Introduction to Logic

Prof. A.V. Ravishankar Sarma
Department of Humanities and social sciences
Indian Institute of Technology, Kanpur

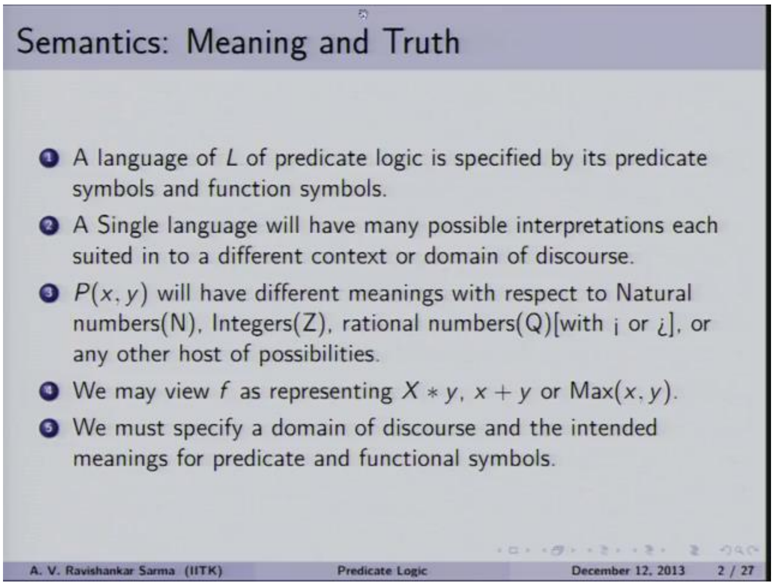
Lecture - 37
Semantics of predicate logic

Welcome back. So, far we discussed syntax of predicate logic where we discussed about what we mean by terms quantifier etcetera and all. What is a scope of quantifier and when, we also discussed about particular things that a given well form formula comes up with unique formation tree and each term also comes up with a formation tree etcetera. So, now, what we will be doing is we will be talking about the semantics of the predicate logic. The semantics means giving the meaning of a given formula. So, meaning of a given formula means; giving truth conditions of that particular kind of given formula following frugal we will be taking that into consideration.

So, semantics of predicate logic is little bit different from that of propositional logic, in propositional logic the meaning of a given complex formula a molecular formula is only determined by the meaning of it is constituents; that means, whatever values that the individual constituents takes and in whatever way the connectives or and implies etcetera behaves, based on that you can talk about a meaning of particular kind of given formula; that means, you are giving the truth condition of a given formula for example, if you want to know the meaning of p n plus q; that means, the truth condition of that 1 is whenever p is true q is false p implies q is going to be false. So, things are not say as simple as in the case of propositional logic, because in the predicate language we will be we will be using variables, Predicate, functional, symbols etcetera.

So, now, once you give the meaning of a given formula we need to take into consideration all this symbols that we are trying to use we need to assign some kind of values, to these symbols that you commit cross in the predicate logic.

(Refer Slide Time: 02:16)



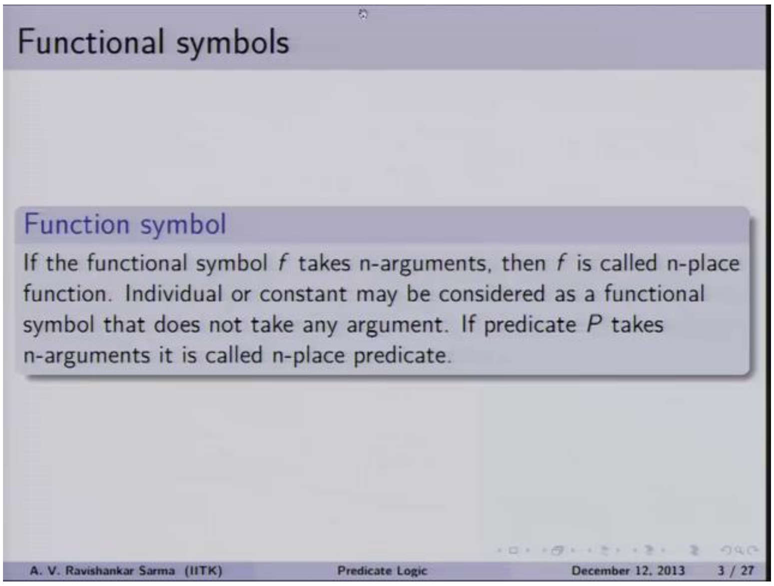
So, a language of L of a predicate logic specified by it is predicate symbols, functional symbols, variables and constants. So, functional symbols, variables, constants and predicates they are the 4 building blocks of predicate logic, and we need to have quantifies the exist some x for all x etcetera. So, a single language will have many possible interpretations each suited to a different context are domain of discourse for example, if you have a particular kind of formula the same formula can be true of natural numbers and there is the same formula that we are trying to in terms of a different domain, in let us say real numbers other than natural numbers whole numbers etcetera which includes 0 also.

So, if you talk about whole numbers the same kind of formula may turn over to be false or if you are talking the same kind of formula the meaning of formula with respect to integers and the meaning might change. So, it is dependent on the domain that you are using. So, suppose if you have P of x y; it will have different meanings with respect to let us say natural numbers may be it might be true it may false in in natural numbers. The same thing might be true in integer's etcetera or may be in the rational numbers it might be something else. So, what we need is, we need to fix some kind of domain see in order to talk about the meaning of a given formula in the predicate logic, 2 essential things are important first we need to fix the domain.

We need to fix the domain consist of a let us say, if you are talking about numbers need to talk about either natural numbers or whole numbers or integers non-negative numbers positive numbers etcetera and all and the real numbers of course, it includes all this things or if the same thing might be false with respect to irrational numbers etcetera. So, it mightily to multiple numbers of possibilities the same formula can be true in different interpretation. So, we may view the function f as representing kind of multiplication or plus or something relation such as x and y whatever is the maximum of x and y etcetera. That will be a considered as a function functional symbol and essentially what we require is, a domain and there is some kind of thing which you require that is interpretation function which assign some kind of values to constants, variables, predicates and functional symbols.

So, the first and 4 most things which is essential for the semantics of formula in a given predicate logic is the domain of discourse and the internet meaning of a meaning for predicates and functional symbols. So, that is taken care by interpretation function. So, the first essential thing which you need is the domain and the other thing which you require is the interpretation function.

(Refer Slide Time: 05:35)

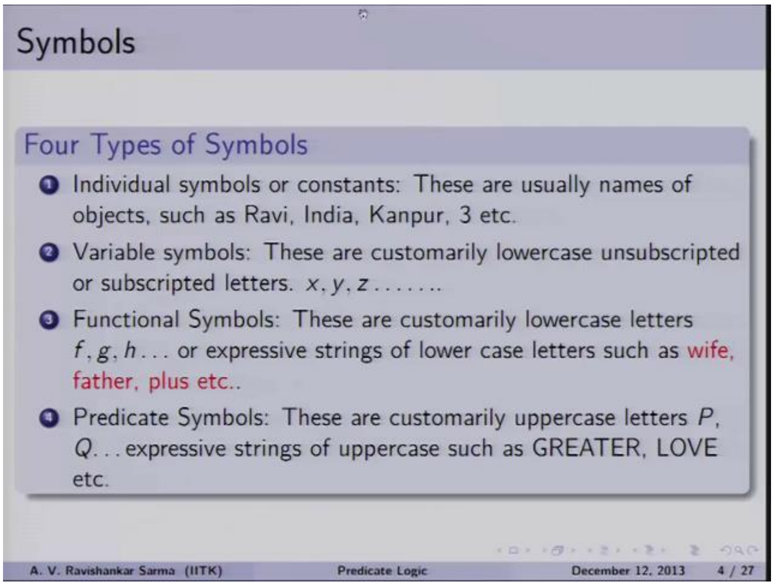


So, let us talk about what we mean by these functional symbols. There are 4 symbols that we are essentially talking about, first is functional symbols f, g, h it is represented by f, g h etcetera. And we have constants which represent some kind of individual objects in the domain like Ravi, Raju etcetera and all. They are referring to some kind of individual entities and we have variables such as x y z etcetera. It stands for anything as a anything, and then we have predicates it which talks about some kind of relationship between some kind of objects. Like something is red something is white beautiful etcetera and all. Are x brother of y or y father of z etcetera all this things are predicates, which essentially have some kind of property.

So, know let us talk about what you mean by functional symbol. So, functional symbol f takes n- arguments, then f is called as emery function if it takes only 1 kind of thing it is called a unary function f of x is equal to y and f of x square is equal to z etcetera and all. Suppose, if you are talking about binary function like x plus y for example, it is a binary kind of function. So, if there are n kinds of arguments and all it is called as enery function. So, no individual or Constance may be considered as functional symbol that does not take any argument. So, these things are considered to be individual Constance.

If predicate P takes n-arguments then it is called as n place predicate. Example unary predicates are for example, x is mortal. So, that is x suppose if you want to say that x is brother of y; so b x y. So, x and y are in some kind of order or if you want to talk about the treasury predicates and all 3 place predicates, then we can give some examples for treasury predicates etcetera. So, if use n kind of arguments and all it is called as n plus predicate.

(Refer Slide Time: 07:50)

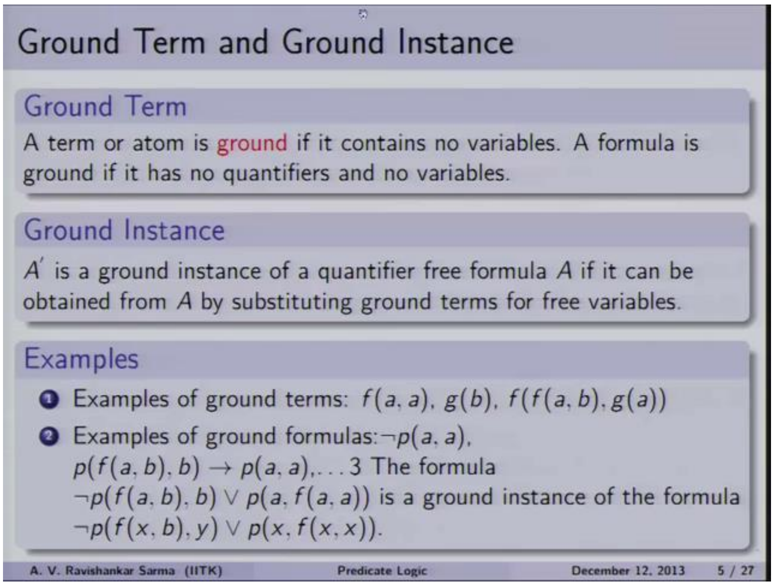


This is the thing minimal things which we need to note these are the 4 kinds of symbols that, we are using individual symbols are constants, which we have discussed just now there are usually names of objects such as duster chock piece etcetera Ravi India Kanpur all this things comes under a referring to specific kind of entities in your domain. So, they are called as constants individuals. So, usually variables are replaced by these individual constants.

So, now, there are other kinds of symbols in the domain. So, they are variables symbols why we are discussing all this things, because for interpretation for giving the meaning of a given formula what you require is a domain, and then we need to talk about, assigning some kind of values to these 4 kinds of symbols. So, variables are represented by x, y, z etcetera x can stand for anything. So, we are not just specifically mentioning what x is all about? So, they are all variables. So, now, the third thing is functional symbols represented by f, g, h usually plus minus multiplication all this things are called as functions and the predicate symbols.

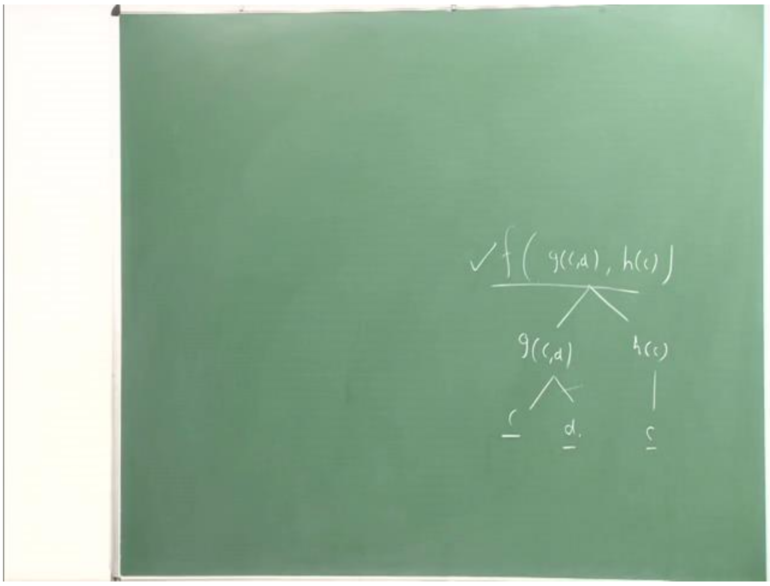
Usually they are represented as capital letters love greater then etcetera and all, beautiful all these things are predicates mortal all these things. So, this is the 4th symbols that you come across you need to when you talk about meaning of a given formula, you need to take care of all this symbols. And we need to talk about some other things which are important for this 1 for defining the meaning of for all x f x and all.

(Refer Slide Time: 09:32)



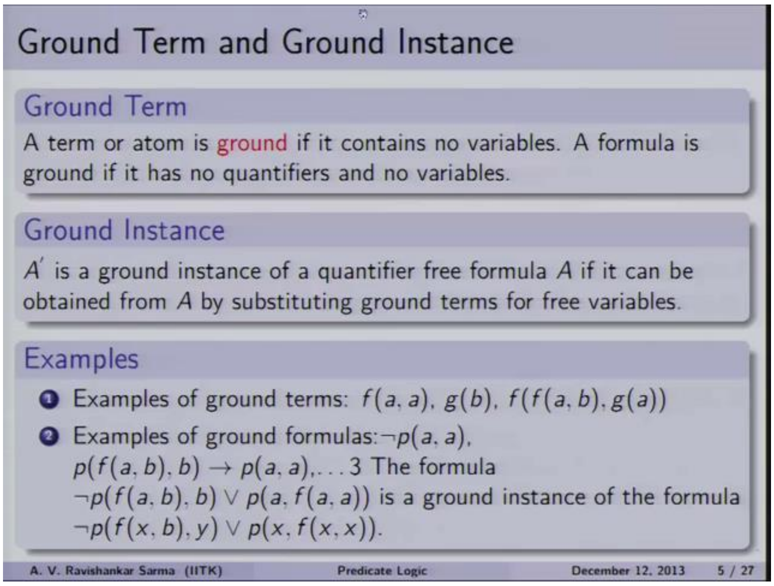
So, we require some of the basic concept such as ground term. So, in the last class we have seen that, in the formation of formation tree for a term we have seen that in that particular kind of formation tree for that thing, we do not have any free variables and that particular kind of term is called as a ground term. So, a ground term is considered to be a term or an atom which is said to be ground if it contains no variables. So, if formula is ground, if it has no quantifies and also no variables.

(Refer Slide Time: 10:11)

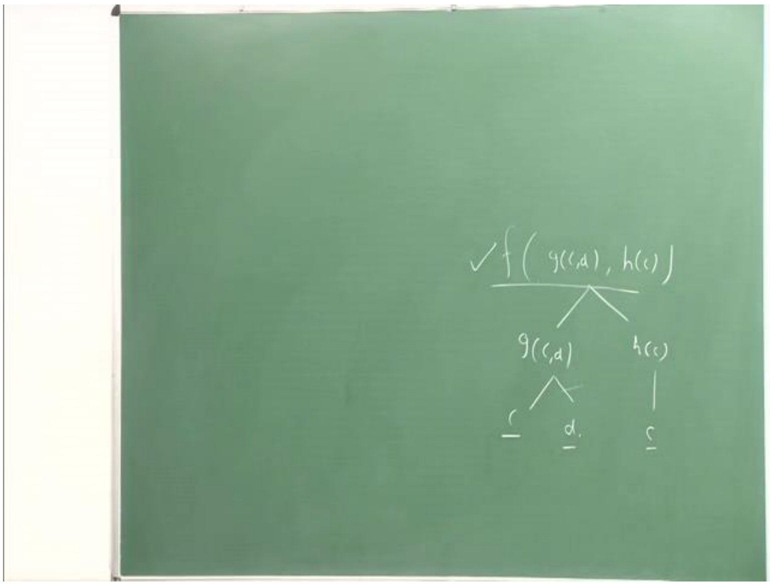


For example, if we say something like f of g of c, d and h c etcetera and all. And if you draw the formation tree for this 1 it is going to be like this g c, e t and h c and then it further reduces to c and d and c. So, now, here all this terms are going to be constants. So, you do not have any variable here. So, that is reason why this term is called as a ground term.

(Refer Slide Time: 09:32)

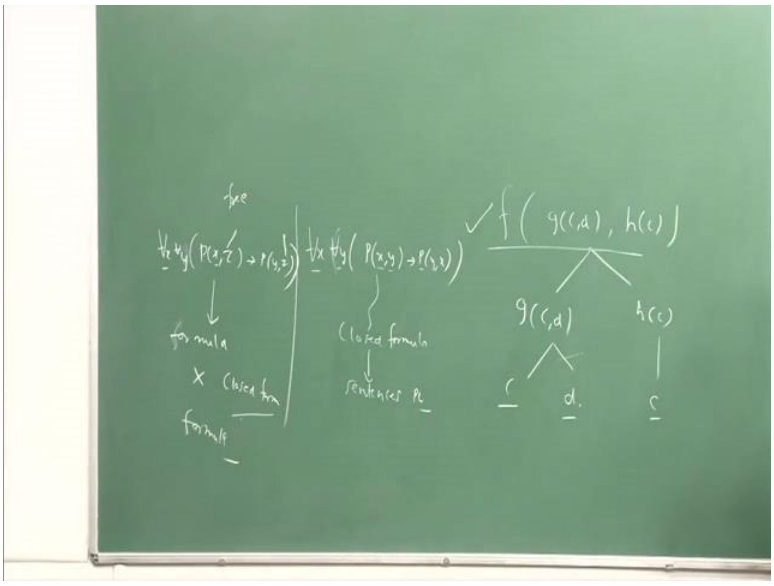


(Refer Slide Time: 10:11)



A ground term is a term which does not consist of free variables and a formula is set to be ground if it has no quantifies as you see here, it does not have any quantifier. And it has even know free variables and all free variables are x, y, z etcetera and all study the thing which is considered to be ground formula or term is said to be ground in that particular sense, which has no free variables it has no variables that is a thing which you need to talk about not free variables a term a formula is said to be closed when it has no free variables.

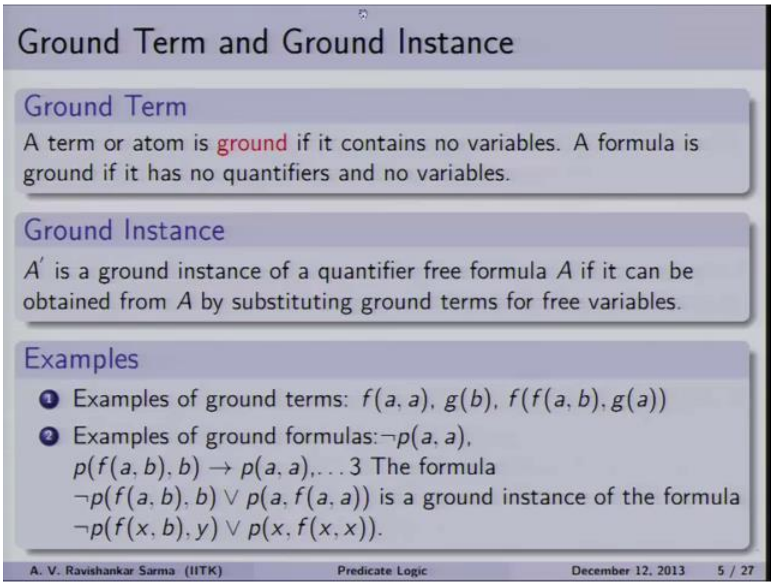
(Refer Slide Time: 11:41)



So, this is this is the difference between close term and the ground term ground term has no free variables no variables at all whereas, closed formula does not have any free variables like for example, in this case this is a formula example if you say for all x, for all y; p, x, y implies p, y, x and all something like this 1. So, all this variables are bounded by this 2 quantifies. So; that means, there is no free variable in this particular kind of formula. So, it is in that sense it is called as say close formula and all the closed formulas predicate logic they are considered to be sentences in the predicate logic and there are some formulas; such as this 1 for all x for all z for example, if you write like this p, x, z for all x and for all y p x z implies p something like y z and all.

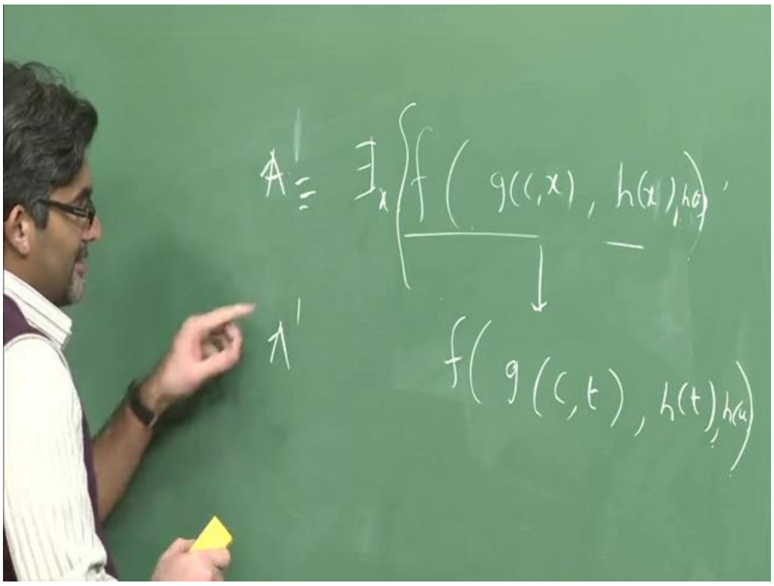
So, now, if you observe this particular kind of formula x is bounded by this particular kind of quantifier whereas, the occurrence of z in both the terms is considered to be free. So, now, it is in this sense it is called as a formula in predicate logic, but it is not considered to be a closed formula. Because it has free variables whenever a predicate logical formula has free variables, then it is considered to be it is it is not considered to be a close formula, it is consider to be a just a formula and all and this is also not considered to be a sentence in predicate logic only close formulas are going to be considered to be as sentences in a predicate logic.

(Refer Slide Time: 09:32)



So, this is one of the important distinction that we need to make out. So, the other thing is what do we mean by saying that something is considered to be a ground instance for examples there are 2 formulas a and a prime and a prime is considered to be ground instance of a quantifier free formula a; if it can be obtained from a by substituting ground terms for some kind of free variable.

(Refer Slide Time: 14:13)

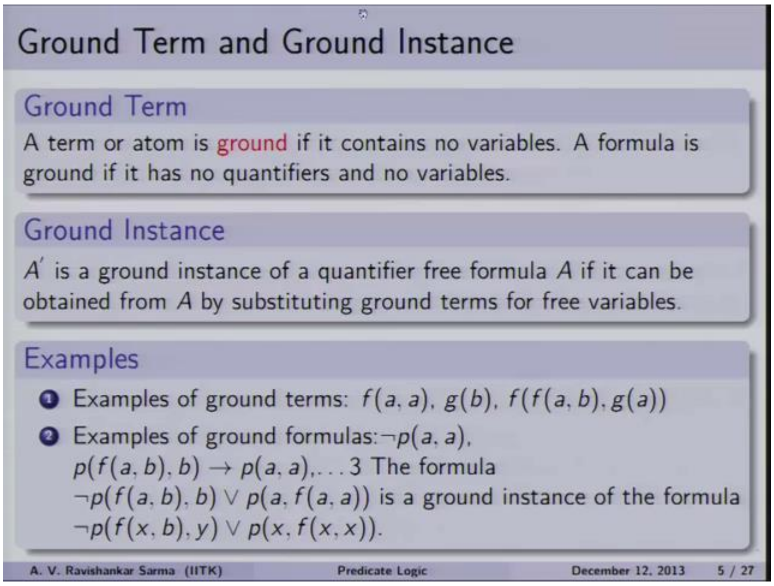


For example: if you have something like this 1 the same formula which you can take into a consideration. Now, imagine that you have some kind of free variables like this, some formula which is there like this and h x and all; for example, if you take this into consideration. Now, 1 of the instance of this 1 is this if you replace this particular kind of variable x with some kind of ground term just t, e, r anything which you can use, then 1 instance of this 1 is like this. So, this is a formula A; let us assume that this is a formula a and 1 instance of this 1 when you remove this existential quantifier, and then substitute x with some kind of ground variable like s t u v whatever it is, then it is considered to be the ground instance of this particular kind of formula.

So, now, this will become let us say you are uniformly replacing x with t; now h t. So, now, this is considered to be a ground instance of this particular kind of a formula. So, since it is properly it is not called as a formula because, x here there is not variable which is it has not variables and all. So, we can introduce another thing called h z or something like that. So, now, h of z you can replace it with something like u or something like that. So, this is 1 of the instances of this particular kind of formula.

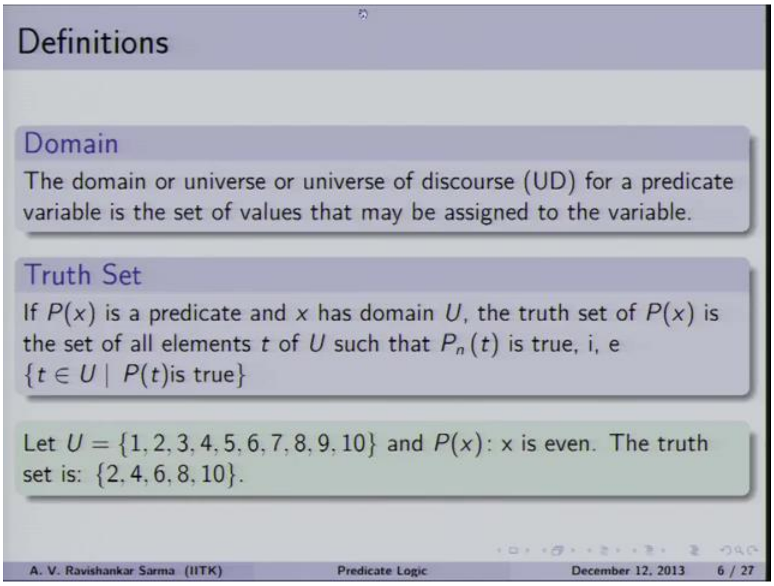
So, it is in that sense A prime is an instance of ground instance of a formula e. So, what happened here is simply, this that the free variable x is replaces by some kind of wrong term. So, when a free variable in a formula is replaces by some kind of ground term then it is called as a ground instance.

(Refer Slide Time: 09:32)



So, now, these are the examples of ground terms f a, a it does not have any free variables. So, that is why it is ground term f a,a; z b, f of f of a,a b z,a all; this things are ground terms examples of ground formulas are like this not of p a implies p f of a, b b implies p a, a and the formula the whatever is there down, it is not of p f of a b, b or p of a f of a etcetera is a ground instance of this particular kind of formula that is So, in not p f of x b y or p x f of x x in that particular kind formula x is replace by a and y is replaced by b. So, it is in that sense it is a ground instance of the particular kind of formula. So, this is considered to be a ground instance of a given formula.

(Refer Slide Time: 17:34)

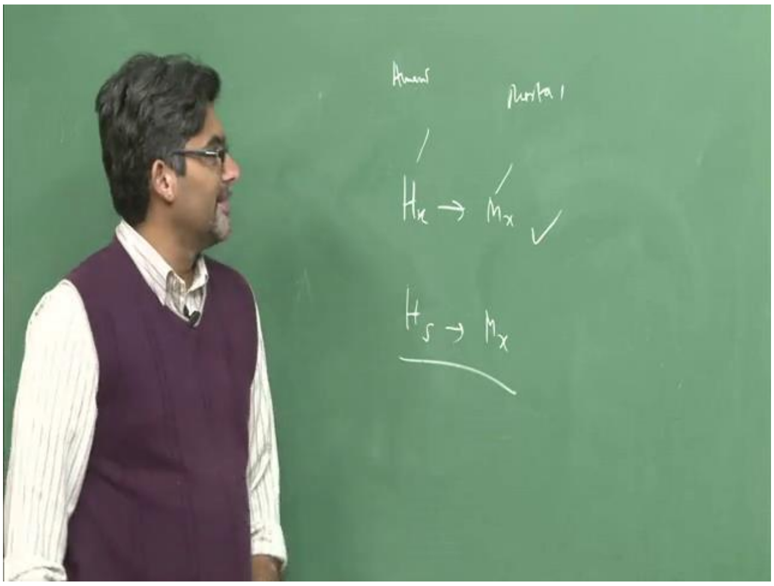


So, the variable is replaced by ground terms then it will become ground instance of a given formula. So, now, these are some of the definitions that we need to use before talking about the meaning of a meaning; that means, truth condition of a given well form formula in the predicate logic, first we need to have a domain. So, domain is usually considered to be a universe or you can also talk about domain the name or universe of discourse sometimes in some text book its writ 10 as universe of discourse etcetera for the predicate variable predicate variable is some set of values that, may be assigned to a given kind of variables. It can be natural numbers a domain can be natural numbers a domain can be set of people a set of animals etcetera or set of a rivers etcetera is all considered to be 1 particular kind of domain.

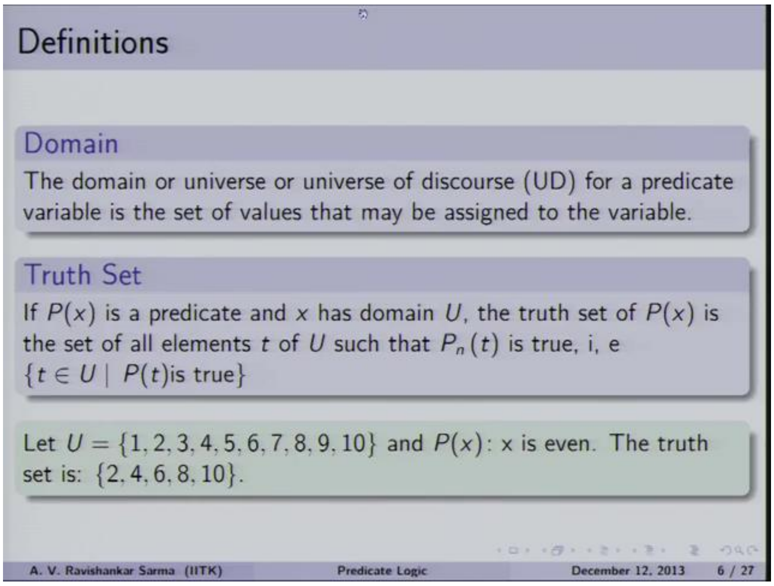
So, x stands for variable which stands for rivers that Ganga, Krishna and all this things come under that particular kind of category. So, this is what we mean by domain it is consider to be an universe of discourse. And the second thing which we need to notice something called truth set for example, if p x is consider to be predicate where, x is an individual entity which has that particular kind of property p like x is mortal etcetera is a predicate and x has this particular kind of domain u. You can be anything it can be natural numbers it can be set of people etcetera and all.

So, then the truth set of p x; that means, we are talking about when this formula p x is going to be true. Example, if you say that all humans are mortal it means all will die at some day or other you represent it as h x or something like that, if x is again human being then x has to be mortal h x implies m x. So, that particular kind of formula when that that is going to be true, when you need to have domain u, the set of people in that context the set the truth set of p x is considered to be set of all the elements of t of u such that, the p and t has to be true; that means, a truth set is considered to be any term which belongs to the universe of discourse u, such that the p it is replaced by a ground term t and that has to be true.

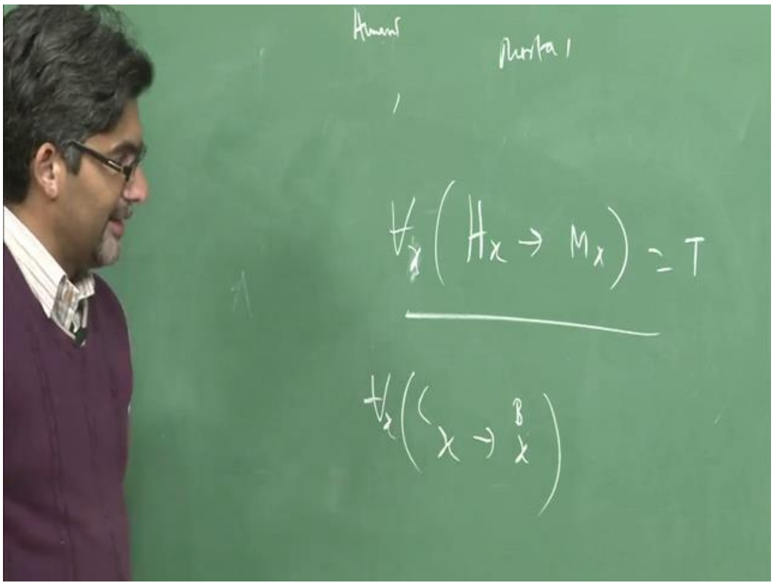
(Refer Slide Time: 20:02)



(Refer Slide Time: 17:34)



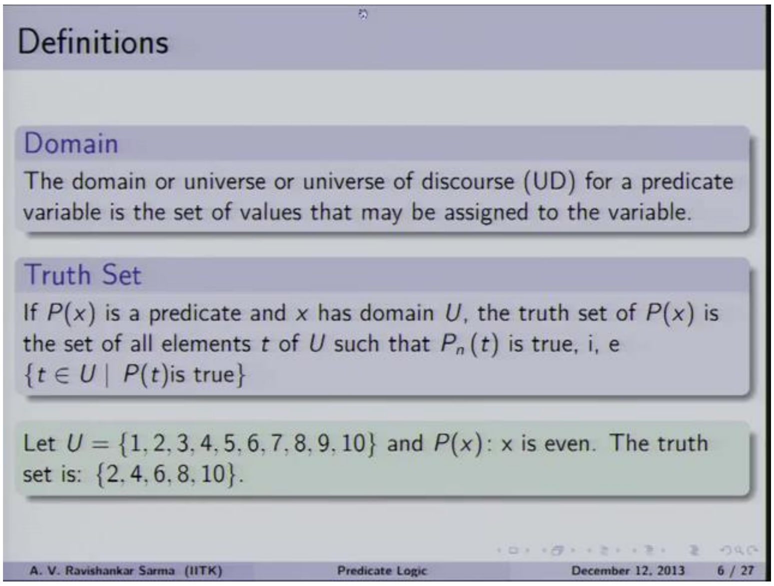
(Refer Slide Time: 20:53)



For example, if you say all human are mortal suppose x is consider to be all human and mx is consider to be H is consider to be humans and m is considered to be mortal. Now, this is going to be true when you have an instance where let us say something called Hs is stands for example, is. So, happen that is human being and then is also mortal in that, case this is going to be true. So, this is this constitutes the truths of particular kind of predicate p x.

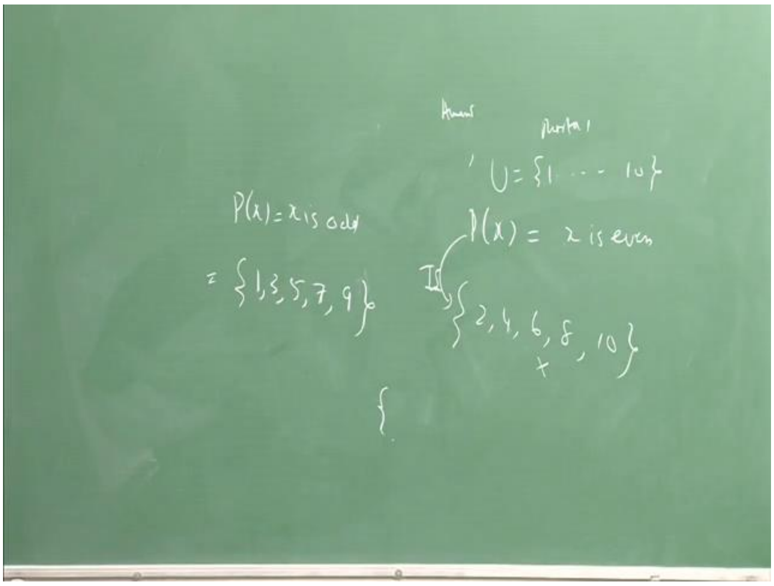
So, when it is replaced by ground term that p t has to be true; it has to be true in all the cases, then we represented it as this thing Hx implies mx for all such kind of substations of x. If this becomes t then we write it in this way for all x, if x is human being then xs has to be mortal in same way all crows are black. So, if x is a crow then x has to be black if it happens for all the crows that you have seen.

(Refer Slide Time: 17:34)



So, far then it will be it should be written in this particular kind of sense. So, this is what we mean by truth set for example: if you say, if you take the universe of discourse as a natural numbers from 1 to 10; 1, 2, 3, 4 to 10. Now, P x is consider to be some kind of property which x, has that is x is consider to be even number.

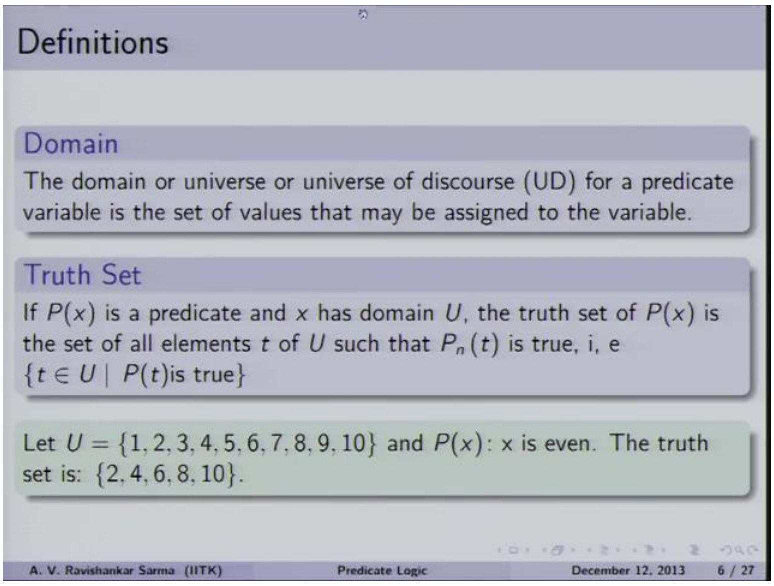
(Refer Slide Time: 21:47)



So, then if you take this particular kind of thing P x is consider to be x is even and then we have, I said such as universe of discourse is 1 to 10. So, now, the truth set; that means, when this P x is going to be true only when, you take this particular kind of numbers. So, when 2 when that particular kind of P x that it satisfies this particular kind of thing x is even then only this is considered to be truth set; 2, 4, 6, 8, 10 all satisfies this particular kind of property that x is going to be even. For example, if it P x is x is odd, then that particular kind of set is going to be this is consider to be the truth set of this 1.

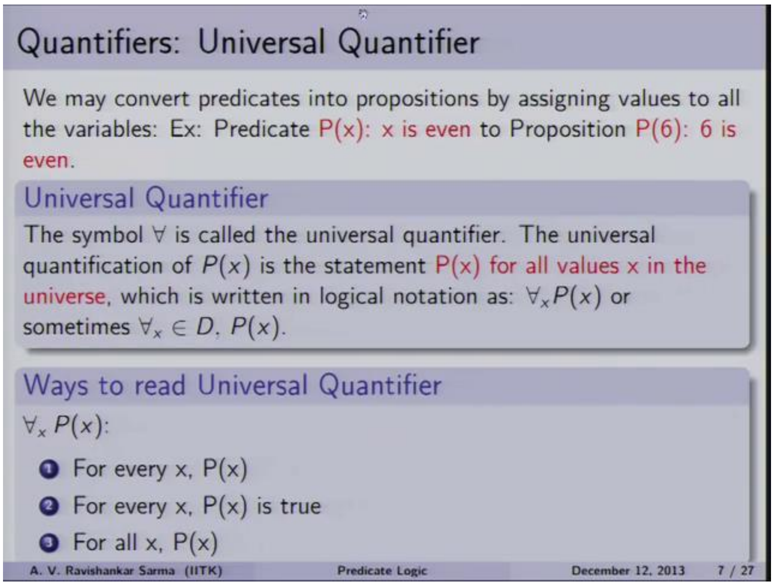
Suppose, if you take the predicate as x is odd, then all this things will come into x 7 and 9 and that is it. Suppose, if you take this particular kind of a set the same set 2, 4, 6, 8, 10 and all and then you are a predicate is that x is odd. The this is not consider to be the truth set with respect to P x is going to be false. In that case because, it is any number that you take into consideration is not even it is not odd all are even. So, that is why that is not consider to be the truth said with respect to P x x is odd.

(Refer Slide Time: 17:34)



So, truth said in the sense that when a given a universe of discourse U and a predicate property which contributed to the some kind of individual x, then under what conditions P x property satisfies and all. So, then based on that you can talk about the truth set in some context it is truth, some other context it is going to be false. If it is called odd numbers, then suppose if you have universe of discourse as all natural numbers till 10 and P x :x is even then this particular kind of set 2, 3, 1, 3, 5, 7, 9 etcetera that is going to be false.

(Refer Slide Time: 24:05)



So, this is what we mean by true set when the predicate is going to be true is a 1 which taken into consideration. Now, let us talk about the quantifies now. So, basically essentially what we are trying to do is, the building blocks of predicate logic or variables constance, functions and predicates. We need to address all this 4 things before talking about the meaning of it given formula. So, now, quantify is we may convert predicates into propositions by assigning values to all the variables; that means, suppose if you have some predicates such as, P x such that x is even we can convert it into some kind of proposition. Suppose if replace it replace x with a ground term then that P x P 6 and 6 is even that will turn out to be a proposition.

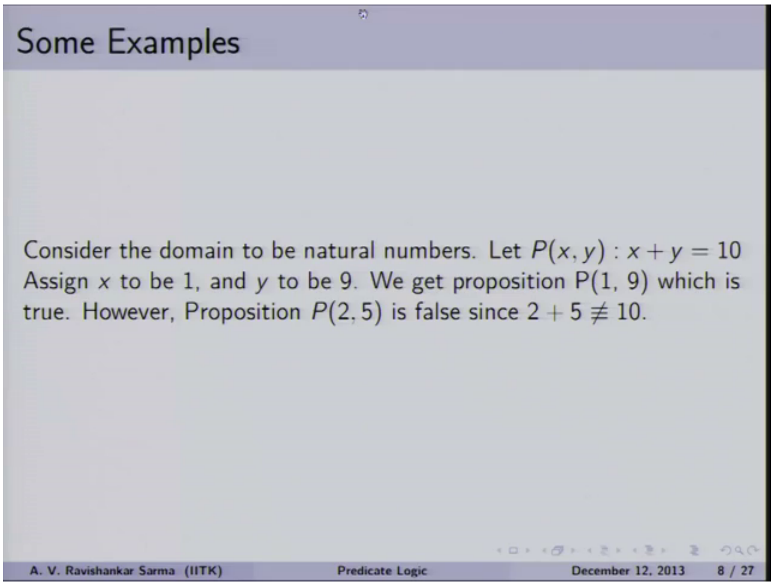
So, all the predicates are reduced to propositions when you replace x with some kind of ground terms and like 6, 7 etcetera and all. So, now, universal quantity it is represented as for all x, universal quantification on P x is consider to be statement, which is writ 10 in this sense it is consider to be a predicate P x and p x holds for all values of x; in that particular kind of universe. Suppose, if you take the universe of discourse of crows, all the crows and then P x is consider to be something like x is black, then that for all x P x has to hold for all the crows that you have taken in the inverse of discourse, even if for 1 particular crow which is stand out to be white and all then P x will not hold.

So, that property P x holds for all values of x then we call it as for all x, P x and It is represent as universe quantifier. So, which is writ 10 logical notation as for all x; P x r sometimes in some other text books, it is written as for all x were x belongs to some kind of universe of discourse D. So, that p x; that means, P x holds for all x. So, different ways of reading this universal quantifier that is same thing it stands for the same thing for all x, p x sometimes you can say that for enery x, P x for every x, P x is consider to be true; that means, P x satisfies or the other way of saying is for all x some P x holds and all.

So, there are some terms such as n e term n e phrase n e, sometimes it act like universal quantifier sometimes it acts like a existential quantifier. So, depending upon the thing 1 may use it as universal or 1 might use it as existential quantifier is consist some example and consider the domain to be natural numbers. Natural numbers are 1, 2 all positive kind of numbers 1, 2 infinity. And if you add 0 to it will become whole number and then if you add all minus 1 to minus infinity that will become integers. And then if we add all the rational numbers to that particular kind of thing all the fractional numbers including minus etcetera and all.

So, then it will become q rational numbers and if you have real numbers, rational numbers, natural numbers and integers etcetera and all. And that will constitute real numbers, and then if you if something is called as complex number which is different from the real numbers. So, that is a different kind of domain and all other than, real numbers. Real numbers has all this things whole numbers, natural numbers, integers rational numbers etcetera.

(Refer Slide Time: 27:02)

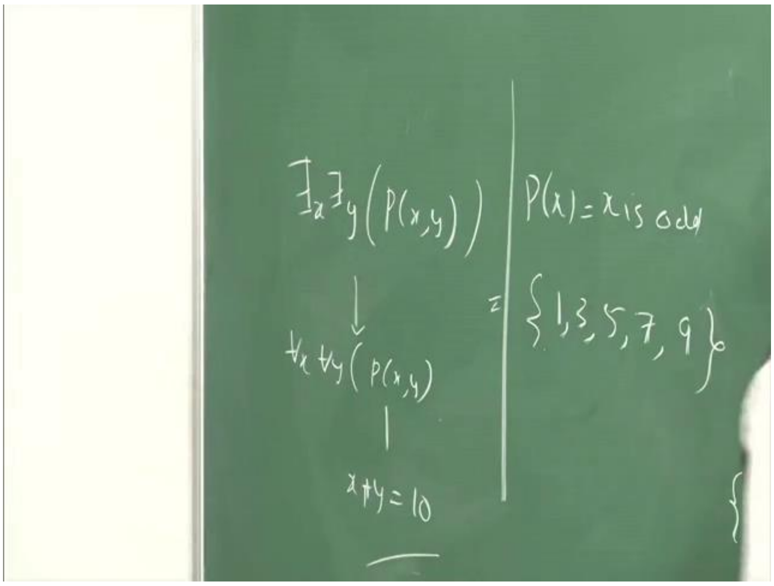


Consider a domain to be natural number now consider a predicate P x, y: x and y are related in this sense, in this way. Were P x, y is represented in this things if x and y are added to each other and it will you will get a value 10. Now, assign value x to be 1 and y to be 9 and you have taken 1 and 9 from this domain, set of natural numbers 1 and 9 are consider to be natural numbers only. If you add 1 and 9 it satisfies this particular kind of property P x plus y, x plus y is equal to 10 so; that means, P 1, 9 satisfies this particular kind of thing that is consider to be true.

So, now, if you take another proposition in another in thing into consideration another values 2 and 5 adds to 7 only is not equal into 10; that means, P 2,5 does not satisfy this particular kind of formula that is; that means, if you take the values 2 and 5. P x y: x plus y is equal to 10 is not going to be satisfied in that sense it is false. So, if you take 5 plus 5 then of course, that is going to be true. So, now, if we change the domain to be negative integers also then for example, if we take x as minus 1, and y as 9.

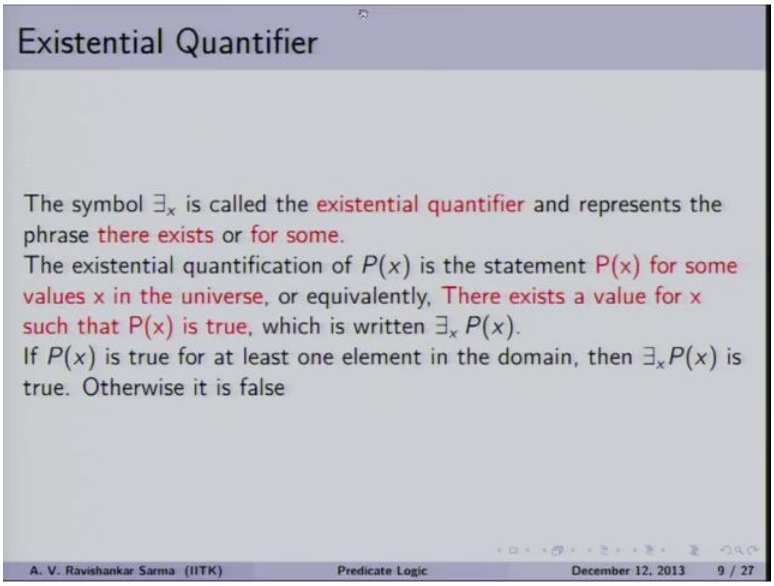
In some cases it might be either case that suppose if you take natural numbers in some cases this holds and all, but this is not going to hold for all the all x and all. So, whatever x that you are going to take in consideration and whatever y that you are going to take into consideration, P x y that is x plus y does not add up to 10.

(Refer Slide Time: 29:55)



So, it is in that senses you write it write this particular kind of formula as this thing since it holds for only some kind of properties. So, P x and y if it holds for all the properties and all which is not the case in this 1, then you can write for all x and for all y, P x ,y. Where P x ,y is defined as x plus y is 10 it holds only if it holds for at least 1 particular kind of values of x and y, then you write it in this sense the exist x that exist some y P x, y; otherwise if you write it as for all x and y for all y; P x, y that is going to be false.

(Refer Slide Time: 30:37)

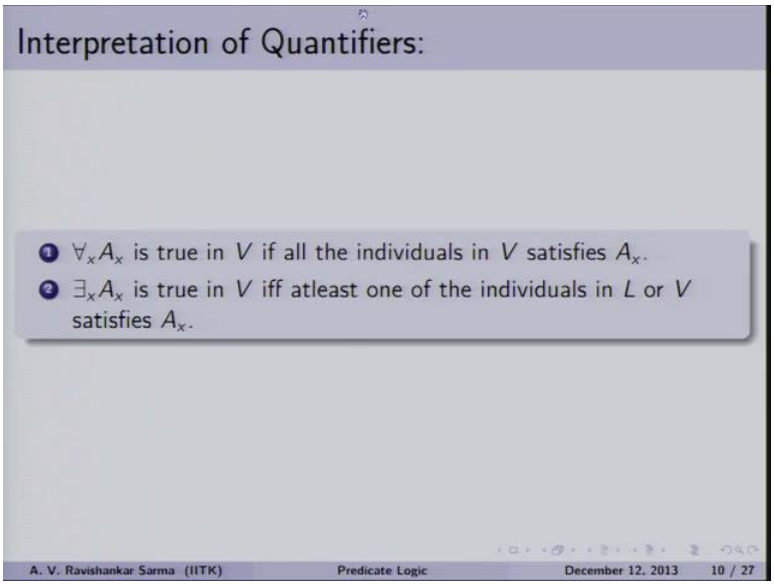


So, let us talk about the existential quantify is represented as that exist some x usually if you write it in the sense existential quantifier is usually consider as a disjunction whereas, universal quantifier is consider to be conjunction of all the formulas and all; that means, at least 1 particular kind of thing is false, the entire conjunction is going to be false. So, in the case of existential quantifier even if 1 disjunct is false if at least 1 con 1 disjunct is true, then it is enough for us to say that that particular kind of formula is true.

So, the symbol it is represented by symbol there is some x sometimes it you represent it as for some x etcetera. Existential quantification P x is consider to be a statement which needs to be read like this, a particular kind of property P x which holds for some values of x in the universe; that means, some values of x means, if there is at least 1 x which satisfies this particular kind of property, then that will serve our purpose are equilently. You can also say that there exist a value for x such that, that particular kind of P x is going to true.

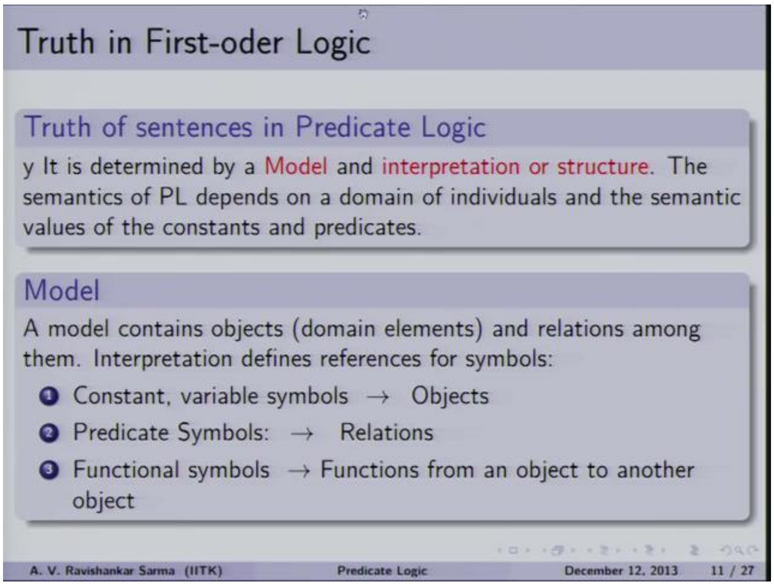
So, in the last example P P x, y e where x plus y is equal to 10 that is going to hold at least for some values of x and y. There exist some x there exist some y, P x y that is going to be true, but the same formula may not be true for all the values of x and for all values of y for example, if you take x as 7, and then y as 5 then 7 plus 5 is equal to 12 which is not equalent to 10 which does not satisfy that, P x, y is equal to x plus y. So, that formula can only be written as there exist x there exist some y; P x, y : x plus y is equal to 10. If P x is consider to be true for at least 1 element in the domain, then there exist some x P x is going to be true otherwise it is going to be false. In the case of for all x, it has to be true for all the elements of domain otherwise it is going to be false. So, that is a difference between existential and the universal quantifiers.

(Refer Slide Time: 32:56)



So, now this is a way to interoperate the quantifies for all x Ax that is going to be true in in V. If the domain some kind of domain if all the individuals in V satisfies Ax that particular kind of property Ax in the same way there exist some x Ax going to be true in a domain D r V if and only if at least 1 particular kind of A1 of the individuals in A l or v satisfy your property that Ax is the case, A x hold for some at least 1 value then you call it as there exist some x Ax is true.

(Refer Slide Time: 33:39)



So, now, in a very informal way we discussed about the truth values of a quantifier etcetera and all or a given formula and we just indicated that same formula is going to be true with a respect to can be interpreted in different ways; that means, same formula sometimes it can be true in some domain. Like if it takes only natural it might be true or in some other cases if you take the real numbers into consideration; that means, all the whole numbers etcetera and all then the same formula might be false.

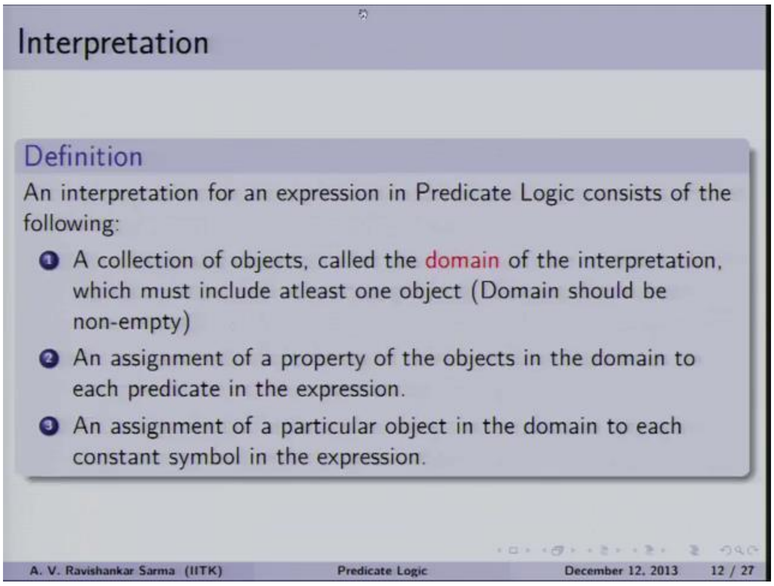
So, how to formally talk about how we can formally express a truth of given formula in the first order logic. First order logic is also called as predicate logic a quantificational logic and all; were variables are ranging over individual sentences which are there in the domain it is not we will not mean by predicates and functional symbols etcetera and all. Variables are not ranging over predicate functional symbols etcetera and all. If you talk about those things you are talking about second order logic. So, truth of sentences in predicate logic it is determined by something called as model. We use this words interchange you will and all model structure interpretation this 3 terms of another.

So, some formula is going to be true with respect to a model in the same way we discussed the propositional logic we discussed about a given formula with respect to a model. So, in the same way we can talk about the truth value of a given formula with respect to a modular structure. So, we need to define what you mean by model interpretation structure now, the semantic of predicate logic depends up on 2 important things are important 2 important things which we need to note.

So, they are first id domain and second 1 is interpretation function i it depends upon the domains of individuals and semantics value of the Constance, predicates, variables etcetera that is going to take. So, now a model consist of; obviously, the objection the domain and the relationship between these objects within the domain and interpretation function. So, first of all what constitute a domain a domain constitute of the objects which are there in the domain for examples set of people of all inanimate things etcetera and all for example, those who does not have life etcetera; chock piece, duster, tables chairs etcetera and all are set of trees for example, it is constitutes some kind of set a plants for example, all the trees etcetera and all come under that kind of category.

So, a model consists of an objects and relation among them and then we have a interpretation function which defines references for this particular kind of symbols. So, what are the 3 things which are there in the predicate logic Constance, predicate, symbols and functional symbols and variables? So, now, Constance and variables symbols should find out, some kind of object in the domain and predicate symbols have some kind of relations in the domain D and the functional symbols have some kind of corresponding thing in the domain that is functions from an object to another kind of object.

(Refer Slide Time: 37:31)

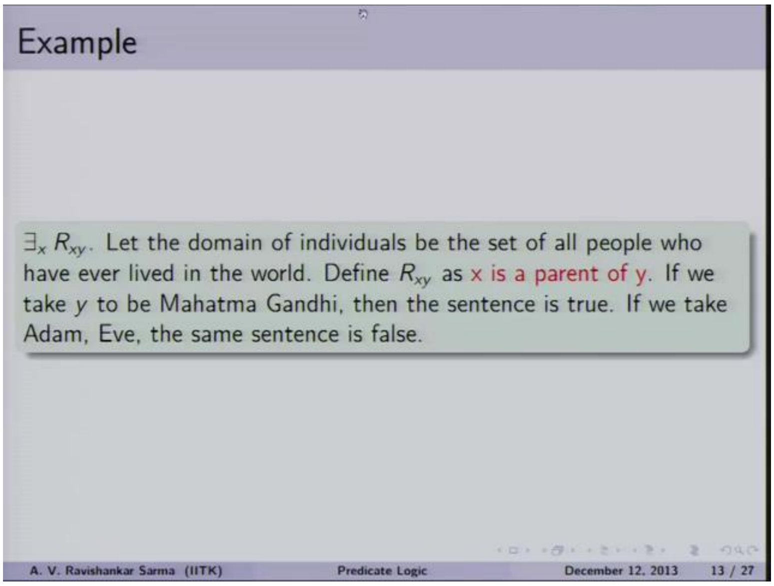


So, it takes these values and all now the definition of interpretation is like this. So, interpretation means giving assigning some kind of truth vales to a given formula in the case of propositional logic. So, it is not as simple as in the case of propositional logic, in the predicate logic when you say that, interpretation need to take into consideration what values that we are variables etcetera constant and functional, symbols are going to take that is also we need to take into consideration and interpretation for an expression in a predicate logic consist of following things. First start with we need to have a domain of the interpretation which must include at least 1 particular kind of object sometimes a domain can also be empty.

So, in the empty domain suppose if properties such as P x for all x, P x is going to be true is going to be vacuously true whereas, if you talk about there exist some x P x with respect to empty domain that is going to be false we've going to see in a while from now, the difference between this things. So, in general if you talk about domain it is usually taken into a consideration that the domain is non-empty. You do not talk about domain such as, set of a suppose if you are talking about particular kind of formula that all men are mortals. So, called is man. So, called is mortal and we do not we do mean, by saying that at least some kind of objects exist in the domain; that means, we need to take into consideration some kind of domain which consist of some people at least.

So, if you do not talk about any kind of people you know if you talk about animals etcetera and all, that does not makes any sense to talk about particular kind of thing all the formulas are going to be vacuously true; that means, all the universal formulas which are expressed by universal quantifies are; obviously, going to be vacuously true. So, now, usually domain is considered to be usually non-empty at least 1 or some object needs to be there in the domain, and then an assignment of property of the objects in the domain to each predicate in the expression and you need to have an assignment of a particular object in the domain to each constant symbol in that particular kind of expression and that constitute what we call it as interpretation for example, if you say that there is a formula such as there exist some x r x, y.

(Refer Slide Time: 40:02)



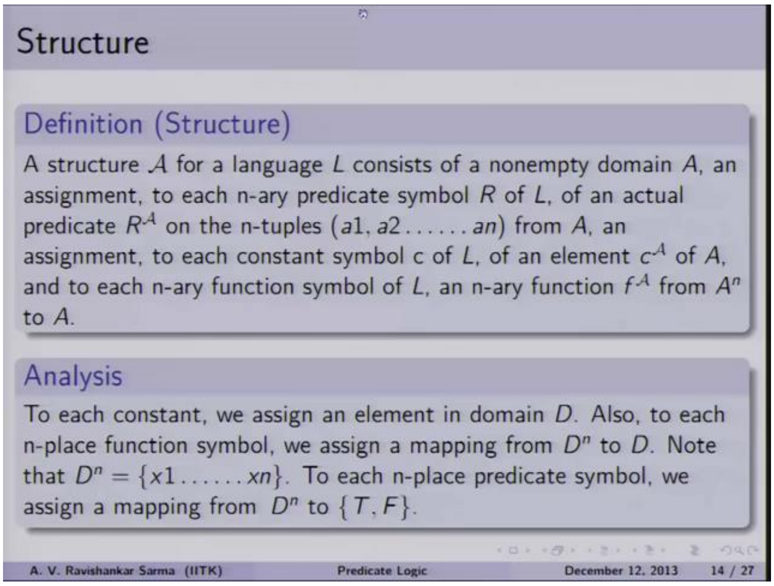
Now, the same formula in some domain is going to be true some other domain it might be false. Let us consider a domain of individuals to be sat of all people and then we are trying to evaluate this formula there exist some x Rxy we need to talk about what we mean by Rxy also the set of all peoples who have who have ever lived in this world whose ever is not lived in this world does not makes any sense to talk about this particular kind of formula. And then we are also taking into consideration Rxy, then is a relation between x and y it is like this x is consider to be a parent of y.

So, now, if we take y to be mahatma gandhi then we usually we call him as father of the nation etcetera and all father of every 1. So, x is a parent of y in that sense there exist of course, we are not talking about for all x Rxy we are just talking about there exist some x such that, x can be Ravi or something like that x can be Mahatma Gandhi and x Rxy stands for x is a parent of that particular kind of y, y can be treated as Mahatma Gandhi what it essentially says is that every person who is who existed in this world have at least father and all.

So, x is considered to be parent of y in that sense there exist some x or x, y is going to be true. So, now, if you take y to be Mahatma Gandhi then the sentence; obviously, going to be true and anything which you put it for y every 1 has a parent. So, that why there exist some x Rxy is; obviously, going to be true. Suppose if we take for the sake of fun we can take into consideration r m e u x etcetera and all. We do not know, whether parent etcetera and all.

So, the same formula there exist some x Rxy in that particular kind of domain in where you have these objects a Da m eve etcetera and all that, sentence may probably be false. So, what I am essentially trying to say is that the same formulas have different interpretations. So, in some depending upon the domain and the interpretation. So, now, let us formally define what we mean by structure or interpretation or model etcetera all.

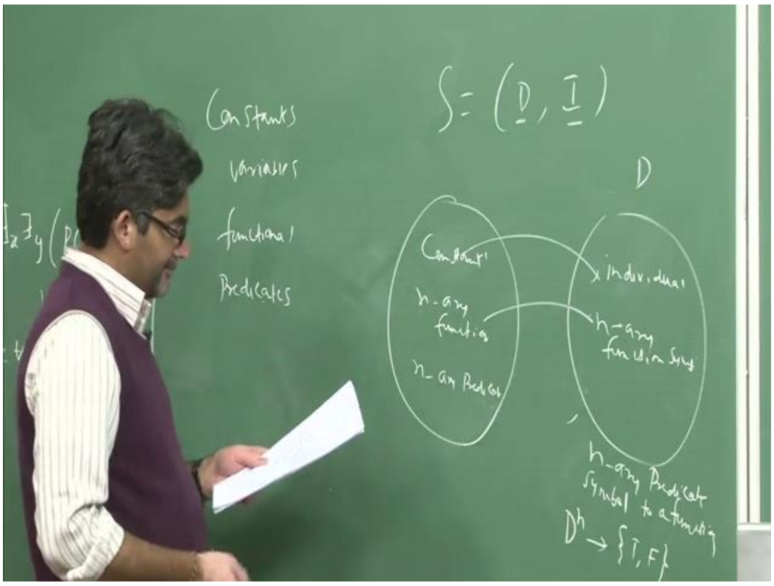
(Refer Slide Time: 42:28)



So, these things terms are 1 and the same this is somewhat technically little bit complicated kind of definition. This definition is usually taken is taken from task keys work, talks keys come up with particular kind of definition which is been changed into or concern and this definition is like this. A structure a or model which consist essentially consist of domain and set of interpretation function.

A structure a for a language l that is a language of predicate logic consist of nonempty domain; that means, domain has to be a nonempty at least 1 particular kind of object should exist in the domain and an assignment that is interpretation function which assigns to each every predicate symbol r of l that kind of predicate logic of an actual predicate or A on the n these are the terms a 1 A 2 to a n from a and this going to be an assignment to each constance symbols c of l to an element a of that particular kind of a domain a and to each every function symbols l there is an every function f a from d to the power of n that is a domain to power of n 2 t. So, what essentially we are trying to say here is like this. So, we have this particular kind of that you know domain this structure consist of domain and interpretation function.

(Refer Slide Time: 44:15)

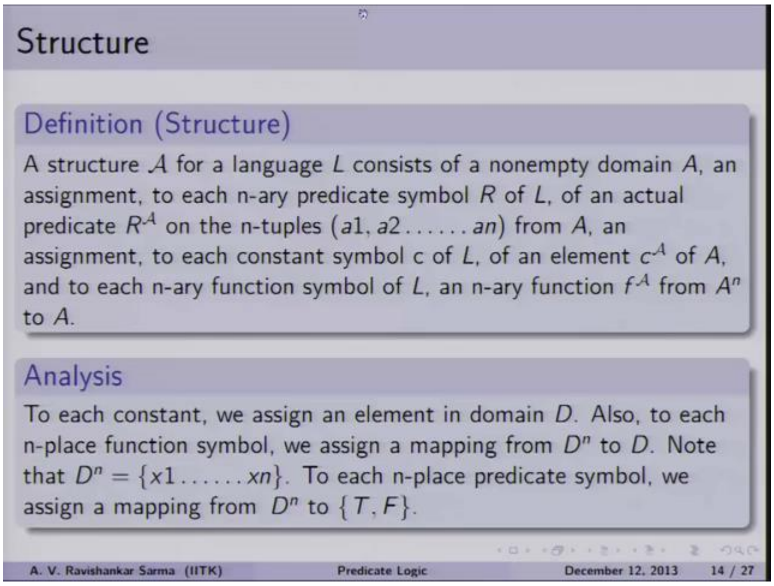


So, this domain has to be non-empty; that means, at least some set of a some object needs to be there in domain it can also be we can also take into that domain is empty, but in general we take non-empty domain and all and interpretation function is represented as I. So, now, So, what we are essentially saying is we have constant, we have variables, we have functional symbols and we have predicates. So, now, this predicates have to be map to something in that domain which has to be either true or false.

So, say in the case of that we have seen P x; were x is even that particular kind of P x has to be true is going to be true when you take all the even numbers and all and if you take all the odd numbers p x is going to be false. So, that has to be map to do something such as t and f. So, now, you have constant this is domain enery functional symbols, we have seen what we mean by this constant enery functional symbols and enery predicates. And each 1 is map to some kind of individual in the domain; that means, we are assigning some kind of values to this 1 constant enery functions and enery predicates.

So, in enery functional symbols for each enery function that exist in the domain you have corresponding enery functional symbol in the domain and each enery predicate symbols, you have enery predicate symbol, symbol to function that is, D n2 it maps to some kind of there are only 2 entities here as it has to be true or it has to be false, P x is false or p x is true that particular kind of thing and all. Usually the interpretation over domain is considered to be a assignment of entities of d to each of the constant variables predicates, functional, symbols.

(Refer Slide Time: 42:28)

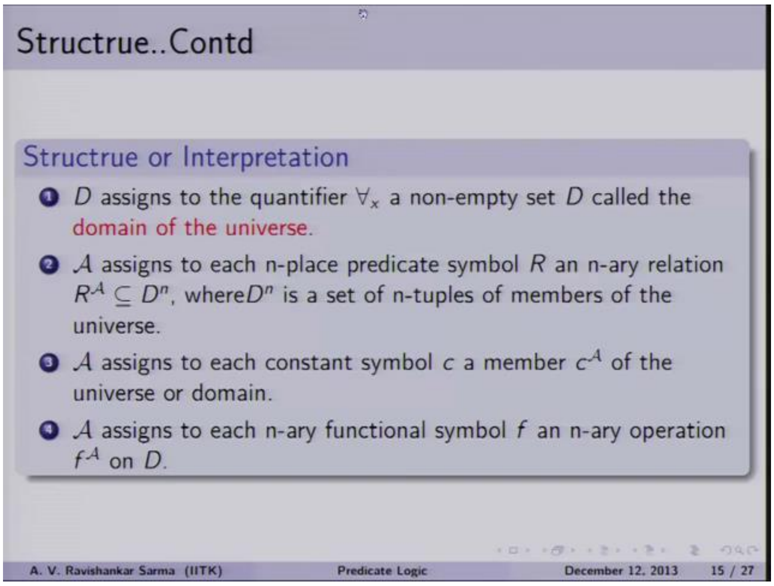


They predicate calculus expression such that here what we are trying to do is, each constant is assigned to some kind of element in the domain d. That is we are basically assigning some kind of entities to constants variables functional symbol and the predicate symbols; that means, we are assigning this some kind of list to this things. So, now, each variable x, y, z etcetera is assigned to a non-empty set of domain, where this are the allowable substitutions for that particular kind of variable for example, x, y, z it can be substitute by or Ravi, Raju, Rajesh etcetera and all. And they all should exist in the domain and all that particular kind of domain.

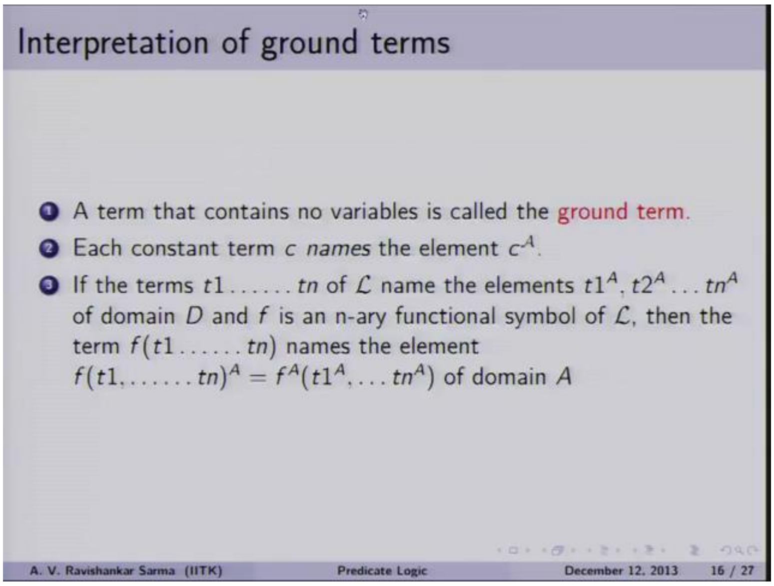
So, now, each function f of m enery function is defined on m assignments of the and defines some kind of mapping from Dn to d power of n to D that is m stands for the number of arguments it of m maps to D. So, we have 0 arguments it will be d of 0 to D. So, each predicate p of L t n is defined as a argument from D and defines a mapping from D n to some kind of set of values D and f. So, now, what we are essentially doing here is like this to each constant we assign some kind of element in the domain D and also to e which n plus functional symbol we assign a mapping from g to the power of n were n is consider to be number of arguments n depends upon the n plus functional symbol and where d n is consider to be 1 into n etcetera d1, d2, d3, dn to each n plus predicates symbol.

So, we assign a mapping from a d power of n 2 some kind of value 0 and 1. So, when is the predicate going to take some kind of value either 0 or 1? So, this is now what we have done so far. D assigns it is a quantifier for all x in non-empty set d which is called as a domain of the universe first thing and structure a assign to e which implies predicate symbol or a enery relation or a is the subset of dn where d n is consider to be of members of the universe and a assigns to each constant symbol Ca member of C is power of a of the universe or the domain and a assigns to each enery function symbol which you have been discussing for ennery operations f to the power of a on d this is much more formal way of saying the same thing.

(Refer Slide Time: 49:25)

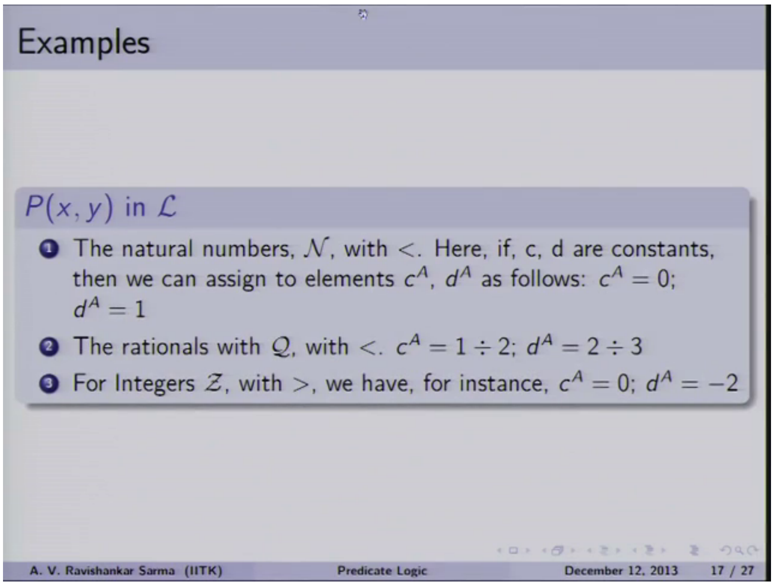


(Refer Slide Time: 50:14)



So, essentially what we are trying to do is we need to have a domain and we need to have a some kind of interpretation function, which interpretation mapping or something like that which assigns some kind of values to variables constant predicates and functional symbols. So, in this let us talk about interpretation of ground terms a term that contains no free no variables consider to be a ground term and each constant term is a names that particular kind of a element as C to the power of a is consider to be structure a domain, if the terms t1 to t n of l name the element such as t1 to the power of a t to the power of a t to the power of n a is a domain D and f is ennery functional symbol l, then the term f of f of t1 to tn that is also consider to be term which names the element f of t 1 to tn to the power of a as f to the power of a t1 to t n to the power of a of a domain.

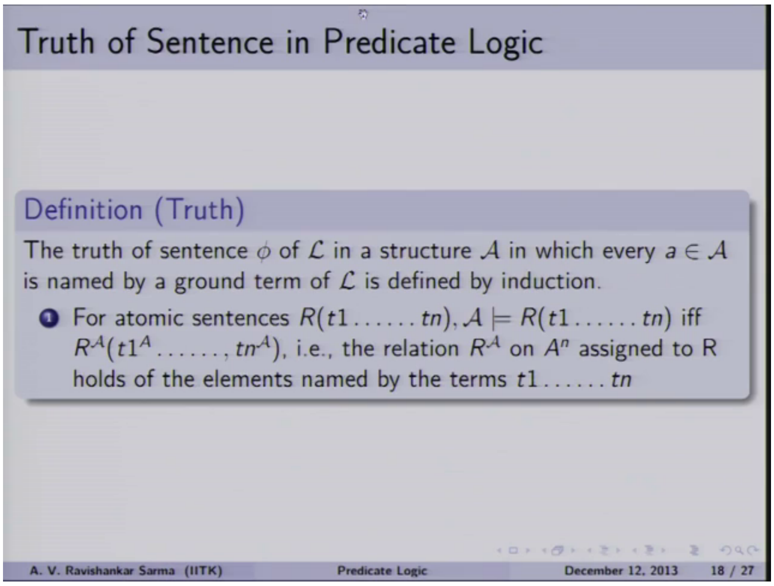
(Refer Slide Time: 59:09)



So, now, just 1 let us consider 1 particular kind of a example and then we will close this lecture, then we will talk about some more examples little bit later. So, N j o w let us consider predicate P x y in a language L. Now, take the domain to be natural numbers n and we have a functional symbol that is less than here C and d are consider to be constants. For example, then we can assign to elements C to the power of a D to the power of a as follows; C to the power of a is consider to be 0 and D to the power of a is consider to be 1.

Now, if we take the rational number with the q which is again presented as less than that context were the constant are represented in this sense 1 divided by 2 d to power of a stands for 2 by 3 etcetera. And if you take the integers into consideration with the relation of functional symbol greater than we have constant represented as C to the power of a is minus 2. So, in this lecture what we have done is that we just talked about what we mean by a structure or a model and we have said that depending upon the model structure a given predicate logical expression will find its meaning we find its meaning in with respect to a model.

(Refer Slide Time: 52:14)



The same kind of formula can be true in some structures same kind of formula can be true false in same kind of structure. So, what matter to us is the most is the domain that you are trying to take into consideration same formula can be true with natural numbers, but it can be false with respect to integers with respect to some other kinds of things. So, in this lecture we define what we mean by giving interpretation or a structure or model a given predicate logical expression. So, in the next class we will be considering some more examples, and then we will be talking about some of the important decision procedure methods in the predicate logic and to start with we use semantic method because, which occupies the central position in our course.

So, we will be talking essentially about the semantic method and in that context will be talking about different logical properties such as when a given formula is valid, and when a particular formula is considered to be a consistence satisfiable and all this things which we will be talking about in greater detail in the next few lectures.