its considered to be binary operator, the binary operators are there foreign number like r and implies and if and only if. If A and B are considered to be formulas then A circuit B were circular is a presented as r and implies if an only if also and we considered to the well form formula.

These are does not tell us much are except that is this going to be useful whenever, feeling some kind of information in the machine, in particular machine should know, how which 1 is a called is a syntactically correct kind of formula; which 1 is syntactically incorrect formula. This happens in the case of ah programing language as 1 value writing a program me there is a syntactical error, it will clear show that there is a error in you in our program.

In the same way, this is the things which are in the important contest of machines in particular. So, the forth role is that forth role is 1 only thing in which is new here in the case of predicate logic. If A is a considered to be formula then x is variable in the particular kind of formula that means, ex etcetera. Then for all x Ax is also considered to be well form formula in the same with there exists some x Ax is also considered to be well form formula.

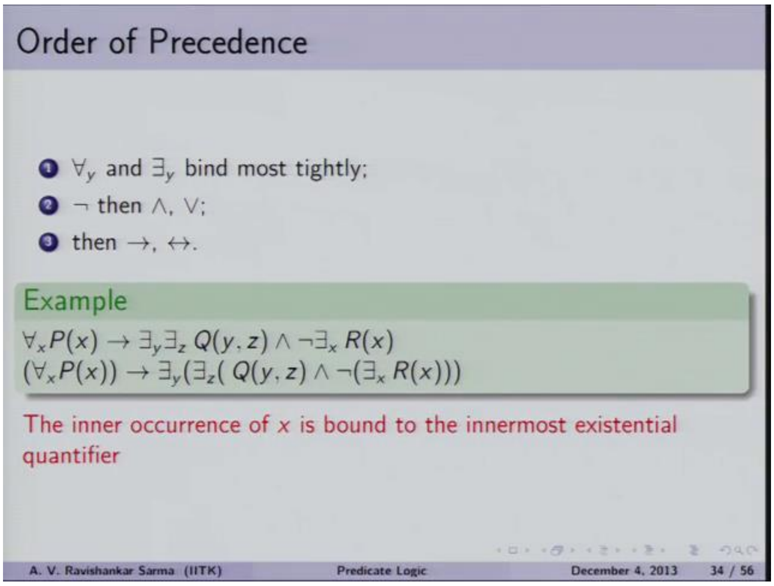
Whereas, Ax there exists is not considered to be well form formula is just tells us, how this formulas how various strings are combined and forms some kind of well form formulas in the thing is not tell us anything extra and all . So, now anything the fifth rule is like formality and all. So that is says that, all formulas generated by the finite number of applications of above rules is automatically it will be treated as a kind of well form formula.

It is talking about all the formulas that you have to judiciously use the above formulas and all is there talk about anything new 1. For example, if you say just px and all it let us say let represented some predicate x let say human. So, criticize mortal for example, p is considered to be predicate that is mortality is attributed to some kind of x that is x is considered to be in some kind of etcetera.

So, that is considered to be a well form formula there exist some x q x c etcetera there all considered to be well form formulas. So, just like in the case of prepositional logic suppose if the parenthesis is not given then we need to follow our own conventions. So, there is an order of residence which is used widely in most of the text books.

So, the order of residence it's slightly defined in the case of predicate logic, when compare to the prepositional logic. The first preference is usually given to the universal quantifies, so we have to put brackets whenever you come across this particular kinds of symbols. For all y there exists some y bind most y within followed by that rest of the things are same as in the case of prepositional logic negation and or implies n double implies.

(Refer Slide Time: 47:10)



For example, if you take into consideration this particular kind of example for all x Px implies there exist some y their exist some z Q y z and they resent exist some x Rx. So, that no brackets nothing is given here. So, now in that context the first preference for this 1 you have to look for universal quantify. So, the universal quantify occurs in the first string first format of formula this for all x Px that is why you put brackets there.

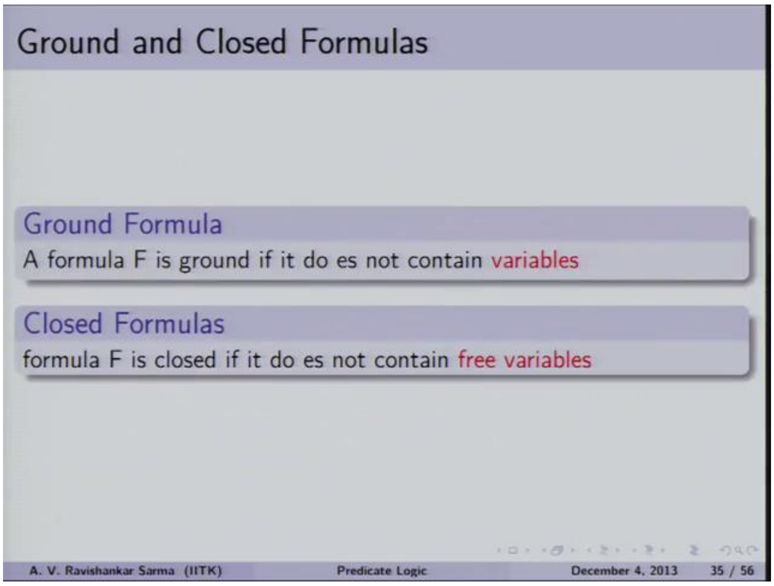
So, now that is taking care of now we need to come to existential quantify that is the 1 which is needs to be preferred. So, now whatever follows after there exists some you have to put bracket. So, that is what is happened here in the second stage. And then again there is another existence operator which exist in the inner sub most formula and all.

So, that is there exists some z etcetera, so where that is where you have to put brackets, and then another 1 which exist inside. So, the inner most you have to note that inner occurrence of x is bound to the inner most existential quantify, not by the other external kind of quantify. So, that is the reason why we have put there exists some x Rx bracket there. And the whole thing there exist some z and the whole thing is in the brackets and all.

Although what you get it from this particular kind of formula is that, the first preference is given to universal quantify forward by that existential quantify and you need to operate with all the existential quantify. Once it gets over then we move to negation and you put brackets there, and then followed by that as usual in the case of prepositional logic you follow and or etcetera and all.

In most of the good text books usually this parenthesis already given. For in some text books suppose if it is not given to you then we need to follow our own this convention that you know first you need to take into consider you know quantify, existential quantify and followed by this particular kind of rules is more less similar to that of prepositional logic except that we have a 2 more operators they are for all y there exists some y universal and existential quantify.

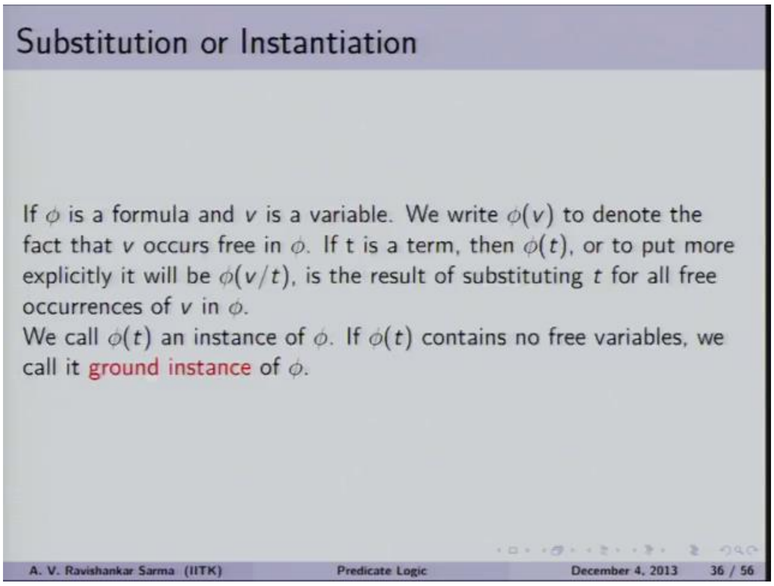
(Refer Slide Time: 49:25)



So, we will talk about what you mean by saying that a given formula in the predicate logic is considered to be down formula, when it is considered a closed kind of formula. A formula F is considered to be ground if it does not contain any variables. So, like you know usually you reflate as constants etcetera and all; a b c etcetera and all. These things are a they are not considered to be variables and all they are referring to fixed individual on the domain.

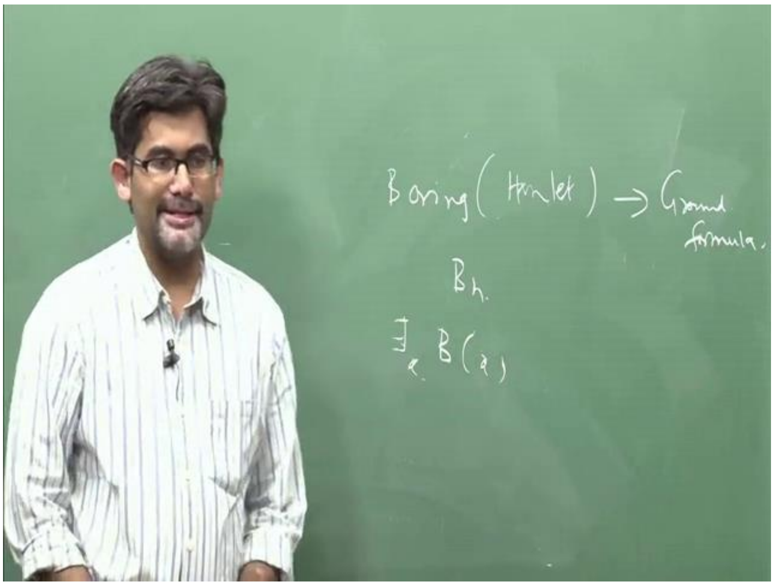
So, there it do not contain any variable that formula is called as ground variable r c r b etcetera. All these things this close formula are those formulas, if you does not contain free variables. So, those things which do not have some kind of free variables means, all the formulas, all the variables that exist in the given formula are considered to be ground.

(Refer Slide Time: 50:39)



Then, these are considered to be placed formulas; that means, you do not have any free variables which exist in the particular kind of formula. So, let us talk about some examples of this ground and close formula, and then we will close this particular kind of lecture. So, these things are important later we will make use of this things little bit later.

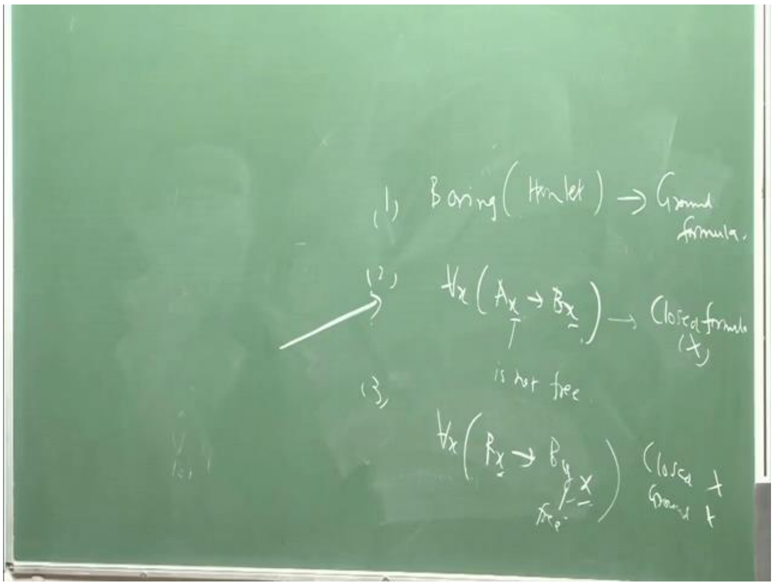
(Refer Slide Time: 50:58)



So, for example, if you say that some book that you are trying to read so that name of this book is something like Hamlet or something and all. If you want to represent as this thing, so this is simply represented as Bh and all. The book hamlet is boring for you, so this is considered to be a ground formula it does not consist of any variable at all. Suppose, if you have represented in this thing boring and some kind of x and you will presentation this way.

There exist some x such that that particular kind of book is called as boring that. Particular kind of thing can be any other thing can be Ramayana, Mahabharata or hamlet any other book and all. So, x is considered to be variable here, but here is the fixed kind of thing. So, it is in that sense this particular kind of thing is called as a ground formula it has no variables at all.

(Refer Slide Time: 52:04)



So, now this is considered to be ground formula. So, now, suppose if you represent some other sentence like where there are variables here in this particular kind of formula usually, you will see it here x x and all here x is considered to be a variable. So, now here x is not free, so now this particular kind of formula; a formula which does not consist of free variables is considered to be a closed formula.

There are some other kinds of formula which are considered to be neither close nor ground kind of formulas. For example, if you say this particular kind of thing Rx By x; B is a predicate and then this Yxx are variables and then r is some kind of x is having some kind of property r. So, now in this 1 there are 2 occurrences of x here and here. So, this is bounded by this particular kind of quantify x for all x.

So, that is why x is not free here, but what about y here? y is considered to be free. So, it is in that sense whenever you have some kind of free variables which exist in a given formula. So, this is not considered to be a close formula closes formula is not the case it is a closed formula. So, now is it is it considered as a ground formula; that means, a ground formula is a 1 which does not consists of any variable, but you have variable here x y etcetera and all.

So, in that sense it is not event called as a ground formulas. So, now usually what you would be interested in is this particular kind of formulas and all. So, mostly these formulas can be you can discuss about it as a tautologies etcetera and all. You can talk about, whether or not these formulas are true or false. So, in the next class what will be doing this will be discussing about how the substitution instances of it.

Then, we will also talk about every formula we come of with some kind of diagram tree diagram unique kind of tree diagram with which you can read the particular formula and all. So, what we have discussed in this class is simply is that we discussed about what you mean by a quantify. So, and then we introduce 2 quantifies for all x there exists some x if you do not have this particular kind of quantifies things will be very difficult.

Because, you will be keep on writing it recursively n number of times and all without even coming to know what it says. So, we need this universal quantifies and existential quantifies, and then we discussed about relationship between universal and existential quantifies. And then we discussed about when a given formula is within the scope of the quantifier and based on that we can judge whether given formula is given variable in that formula is free or bound etcetera.

Then based on whether or not you have free variables and variables etcetera and all then we discussed about what you mean by grounded and close formulas and all. Then, we said that close formulas are of some kind of interest to us. Because, you can discuss many interesting things about satisfiability tautology, validity etcetera with respect to the close formulas. So, in the next class we will continue with the syntax only we will finish with syntax and then we will move on to symmetries.

Then, we discuss about all the important decision procedure method which exist in the predicate logic. They are first we start with symmetric method that is the 1 which we have discussed in the in the case of prepositional logic. And then we move on to one of the important proof procedure method that is the natural deduction method. Then, as usual in the case of prepositional logic we use resolution reputation method. So, we will be talking about the same thing little bit in the next few classes.