Introduction to Logic
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Lecture - 42
Natural deduction in Predicate Logic

Welcome back. In the last lecture, we discussed some of the important decision procedure methods; one such method which occupied the simple position for this course that is, semantic tableaux method. So, using the simultaneous tableaux method, we discussed about the validity of a given well form formula in the predicate logic. And we also discuss something about, when to we say that 2 statements are sentences in the predicate logic, are set to be consistent.

So, in this lecture, what would be doing is you will be taking up another important proof procedure method, which all those serves as a kind of decision procedure method; so that is the natural deduction method. Natural deduction method simply involves some principles logic. This is what we have already discuss in the contest of prepositional logic, were we used some of the important valid principles of logic such as, mod penance, such as mod stolance, constructive of dilemma, distractive dilemma etcetera, they all valid principles, which we have taken into consideration. And then we proved some of the important theorems. If something has a valid formula, it has been proved.

So, in that contest, in order to prove that particular kind of valid formulas, all the valid formulas are in your formal system. So, we have used natural deduction as 1 of the important decision procedure method, for proving a particular kind of well form formula. So, natural deduction in the contest of predicate logic is slightly different from that of natural deduction in case of propositional logic. Although it is consider to be extension of natural deduction in case of propositional logic. So, in the predicate logic we have quantifies and enhance we have some new set of rules in the contest of predicate logic.

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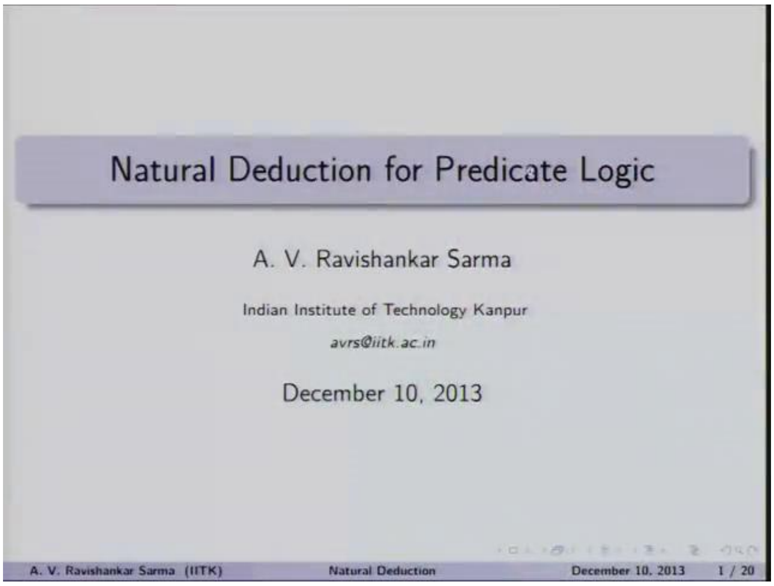

So, I will be discussing rules first, then we will talking about some of the examples so that, we will get our self familiarize with this particular kind of technique, that is a natural deduction method. So, another important thing which we need to notices that, natural deduction is some are closer to the ways humans reason because, it involves simple principles of reasoning such as, mod resonance, etcetera and all, which have which comes closer to the human common seneschal reasoning law. So, that is why they find important, in important especially in coming of its some important kind of decision procedure method.

So, all these methods are has is own importance, but some methods are closer to human reasoning and there are some which close to in to the implementation of machine, automatically reasoning etcetera. So, this based on our convenience, you will be using this particular kinds of methods. At least 4 5 method we have discussed in this course, we started with truth table method and then we mood simultaneous tabular method and then we discussed some about resolution reputation method, and then we have also discussed something like reducing the given formula into conjunctive and designative normal forms. And we can talk about whether the given formula is a tautology or not, when than formula is discussed about natural deduction method in the contest of prepositional logic.

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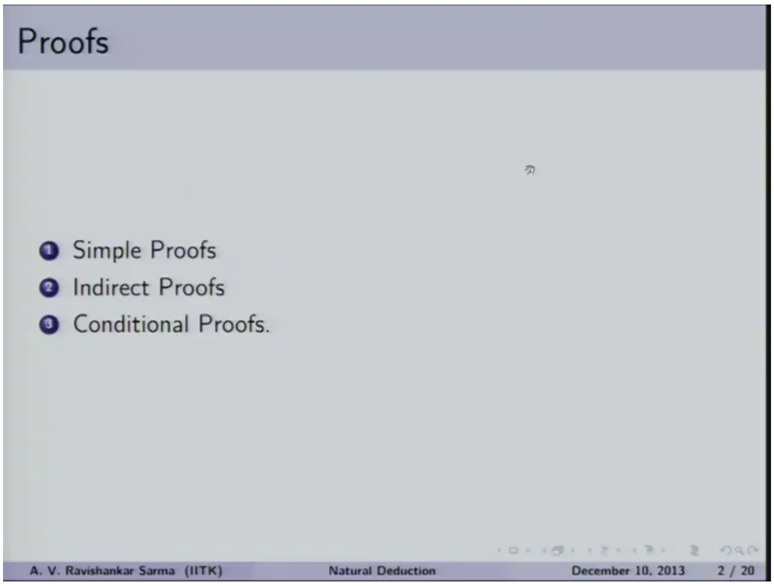

So, now, as natural deduction involves in some kind of you start with your simple proofs in there are indirect proofs and then we have concessional proofs. Usually this divide in to 2 parts; one is conditional proof; another one is based on considered to be indirect proof is, we start with a given formula and what you will do in the negative formula and then will see whether it is contradiction or not. If at least contradiction, then the negation of the formula unsatisfiable; that means, a original formula has to be true; that means, it as to be invalid formula. So, will be talking about is 3 important proofs in the contest of natural deduction

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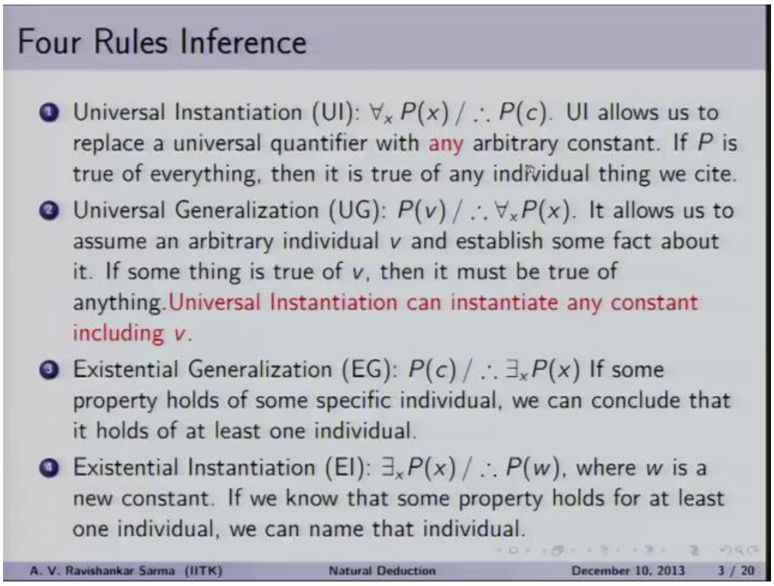

So, apart from all the rules of propositional natural deduction for the propositional logic, where we have a list of rules, sometimes it is remember this rules, but in general simple valid principles of logic like moos exponite, mood tolence etcetera. Apart all the rules is there for the preposition logic, since we have confiscate for the predicate logic and minute formulate some kind of rules, for the rules which respect to the quantifies. The first rules states like this; which is called as universal instantiation. So; that means, you have to find out 1 instant of a particular kind of sentence, such as for all there a x implies P x a x implies d x.

Suppose, if you have a formula like for all x P x, 1 instance of 1 is just P, were x is def lies by some kind of ground term c. So, that is P is considered to be a instant of for all x P x for, universal instantiation allows to replace universal quantify, that is, for all x with any arbitrary constant. So, for any such kind of arbitrary constant, the P c as to be true, you can take P d P c P f anything for all kinds of that arbitrary variables at sentence for all x P x is going to be true, there is P c is going to be true that is, P is true of everything, then it is true of any individual thing we side.

For example, if you say that all coarse are black and if you find out specific kind of crow and that crow also has to be black, that is 1 instant of that particular kind universal proposition. And the second one is this thing which is little bi try key to use particular. Suppose, if you have formula such as p of v, then you can generalize and say that for x p x. So, it allows to assume and arbitrary individual we and establish can some fact about some particular kind of thing. So; that means, if something if true of that particular kind of variable v, then we can say that must be true of anything, just like in a mortality is attributed to single human being for example and then every human being has to dies some day or other. So, that is why motor it is attributed to all the human beings. So, it is in that sense we have generalizing, we are generalizing the particular event, we will generalizing and forming universal generalization.

So, universal initialization can instantiate can any con stating including v, were as in this case, so there is some kind of restriction which we need to impose on this this particular kind of variable v that exists here. So, this is a second rule. So, now, the third rule is existential generalization. Suppose, if you have it term P c, all this terms are consider to be and all, we discuss what we mean by terms in the last 5 lectures. So, if you come across a term P c; that means, something is the case, then you can generalized it and say that, there exists something P x is the case.

Suppose if you say that, this duster is yellow and if can say that there exist some x that is x is consider to be duster and then them that particular kind of duster is yellow in color. So, that is what is existential generalization because, there exists some x P x is true especially when a at least 1 objects satisfies that 1 particular kind of property. And the fourth role is existential instantiation, which cells that, if have formula there exist some x P x. In the proof of particular varying come across of this particular kind of formula in the proof, we can always find out 1 instant of proof that is, you can replace in with P w in the sequence of your proof, but need to be little bit conceals in using this rule. Whenever you reply the quantifies that each may we reply the existence to quantify, you need to use a different kind of parameters That means, you use w earlier in your proof, you are not suppose the same w, which comes next time when you replace this a existential quantify.

So, each time used w as to be new 1. So, this rule tells us that if you know that some property holds for at least 1 individual, then we can we can say that, we can name that particular kind of individual. So, say that something is black in color, then you can say that, specify a particular kind of individual and say that, this object is black in color.

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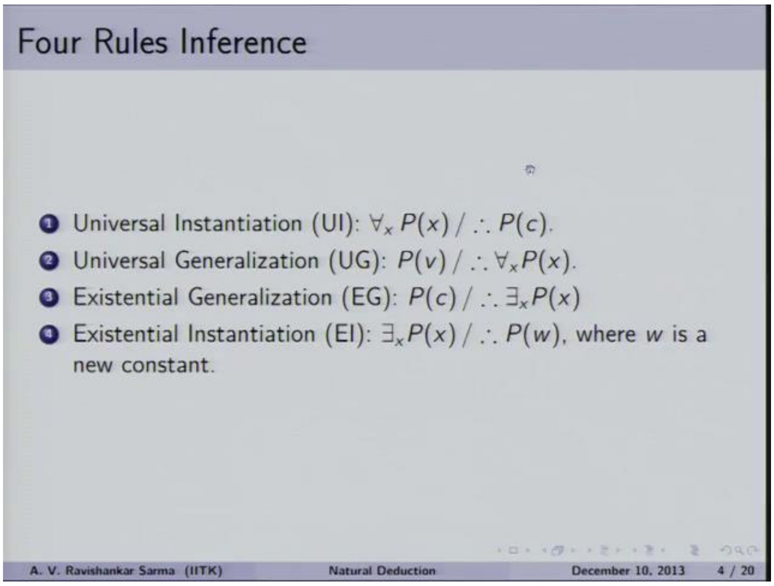

So, these are the 4 rules that we have, which are expressed in this sense universal installation for all x P x, you substitute as substitute P c and if you have universal in the universal generalization, you have P v, then and you generalize it say that for all x P x it is some restriction and existential generalization if you have a P c, then P f c, then you can receipts some x P x etcetera. All these things which have to you are discussed just now.

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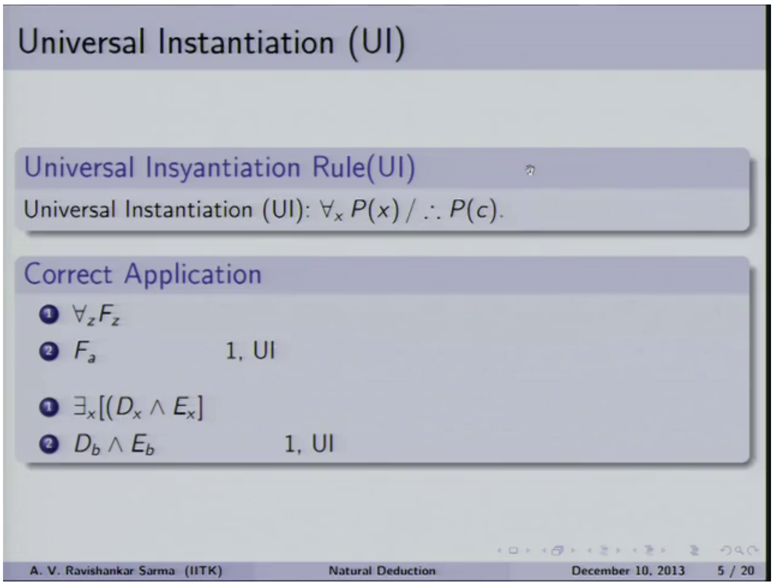

So, now how do you know that we have applied these rules correctly? So, let us consider some examples with which we can, we will come to know whether, we have applied this rules correctly are now. So, now, first thing is universal instantiation rule which tells us that, for all x P x, we can obtain P c. Now, the correct application of that 1 is like this; in the first case for all z f z. So, now, in this 1 z is depressed by a. So, then it becomes f a that is seems to be a correct kind of application. In the same way for example, if you have for all x D x and E x, then 1 instant of that 1 instant of going to be D P and E P, here which will be red us for all x.

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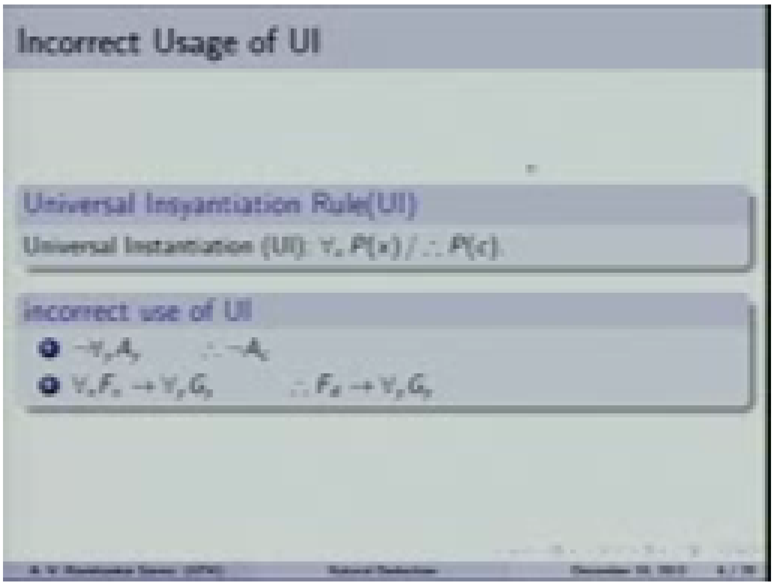

So, what is consider to be a incorrect application of this particular kind of rule? The rule tells us for all x P x, you implies with P c.

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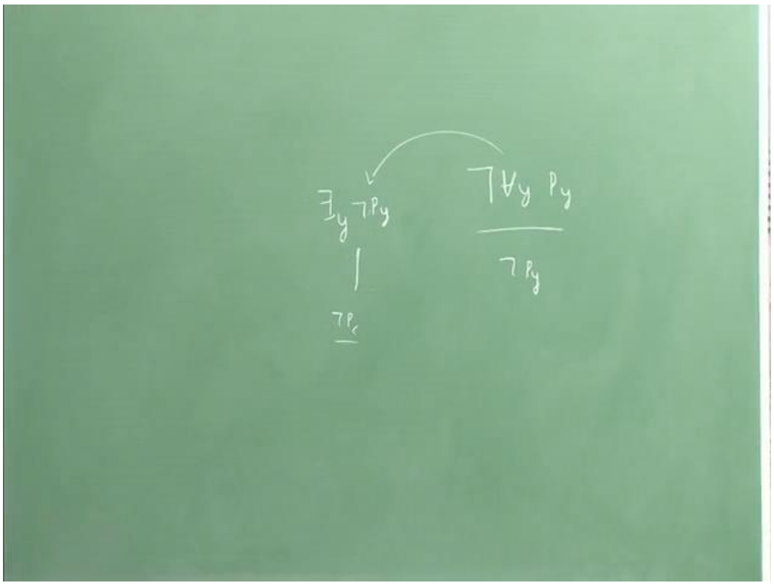

Now for example, if you have a formula like this; for all by P y if cannot simply say that it is not P y. So, why because we have to first transform this particular kind of formula into the corresponding formula, then only you can substitute the variables with some kind of constants. So, this will change to; there exists some y not P y. Then you can substitute eliminates of this 1 and can you can say that, P c or something like that. So, directly you are supposed to substitute, if you come across not for all not well P y, you are not supposed to substitute the inside away not P y. You have change it to the proprietor form then you make substitution, then that is going to be a correct 1. So, that is 1 thing.

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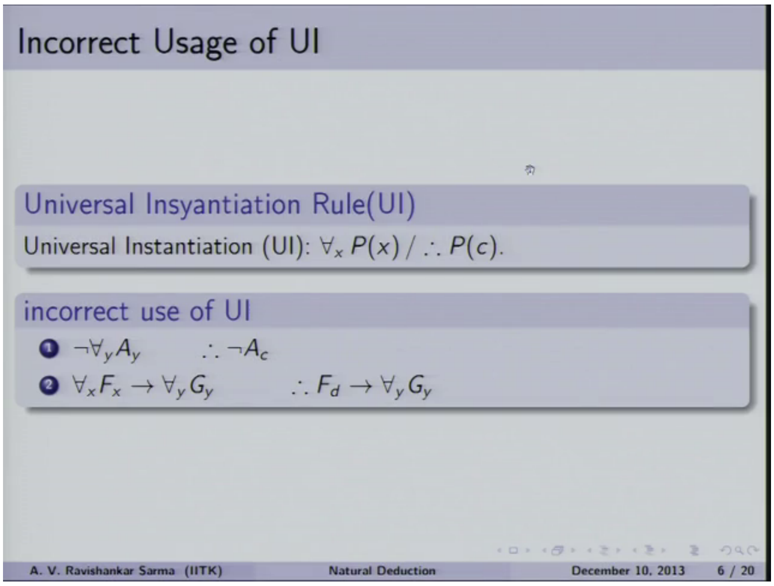

In the second case, that you are seeing here; for all x f x for always e y. Only in the first instant, you find some kind of universal instantiation and the next 1 is did not you did not find any universal instant particular all that kind of thing and then that also with this way of a applying the rule is also a consider to be incorrect application of the rule. That means, you are not suppose to apply universal instantiation only to the part of it.

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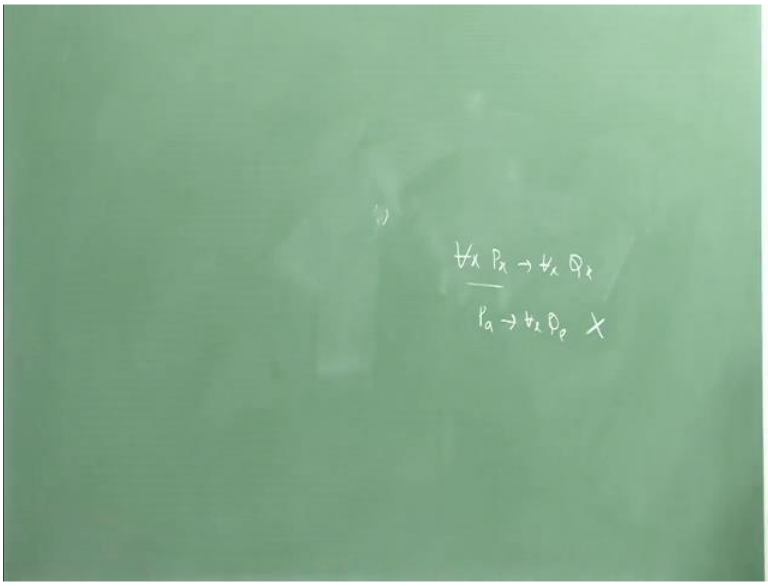

For example, if you have a some formula like A x implies B x and then you this is universal and quantify statements with universal quantify. 1 instant of that 1 for example, if you say that for all … Suppose if you have a formula like this; for all x P x implies for all x Q x. Now, suppose if you replace, if you find only instants of this particular kind of thing and then you keep the other thing as it is, then it is incorrect application of that particular kind of rule. That means, universal installation does not apply to part of the sentences, the whole sentences rather than pats now. In this second case it update only parts. So, it is considered to be incorrect application of universal instantiation.

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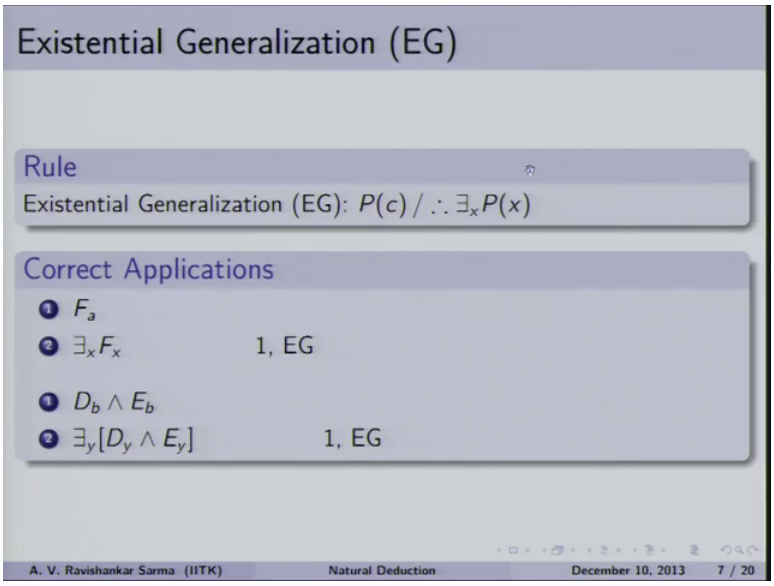

Now coming back to coming to the existential generalization and something in the case, you can generalize it say that there exists some x P x. The correct applications are like this; if you find that something is having some kind of property P, let us say that duster is yellow in color, then you can generalize it you can say that there exists some x. So, that x is green in color. So, the here F stands for predicate and x stands for individual objects within the domain. The domain is considered to be in a nimate objects such as, chock, pieces, pens, etcetera, P c is etcetera.

So, now, if you have term D b and E b, then you generalize it and say that there exists some y D y E y. So, you not to suppose to say that there exists some y D y and just simply E B y something like that. Part of the sentence cannot be generalized and all, if you generalized it has be generalized, it has be apply on whole sentence. So, then what are consider to be the incorrect application of this kind of particular rule.

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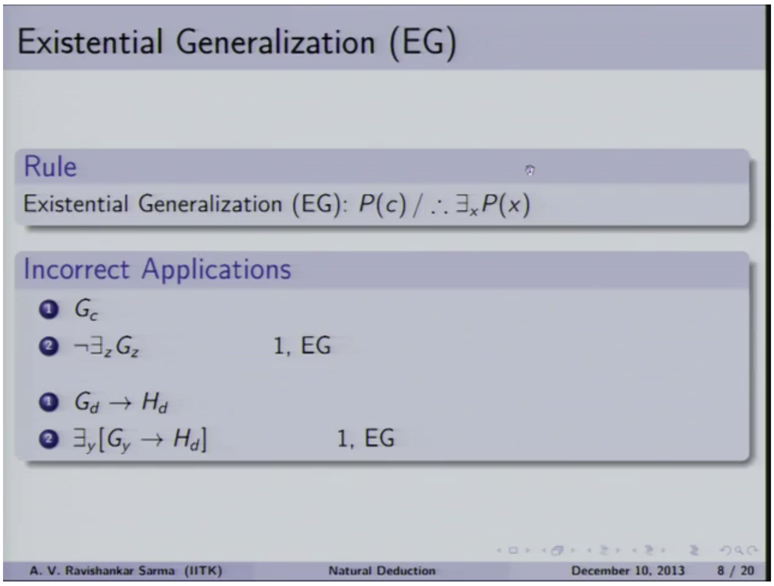

So, why we are discussing all these rules, we would be making this of rules in deriving some of the important valid formulas. So, the idea here is that, all the valid formula should find some kind of rules. One of the important effective decision procedure method that, we are discussing today is the natural deduction method. And the rules what we apply in the natural deduction method, are we come usually come closer to or human reason. So, here existence will generalization the incorrect applications are like this. Suppose if you have not G c example, then you cannot simply substituted as not E Z and G Z.

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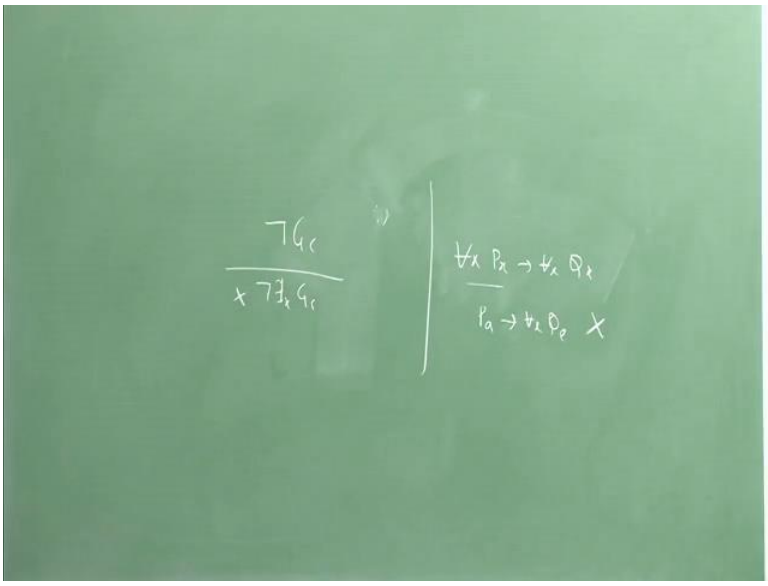

So, here the rule incorrect application of this rule is like this. We have a formula not G c that means, something is not black for example, then you cannot say that, we does not some x G c. So, this needs to be rule. So, that is consider to be incorrect application of this role. In the same way, part of the sentences, we cannot apply universal existential generalization G d implies to H d. Only part of thing you applied this particular kind of rule that is a first part of this sentence. Second part you did not thing that is. So, it is considered to be incorrect application of this rule.

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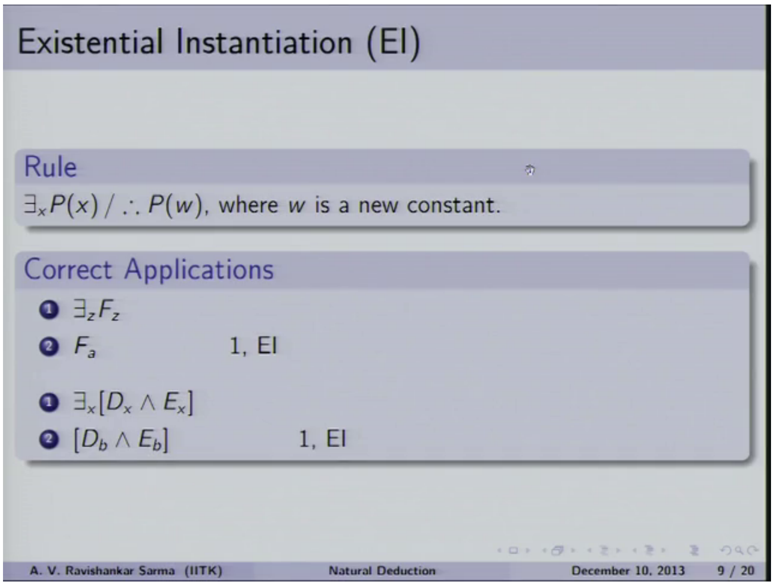

In the same way, you can see what is considered to be correct application of this rule of existential instantiation and even existential generalization etcetera. So, this rule tells us that existential instantiation for all x exists some x P x, from that you can specifically say something is having some kind of property. So, each time see reply this particular kind of thing, then each time they will replace this quantify that, the term as to be it has to be constituents. So, correct application of this truly is there exists some x some z, you simply falsity with f a and them exists from some D x and E x, this simply you are pressing with letter b D x and E x become D b and E b, it is a universal instantiation.

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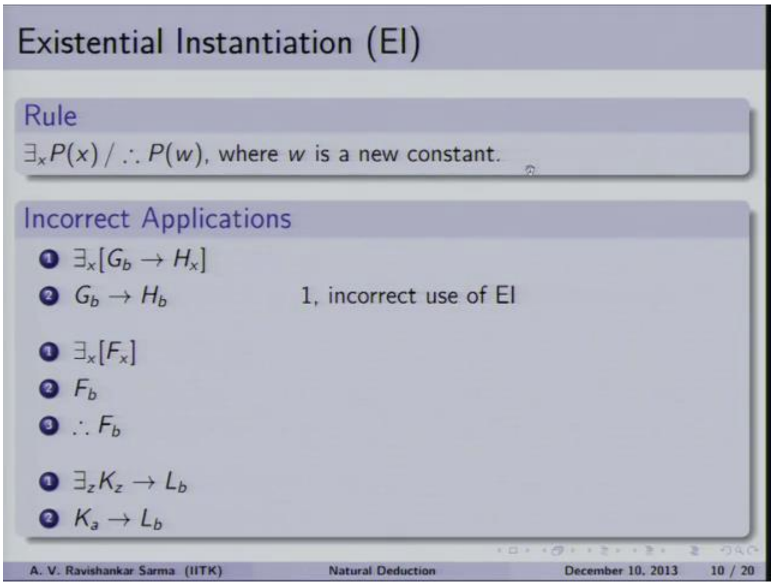

Now existential instantiation there exists some x d x p w and they are these are consider to be the incorrect application of this rules. There exists some next G b and H x and then your replacing with x with b, only second part is applied we applied this particular kind of rule so that will not work here. So, only to the part see cant applied this particular kind of rule. If you apply existential instantiation, you have to apply through out of this particular kind of formula. In the same there exists some x F x F d, F d is formula is already found that.

Now once you remove there exists some x F x in the in the sequence of this particular kind of thing 1. You already have F b here; the b is already the parameter b is already used in 2. So, when your removing some exists some x f x, you have to choose another parameter. That means, it has to be F c rather than F b.

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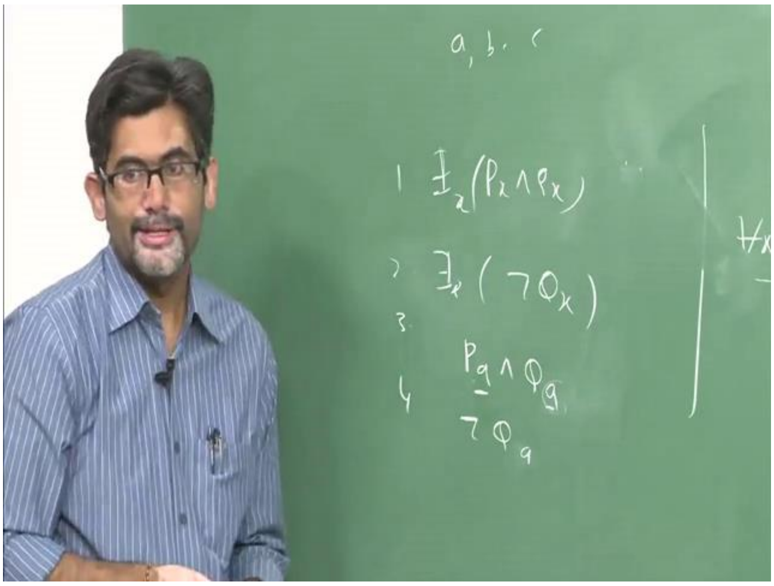

So for example, if you have in the proof we have this particular kind of thing there are 2 different quantifies there example. There exists some x P x and Q x, then exists some x not Q x something like that. So, now, first in menu remove this chose 1 particular kind of parameters this are a b c r the individual consists shall also called as parameters. First in remove this thing used P a and Q a there is consider to 1 instant of this particular kind of thing. And the second time will you remove this particular kind of thing, when you instantiated you have to choose another formula and which is other than is a.

Now this is to going to become Q d or any other we can use any other letter, other than whatever letter that you have to used in the earlier step of here proof. Suppose if you are used the same kind of parameter a, then that is consider to be incorrect application, now this existential instantiation true.

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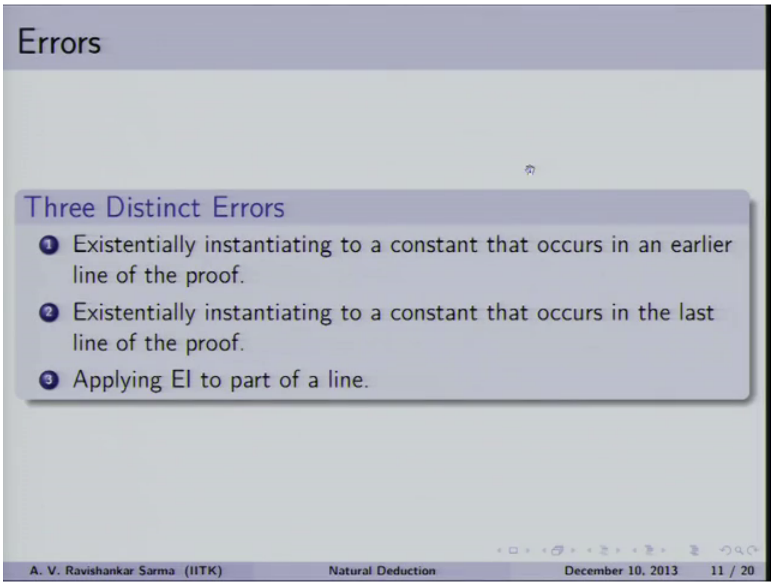

So, there is there are 3 errors that are possible here that is, existentially instantiating to a constant that occurs in the earlier line of the proof. If use a same kind of parameters, then that least to error. And the second 1, second way this errors reserves in is like this existentially instantiating to a constant, it occurs in the last line of the proof are third 1 applying existentially instantiating rule to only part of a line, but also leads to error.

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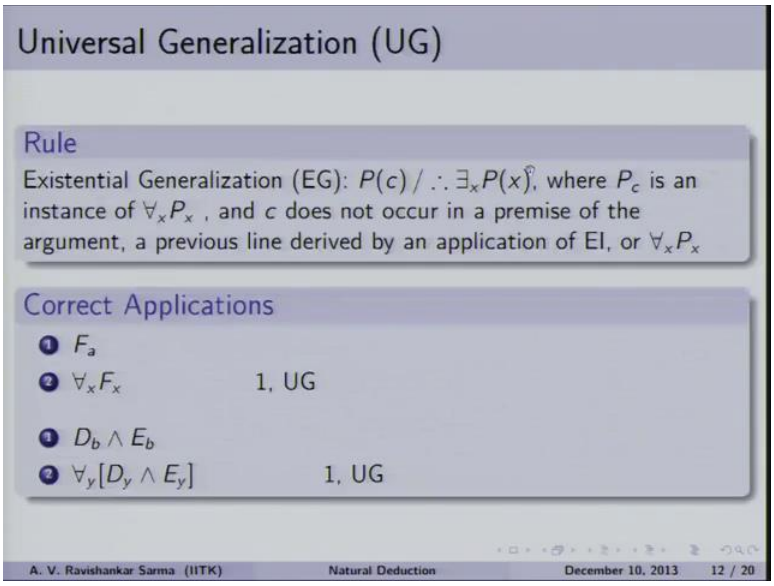

So, now, coming back to coming to the fourth rule; universal generalization rule. So, we have piece P v and then we generalize it and say that for all v P b were P c is an instants of rule x P x. So, now, correct application of this rule are like this F a generalize it, and then we say that for all x F x. Somebody is motor means only with some kind of restriction whose particular kind of thing. So, D b and E d and then correct application of this role is for all b y and v y.

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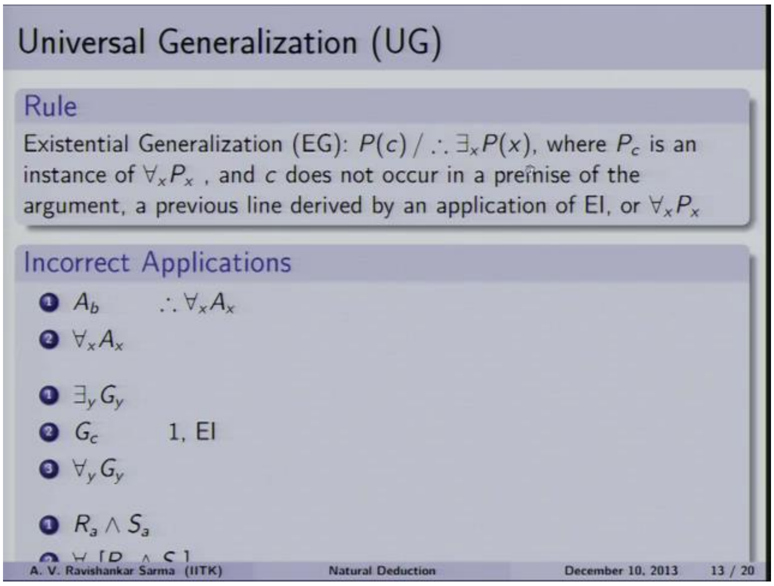

Incorrect applications are like this. So, above the role should be treated as universal generalization, I have return existence generalization; that means, to be corrected here. The rule needs to be red like this; P v are implies for all x P x.

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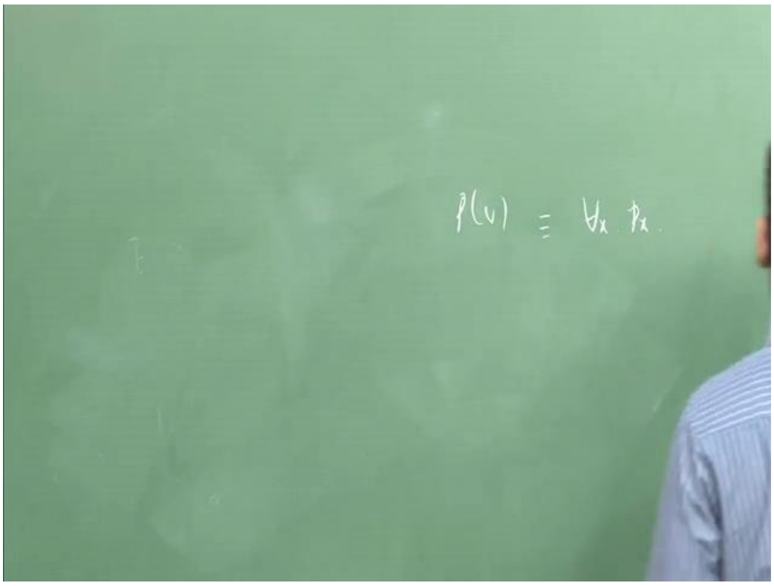

The rule should be red like this on instants of this 1 is for all x P x. So, forget about the rule which is stated about. So, the in connect are like this; suppose if have A formula a b means something apply only is some kind of specific situation. Then you can generalized if say that for all x P x. All animals for examples if cats, dogs have some 4 legs know, you cannot say them all animals have 4 legs and all, it may be some animals which may be have in 2 legs, may be 1 leg etcetera and all. So, we can generalize and say for all x A x this is incorrect application of this rule. In the same way there exists some y g y g c. And then you will generalize it and say that for all y g y. That is also considered to be an incorrect application of this particular kind of rule.

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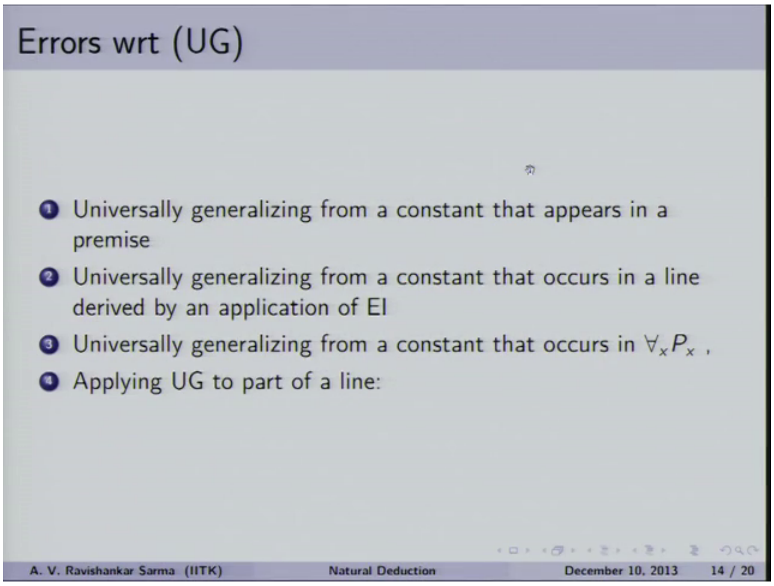

So, now errors with respect to universal generations are like this. Universally generalizing suppose if you universally generalizing from a constituent that appears in the premise, are it may results in because of this universally generalizing from a instant that occurs in a line, derived earlier by an application of existential instantiation, that is a role number 2; incorrect use of role number 2, which we have seen earlier in the ... Third 1; universally generalizing from a constituent that occurs in for all x P x. And then fourth 1 applying universal generalization to only part of the line, rather than full line full formula, we need to apply universal generalization, if you do not apply that is also considered to be incorrect application of this row.

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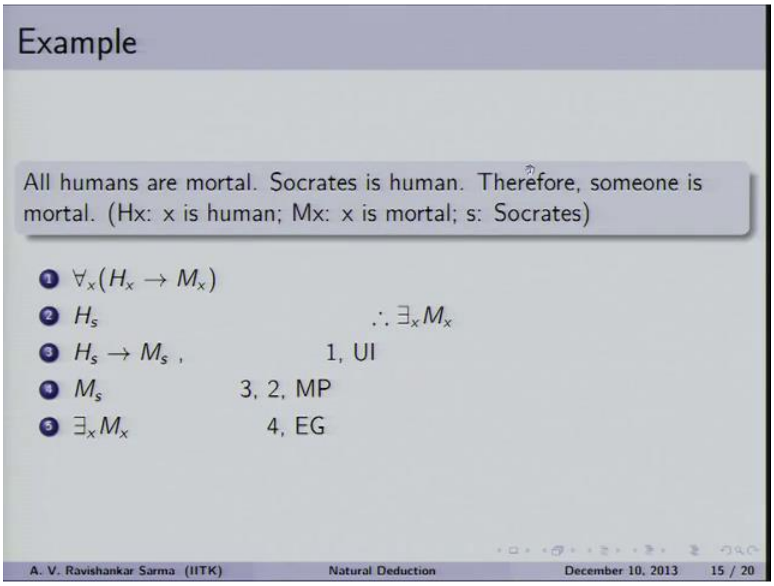

So, far you have discussed something about the rules. Now we have apply this rules to some kind of examples. To start with be use in simple kind of example, all the logic courses begin with this the particular kind of example. All humans are mortals are sub credits in human, therefore, you trying to derive some human considering mortal. So, here H x represents x is human, M x represents x is mortal and then someone is considerable soc rates. So, how do we reduce this particular kind of thing; using these rules.

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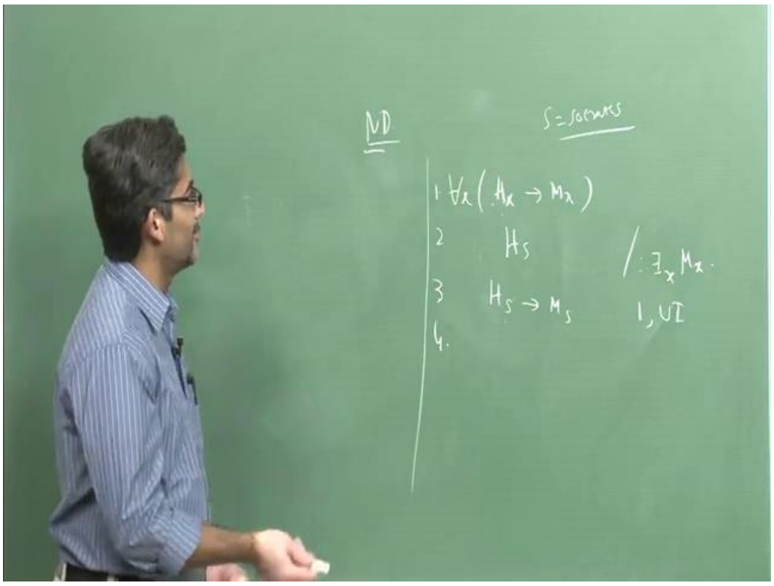

So, now, as a first step all humans are mortal for all x H x implies M x are evens are mortal. And second 1 is H x there exists some persons, so grades it is consider to be human being. And now from this we need to dues recessing; there exists some x that the particular kind of x is consider to be mortal. So, now, how to be used natural deduction methods is solved this particular kind of problem. So, now, we need to this only the 2 premises and then these 2 premises these need to these 2 promises together with rules at we have discussed, then here should need to the conclusion. That is 1 way of curving this particular of thing. That is called as direct prove.

There is another kind of method which we can use. So, we can negate conclusion and see whether which results in a contradiction or not. So, that is considered to be redox add axiom kind of proof, this is consider to be indirect proof. You will see both the proofs now. So, now, these are the things are given to a 1 2 reliable this 1 and 2. So, now, so this is H x implies M x happens for all x 1, instants of that 1 is like this. You take s he replace h with s that means, one particular kind of individual s is consider to be. So, crates are any 1 in human being. So, now this m1y instant have this is like this h s are M s solved get. This 1 you have used universally instantiation proof.

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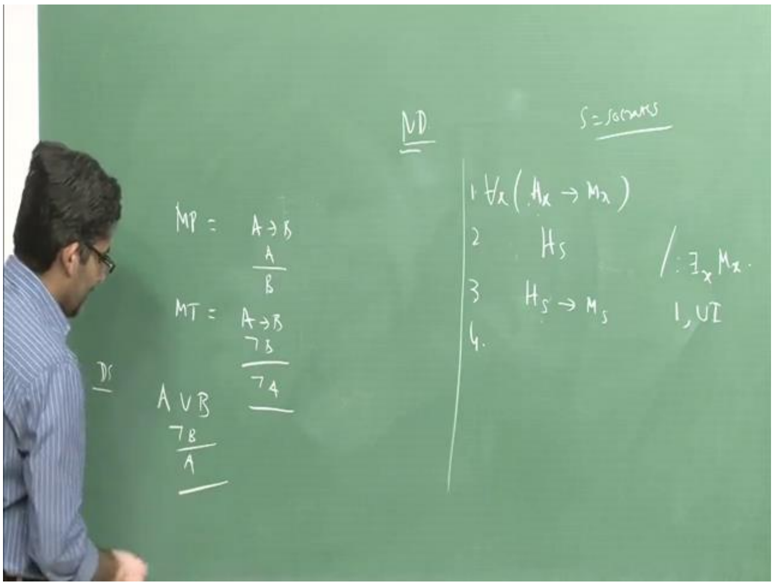

Now fourth 1 we use all the basic principles that we have already use in the contest of prepositional logic, like mode penance, mode tolence always things which we will be précising. Modo ponance is simply like this A implies B and then A and then results in B, in mode tolerance A plus B not B and usually you will denied the ancient as well as. And then there are kind of other rules which are frequently used, the list of rules which have we which have already menti1d it in the contest of propositional logic, when I discussed natural deduction in the contest of prepositional logic, I discussed several rules. Just I am writing very few of this rules which frequently occurred.

If you have A have B and it will not B and A etcetera, there are some lots of other rule and all, there are considered to be valid principles in the sense that, the conclusion is follows from the premises. So, now, coming back this example; so now, H s H s implies M s are now 2 or 3 modosponance we lead to M s. So, now, this is what not exactly we are supposed to prove, but we need to prove this thing; there exists some s M s.

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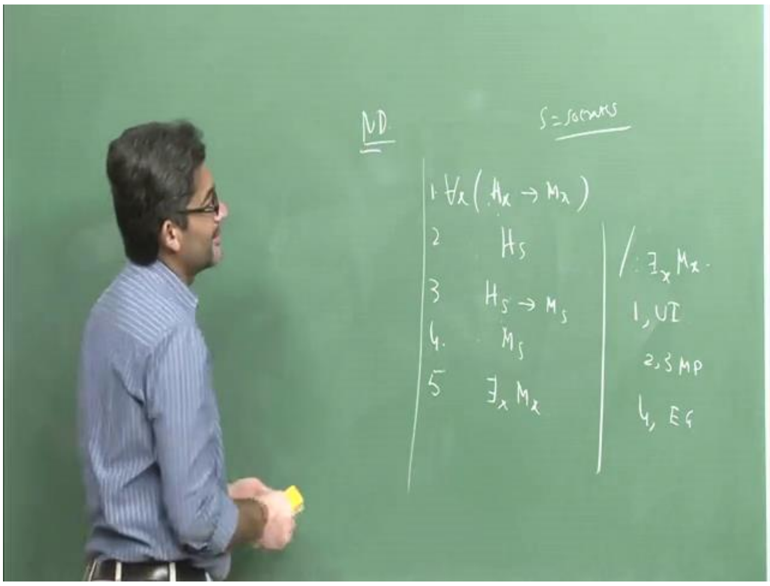

So, s is having some kind of property M, that means, at least 1 or 1 object is in your domain is satisfiable having this particular kind of property t. So; that means, you can use 4 existential generalization and you can say that there exists some x M x. So, how would be get to this 1? Suppose if you say that this chock is white in color, you can generalize it and say that there exists some x. So, there is a x is considered to be white in color, is nothing in wrong in saying that particular kind of thing, will satisfy this particular kind of property. There exists some x F x is going to be true if at least 1 object is having that particular kind of property f r g.

So, now, this is what we got it now for all H x M x H x and these things we got what we wanted, there exists some x M x. So, 1 thing which need to know this is that, we need to write justification here, immediately following your sequence of your proof, otherwise it does not make any sense to talk about, it does not make any sense to talk about what we called it has correct proof. If it has to be correct and rigors proofs, it has be find justification. So, usually we get it on the right hand side, extreme right hand side of each tom in your proofs. So, now, this is the direct method, there is another way which you can proof this particular kind of thing.

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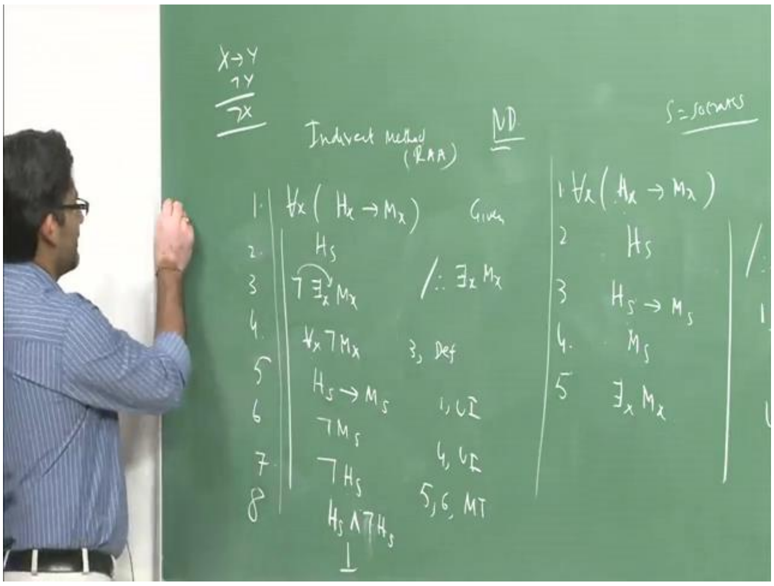

So, that is indirect method that is reduction and oxedo. So, what you we will do here is that, you take the premises and consideration; you take the negation of this 1 and then see whether it is to contradiction and not show. So, for all x that is looking for a counter example, were the premises are true and the conclusion is falls. So, instant looking for the validity, what you are trying to do is here ruling out instances which are considered to be invalid. So, when you say that argument in a invalid, in that true premises and falls conclusions. When you can cook of some example, were you have true promises at a falls conclusion in a; obviously, that arguments is invalid. If you rule out all these cases have been invalid thing and all; obviously, under with validity.

So, now, H s saying this is same thing like this, this is what is given to you premises. And second 1 you same thing, third 1 this is a conclusion separated by a n epic, there is a some x M x. So, now, what will you do is look for the countering sample, assume that the 2 promises are true and then the conclusion is falls M x. Then this leads to, you can directly apply existential instantly rule and say that this M b and all this is wrong. So, first you need to transform into the appropriate form that is, not of there exists some x M x leads to this in for all x this negation goes inside and this will be like this. So, this is 3 by definition. So, this leads to this 1.

So, now, 1 strategy of again in the natural deduction is that, first you deal with existential quantifies and then you move to the universal quantifies. So, there are no existential quantifies here. So, now, we need to insistance of this 1 and then we can find out contradiction and this 1. So now 1 instant of this 1 is like this; H a implies M a. So, you can take a s also, but I am taking into consideration a does not make any this differential.

So, here makes difference because, we have used s here. So, we use capital letter stands for the predicate and s for the individual objects here s refers to so critics. So, 1 instant of 1 is this. Another instants of this 1 licks not a let for wholes for all x, that means, it might be true even for scrotes also. Now how did we get this 1 5 1 universal instantiation and then 4 universal instantiation, we got this particular kind of thing.

Now 7; we have a rule which have be discussed yes now x implies y and not y implies not x. So, this is x implies y and not of these things, denied of the consequent, we should denied of the anticipate. So, now, this is H s, how would be get this 1 5 and 6 modes tools. So, this is a rule that we have used here. So, now, observe this draw align like this and then in the eighth step what we got is we have H s here and we not H s here. So that means, so crates it is human and them so crate is not human, that is what we got if you denied is particular kind of conclusion. So, now on this will drawn alive this, you sat that using deduct and appoxidiam method, what you got is contradiction because, H s and not H s is contradictory to each other.

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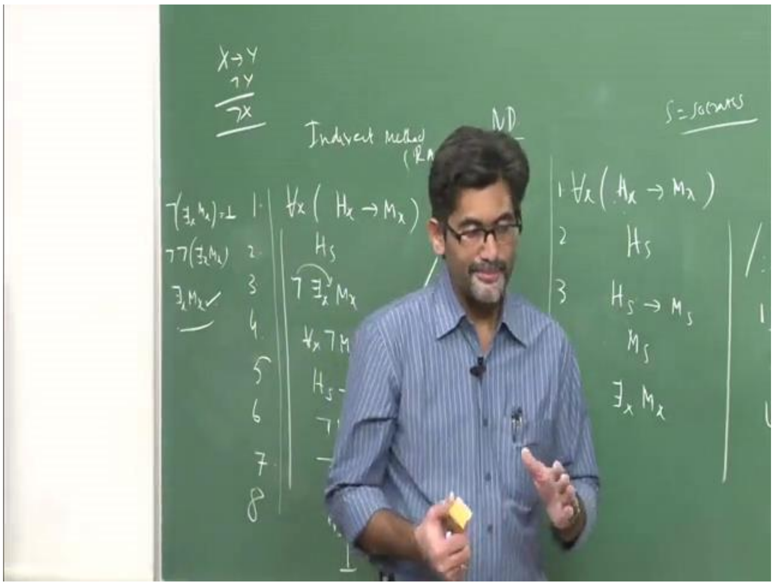

So, now, denied of the conclusion leads to contradiction; that means, a this is what in unsatisfiable, there in the exists some M x leads to this 1; that means, the actual thing that has to be true is not of not of there exists some x M x; that means, there exists some x M x has to be true, this is consider with the original conclusion. So, what essential we have done here exists that simple example, then we have applied natural the principles of natural deduction and when then we showed that, the conclusion follows from the promises; that means, argument is considered to be invalid argument, by using the both direct method and indirect method.

Similar is method is considered to be sometimes, it would be more affects and the sense that suppose will arguments is invalid, then you keep on a applying rules and all in a end with because, it is an invalid formula, you will never able to derive that particular kind of formula and all. So, indirect method will come to or rescues to any many occasions, indirect method is a 1 which an is often widely used and all. But moralist the both the proofs are have or make or making use of this rules, universal instantiation existence instantiate etcetera. So, let us consider some more proofs so that in uncertainty will understand this thing from line better way, this avail go through this proof.

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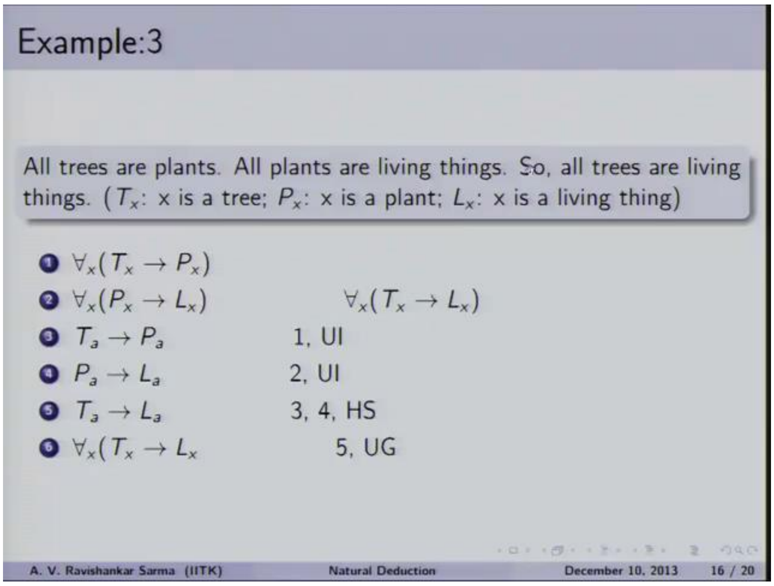

These are the things which may have already use in the contest of theory philoism. Aristotle come of a this thereof philoism, were all the sentences begins with some kind of special kind of prepositions, which are consider to be categorical prepositions. So, they all begin with all some n1 etcetera. There also like this only, so but not all the sentences is can be that particular kind of form; the sets limit to Aristotle theory silvisim, but in the predicate logic 1 plus can express relations all these things, when a better way. So, we can our come some problems which we have faced in a constant of Aristotle theory of silvisum.

Now, let us consider this particular kind of things and all. All trees are plants, all plants are living things. So, all trees are living things, it is simple same the movement is see this particular kind of formula it is clear that, some kind of u properties P x implies P x and P x implies l x and then; obviously, T x has to be L x. So, now, what we have done is we have yesterday all the truth promises; all x T x implies P x, for all P x implies L x and then the conclusion is all x P x implies L x.

So, now, 1st you apply this universal instantiation rule for number 1, then let us become P a implies P a. Now next time again here universal suggestion rule and to that is for all x P x implies L x, since P x implies L x studies for all x. So, you can use the same parameters, but if you have different quantify here then you are to use different kind of thing. First we need to handle that thing and then move to universal kind of quantify.

So, now, if you apply universal installation, it has become P a and plus L a and now the used again the valid principles of logic in the contest of natural deduction, then there is a rule called as a x implies y y implies z and x implies z and all it implies. This is this rule as called as hypothetical siloism, there is what we have used in the fifth step. So, once we have in the particular kind of thing, T a in plus L a which we did not come across with an application of existential with a elimination of existential quantify, but we got it as an instance of universal statements with a universal generalization.

So, in that contest T a implies L a can be generalized and you can say that, for all x T x implies L x, there is perversely what we wanted to prove. Same thing can be proved by using indirect method as well. So, what you do is for all x P x implies P x for all a P x implies L x, you destroyed the same thing and you negate the conclusion and then we conceal whether, it leads to contradiction and not.

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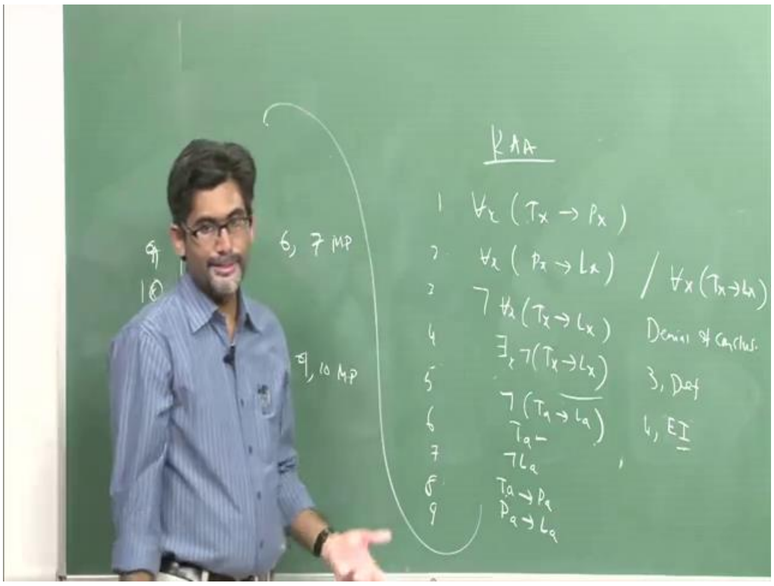

So, let us consider particular kind of thing and we apply indirect method on this particular kind of thing and then see whether it follows or not. So, what we have is like this; for all x T x all trees are plants and then for all x P x what is that here for all x P x L x or living beings then for all x T x, T x implies L x. So, what we have trying to do is we are applying redactor add oxide method, which is consider to be the indirect method. So, now, what you here is; now you denied the conclusion T x implies L x. So, this is denied law conclusion denied of conclusion.

So, now, I will fourth step, what to do is; we have some rules for all x P x is same as there exists some x not P x. In same way, there exists some x P x is same way there exists some x P x is same as not for all x not P x. So, now, this will become there exists some x not of T x implies L x. So, now, this is 3 by definition. So, now, we need to look for quantifies. This statements starts with existential quantify. First you handle the particular strength end of thing, you eliminate this 1 and then we remove to particular kind of thing T a implies L a were x is replaced by a. So, this is what we have and then you furthers implies then you to become T a and not L a.

So, now, we need to talk about universal, this is 4 existential initialization, when we need to write justification here, otherwise it as make any sense. Nobody will understand what we have done here, if you do not write the justification for this 1 here on the right hand side. So, now, 8 define 1 instants of this 1 anyone of this you can handle now. So, 1 instant of this 1 is; since it happens for all x it happens for even a also T a implies P a and then in the second case, for all x P x implies L x, that means, 1 instant of this 1 can be P a implies L a.

So, now, what we have is like this; this z1 like this P a not L a now, you are not supposed use this particular kind of thing. So, now, you have T a and P a implies T a, that means, 6 and 7 modosponance we will get you have T a here and T a implies P a. So, you will get P a. So, now, and we have P a implies L a in the eighth step. So, now, these 2 modosponance you will get L a, so 7. Ninth, tenth for example, 9 and 10 modosponance, we will get this 1. So, now, you are observe here in proof, what is a proof first of all a proof is a sequence of steps and all, if ends in intervals of time.

So, the each step is consider to true and all. So, the final step is also consider to be truth.

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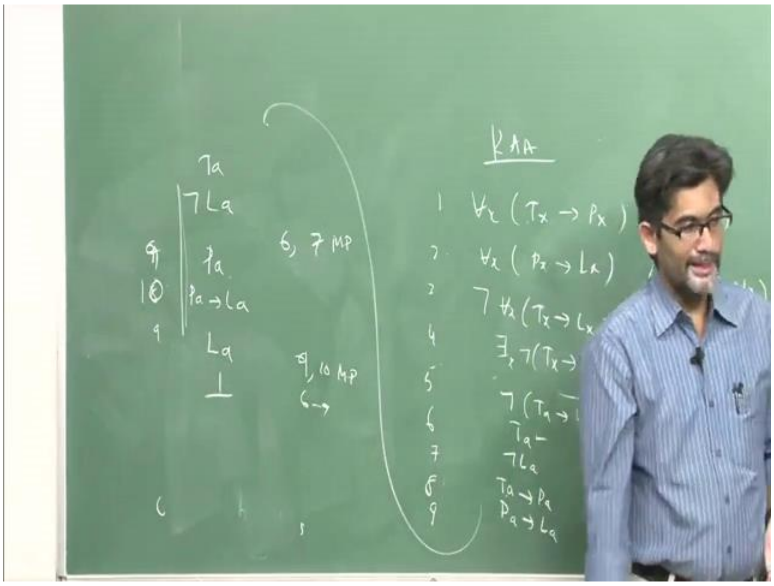

So, now, you have L a here and not L a here. So, you draw a line like this and then from whatever it is 6 to 9, it relate to contradiction. So, how did we end of the contradiction is; we denied this conclusion, if you denied the conclusion, we end of the contradiction. Suppose if it has not denied is they did not let to contradiction, let means negation of conclusion these to unsatisfiablility, unsatisfiablility is the sense that, it relates to contradiction. So, in that sense the original conclusion is the 1 which holds. So, like this simple examples with which 1 can solve this particular kind of thing.

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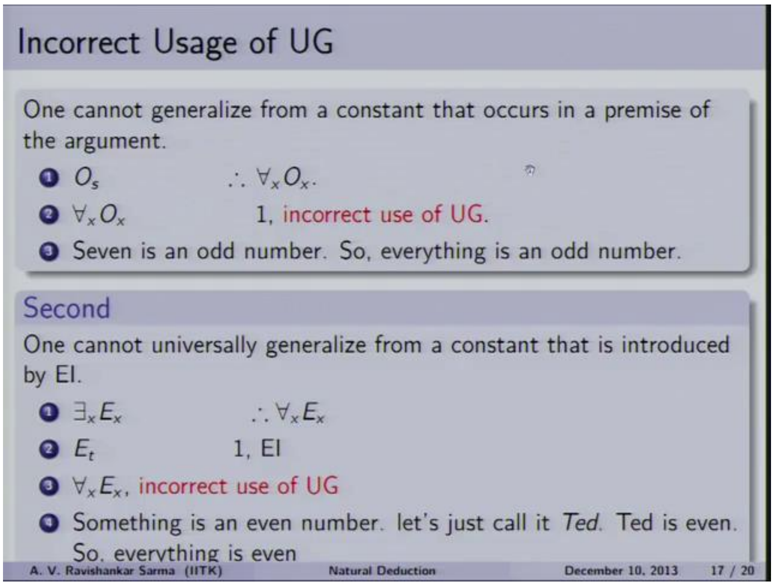

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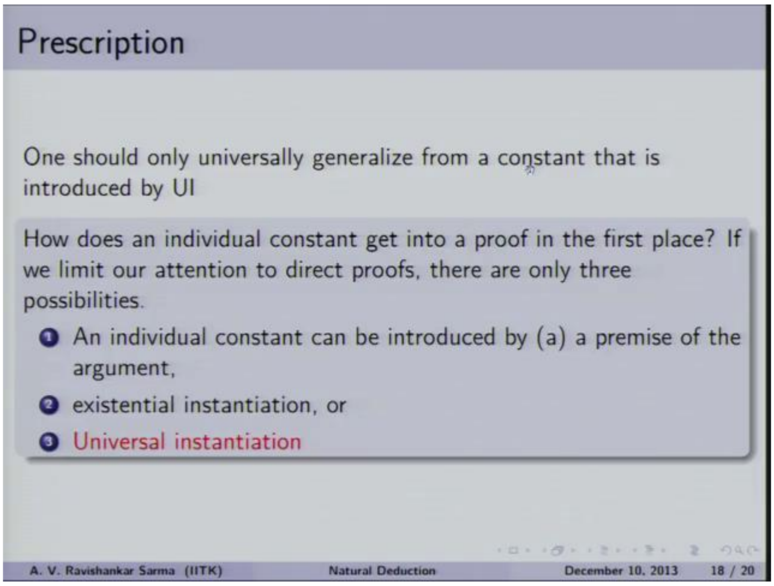

But one can if use of for the complicit cases also, 1 can use this natural deductions method and 1 can solve the problems. So, there is 1 particular kind of restriction 1 uses 1 have make use of it as a strategy that is, like this. Once you would only universally generalization from a constituents that is, introduced by universal installation. For example, if you have this is 1 of the important strategies have that we need to use.

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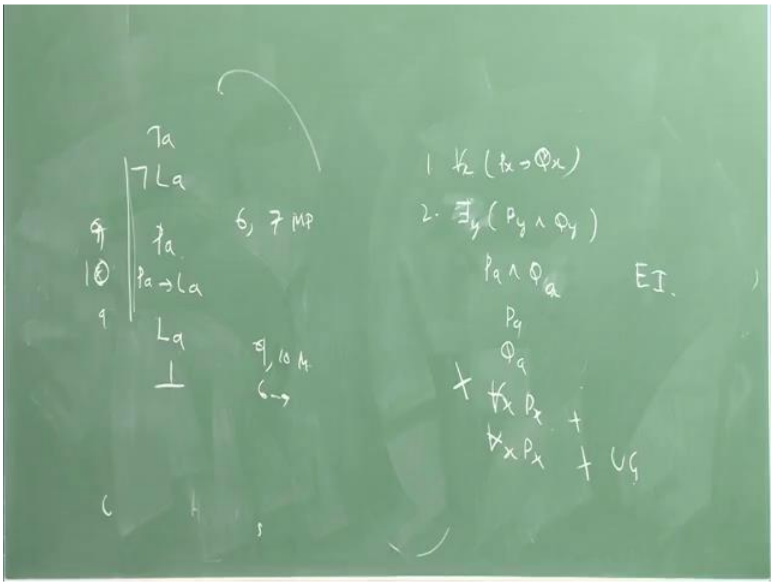

So, let us say for example, you have a formula like this for all x P x implies Q x for all y there exists some y P y and Q y example these are the 2 formulas let us there in your proofs. Just we are trying to talk about some kind of strategies so that, you can make use of this rules correctly. So, now, 1 instant of this 1 could be like this; P a implies Q a etcetera. And another instants of this formula for example, if you say it is P b and first usually, we handle this and existential quantify. So, let us say this is P a and Q y. So, this is an instants of P a and Q a existential instantiation.

So, now, this can be return has P a and Q a. So, now, strategy tells us that, if you got this formula out of existential instantiation, you cannot generalize it and say that it is for all x P x. So, this is wrong. In the same way you cannot generalize Q a and then say that, P x by using this particular kind of rule. This is a incorrect application of rule because, it you got this P a and Q a of out of the existential instantiation, rather than the universal instantiation. You need to apply this universal generalization, only when you come across you came across that particular kind of instance to the application of universal instantiation rule, otherwise you are not supposed to do it.

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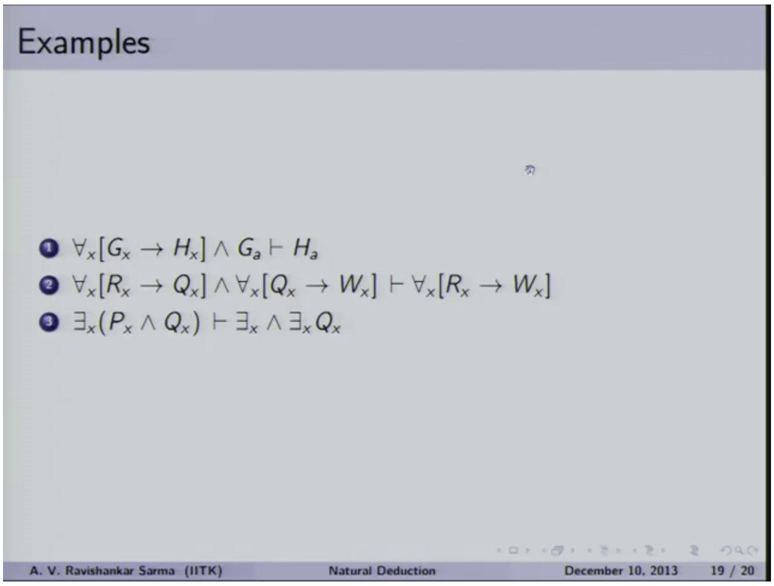

So, how does an individual constant get in to the proof in the first place if you limit or attention to direct proofs. There are only 3 possibilities that is, an individual constant can be introduced in the premise of the argument. That means, a universal instantiation you will you might do it are you night apply existential instantiation in come across that particular kind of thing. The third 1 is universal instantiation. That is what you are done in the first step.

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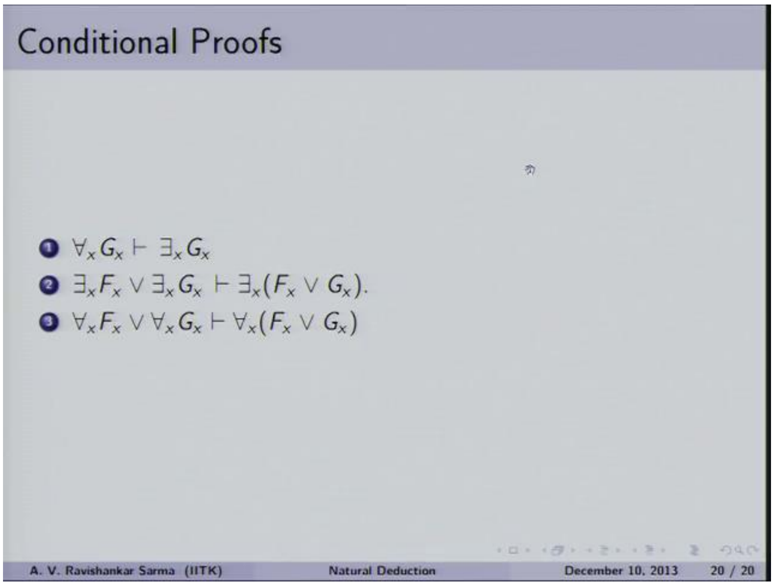

So, several the some other examples conflicts examples 1 can take in to the consideration and then you applied this particular kind of rules and you can reduces the particular kind of theorems. So, 1 can use both the direct and indirect proofs to handles particular kind of situation. We will end of with 1 simple example, then will see will end this lecture.

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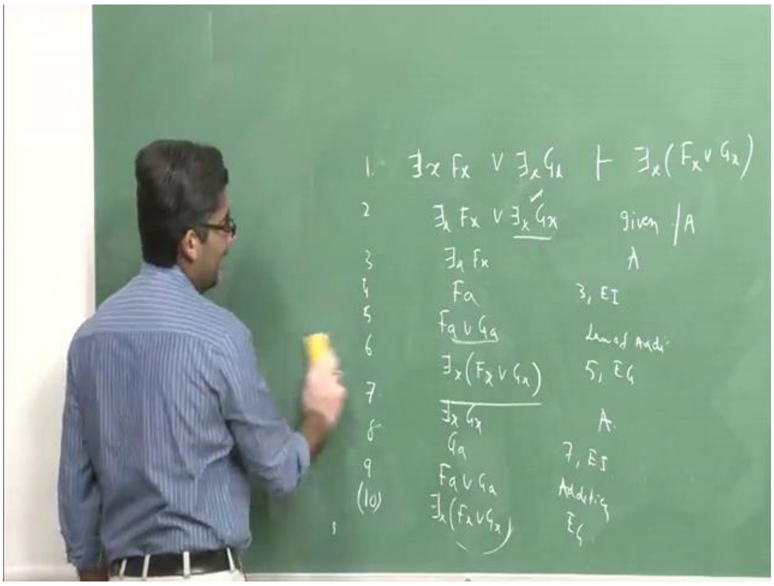

So, we talk to about distribution of quantifies, with respect to convention. So, we know that this particular thing holds that, there exists some x F x are there exists some x G x and some this get F x or G x. So, this is what we are trying to prove. Again 1 can use any 1 of this methods, you can derived this particular kind of conclusion and then start constructing using the universal instantiation etcetera, all these rules, then you will come of this contradiction and you say that negation of this 1 leads contradiction, hence negation of negation of this formula is going to be the case. That means; the actual consequent or image.

So, now, let us consider this proof of this 1 quickly and then you will and this lecture. So, we have to solve many problems to get a outside familiarize with this particular kind of technique. So, in that contest, I could only discussed how to judiciously use this particular kind of rules and when it comes to the applications in particular, by solving the problems, that that is going to help us.

So, now, you list out this thing for all x for all G x is what is given to us, this is given or you can write it has assumption etcetera. So, now, from this if you take onto consideration, there exists some x F x, just 1 part of it you take into consideration that is also consider to be an assumption. Now fourth 1 just certainly write it all. So, now, 1 instants of this 1 is this; 1 3 existential instantiation is this. Now fifth step F a G a. So, how did we do this thing, there is something called law of edition. Since F a already true, you can add anything true thing particular kind of thing. without disturbing truth value of that. So, the admins you can write F a r G.

So, now, it can apply existential generalization rule and this can be this holds for a at least 1 particular kind of situation. So, you can generalize it and say that some x or some x this is a case; that means, F x are G x. So, now this is what got it we take it a particular kind of thing, there exists some x F x, but you may if take of this particular kind of thing, you will get the same kind of preserving. So, this is what we have suppose to proof and then we proved it. But, this might this proof you might get come across this particular kind of thing, even by using by taking this particular kind of assumption also. We have either P or Q kind of situation. First you are take in this consideration, you proved this 1 and the same way you take if you take this also it should be a position to derive this particular kind of thing. So, that is, you take this particular kind of thing, there exists some x G x this is again assumption and all.

Now, this you take into consideration again, you will prove to the same thing and that hence, that gives the complete description of your proofs. So, now, 1 instant of this 1 is G a 7 existential instantiation. Now 9 again you can add same thing F a r G a, this is edition law of edition. So, like suppose if you have formula a, which is already true in law, then you can add A and B. Of course, use an commuted property and you can say that this same as B are A, this make any big difference. Actually it should be G a are F a and that is same as F a are G a.

So, now, since you got this 1, 1 instants of if you existential generalization rule, then this will become F x are G x. So, the way may if you take this into consideration as assumption, we could true this particular kind of thing, in the hence, there exists x F x there exists some x G x and you got this particular kind of thing. 1 final thing remark is exists that, you can use indirect proof also to solve this particular kind of property. So, that is likes this.

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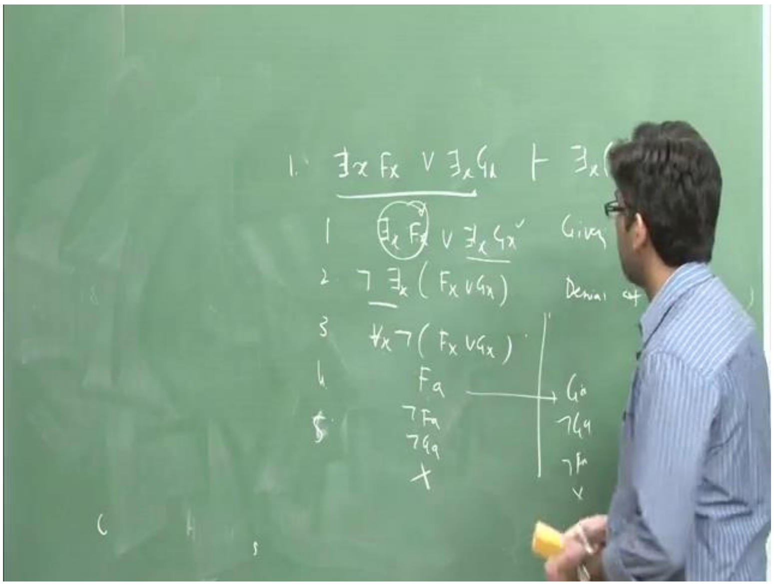

So, this is considered to be ancient and this is consider to be consequent. Now what you will do here is that, you list out premises like this there is a some x G x and now denied the consequent. And then you will end of with the contradiction F x are G x, this is of consequence and something like that, 1 2 this is given. So, now simplify this particular kind of thing, this will for all x not of F x are G x. Now first we need to handle the existential quantifies, then in more to universal quantifies.

Now once you replace this thing, you take this as your assumption there exists some x F x and this will become F of a, you request x with a. In the same way, contact there exists some x G x and then you start G a and you can find the proof of G. Now fifth 1 not of apex G x, 1 instant of that 1 is this thing not of F a negation of distinction is consumption. So, it will b not G, since you have F a and F a, it closes here itself. So, now, this is when you take this in to consideration.

Now you can take this also it consideration and the proof will be little bit different and now it will be G inter thing will be same, it will be G a not G a some not F a, something like that. So, now, G a and not G, it closes and all any 1 of thing. You take into consideration; it leads to branch closure. So, this is another way of proving the same formula, by using natural deduction method. Then you will stop here and then you will what essentially, we did in this lecture is simply this that, we discussed some of the important principles of natural deductions method, with respect to quantifies in the contest of predicate logic. We introduced universal quantify, we introduced universal generalization and universal instantiation and with respect to existential quantify, we introduce existential installation and existential generalization.

Then we discussed to different kinds of proofs 1 is considered to be conditional proof. Another 1 is based on a indirect proof which is called as redexio add append kind of method. So, this method is closer to human reasoning in a sense that, we are familiar we with modosponance modostolence etcetera and all, rather than proving formula. Now nobody usually, you do not prove particular kind of thing, we denied the formula and then see the contradiction etcetera and all.

So, in a sense natural deduction method comes closer to our human sense, human reasoning, but sometimes some other method might be knife is better than natural deduction method. So, 1 important method which unfortunately, not able to discuss in the contest of predicate logic, which is very essential in a contest of computer science that is, the method is called as resolution of reputation method. We discuss in particular kind of method is in the contest of prepositional logic, but lack of time will not able to deal with the resolution reputation method. So, in the next class, you will be dealing with some of the important theorems of first order logic. There will discussed about some of the important theorem such as, completeness, compactness and the celebrate results in the first order logic, that is in completeness scene.