Introduction to Logic
Prof. A. V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 41
Semantic Tableaux Method: Satisfiability, Validity

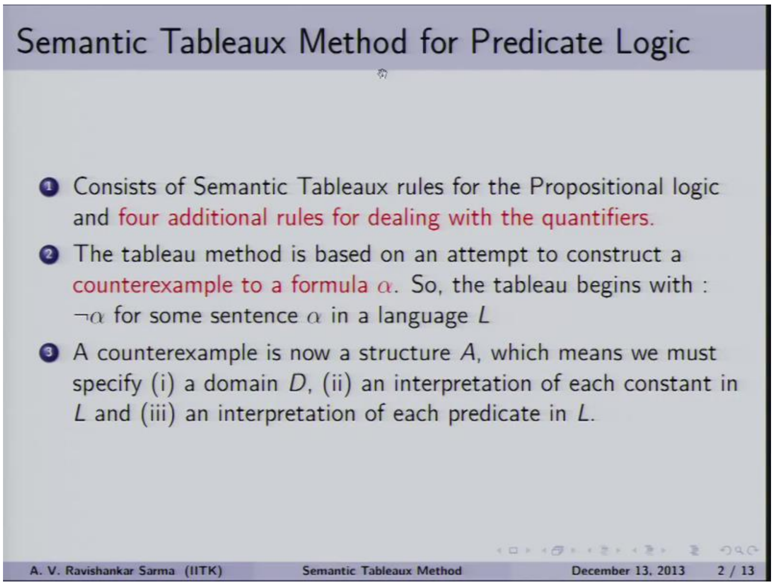
Welcome back, so far we are discussed about syntax of predicate logic. And partly, also discussed semantics of predicate logics, were we discuss that a given well form formula. It is true with respect to domain and it depends upon the domain; that you have taken into consideration. That is, the same kind of well form formula can be true with respect to, suppose, let us say you take the natural numbers, it might be true.

If you take the other numbers into considerations integers, etcetera same formula can be false as well. So, you can talk about truth value of a given well form formula, only with respect to a model. A model consists of a domain and an interpretation function I. So, what we will be talking today is one important decision procedure method, which we already discussed in the case of propositional logic. So, that is the Semantic Tableaux Method.

So, this method is due to first originated in the works of Indica. And then later it was reformulated by a Raymond’s Smullyan, etcetera. So, these are the people, who are responsible for this particular kind of method. And using this method as in the case of propositional logic, we can find out whether a given formula is a valid formula. That means, all the tautologies are valid formulas, just as in the case of propositional logic.

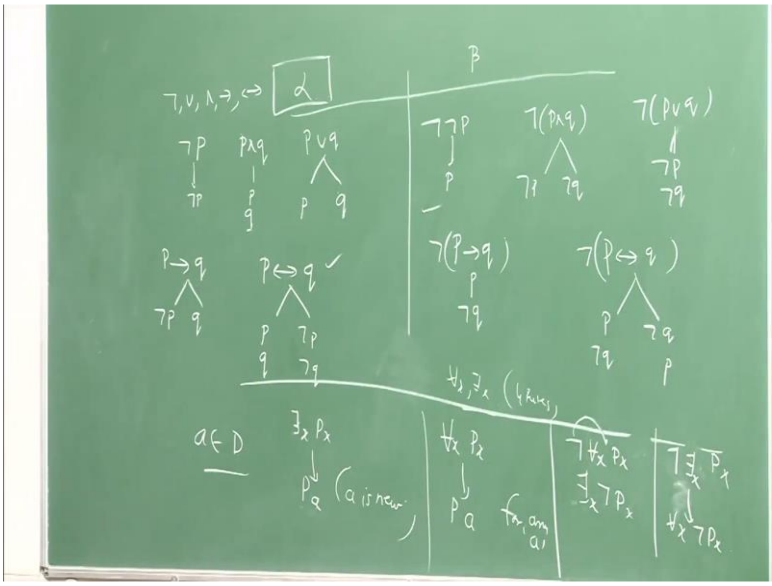
And you can also talk about, when two sentences in a predicate logic are consistent to each other or when a given formula satisfies within a domain. So, we will be talking about this particular kind of method in some greater detail with respect to the predicate logic. And then we will consider some examples. So, that, we can get familiarize ourselves with this particular kind of method.

(Refer Slide Time: 02:19)



Semantic tableaux method, it consists of some kind of semantic tableaux rules for the propositional logic. And in addition to those a rules for the propositional logic, we have four additional rules for dealing with the quantifies. What is extra in predicate logic is simply, the quantifies. So, we have all the connectives and negation, implies, if and only if etcetera. Plus in addition to that in our language of predicate logic, we have quantifies. That is for all x and there exist some x.

(Refer Slide Time: 03:01)



In the case of propositional logic, we know that, these are the rules that, we used in case of propositional logic. In the case of semantic tableaux for the predicate logic, you have all the rules; that are already there in the case of propositional logic. Plus in addition to that, we have some rules for quantifies. So, there are four such rules. So, I will be discussing these four rules in greater detail in avail from now.

But, before that, so there are something called alpha and beta rules in the context of propositional logic. So, we have these connectives, negation r and implies and if and only if. Now, suppose, if we have the formula like this thing not P and you simply write it as not P only, and then P and q. So, this is written as P and q. So, we are constructing a tree diagram for this particular kind of formula. So, this looks like a trunk of a tree.

So, whereas P or q branches out, it will be like the branches of the tree. And only thing, which you need to observe here is this that. It is an upside down kind of a tree. A tree will be like this, trunk will be like this, and then branches will be there. But, for our convenience, we are taking the up side down kind of tree. So, now, P or q, the tree structure for this one is P q. And then we have other kind of connectives implication.

So, we have not P or q. So, this is the structure for this P implies q and whatever is left is, P if and only if q. So, it is like this, both P q are true and not P or q are true. So, these rules are based on the semantics of this propositional logic. We know that, a conjunction is a going be a formula with conjunction is going to be true, when both the conjunct are true. That is why; it is sitting at the trunk of the tree. If any one of this thing is false; that is going to be false.

So, these are considered to be alpha rules. So, now, beta rules are with respect to the negation of all these things. Suppose, if you come across negation of negation of P; you simply substituted with P, and then negation of P and q. So, negation of conjunction is a distinction. So, it is not P not q, and then negation of P or q. So, it is negation of distinction is a conjunction. So, it will be not P, not q.

And then there are three other things, which are left, a two things other things, which are left not of P implies q. So, this is simply p and not q, and then P if and only if q negation of that. So, this is going to be P and not q and not q and P or you can ever write it as not P q and not P q not q P does not matter, what way, you write it and all. So, these are the rules for the propositional logic.

So, now, we needs some more rules to deal with the quantifies. That means, the formulas are begin with the quantifies. We need to have a few more rules ((Refer Time: 06:37)); that I will be talking about in a while from now. So, the tableaux method is based on an attempt to construct a counter example to a given formula. The main idea of behind this method is this that, given a well form formula, instead of checking for it is, whether or not, it is true and all.

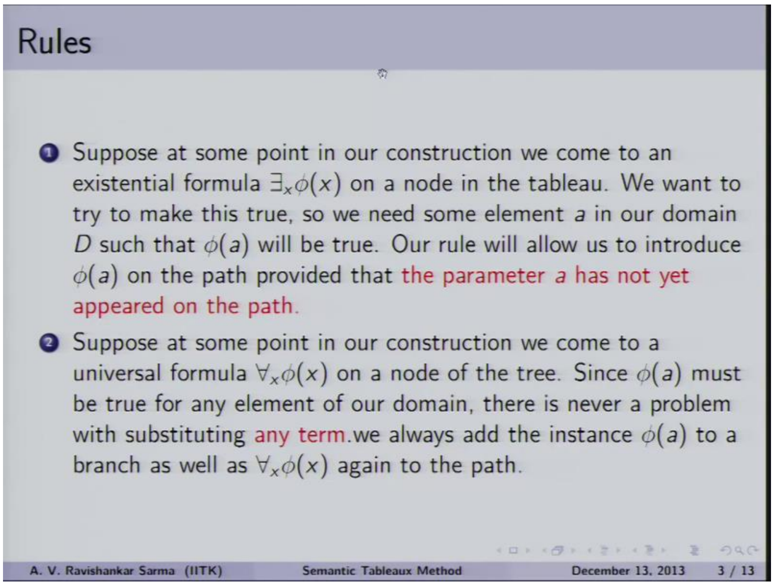
What you look for is, You look for a counter example, were that formula is going to be false. So, if at least one instance the formula is going to be false at least. That means, you are come up with a counter example. So, in the beginning of this course, we discuss that, an argument is considered to be invalid. If you have an especially, when, it is a possible for the premises to be true and the conclusion is false, if it is impossible for the premises to be true and the conclusion to be a false. Then, that is called as a valid argument. This is considered to be a valid argument. So, in the semantic tableaux method, the main idea is this that, you try to look for a counter example, because that is going to make it invalid. So, the tableau begins with the formula not alpha.

Suppose, if you are given a formula alpha, which expresses some kind of formula in the predicate logic. And you start with the negation of that formula. And then you start constructing the tree, based on the rules, which I am going to discuss in a while form now. So, the tableau begins with not alpha for some sentence alpha in a language L. So, now, a counter example is now is consider to be another kind of structure A, which means, we must specify again the domain D.

And interpretation of each constant in particular kind of language L and interpretation of each predicate in that particular kind of language L. Usually, once you construct the semantic tableaux, a tree for a given formula. Suppose, if you come across, if we negate the formula, and then your branch is open. At least some of the branches are open and that, will serve as out counter example. That means, you are come up with an instance where, you are true premises and a false conclusion.

So, from the open branch, we can construct a counter example, and then you can cook up a domain. And then based on the open branches, you can come up with a structure in which this formula is considered to be invalid.

(Refer Slide Time: 09:16)



Now, suppose, these are some of the rules; that we will be a making use of, with respect to quantifies. Suppose, some point in our construction of your tree. That means, you are trying to construct a tree for the even predicate logical formula. So, we come to an extent were, let us say, you have a formula there exist some x, phi x. So, there exist some x is an existential quantifier.

So, this suppose, if you find it, on the node of the tableau. So, then you want to make this thing true. So, we need some element a in our domain D. So, some element a, has to exist. So, that, it will become an instant of that particular kind of thing. Such that, phi of a, has to be true. So, our rules should allow us to introduce phi a, on the path provided that. The parameter a, has not at appeared on the path.

So, now, there are four more rules that, we will be using with respect to for all x and there exist some x, etcetera ((Refer Time: 10:26)). So, these are the four rules that we will be using. So, they are all sitting at the background and all alpha and beta rules and all. So, similarly, with respect to quantifiers, suppose, if you come across a formula like this in a given tree and all. So, now from this, you need some kind of an object a, parameter in your domain. That means, a has to belong to a domain D.

So, now we should be in a position to say that, it is P a, there exist some x, P x is true. When, obviously, one of the instances is also true. So, now this is, where a is considered to be new. So, this rules says that, for example, if you come across two existential quantifies like this, r, q, x, etcetera. Once, you eliminate this quantify, you use an individual letter a. Another occasion, if you remove this existential quantify, you do not use this parameter a, but you use b. Any other things, which is other than is a.

So, whatever you use just below this one, it should not figure out in your branch L here. So, this is one of the important rules, with respect to existential quantify. So, each time, you remove this existential quantify, you have to use a new parameter. That means, this a should not exist anywhere else in the tree, at the earlier parts of your tree diagram. So, now, the other rule is this thing, for all x, P x, if it has to be true. So, this says that, for any x, at P x is going to be true.

So, we can simply substitute as P a, you can say for any arbitrary value a. We can freely substitute any value for this particular kind of thing. It is true for a, it is true for b, it is true for c, etcetera and all. It is as bit as saying that, all crows are black. Saying that, particular crow a, is black, another crow, which you are taken into consideration b. That is also black, etcetera. So, now, these are the alpha rules with respect to the quantifies.

Now, the other rule, which we have is beta rule. It is talks about the negation of disquantifies. So, now negation of for all x, P x, so this is nothing but, if you push the negation inside and negation of universal quantify will become existential quantify and you push the negation inside. Then, if you remove this particular kind of thing, then you use the same kind of rules. You cannot simply apply rule for this one and saying that, for not for all x, P x, you cannot simply say that, it is not P x and all.

So, this changes to their exist some x not P x. And then you can eliminate this existential quantify, using the same kind of rules. Each time, you eliminate the existential quantifier; you have to use a new parameter. So, now, the other rule is this thing not for all x, P x. Suppose, if you come across this particular kind of formula in the tree, in the construction of your tree, you come across this one. Then, it is nothing but, this is there exist some x, P x. So, this changes to for all x not of P x.

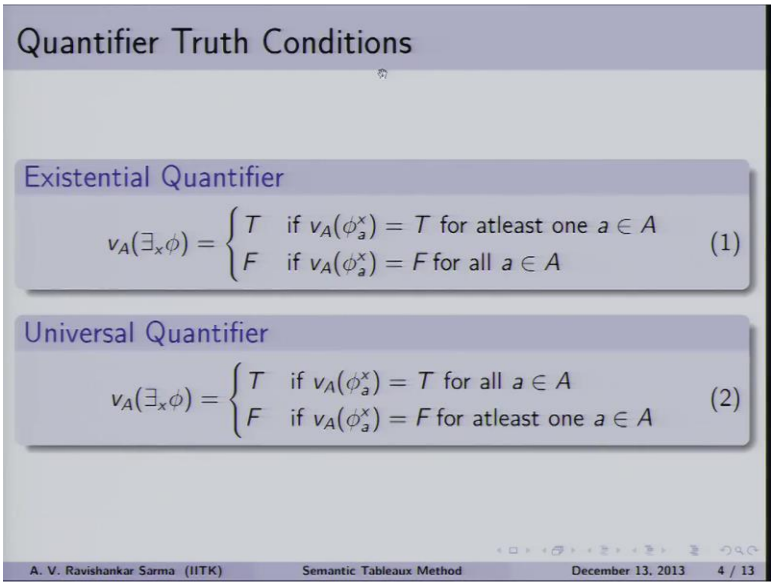
So, these are the rules; that we require to construct tree diagrams for any given predicate logical formula. So, the only thing, which we need to notice is that, when you are try to eliminate this existential quantifier, we need to use a new parameter. Each time, when you remove this existential quantifier, we need to use a new parameter. So, this, a, b, c, etcetera has to exist in the domain. They are the objects in a given domain.

So, essentially, what we are trying to see is this that, we are trying to construct a tree diagram, based on, we are looking for a counter example by negating the given well form formula. And you are constructing a tree, based on these particular kinds of rules. So, now, this is what we have explained already. Suppose, at some point, a second rule says that, these also consider to be in alpha rules. Suppose, at some point in our construction, we come to an universal formula, for all x, phi x and the node of the tree.

Since, for all x, P x is true, phi x is true, phi of a, also has to be true, for any element of your domain. So, there is never a problem with substitution of any term into this particular kind of thing. Unlike, the case of for there exist some x, P x, we have some restriction. Once, use one particular kind of parameter you are not suppose to use the same thing.

When you are eliminating some kind of existential quantifier in the next time, when you come across, existential quantifier in your tree. But, there is no such kind of restriction in case of universal quantifiers. They are readily available for, it is true for all kinds of a's etcetera. So, we always add the instance phi of a, to a branch recursively. And n number of times, you can use this particular kind of thing, as well as for all x, phi x, again to the path. Whenever you want to use again this thing, you can reintroduce the same formula for all x, phi x, again into the path.

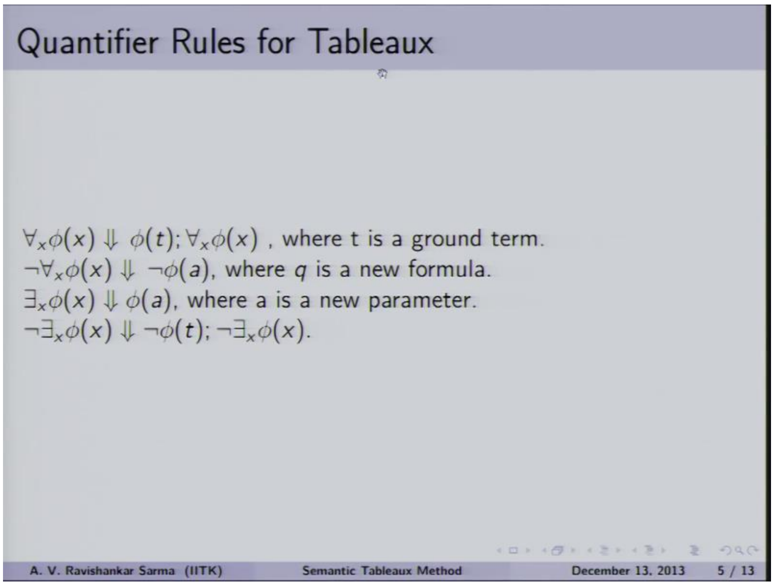
(Refer Slide Time: 16:37)



So, now, these are some of the truth conditions which respect to quantifies. It tells us, when a quantifies is going to be a valuation of a given quantifier is true or false. So, valuation of a, there exist some x, phi is going to be true. Especially, when, valuation of a, phi a x; that has to be true. That means, this formula phi a, has to be true for at least one particular kind of a.

If at least one a, satisfies this particular kind of thing, then it is called as there exist some x, phi, otherwise, it is going to be false. The same way, for all x phi, this has to be for all x phi, it has to be represented in that way. There is a mistake here. So, for all x phi has to be true, especially, when a phi a, has to be true for any kind of a, that you what taking into consideration. So, in the second case, the particular kind of phi a, has to be true. In the first case, it has to be true in at least one kind of occasion. This is the only difference between these existential and universal quantifier.

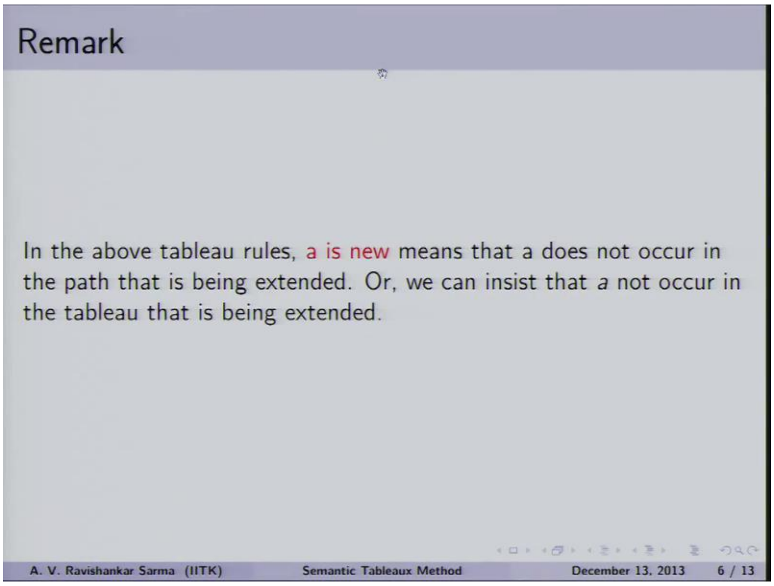
(Refer Slide Time: 17:53)



So, these are the quantifier rules, which we need it, while constructing the semantic tableaux trees. Example, if you come across a formula for all x phi x. Then, you simply substituted with phi of t and for all x phi x, where t is considered to be a ground term. For example, if you come across not for all x, phi x, then you simply substituted it as not phi a.

Where, if you come across there exist some x, phi x, you simply substituted as phi of a, where a is always considered to be a new parameter. In the same way, it is not the case that there is exists some x phi. That is simply replaced by not phi t. So, these are the things, which we have explained already.

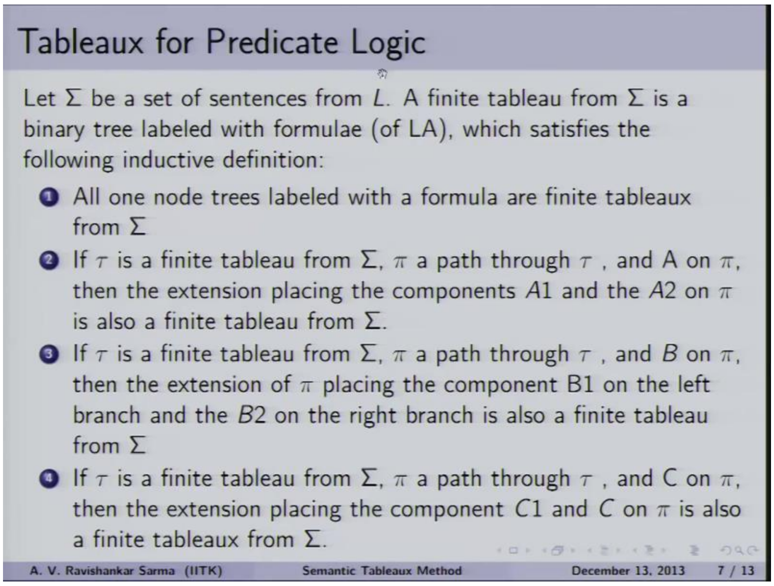
(Refer Slide Time: 18:53)



So, now, one important remark is this that, we are saying that, which time, when you are removing the existential quantifier, we are using a new parameter. So, what do you mean by saying that, a is new? A is new means, that is particular kind of a, does not occur in the path; that is being extended. So, suppose, if it is used earlier, then you are not suppose to use the same kind of literal a. We had used some other kind of thing, it can be a prime or b or some other thing.

Just to make distinction, you are using a different kind of parameter or we can insist that, a not occur in the tableau; that is being extended. So, you have to ensure that, nowhere else in that particular kind of tree diagram is particular kind of a, occurs. Then, you can use this particular kind of a literal a.

(Refer Slide Time: 19:56)



So, little bit more about these tableaux of predicate logic, this we look into it quickly. All this things will be very clear, once we talk about some kind of examples. So, now, let us consider some kind of formula analysis of tableaux for the predicate logic. Let sigma be a set of sentences from L. So, a sentence in predicate logic; that will be of defined earlier. A sentence in predicate logic is a one, which does not have any free variables. If it is free variables, it is considered to be a formula in the predicate logic.

So, if suppose, gamma is considered to be set of sentences from in the language of predicate logic, it can be axioms, it can be other thing and all. And let us considered a finite tableau from that sigma, you are constructed a finite tableau from sigma. It is considered to be a binary tree label with a formula L A, which satisfies the following kind of properties, a following definition on, it goes like this.

All one node trees are label with a formula are finite tableaux from gamma. Obviously, after a path ends in finite steps and all. So, now, second thing is that, if tau is considered to be finite tableau from sigma, we constructed a kind of tree from sigma. And phi is a path through is tau, I means, it is extended in a tree a starting the main formula at the node.

And then you are started expanding that particular kind of formula in a tree. And a is on that particular kind of path of the tree, then the extension placing the components A 1 and A 2 on that particular kind of path is also considered to be a finite tableaux. This tells us that, what is considered to be a finite tableau. Third thing says that, if tau is considered to be finite tableau from sigma; that means, it ends in some finite steps and all, finite in duels of time.

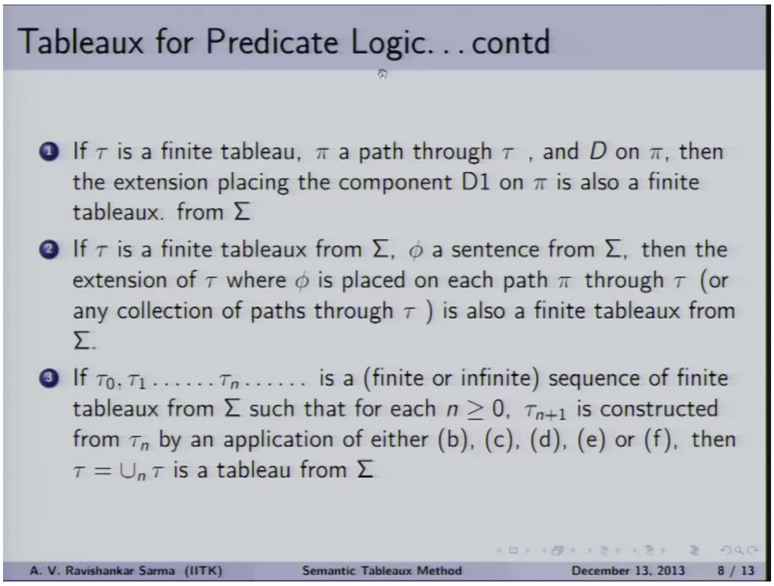
So, phi is a path through tau and B is on their particular kind of path. Then, the extension of phi placing the components of B 1 on the left hand side the branch, at B 2 on the right hand side the branch is also considered to be finite tableau from sigma. Essentially, what we are trying to talk about is that, you have some sigma, which consist of some basic formulas, etcetera and all, which we know that, there all a sentences in the predicate logic.

And together with that, we have a given formula a, we add to that particular kind of thing. And then we are trying to construct a tree. So, now, we are telling that, so when that, construction after constructing the tree is it going to be a finite tableau or not. So, these are some of the things, which tells us that, this is going to be a finite tableau, is not going to end forever and ever and all it will end at some point.

So, ending means in the sense that, once you end of with only atomic kind of propositions. Then, you have to close the tree or whenever, you come across a formula x and it is negation, then also you will close the tree. I means, a path ends there itself, that is considered to be considered to be contradictory path. So, now, the 4th rule tells us that, the tau is considered to be finite tableau from sigma. That means, you constructed a tree from sigma and phi is a path through tau and let us say another thing C on that particular kind of path phi.

Then, the extension placing the component C 1 and C on phi is also considered to be finite tableaux from sigma. Is the rule tells us that only, we are just try to construct a finite tableau, based on, it given any formula, we have only finite kind of tableau.

(Refer Slide Time: 24:15)

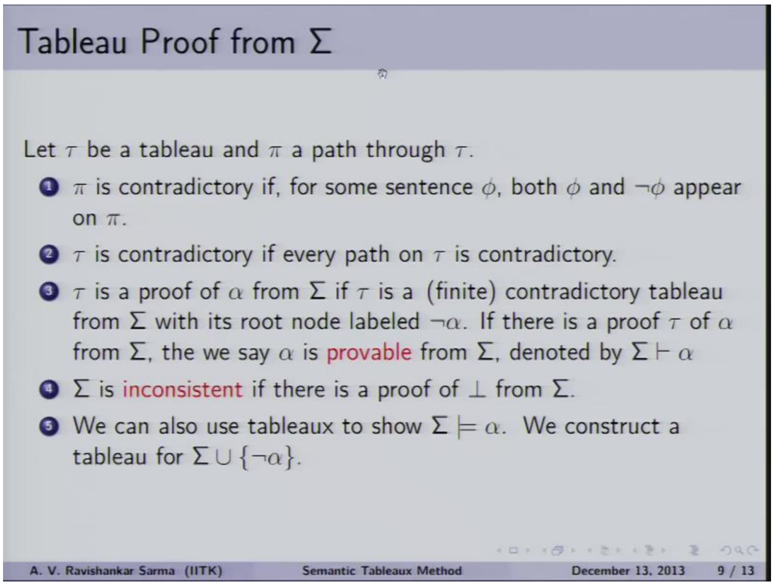


Because, once you have an atomic formula at the end of the, somewhere else in the branches, tableau will end there itself. So, there is no way in which you can extend the tree. So, all the tableaus end with atomic propositions. So, now, tableaux for predicate logic, if tau is considered to be a finite tableau and phi are considered to be a path through tau and D on pi. Then, the extension placing the component D 1 on pi is also considered to be finite tableaux.

In the same way, tau is considered to be finite tableaux from sigma and phi is a sentence from sigma. Then, the extension of phi, were phi is placed on each path phi through particular kind of format tau is also considered to be finite tableaux from sigma. So, like this, if tau 0, tau 1, tau n, etcetera is the sequence of finite tableaus from sigma. Such that, for each n greater than 0, tau of n plus 1, the next one is constructed from tau n by the application of all this rules, which we have mentioned so far.

So, then tau is equivalent to union of n tau. That is also considered to be tableau from sigma is also considered to be in a finite tableau. So, we are not said anything great about this thing except that, tableau ends in finite steps and all. So, for any given formula, we have a finite tableau. So, now, there are certain things, which we can talk about with respect to this tableaux construction.

(Refer Slide Time: 26:02)



So, let us consider the tau v a tableau. That means, you construct a tree diagram for a given formula. And pi is a path through tau; that means, sometimes, it will have branches. Sometimes, it will be only trunk and all and etcetera and all depending upon the formula; that you have taken into consideration. So, now, the path pi is considered to be contradictory.

If for some sentence phi, a sentence that you are taken in to consideration phi, both phi and v phi appears in the same path of your tree. Pi and psi not phi occurs in the same branch, then it close. So, then the path is considered to be contradictory. So, tau is considered to be contradictory, if every path on tau is considered to be contradictory. So, suppose, if you a have a branch, and then you have two paths in particular. One is going to the left hand side and the right hand side.

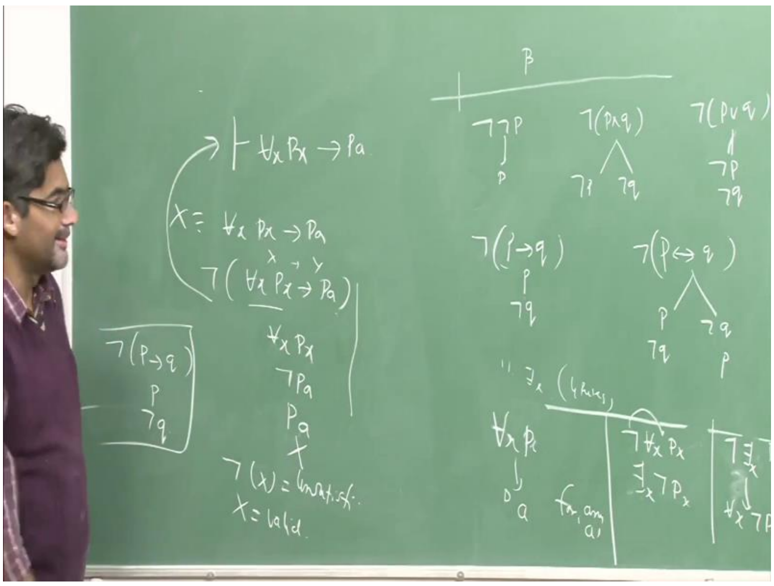
In both the path, you come across a literal in it is negation and obviously, that is contradictory. And then entire path has as considered to be contradictory. So, if tau is considered to be proof of alpha from sigma and tau is a contradictory tableau from sigma. It is root node labeled not alpha. If there is a proof, tau of alpha from sigma, we say that, it is considered to be alpha provable from sigma. And it is denoted by alpha is prove from sigma.

When, do we say that, sigma is considered to be inconsistent. If there is a proof of a contradiction from sigma; that means, you come across a and not a in a given branch. So, we can also use tableaux to show that, alpha is true in a sigma. So, whenever, sigma is true alpha also as to be true in that sense alpha is the logical consequence of sigma. That means, what essentially, we are trying to do is this that, a given sigma, where adding not alpha to it. And then we are trying to see when the branch closes and all.

When, the branch closes, then not of alpha is considered to be a contradiction, then; obviously, alpha has to be true. That means, you cannot deny the formula and all, because deny of the formula leads to the contradiction. So, what essentially, we are trying to do is, we are adding not alpha to sigma, and then we are trying to show that, it is unsatisfiable.

So, if all the branches closes and all, if it becomes unsatisfiable, then the original formula is going to be valid. Otherwise, it is going to be invalid, because at least, tell me one particular kind of counter example. So, now, a thing which will make use of in constructing proofs of some kind of formulas and all. So, now, let us consider some examples. So, that, we can understand this particular kind of method in a better way.

(Refer Slide Time: 29:22)



So, let us try to say that, you take some formulas into consideration, and then we will see whether, this is considered to be provable in the predicate logic or not. So, for all x, P x, implies P a. So, now, this is a formula, which is given to us ((Refer Time: 29:36)) this is the formula in the predicate logic. So, in the semantic tableaux method, what one essentially does is this that, you take the negation of the formula. And show that, this is considered to be unsatisfiable.

If negation of x is unsatisfiable, that means, all the branches closes, then at implies x is considered to be a valid formula, valid formula are tautology something that. So, that is what, essentially we are trying to do. So, now, we negate the formula, and then we need to use is alpha and beta rules; that we have mentioned here, so in this format, x and y. So, this can be like this, for all x, P x and not P a.

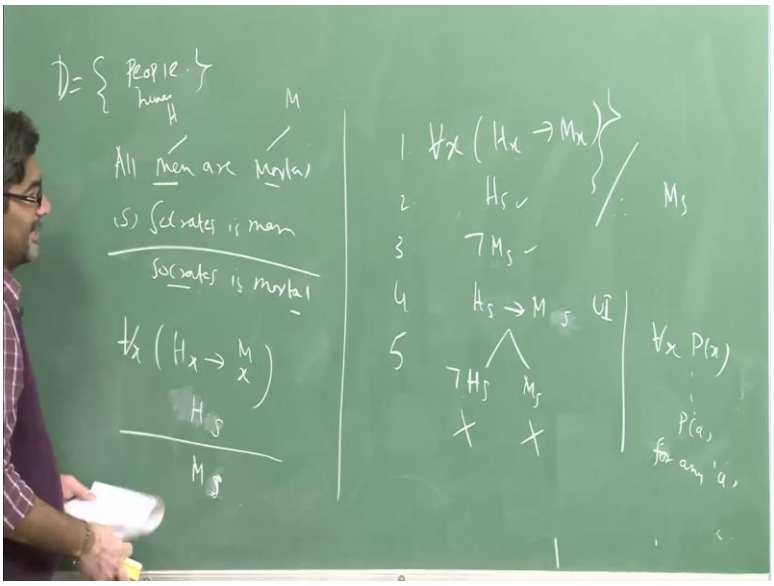
So, how did you get this to this one, not of P implies q is simply P and not q. Somewhere, else, we have use this particular kind of rule, negation of P implies q is P and not q. So, what essentially, we are trying to say is this that, whether, it is considered to be valid formula or not that is what, we are trying to check.

So, now, you have something called for all x, P x at the node of somewhere else in this of kind of formula. Whenever, you come across this particular kind of formula for all x, P x means, it instance is also true. That means, it has to be true for that formula a also. A as to be having this particular kind of property P. Suppose, if you say all are happy or something like that and if you take a particular kind of individual, the individual also has to be happy has to feel happy.

So that means, for all x, P x means one instance of that one is P. So, now, you come across P a not P a in the path of this particular kind of tree. So, this particular kind of thing closes, because this is a contradiction not P a and P a leads to contradiction. So, that means, what essentially, we have shown is simply leads that not of x is unsatisfiable. Because, it takes the negation of this particular kind of given formula, it will use the closer of branches.

That means, it is unsatisfiable, that means, x has to be a valid kind of formula. So, that means, we have derive particular kind of thing, for all x, P x implies P a is the theorem in the language of predicate logic. So, let us consider some more example, so that.

(Refer Slide Time: 32:41)



For example, if you say at this particular kind of this famous example that we have been trying to talk about right from the beginning of this course. That is thing all men are mortal, a Socrates is man. So, Socrates is mortal. This is the famous example, which is given in all the introductory logic courses and all. So, now, you represent it as H humans, and then this as mortal as M. Because, if you represent it with same letter M will be no restriction between these two predicates.

So, now the first sentence will become like this, if x is man, of course, we are representing the H as men, if x is a human being, then x as to be mortal. Now, Socrates is mortal; that means, M. Socrates is a man means is this, H x. This has to be written with the individual letter S. Let us say S stands for Socrates, this represent some specific kind of objects in the domain.

So, what is our domain? Domain consist of people are human beings, etcetera. So, I don not to have to take consideration animals, trees, plants, etcetera into that one. It is not in this particular kind of domain. So, now, this is H s and then Socrates is mortal. So, now, this is considered to be this thing. This predicate is written the capital letters, whereas the individual Constance; that you come across, this particular kind of formula are written in terms small letters. So, now, this is the argument that we have.

So, now, we want to see, whether this particular kind of argument is valid or invalid. We know that, this valid argument. So, now, we are trying to establish it with the help of predicate logic. So, what essentially, we are doing in the second part of this lecture is this that, we are trying to consider, some examples like this. And then we are trying to see, we are trying to make use of the semantic tableaux method, which severs as some kind of decision procedure method.

It tells us, when, if the argument is valid, tells us, it gives us some kind of a proof per this particular kind of thing. That this conclusion follows from these premises and all. So, now, let us take this argument in detail, H x implies M x and second be is this thing, H s and this is going to be your conclusion M and S. So, in the semantic tableaux method, what you will do is, we will start with the negation of the conclusion.

So, what essentially, we are trying to do is, sigma is this particular kind of thing. So, these two premises constitute sigma. So, now, what you are doing is, you are adding not alpha to, where alpha is considered to be the conclusion. So, now, if sigma union not alpha is unsatisfiable. Then, so how did we come to this unsatisfiable thing, we have we landed into this particular kind of problem.

Especially, when you are taking the negation of the conclusion, that means, negation of alpha is unsatisfiable. That means, alpha has to be valid; that means, alpha has to be true. So, now, we take the negation of the conclusion like this, and then you try to construct a tree for this one. So, there all atomic formulas and all need not to worry much about it. So, now, H x implies M x happens for all x m. So, that means, this has to be true for even this particular kind of thing.

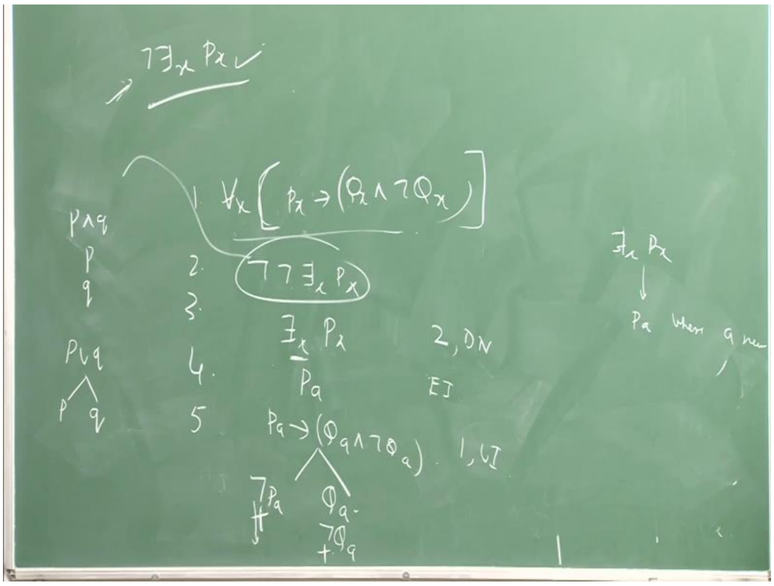
For all human beings, if x is a human being, then x has to be mortal. That means, all human beings have to die, some day or other. So, that, happens we even for the Socrates or even for anything which is substitute it into this one. If it all is human beings, then it has to die. So, now, this S stands for Socrates. So, one instance of this one is this. So, we have use this particular kind of rule for all x, P s if you come across is particular kind of formula in your tree. Then, we can simply represent as a, this is happens for any a.

So, now, H s implies M s. these thing, which is called as universal instantiation of this. One instance of this one is this. Now, this can be represented as not H s and M s. So, this is simply P implies q nothing but, not P and q. So, it is in that sense, we need to write like this. So, now, we have H s here and not H s here and this branch closes. Now, there is another path like this H s not M s etcetera and all, and then M s here and not ms here and this also close.

So, that means, negation of this particular kind of thing, leads to contradiction. So, that means, it has to be M s rather than not M s. So, this is the proof, which is based on a something called as ((Refer Time: 38:24)) absurdum kind of method. So, this is that means, the original conclusion is M s rather than not M s. So, in this way, we can a guarantee that Socrates is mortal, necessarily follows from all men are mortal Socrates is man.

So, like this, you can translate a many formulas into the language of predicate logic. Then, you can talk about the validity or truth the tautology of a given formula. Let us consider some more examples. Since we have sufficient time, so we can consider some more examples.

(Refer Slide Time: 39:07)



A let us say a formula like this, for all x, P x implies Q x and not Q x. So, this close by this, particular kind of thing. So, this is one particular kind of formula and from this, whether or not, we will be able to deduce this thing, they does not exist x, such that, P x. So, now, we are trying to see whether, this can be derived from this particular kind of thing or not.

So, in the semantic tableaux method, what you will do here is this that, you just take this as it is. And then you add the negation of the conclusion. So, this is what you have trying to do that; that means, is a conclusion and all. So, now, what you will do in the second step is, you negate this particular kind of thing and add to your sigma. So, now, you start constructing a tableau or this one.

And then we will see whether, the tableau closes or not, not, not of there exists some x, P x, it is not, not, not of P. Whenever, you come across a formula and not, not P. You simply represent it as P on. So, same way, you have this particular kind of, there exist some x, P x not of not of there exist some x, P x is simply this one.

So, how did we get this one, two double negations, need to give the justification, the right hand side. Otherwise, it does not make any sense to talk about this particular kind of thing. We need to say, how did, we come to this particular kind of formula. So, that is why; we need to write justification on the right hand side, followed by this particular kind of formula.

So, now, we need to talk about one instance of this particular kind of formula. So, P x implies Q x and not Q x is true for all x and all, I mean it has to be true for some kind of a also. So, before that, there is one strategy we need to follow, always eliminate the existential quantifies first, rather than dealing with the universal quantifiers. First, we handle with the existential quantifiers, eliminate those existential quantifiers first, and then move on to the universal quantifies.

So, now, there exist some x, P x; if you come across this particular kind of thing, there exist some x, P x. In the tableaux rule, we can replace it with P a, where this P a should not occur anywhere in the branch, which is above this particular kind of formula. Nowhere, that a has to exist; that means, a as to be new. So, now, whenever you have a formula, there exist some x, P x, you can simply represented with P a. So, now, that is what, we are trying to write.

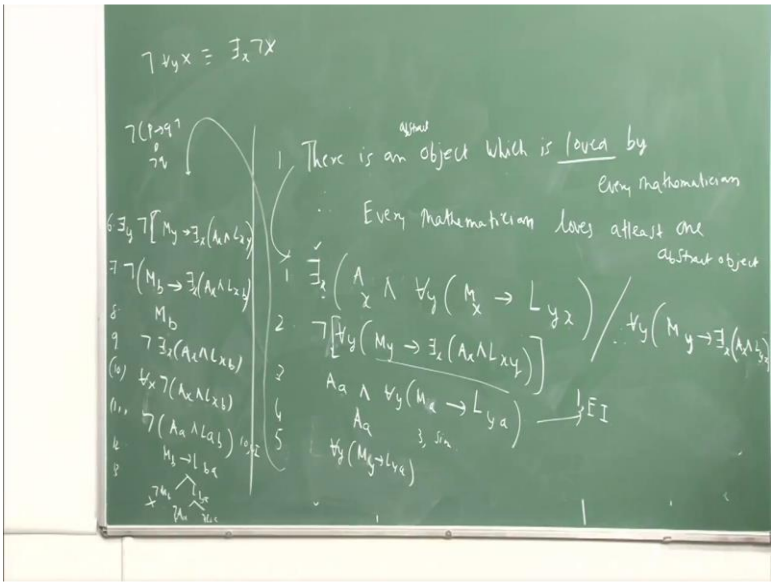
So, now, this is tableaux rule, you can say existential instantiation something like that. So, this P x implies Q x and not Q x, it is true for any kind of a and all. That means, it has to be true for a also. So, that means, it is P a implies Q a and not Q a. So, now, if you further expand this particular kind of thing. So, this is one universal instantiation. So, one instance of this one is this. So, now, this is going to be in not P a and Q a and not Q a. You can write it as like this, because P and q can be simply written as P q, it is like a trunk of your tree, whenever, you P or q, it is a branch P q.

So, now, you have P a here and not P a here. it closes and all this like cutting your own tree and all. You sitting on tree and cutting your own tree, because there is a contradiction P a and not P a, you cannot go further. So, that is why; it closes here and already have Q a and not Q a, it closes. So, negation of this conclusion, leads to the contradiction. That means, the original conclusion votes.

That is, what is the original thing, which we have to deduce? That is, then it is not the case that they exist some x, P x; that has to be true. So, this is the way to show that, this particular thing, follows from the given formula P x implies q x and not q x. So, now, let us try to consider some one more example. Here, what we will do is a given predicate logic sentence will transformation to the language of predicate logic.

And then we are trying to see whether, that particular kind of formula well form formula, if valid or not. So, here is the statement, which you come across in the natural language and that is trying this.

(Refer Slide Time: 44:44)



So, there is an object, which is loved by every mathematician, we do not know, what kind of object it is. We can assume that; can be up site object, it can be anything, which is loved by every mathematician. This is for the sake of for example; we take into consideration this thing. So, therefore, every mathematician loves at least one object at least one. This is abstract object at least one abstract object.

Mathematician does not require any real entities to exist in the world. Again, even consider abstract object, and then they can still mathematics and till talk about the mathematics, there are surrounding that particular kind of abstract objects. So, it looks like that the conclusion seems to be following from the premises and all. We have to establish with the help of some kind of decision procedure method.

So, the first one state that, there is an abstract object, which is loved by every mathematician, and then that implies; that means, every mathematician loves at least one particular kind of abstract object, should follow from, whether or not this follows from this or not is there one, which we are trying to see. So, now, we need to translate this things into the language of predicate logic is using quantifies.

So, the first one, we can be translated in this sense. Suppose, if x is considered to be and abstract object. Abstract object is represented as A x and for all y, if x is a mathematician. Then, it is one more predicate that is there here, loved by every mathematician. So, that means, L y x; that means, for all the mathematicians, there is at least some kind of object x. And all the mathematician, whatever is considered here, mathematician loves this x. So, there is an order, which we need to follow.

Suppose, if you write x and y; that means, x loves y and Y x means y loves x. So, there is order that, we follow predicate logic. So, this is the translation of the first letter. So, there is an object means, there should be at least one object that should exist. That means, there exist some x and whatever is there here is the one, which we have written. So, now, this is the formula, which is represented by a premise.

And then the conclusion is there every mathematician; that means, you need to start with the quantifier for all y. If y is a mathematician, which is written in the sense, y is mathematician. That means, there exist some x, such that, A x; that means, x is an abstract object and that mathematician has to love that particular kind of object x. So, for all y, if y is a mathematician and there exist some x. Such that, x is an abstract object and that x has to loved by the mathematician; that means, y has to love x.

So, this is the conclusion, once you represented in terms of the language of predicate logic, then need not have to worry about, what is the content of this argument and all. Then, because will be handing only the symbols; that you see here. So, now in the semantic tableaux method, as usual we start with the negation of the conclusion. So, this is the first thing, where exist some x and for all y, M x implies L y x etcetera. And then negation of the whole thing negation of for all y, M y implies there exist some x, A x and L x y. So, that means, you are denying the whole formula.

So, now, once you deny this, suppose, if the conclusion indeed follows from the premises. Then, if you take the negation of the conclusion, then the branch should close and all. That means, for example, if you take sigma into consideration, sigma as your premises union not of alpha, should lead to contradiction. So, now, this is the second one, now we need to use different rules and all.

So, now, our strategy is always this that, first you eliminate this existential quantify. That means, we need to find out an instance of this one. So, now, if you eliminate this existential quantify; that means, wherever you find x, we need to replace it with some kind of a parameter a, b, c, whatever if we like replacing. So, now, the first one existential instantiation will become this. You taking replace x with a and this becomes this, for all y remains the same, M x, x is replaced by a, and then L y remains the same and the next is replaced by a. So, this is what is, existential instantiation of one existential instantiation, one instance of this one is this. So, no one extent, when you remove this existential quantifier, you should ensure that, you are not using this particular kind of thing a parameter a.

So, you have to use another different kind of parameter. So, now, this can be different thing. So, this is for example, you have formula x and y in the tree, you can simply write x and y. It is like a trunk and all. So, that means, three simplification are, so conjunction rule. So, it will be like this, for all y, M a implies L y a. So, this is the 5th step.

So, now, we have this particular kind of formula, we need to simplify this one. So, this will become like this. So, it is not for all y, M y implies write this some x. So, we have this particular kind of formula not for all y, x suppose if that is there like this and it will become there exist some x, not x. So, the same way, it is like not for all y; that means, there exists some y.

And then you push this negation inside, and then this will become the entire thing M y, whatever is say, M y implies there exist some x, A x and this formula L x y. So, now, this is coming like this. So, this is considered by the 6th step. So, this is simplification of this particular kind of thing. Now, one instance of this particular kind of thing is like this, just getting now.

So, now this formula can be write in this sense, if you replace y with this thing, you will become b, you are not suppose to use a here, it will become b. So, now, this is same, there exist some x, A x and L x, and then you replaced y with b. So, this is one instance of 6. So, now, 8 we have M b. So, this is double negation of a 7, negation and implication will become this thing. Negation of this particular kind of thing is like this.

Negation of P implies q is nothing but, P and not q. So, this is first one is M b and the second one is not of there exist some x A x and L x b. So, this is the simplification, and then not of for all x, this one will become a negation of existential quantify will become for all x not of A x and L x b. So, now, one instance of this one, because, it happens for all x and all, it has to be true for this one also.

That means, not of A a and not of L a b, it has to be true for all kinds of anything, which you take into consideration for x, it has to be true. That means, for a also, it has to be true. And all in that sense, we have written like this 10 and universal instantiation. Now, we are getting closer to our proof a 12th one is M b, L b a; that has come from this one, M a, L y a; it happens for all the thing and all, 5th one M y, L y a is getting over.

So, now, 13 step, if you expand it and all, it will become not M b and L b a and then this branch closes because not M b is a and M b is a, and then this branch are it can be further expanded to not A a, and then not L b a; this and this closes and all. So, ultimately, what will happen is, if you take the formula, if you take this is premise and this is a conclusion. So, negate the conclusion, it leads to closer of all the branches and all.

So, with this we line this lecture and all. So, what we discuss in lecture is simply is that, we discussed about semantic tableaux method, which serves as some kind of decision procedure method. With which, you can find out whether or not, a given well form formula is valid or we did not talk about of the consistency satisfiability etcetera and all. More or less, you know in the case of consistency, if you have given two sentences in the predicate logic and you constructed tree using the same kind of rules and all. If at least one branch is open and that is considered to be in this two formulas are considered to be consistent. So, in the next lecture, we will be talking about some more examples. So, that, we get a familiarize ourselves with this particular kind of method.