Introduction to Logic
Prof. A. V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

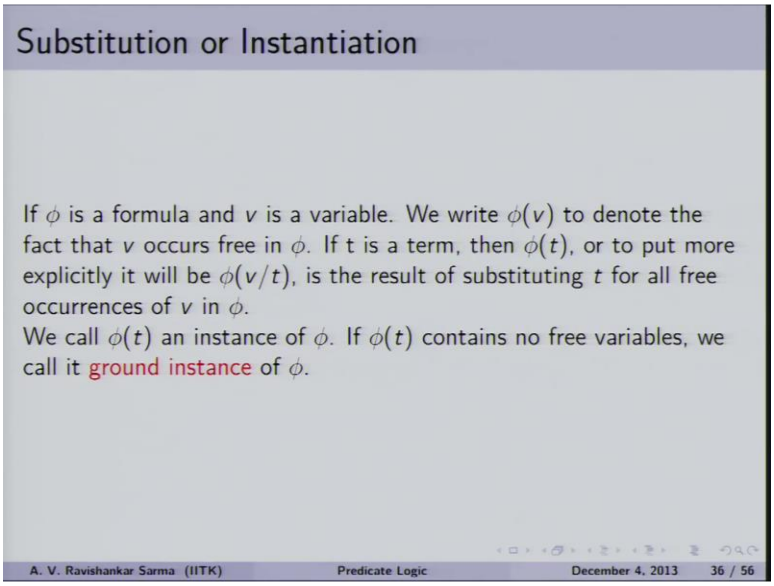
Lecture - 36
Translation in to Predicate Logic

Welcome back, so far we are discussing about the basics of Predicate Logic. And we started with, what do you mean by quantifier. And we introduced to define quantifies, that is, one is for all x and the second one is there exists some x. So, in a way, we are trying to extend a preposition logic with these two quantifies. Then, in the last few classes, we discussed about various properties of quantifies.

And then we introduced a concept called as scope of the quantifier. When, a particular kind of variable is considered to be free. When, a particular variable is considered to be bound. And these are the things; that we have discussed in the last few classes. So, today, we will be talking about, some of the other important properties of quantifies. And then basically we will be talking about the syntax of predicate logic.

So, what, we will be doing today is, we will be talking about various things related to quantifiers. Like, what do you mean by saying that, substituting a term with variable when do we say that variable is substituted by another term, a constants, etcetera. So, what are the ways to substitute etcetera?

(Refer Slide Time: 01:31)

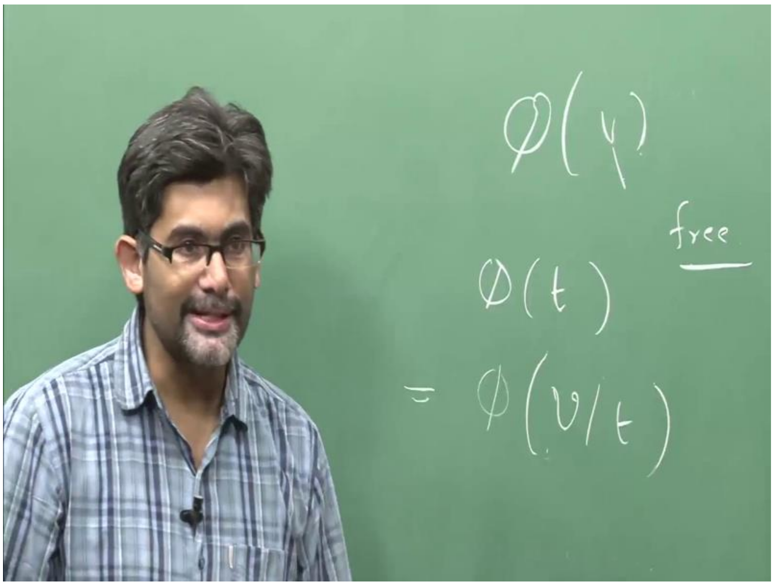


And then we will talk about instantiation, etcetera. And then we will go into the details of various kinds of translations, so translations in predicate logic. So, given in English language sentence, how do we translated it into the language of predicate logic is a one which, we are going to see in this class. And then at the end of this lecture, we will be talking about particular thing, which is called as, I mean, each and every formula will have it is own corresponding tree diagram.

So, each and every formula comes up with it is own, unique tree, diagram which we will be drawing in a while from now. So, to start with, we will begin with concept of substitution or instantiation. So, these are the instantiation is the one which you often come across in a next few classes. Especially, when a universal quantifier is instantiated, then we call it as universal instantiation.

And then the same way existential quantifier is instantiated. That means, one particular instance of this existential quantifier. We call it as instantiation. So, what do you mean by saying that, you mean by saying that substitution. So, let us consider a simple formula phi. A phi is a formula and v is considered to be a variable. So, you have to note that, we have constants, we have variables, we have predicates terms etcetera and all. So, phi is a formula and v is a variable. Then, we usually write phi of v to denote the fact that, v occurs free in phi.

(Refer Slide Time: 03:10)



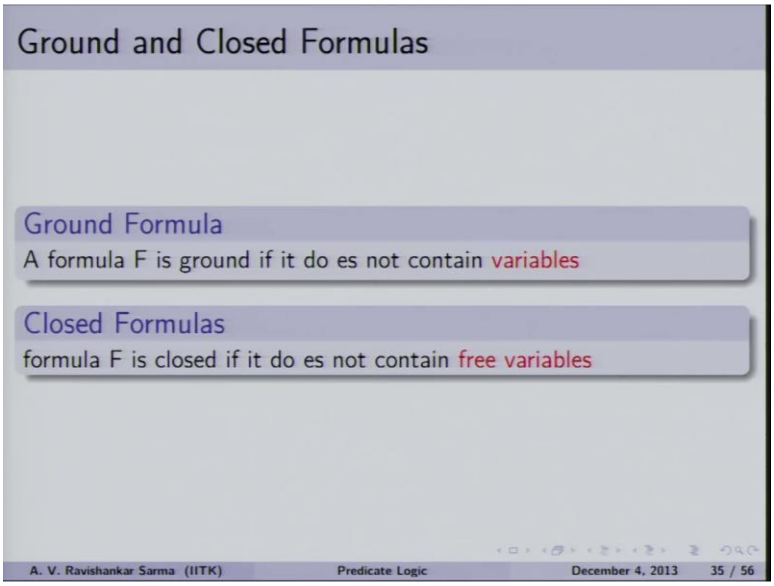
Suppose, if you write like this, phi and v, then this v occurs as free. So, only when this variable occurs as free. Then, we can substitute it with another term, another constant a, b, c etcetera and all. So, phi of v denotes a fact that, v occurs free in ((Refer Time: 03:31)) the particular kind of formula phi. Suppose, if you take t as your term. Then, phi of t are to put it more explicitly, it will be phi v given t.

So, it is a result of substituting t for all the free occurrences of v in phi. So, this is considered to be a free variable. And this variable, whenever you have a variable like x, y, z etcetera and all, just like saying that some men, all men etcetera and all. So, that, if represent with some kind of constant, such as securities or Manmohan Singh or anything. So, then it will become t. So, usually, we represent it as this thing, a variable v is represented by another term t.

So, usually, we write it like this, v given t. This means that, the variable v is substituted by t. So, this is to be present, we will write in this particular kind of way. So, this means a formula, which consist of a variable v is substituted by a term t. When, you can substitute a term t, especially, when this variable is considered to be free. So, when do, we say that a variable is consider to be free, within the scope of quantifier.

A variable is considered to be free, if it is not within ((Refer Time: 05:00)) the scope of this particular kind of quantifier. So, then that variable is considered to be free. So, now this is substitution instance of this thing. Now, we will be talking about, some kind of strategy for substituting these terms for the given variables. So, we call that phi t as an instance of phi of v. So, if phi of t contains no free variables, then we call it as ground instance of phi.

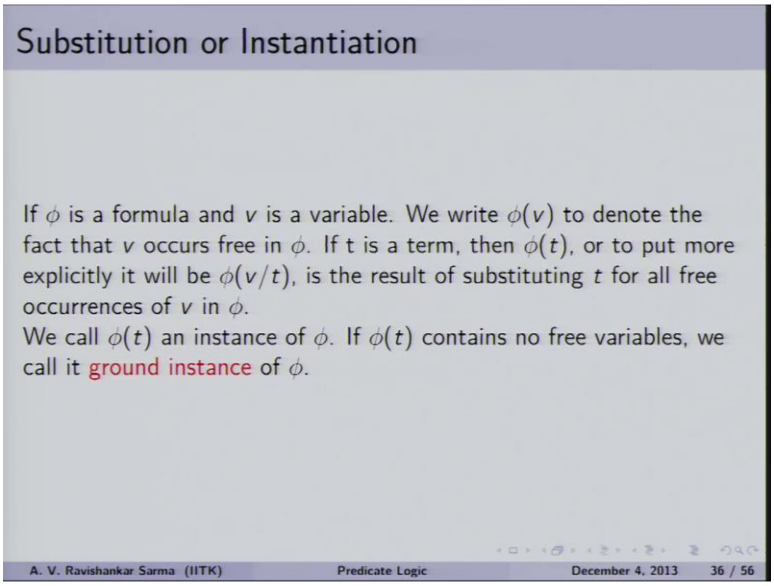
(Refer Slide Time: 05:35)



So, in the last class, we discussed about this particular kind of thing. A ground formula is a formula, which does not contain any variables. So, if phi of v, v is not considered to be a variable. And it is like x, y, z, etcetera and all in our language of predicate logic. Then, we call it as a ground kind of formula or the term exist in that kind of formula is called as a ground term.

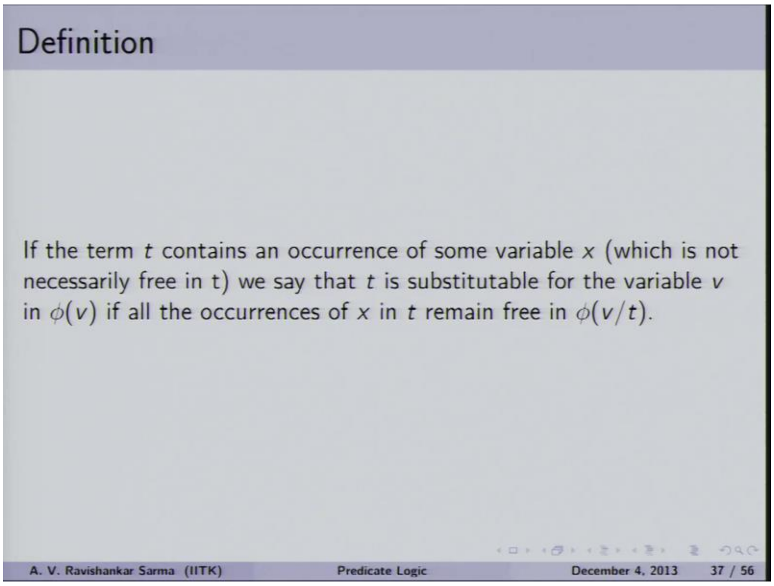
So, in the same way closed formula is a formula, which does not contain any free variables. So, that means, you can make substitution, only when it is not a ground formula are a closed kind of formula.

(Refer Slide Time: 06:16)



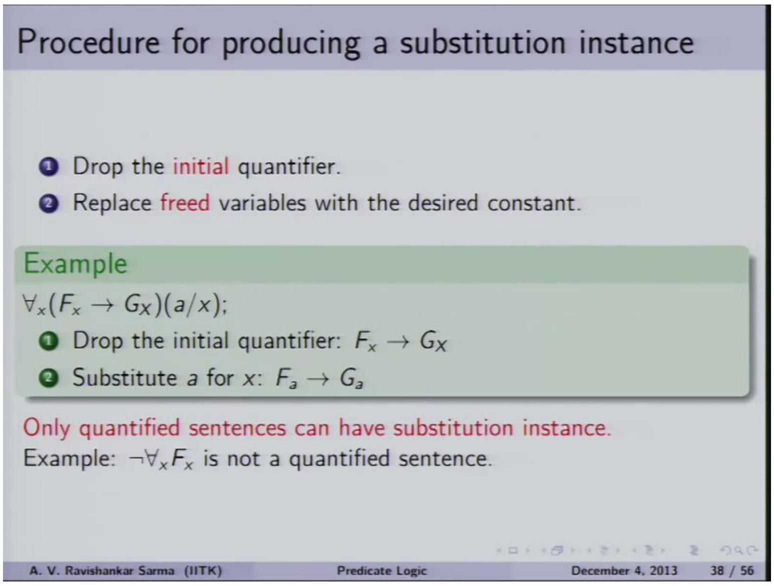
So, it has to have a free variable and that, free variable, well is going to be substituted by a term t. And that is represented as an instance of that particular kind of formula phi of v. So, now, if phi of t contains no free variables, then we call it as ground instance of phi. It would be like another constant phi of c, etcetera and all. So, that is called as a ground instance of phi.

(Refer Slide Time: 06:47)



So, now if the term t contains an occurrence of some variable x, which is not necessarily free in t. Then, we say that t is substitutable for that particular kind of variable v in that formula phi of v. If all the occurrences of x in t remains free, in that particular kind of formula phi. Usually, we represent it as phi v given t. That is what we have discussed earlier in the last slide.

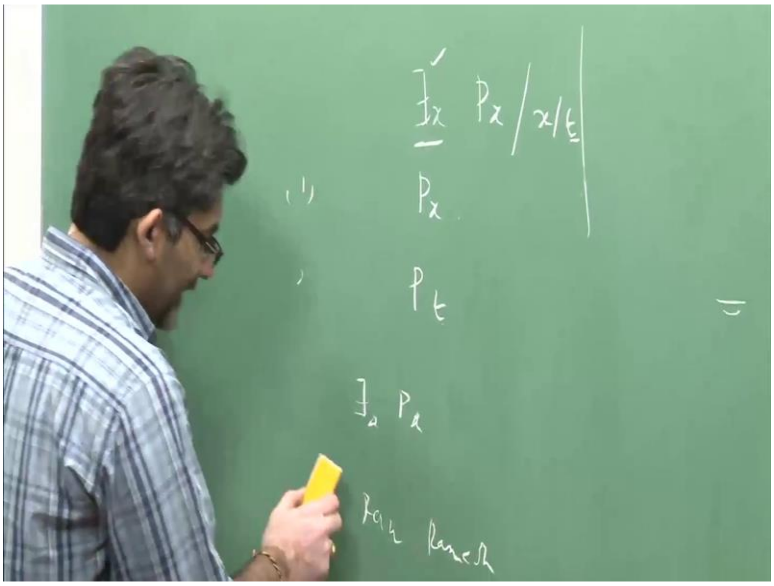
(Refer Slide Time: 07:17)



So, now there is some kind of procedure, which we follow for making this kind of substitution instance of a given formula, which consist of a free variable. So, the first step; that we will be following is this, that first, we will be dropping the initial kind of quantifier. And then after dropping that quantifier, then we will be talking about the instance of that particular kind of quantifier.

So, now we replace all the free kind of variables with some kind of desired constants. For example, let us consider this particular kind of formula. So, we start with simple kind of formulas.

(Refer Slide Time: 07:55)

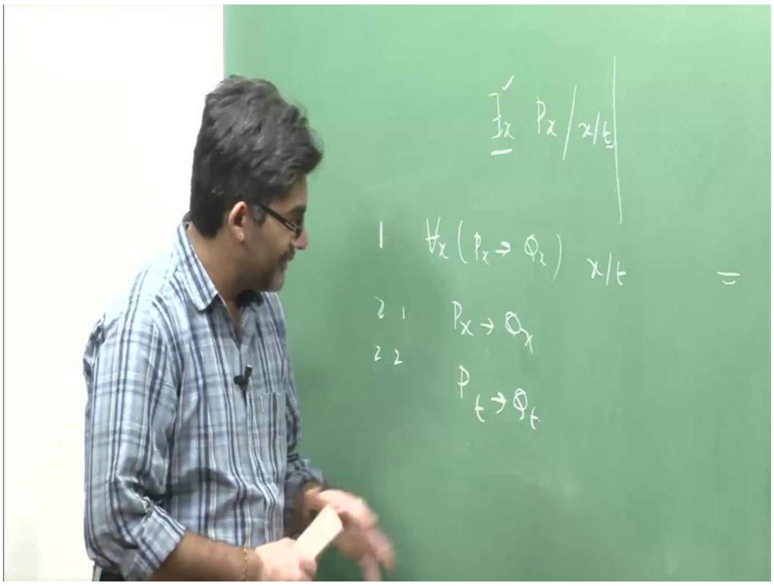


Let us say you have a formula, there exists some x, P x. So, now, one instance of first what we will do is, we will drop this kind of quantifier. Then, we will talk about this particular kind of term, which follows after this quantifier. So, now, what we are doing is, you are replacing this x with another term t. So, now, in the second step, what happens with this kind of formula is, this thing P t.

So, it is like, there exists some x, such that, some P x, x is intelligent, some IIT case students are intelligent, for example. So, if you one instance of that one is, this that some Ram, Ramesh, etcetera are considered to be intelligent. So, what exactly, we are trying to do here is this that, first what we are doing is, we are eliminating this quantifier. And then we are substituting the variable that occurs here. That is x with some kind of constant.

Usually, all constants are also considered to be terms here. So, that is why; x is replaced by some kind of constants t. These constants represent some kind of individual objects in the domain.

(Refer Slide Time: 09:17)

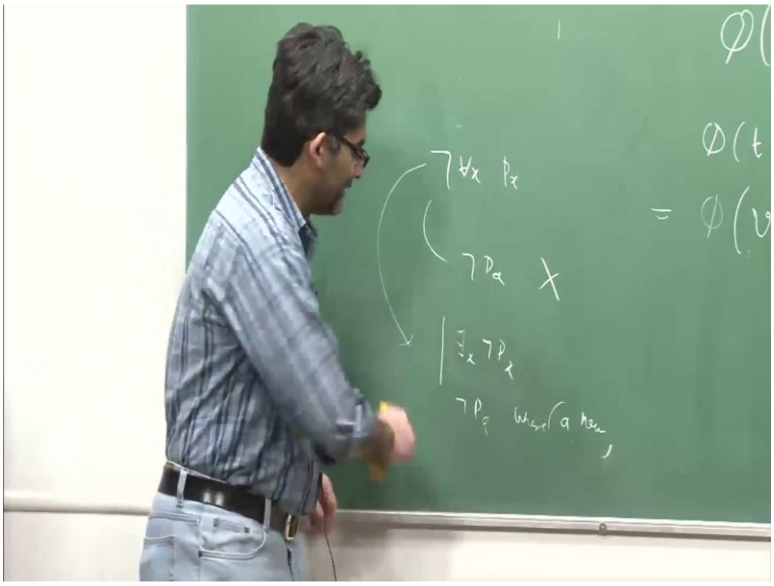


So, for example, if you wanted to do this particular kind of thing P x implies Q x. And then one substitution instance of this one is like this, x is substituted by t. So, first what we will do is? We will do like this, will eliminate this quantifier, and then it is going to be the thing. And then in the second step, what we will do is, we substituted it with some kind of constant. So, this is what it becomes.

So, now this is considered to be an instance of this one. It is like, saying that, all crows are black. And one instance of that one is, like this, you might have seen. You have seen one particular kind of crow, which is considered to be black. So, that is considered to be one instance of that one. In the same way, if you say, all metals expands upon heating, you observe one particular kind of metal and that started expanding. And that is considered to be instance of all metals expands upon heating ((Refer Time: 10:20)).

So, in this way, we can substitute it with this particular kind of in this way. First, you drop the quantifiers, and then replace the variables with some kind of terms. Then, it will become a substitution instance of a given formula. So, only quantified sentences can have substitution instances. Either, the formula should be staring with the either existential quantifier or the universal quantifiers. So, that we can substitute it, for example, if you take this into consideration. This one will not have any substitution instance.

(Refer Slide Time: 10:58)

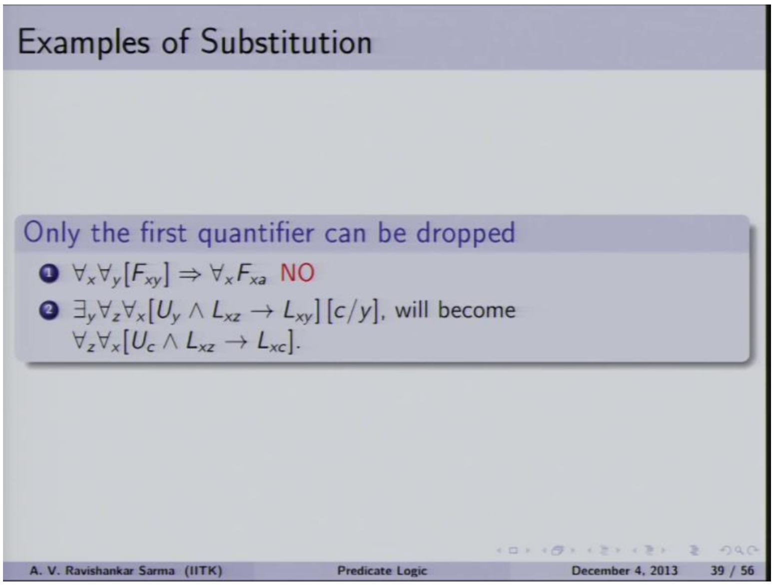


For example, naught for all x, P x. You cannot simply say that, this will become naught P a for. So, this is not permitted here. First, what you need to do is? You need to convert into some kind of standard form. So, this will become there exists some x not of P x. This can be substituted; this will have some kind of substitution instance, but, not this particular kind of formula. So, now, this will become naught of P a, where this a has to be new. We will talk about this rules little bit later.

So, there is lot of difference between naught of for all x, P x. There is lot of difference between this thing for all x, P x and there exists some x, P x and all. So, these kinds of formula will not have any substitution instance. So, you need to simplify this formula. Then, only you can substitute for x with some kind of constant. You cannot straight away substitute and say that, it is naught P a or something like that. So, that is kind of wrong substitution.

So, that is what we are trying to say, not of for all x, F x is ((Refer Time: 12:20)) not considered to be a quantified sentence. You have to simplify that formula, and then it will be become some kind of quantificational sentence, because it is starts with negation of the quantified. So, this formula will become there exists some x, naught of F x. And then you can starts substituting for this ground variable x with some kind of term.

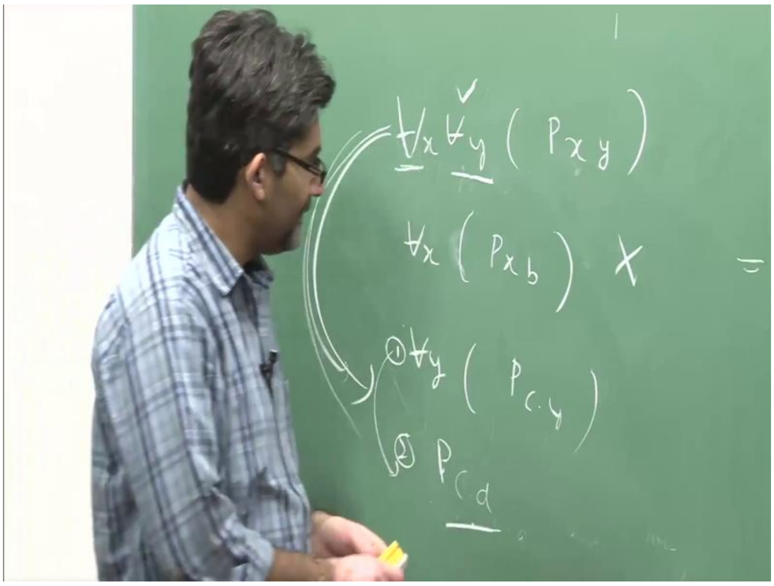
(Refer Slide Time: 12:45)



Now, let us consider some other examples of substitution. There is some kind of strategy, which we follow here. For example, if there exists two quantifiers for all x and for all y, anyhow, F x y. Then, what it is procedure that we followed earlier? First, we need to drop this quantifiers. And then you have to substitute it with some kind of term, which is considered to be a constant.

So, now if you take this example into consideration, for all x, for all y, F x y. And then you drop the second quantifier, and then substitute it y with some kind of constant a. And that is considered to be a wrong kind of substitution. So, why because you need to drop, there should be some kind of convention, that we will be following. And that convention should be like this.

(Refer Slide Time: 13:37)



So, that formula is like this P x y. So, something like P x y and all. So, now, what I am trying to say here is this that. Suppose, if you write like this, you drop this kind of quantifier. And then you substituted, wherever you have y with some kind of letter b or something like that. So, then this is considered to be wrong and all. So, what is a correct kind of substitution is this one.

First, you need to drop the quantifier that exists in the starting point and all, not the inner most quantifier. So, this is the outermost quantifier innermost one. So, you need to drop this one first, you need to move from left to right. So, that, we will be following some kind of convention. So, initially, what we will be doing is, we will be substituting this one for all y. You drop this particular kind of quantifier. And then wherever you find x, you substituted with c, and then you keep it as it is.

Now, in the second step, you can substitute the variable that exist here y with some kind of constant. So, now, this will become P c, another letter d. So, now, this will become an instance of this one. So, for all x, for all y, P x y, x and y are related somewhere. So, that, one instance of that one is P c d. So, there should be some kind of convention that usually, we will be following.

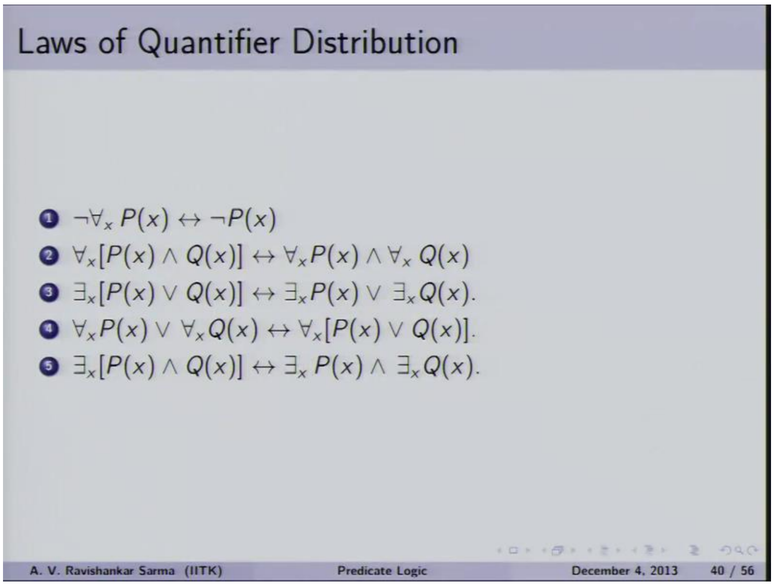
So, that is, first you drop the initial, whatever occurs in the beginning. And then you move towards right hand side. And then you drop these quantifiers, and then make these kinds of substitutions. But, this is considered to be a wrong substitution. So, in the same way, if you consider the second example, there exists some y, for all z, for all x, U y and L x z implies L x y etcetera and all. And then you have given one substitution instance, wherever you find a variable y, you are substituting it with a constant c.

So, now, in this case, what will happen is this that. So, you need to drop the quantifier that you will find it in the beginning of this formula, that is there exists some y. So, when you drop that particular kind of formula, wherever you find a formula with this subscript y, you substituted with this constant c. So, now, this formula will become, first you need to eliminate there exists some y.

And in this formula will become for all z, for all x and U y, y becomes c. Now, that is why; it becomes U c and L x z remains as it is. And then in L x y, you substituted y with a letter c. So, that is why; it becomes L x c. So, this is a way, we substituted with some kind of variables are substituted with constants. So, there is some kind of order, which we follow.

So, what we got from this one is this that, the procedure is simple. So, first, you need to eliminate the quantifiers. And then you substitute the variables with some kind of constants. And that will become a substitution instance of some kind of generalized kind of statement. For example, if you say all men are mortal and one substitution of instance of that one is, there exists some x. So, criticize mortal, for example, is considered to be an instance of that. Where so criticize considered to be the constant, which is substituted for the variable x.

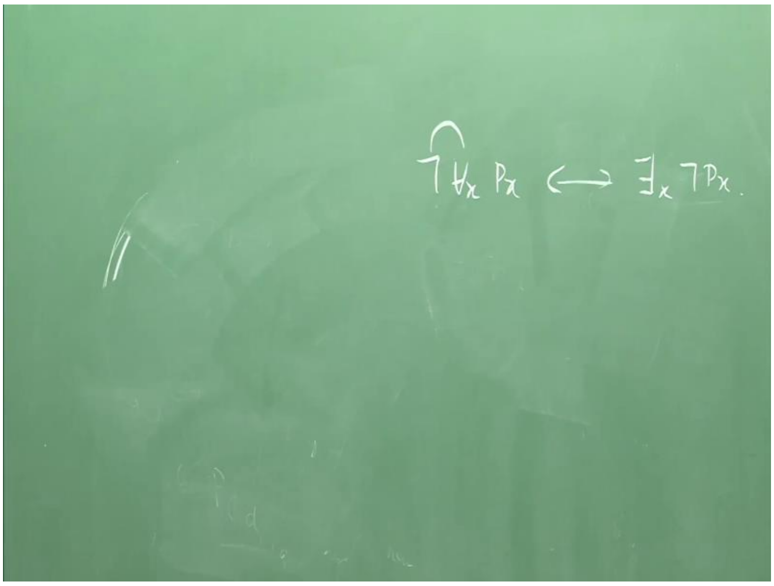
(Refer Slide Time: 17:37)



So, this is what do you mean by substitution, and then will let us discuss something about different kinds of laws of quantifier distribution. So, these laws, we will use it some of the decision procedures; that we will be using it later. Where, we will be talking about validity, consistency, etcetera and all. So, there will make use of this particular kind of laws.

So, the first law says that, if you negate the universal quantifier followed by a formula P x and that is a same as there exists some x naught, P x. So, this formula needs to be written in this way. There is some mistake in that slide.

(Refer Slide Time: 18:16)



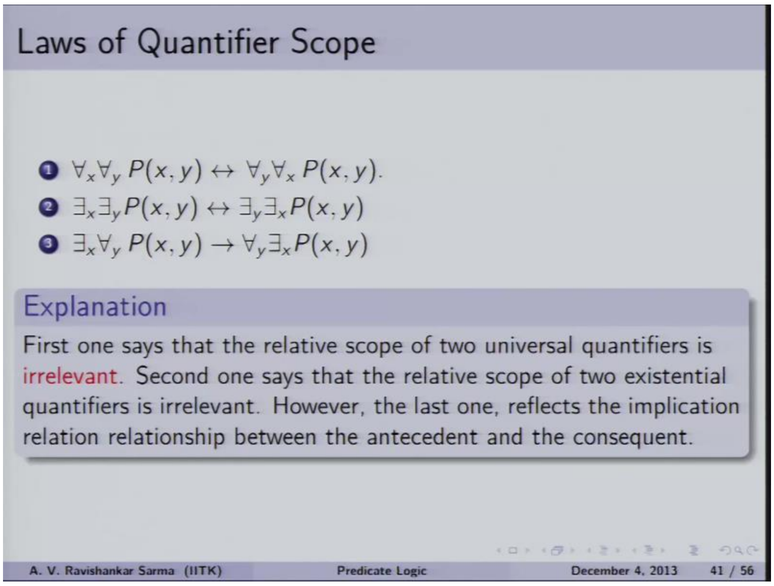
So, for all x, P x, so this is same as, what you do here is this that, you push this negation inside and the negation of the quantifier will become the other one. There exists some x, and then you push this quantifier, push this negation inside and this would become this one. So, now we can write this formula in this way. So, the moment, if I write like this; that means, in both sides it happens.

So, this will become there exists some x naught of P x. So, negation of the universal quantifier will become an existential quantifier with the negation of the particular kind of formula. So, the other thing, which you will going to notice in this formula is this thing. So, distribution over the conjunction, for example, if you say, for all x, P x and Q x, if and only if, this same as for all x, P x and for all x, Q x.

So, it is nicely distributed over the conjunction. Universal quantifiers are distributed over the conjunction, whereas existential quantifier, the next one, third formula is distributed over the disjunction. There exists some x, P x or Q x is same as there exists some x, P x for taken it alone in isolation and is same as this one. There exists some x, P x or there exists some Q x. And the other way around also it happens, there exists some for all x, P x or for all x, Q x is same as for all x, P x or Q x.

So, in the same way, existential quantifier, there exists some x, P x and Q x is same as there exists some x, P x and there exists some x, Q x. So, it is distributed, when you are trying to use the same kind of quantifiers and all. It distributed over conjunction as well as disjunction.

(Refer Slide Time: 20:29)



So, now, there will be some kind of problem, if you take into consideration to different kind of quantifiers and all. If the quantifiers are same, if you are using the same universal quantifiers, it does not make any difference in which order you use. For example, if you say for all x, for all y, P x y is more or less same as for all y, for all x, P x y. So, let us consider domain in which consists of natural numbers and x and y are considered to be any two numbers.

Then, if you take any number into consideration 1, and then let us consider relation as greater than or less than, for example, for time being, you take it as less than. So, now, if you take any number into consideration, let us say 2, which is always going to be less than the other number. If you are taking natural numbers as your domain, so 2 is always less than 3.

So, for all x, if it happens for all y, P x is less than y. Then, if it is same as for all y, for all x, P x y, then this particular kind of property holds. So, the idea here is this that, the order is not going to cause us a kind of problem here. So, for all x, for all y, P x y is same as for all y, for all x, P x y. In the same way, if the quantifiers that you are using are more or less same. That means, there if the same time, either existential quantifier or the universal quantifier.

And that is not going to make big difference, there exists some x, there exists some y, P x y is same as there exists some y, there exists some x, P x y. It is like, let us say x and y are related in this way, x is a brother of y, for example. So, you are saying that, there exists some x, there exists some y, P x y means, x is brother of y. That is same as, there exists some y, there exists some x, again x is brother of y. It does not make any big difference.

When, you interchange the quantifiers, provided, when you are using the same kind of quantifiers. So, if you use different kind of quantifiers, then as you see in the third kind of inference, it happens only in one way. The other way around, it would not happen. So, that is, there exists some x, for all y, P x y implies, for all y, there exists some x, P x y. But, the other way around, it would not happen. That is for all y, there exists some x, P x y does not imply, there exists some x, for all y, P x y.

Again, you take into consider the same example x is a brother of y, P x y stands for, let us assume that, x is a brother of y. So, there exists some x, for all y, where x is considered to be brother of y. It is like in a context of church, for example. That fellow is considered be brother of ((Refer Time: 23:45)). That is why; they call him as brother or father or something like that.

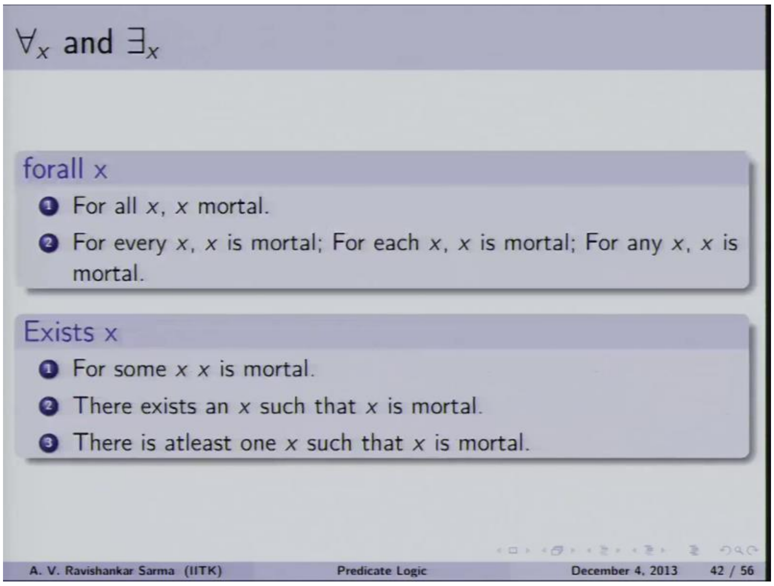
So, that is same as for all y, there exists some x. That is say P x y. But, the other way around, for all y, there exists some x, P x y does not imply, there exists some x and for all y. So, the expression is like this. The first one says that, relative scope of two universal quantifiers is going to be irrelevant. That happens in the second case also as long as you use the same quantifiers is not going to make a big difference.

So, relative scope of the universal quantifier does not make any big difference and all, so there all irrelevant. The second one says that, relative scope of existential quantifiers in the same way is also considered to be irrelevant. What is relevant here is this that, whenever you use two different kind of quantifiers, then the meaning changes. So, that why; it happens only in one way.

In the case of third example, there exist some x, for all y, P x y, implies for all y, there exist some x P x y, but it is not the case that, vice versa is not true. So, that is for all y, there exists some x, P x y does not imply that exist for all y, P x y. Scope is going to be relevant, only when you use two different kinds of quantifiers. Otherwise, there is going to be the same thing as long as we do not change the P x y kind off.

The formula P x y does not change. So, you use in whatever order; that you are going to use the same thing. So, this is what with respect to laws of quantifiers with respect to scope.

(Refer Slide Time: 25:31)

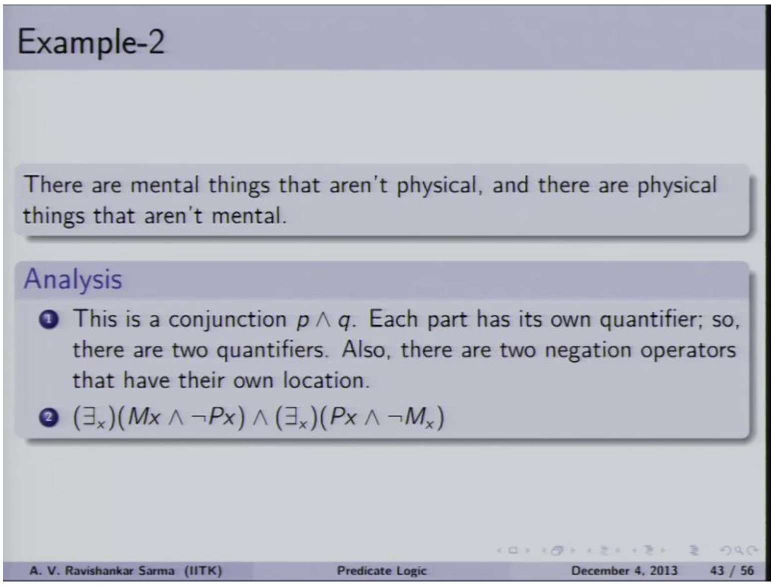


So, let us talk about some kind of translations; that you commonly come across in the language of predicate logic. So, before that, we will talk about the two quantifiers for all x and there exists some x. So, for all x is represented as this thing, for example, if you say that, for all x, x is mortal. Then, you represent it as for all x, just letter P x, so that means, what does it mean to say that, for all x, x is mortal.

That means, for every x, whatever x, that you are going to take into consideration. That x has to be mortal. It cannot be the case that there is one particular kind of x, you have chosen and that x is not considered to be mortal. So, that means, whatever you pick it up and that has to be have this particular kind of property. That is mortality. So, that means, for each x, x is consider to be mortal or the other way round of saying this things is that. For any x; that you are taking into consideration, x has to be mortal.

So, exists x is like this, it happen only for some x at least 1 x is consider to be mortal then you represent it as there exists some x. So, that means, there exists an x, such that, x is consider to be mortal. That is as good as saying the same thing as there is at least 1 x. The sum is usually represented as at least some; at least one particular kind of thing has a property something. Then you say that, we call it as there exists some x, T x.

(Refer Slide Time: 27:19)



So, let us try to talk about some kind of translations. So, we have to familiarize ourselves with the translation. Why, because, given an English language sentence, you should be in a position to an ambiguously transfer the English language sentence into the language or predicate logic. And once, you translated it into the language or predicate logic. Then, all the other things will follow, whether the formula is considered to be valid formula. That means, it is true in all interpretations in a given domain or whether that formula is considers to be contingent sentence, contingent or consistent. All this a kind of things, one can talk about only provided we have some good translation. So, let us consider some examples of this translation.

So, consider this particular kind of sentence. It says like this, there are mental things; that are not physical and there are physical things that are not mental. Whatever is pertaining to physical domain will not be in the mental kind of domain. In the same way, whatever is, there are physical things; that are not consider to be mental. So, now, this consists of two sentences and all. The first one is, there are mental things; that are not physical things.

So, this is a conjunction, if you represent it in terms of prepositional logic. It is simply becomes P M Q. So, that is not going to give us the full information about, what is there in this particular kind of sentence. So, we need to represent it in terms of quantifiers. Then, that makes some sense to talk about the inner structure of this particular kind of sentence.

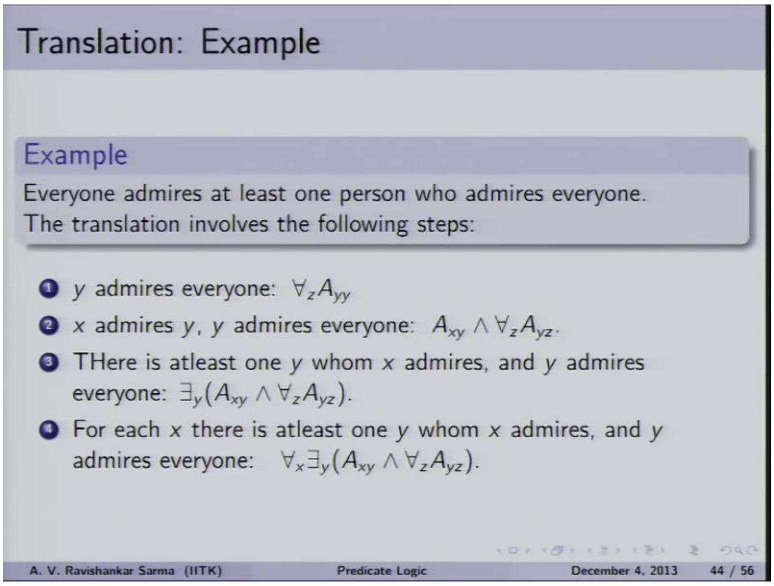
So, now, each part, that is, there are two parts here separated by end. Each part has it is own quantifiers. So, there are two quantifiers, also there are two negation operators, that have their own location. So, one is talking about they are not physical. That means, a negation is already there in that. And the other one is saying that, they are not mental. That means, another kind of operator is there.

So, now, this particular kind of statement can be translated in this. So, there are mental things; that means, not all the things are considered to be not physical and all. But, there are at least one particular kind of mental thing; that is consider to be not physical. So, in that sense, you represent it this sentence as there exists some x, where x is consider to be mental thing. And then that particular kind of x is not considered to be physical. That is represent as not P x and this takes care of the first part of the sentence.

And the second one, there are physical things; that are not mental again. This is represented as the there exist at least one particular kind of x. That x has to be a physical thing. That is P x and at the same time, x has to be not mental thing. That is x naught M x. So, this whole formula is represented in this particular kind of thing. So, why, we are not writing it like, there exists for all x, M x naught P x. because, the sentence is talking about only one particular kind of instance and that instance is like this.

There are some kinds of mental things; that are not physical and there are some physical things; that are not mental. The word sum is not involved in this particular kind of thing. So, basically we will be, since it is not talking about all the things. So, we usually mean it as, there is some kind of usage of the phrase, some in this particular kind of sentence.

(Refer Slide Time: 31:23)



So, let us consider some more examples, and then will talk about, how we are going to translate this things into something. So, let us consider another example, these likes. Everyone admires at least one person; that one particular kind of person admires everyone, so ambiguously stated here. So, now, we need to go little bit slow in this while translating this particular kind of formula by braking the sentence in a appropriate way.

So, now, what this sentence says is that, everyone admires at least one person. Let us say is consider to be the father of the nation or something like that. So, we admire that particular kind of person like, Nelson Mandela or Mahatma Gandhi, etcetera and all, who admires everyone. That particular kind of person admires everyone. So, now this can be broken into different parts and all.

So, that translation, you will get some kind of justification for this particular kind of translation. So, first thing is it, why there exist some kind of y, at least one person is there; that particular kind of is y. So, who admires everyone? So, that means, for all y, A y y, for all z, A y z is the thing, which we need to write it here. It is not A y y, but A y z. So, A y z means y admires z. So, that, z has to be for all z and all. So, whatever z, you take into consideration; that y has to admire that particular kind of z.

So, now in the second step, let us consider there exists some another x and all, x admires y. If x admires y, then y has to admire everyone. So; that means, the sentence is translated as A x y and this particular kind of sentence, for all z, A y y. So, what do you mean by saying that, here in A x y, a stands for the predicate, admires. And then x and y, they are in one particular kind of order.

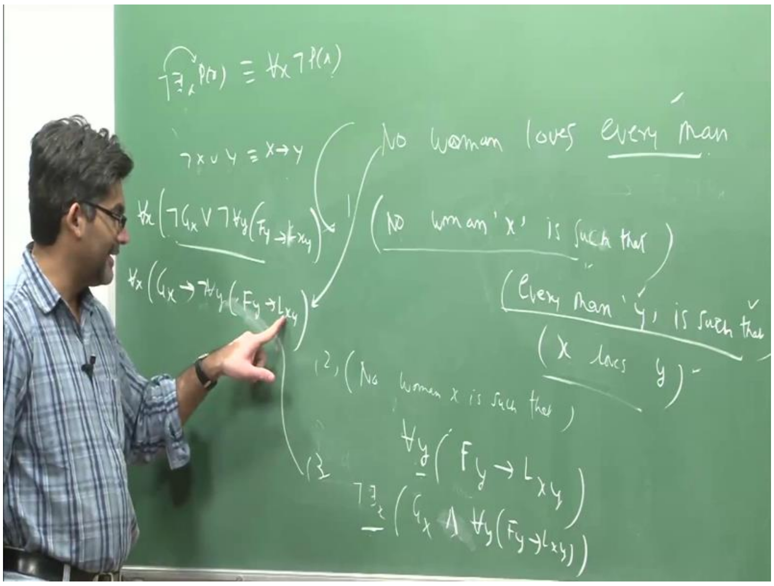
So, x y simply means that, x admires y. It is not the case that y admires x and all. There is some kind of order, which we follow in the predicate law. If it is written as A y x, then we say that y admires x. But, here everyone admires at least one person, who admires everyone. So, who admires everyone is written as for all z, A y z, and then this sentence is a conjunction of this thing, x admires y and y admires everyone. So, that is why; A x y and for all z, A y z.

So, now, there is at least one person y, whom x admires. And in the same way, y admires everyone. So, that means, there exists at least one y is represented as there exists with the existential quantifier, there exists some y. And then whatever sentence that we got it till now, that is A x y and for all z, A y z. So, this will become there exists some y, A x y and for all z, A y z.

So, now, for each x, there is at least 1 y, whom x admires and y admires, everyone. So, now we need to add another universal kind of quantifier. That is for all x, there exists some y and whatever, sentence that is, A x y and for all z, A y z. So, this the way, to translate this particular kind of ambiguous sentence into appropriate form in this particular kind of way.

So, let us consider some more examples. So, that we will understand this idea translation in a better way. So, just we will consider some examples. So, that, we will get used to this particular kind of translation.

(Refer Slide Time: 36:01)



No woman no woman loves every man, need not to be necessary that no woman loves every man and all. He might have some human beings also. So, he need not be necessary that woman always love all the time love every man and all he need not be the kiss and all. So, how to translate this particular kind of thing in various step? So, now, as I first step, you write it like this, no woman. So, that, particular woman, you consider it as x. It is such that, you just put it in bracket.

So, that, the separating the sentence and we are handling the sentences by piece by piece. So, now, we have something called every man and all. So, every such that every man. So, now, woman is represented as x. Now, man you represent it as y is such that. So, this is a second sentence, and then whatever is left here is this that. So, that particular woman x loves y and all. So, now, you write it like this x loves y.

So, what we have done here is this that, there exists some kind of woman, there exists some kind of y; that is considered to be a man. And then the relation between x and y is like this x loves y. It is not the case, that y loves x and all. So, there is one particular kind of order, which we follow. So, this sentence is translated in this particular kind of thing, we are trying to consider it in piece by piece.

So, no woman x is such that, there exists some kind of y. And that y is meant for all the man and all and that x loves all kind of man for all y. So, now, you keep it as it is only, no woman x is such that, you keep it as it is. Now, you translate this thing into appropriately into the language of predicted law. It says that every man y is such that; that means, for all y.

Suppose, if y is consider to be a man, then x is y. If y is considered to be a man, then x loves y and L. L stands for love, x stands for man and all. So, this happens for all y and all. So, that will take care of this particular kind of sentence, these two sentences. So, now, this translation is not yet over. So, now, we need to represent this thing no woman x such that, whole thing should happen and all.

So, now, in the third step, no woman such that means, they does not exist some kind of x, x stands for women and y stands for man. So, this will be like, they does not exist x, such that, you have to take another property into consideration G. So, now, G x and they does not exist x, G x and for all y, the whole sentence is for all y, F y, L x y. So, what it is essentially says is that, which is broken the sentence into this thing.

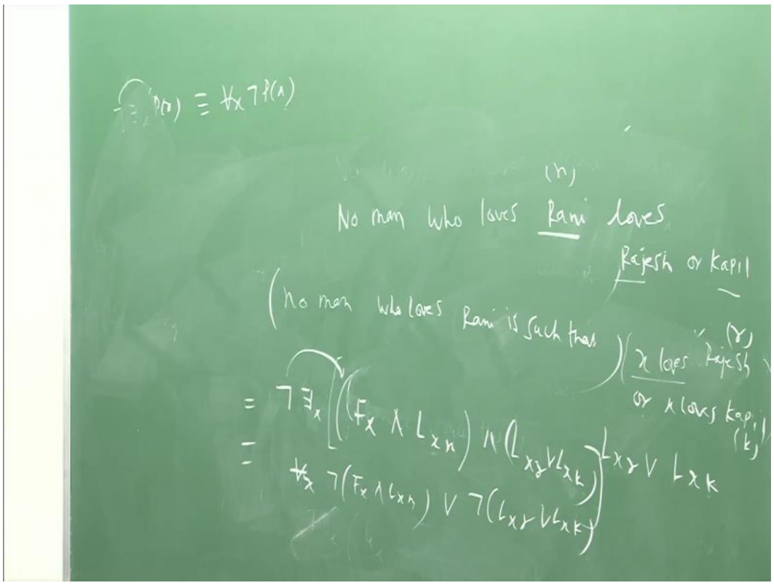
Every man y is such that x loves y, this is represented as this thing, for all y. If y is consider to be a man, then x loves man. So, this happens for all y. And then in the first sentence, no woman x is such that is represented as this thing. They does not exist some x, such that, x is G and at the same time, for all y, if y is a man and x loves y and all. So, now, this can further be translated in this. So, this says that, does not exist some x, only this formula and all.

So, now we have some kind of translations, if you come across a formula like this. There does not exist some x, P x is same as this negation goes inside. The negation of the existential quantifier will become like this. So, now, this will become like this. So, I will write it here, they does not exist this thing mean, for all x, you have to push this negation inside.

So, this is conjunction this will become naught G x and the negation of conjunction will become disjunction, and then it is negation of for all y, F y implies L x y. So, now, you can further simplify it, and then you can write it like this. So, this is like naught x or y. So, naught x or y same as x implies y. So, now, we write it like this G x implies for all y, this will be same a naught F y implies L x y. So, this is the translation of this particular kind of thing.

So, no woman loves every man, some three or four steps, you translated in this particular kind of way. So, this essentially says that, for all x, if x is having some kind of property G. And then does not mean that, x is a woman. They does not exists, it is not for all y, if y is a man, then that particular kind of x has to love this particular kind of man. So, that means, no woman needs to love every man. So, this is a way to translate it. Just one more example, we take into consideration, and then we will move on to some other a kind of translations.

(Refer Slide Time: 43:27)



So, now, let us consider one more example, no man who loves Rani, loves Rajesh are, let us try to translate this one. So, now, we need to represent this constants with there are three people, who exists here, Rani, Rajesh and Kapil. We need to represent it with some kind of symbols. So, now, this is same as, no man, who loves Rani is such that. So, it is going to be the first sentence, we are breaking it into three parts.

So, that, it will become convenient to translate this particular kind of sentence into language of predicate logic, no man, who loves Rani is such that. So, the idea here is just that, no man, who loves Rani loves Rajesh or Kapil. So, that is one which we are trying to translate it. So, x loves Rajesh or x loves Kapil. So, now, this x loves Rajesh is represented in this thing.

L stands for a predicate L, x loves Rajesh is represented as r or in the second sentence, x loves Kapil is represented as this thing, k stands for Kapil and x loves k is represented in this sentence. So, now we are taking care of the sentence, which is on the right hand side. So, now, we need to take care of this particular kind of sentences, no man, who loves Rani is such that.

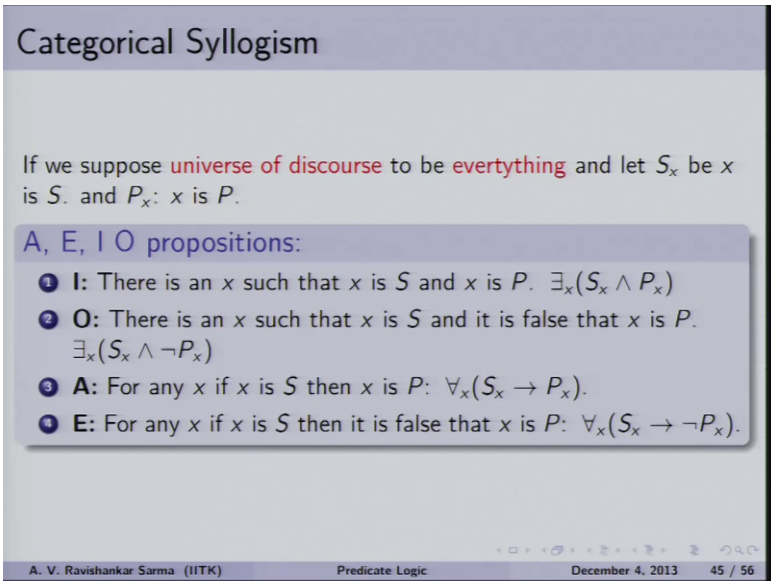
So, this is represented as this thing, there does not exist some x, where F of x and this particular kind of x n. So, where, this represented as n. This represented as r and this is k. So, there are three constants that we have. So, individual which exist in this particular kind of sentence, Rani Rajesh and Kapil.

So, now this says that, there does not exist some x, such that, x is consider to be that particular kind of man. And then x loves n and this happens for and this particular kind of sentences. So, that is L x r, L x k this says that, they does not exist some that particular kind of person x. So, that you know x is a man and x loves Rani and at the same time, he will do this particular kind of x loves Rajesh and x loves k.

So, now, this same as this thing naught for all x, there exist some x will become for all x and you put this negation inside and this will become a F of x, L x m. And negation of conjunction will become disjunction and it becomes L x k. So, this is going to be the translation of this sentence, no man, who loves Rani loves Rajesh are Kapil. So, like this, one can translate given English language sentence into the language of predicate logic by breaking that particular kind of sentence into one or two different parts and all.

First, you manage the rate right whatever exists in the right most of this particular kind of sentence. And then you extend into whatever is there in the left hand side, this particular kind of sentence.

(Refer Slide Time: 48:22)



So, while, discussing the traditional logic, we discussed about four different kinds of sentences. And then it will have it is own translations in the modern logic like this. So, there are four particular kinds of sentences, which we call it as categorical statements, categorical propositions. So, they are A I E and O. So, suppose that universe of discourse to be everything. Let us say, you are talking about people, all kinds of people will come into that particular kind of domain.

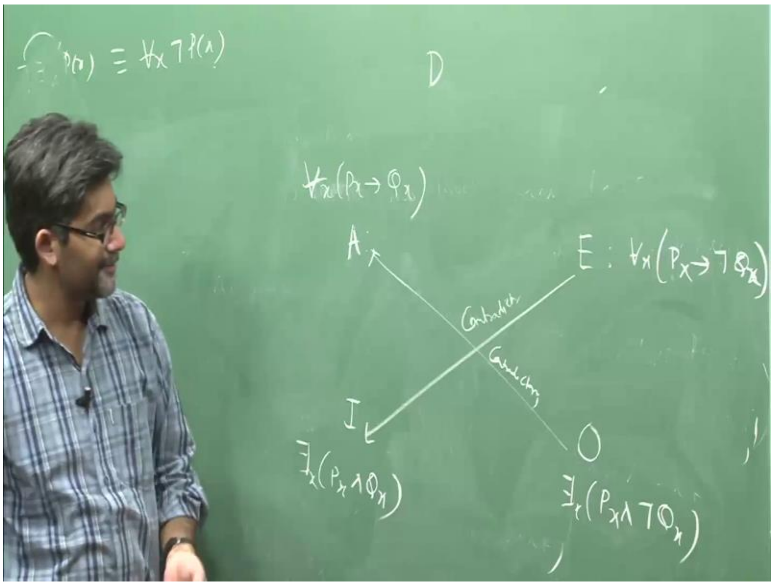
And let, S x be x is having some kind of property S and P x stands for x is having some kind of property P. It can be mortality; it can be beautiful, handsome, etcetera and all these things. So, now A, E, I O propositions are represented in this particular kind of way. I proposition, it is like some man are mortal. So, it is represented in this sense. There is at least 1 x, such that, x is having property S and x is also having property P.

So, it is simply represented as there exists some x, S x and P x. So, this is as bit as saying that some birds flies. More sentence can be represent in this sense, there is some x, such that, x is S, but it is false. That x is P. So, it is simply represented as there exists some x, S x and naught P x. So, these are considering to be particular kind of categorical propositions.

And then there are universal propositions such as A and E. A proposition is stated in this sense, for every x, if is having property S and x is also having property P. We say that, all birds flies, if x is consider to be a bird and x has to fly. It cannot be the case that, x is consider to be a bird and it does not flying and all. So, it is represented as for all x, if x is having property S implies x is having property P.

So, now, E proposition is also consider to be universal categorical proposition. It is states that, for any x, if x is having property S, then it is false that x is having property P. If that is a case and you write it in this way, for all x S x implies naught P x. So, now, let us talk about a relation between these four kind of categorical propositions, and then we will try to stop this lecture. So, this is the square of a position, which we discussed it while doing traditional logic.

(Refer Slide Time: 51:19)



So, it is like this and the one hand, we have categorical universal propositions, A proposition and E proposition. And then we have an I proposition and you have O proposition. Usually, diagonals are considered to be contradictory to each other. So, these are all contradictory to each other. So, let us represent this things, A proposition is represented in this sense, for all x, P x implies Q x.

So, before all this things, you need to talk about some kind of domain, etcetera and all. Where P and Q are considering to be properties and x are some kind of individual, some kind of variables which can be further represented by some kind of individual objects. So, now this is for all x, P x implies Q x and I proposition is represented in this things, existential quantifier P x and Q x. And then O proposition there exist some x, x is having property P.

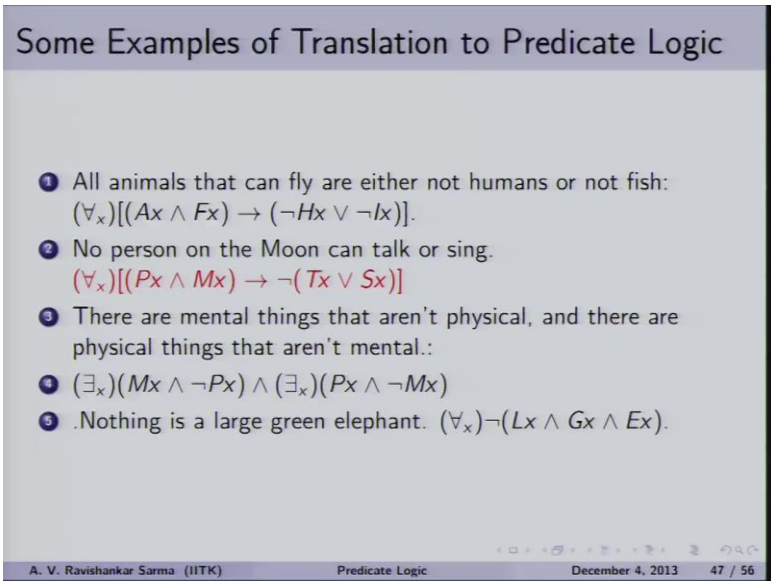
And then this is the thing x is not having property Q and then E proposition is like this for all x, P x is having property P implies does x is naught having property Q. So, these are the things, which we need to note A and O are contradictory to each other. If you take the conjunction of these things; you are going to have the value F. In the same way there exists some x, P x and Q x and for all x, P x naught Q x. So, these are two, these are contradictory to each other.

So, for example, if you say that, all birds flies and all. Suppose, if you come across one particular kind of bird x and x does not fly. Then, that contradicts this particular kind of proposition that all birds flies. So, in the same way, if you say that, some birds flies and all. And then you come across a proposition that for all x, if x is a bird and x does not fly. So, this is exactly contradictory to this particular kind of thing. So, that means, A and O are contradictory to each other and I and E are contradictory to each other.

So, this is what we consider to be square of a proposition in the predicate logic. But, there are some other kinds of inferences, which might of interest towards, whether from A proposition that is for all x, P x implies Q x. Can we deduce that, there exists some x, P x and Q x or for example, if you say that, for all x, P x, you deduce a proposition that you infer, there exists some x, P x.

It looks simple for us, but at least to some kind of problems, which we discussed it partly, while doing traditional logic. That is called as the problem of existential import like.

(Refer Slide Time: 54:44)



This there are some other kind of translations one can do, we need to practice a lot for doing this particular kind of translation. Let us consider some one or two examples, and then we will close this lecture and all. So, let us consider this example, all animals that can fly are either not humans or not fish. So, here, we need to break the sentence like this, either not humans are not fish is represented in the sense naught H x are or naught L x.

So, that is, what is the thing and all, and then all animals that can fly is represented in this things. Animals that can fly is represented in this sense, if x is an animal and x has to fly. So, that is why this whole formula is quantified over all animals. That is why; for all x, A x and F x implies naught H x are naught I x. because, we are talking about not humans and not fish.

So, in the same way, for example, if you want to represent no persons on the moon can talk or sing. So, usually, that should be in this particular kind of format for all x, P x and naught Q x. So, the last sentence should be negation of the particular kind of thing. So, persons on the moon is represented as P x and M x. And then they can talk or sing is represented in this particular kind of naught of T x or S x.

So, like this, one can translate various kinds of sentences that occur and the English language into the language of predicate logic. So, some of the things will look ambiguous for us. But, if you break that particular kind of sentence into two or three parts and all and things will become easy to handle. Some examples, which we have taken into consideration, but it requires lot of practice.

So, this translation is consider to be a very important, because, once you translate given sentence into the language of the predicate logic. And then things will become simpler, and then we can talk about validity, etcetera, consistency, etcetera that. So, in this class, what we have discussed is that, first we began with. When, we can say that a particular kind of formula is instance of a universal kind of a formula, which consists of universal kind of quantifier.

So, we have talked about substitution instances of a given formula. So, we can substitute only, when we have some kind of variables and all. So, then we will move on to some kind of laws of quantifiers. Then, we talked about an interesting observation. We come up with an interesting observation, that whenever, you have two different quantifiers. Then, it makes the scope is going to become relevant scope of the quantifiers is going to be become relevant.

Whenever, we have the same kind of quantifiers, then does not make any big difference is going to be irrelevant. It would not play any role in that particular kind of formula. That means, it is as good as saying that, for all x, for all y is same as for all y, for all x some kind of P x y is a case. So, then we talked about some kind of translations, and then we need to practice a lot with this particular kind of translations. And then in the next class, what we are going to do is, we will be talking about the semantics of a given predicate logic. And then we will move on to various kinds of decision procedure methods.