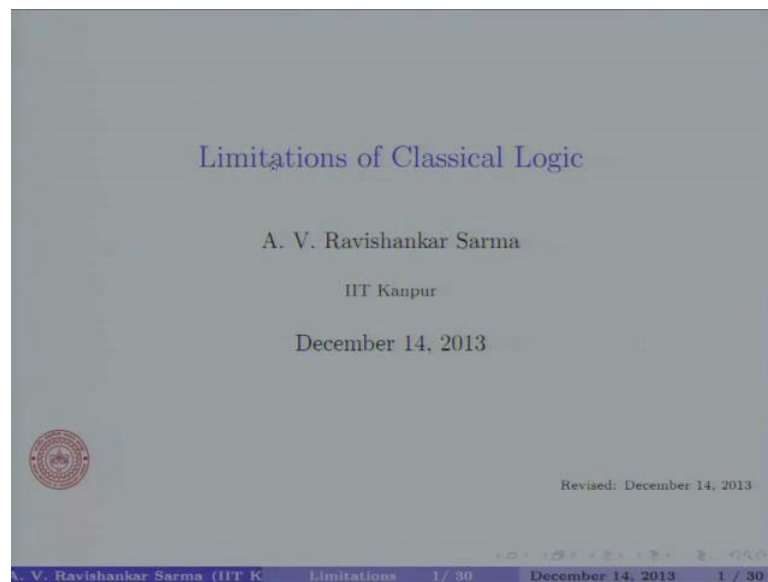


**Introduction to Logic**  
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**Lecture - 44**  
**Limitations of first order logic and introduction to the course**

Welcome back, in the last few lectures and the corresponding lectures we presented proposition logic and the predicate logic. Now, we have come to an important topic of discussion that is, we will be discussing the limitations of classical logic; classical logic we mean, predicate logic and the propositional logic.

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So, before that we have presented in formal logic and fallacies and then Aristotle. Logic etcetera, but mainly classical logic we mean propositional predicates logic. So, other question that we are trying to answer is, this is the case that classical logic explains all kinds of reasoning or it is restricted to just mathematical reasoning. We say that, it is restricted to mathematical reasoning, classical logic as far as possible, it tries to capture mathematical reasoning in a better way. So, in this lecture I will be talking about, some of the important limitations of classical logic in a very nutshell.

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Need for non-classical Logic

Classical Logic

- 1 Logic: A systematic study of *argumentation*, principles of *valid reasoning*.
- 2 Classical Logic (First order Logic): Propositional and Predicate Logic: Good starting point of study of reasoning.
- 3 Most appropriate for mathematical reasoning, it is bivalent (only two truth values), based on material implication.
- 4 CL is not appropriate for formalizing human reasoning.

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So, as we know Classical logic we mean, first order logic and logic is consider to be a systematic study of augmentation, principles of valid reasoning as a deductive or inductive of reasoning. Mostly we focused our attention on deductive reasoning. So, classical logic we mean first order logic that is what, we have presented so far. It is prepositional and predicate logic and they are consider to be a good starting point for the study of reasoning. And it is most appropriate for the mathematical reasoning and it is important property of this classical logic is this that. It is considered to be bivalent and then; that means, a sentence can be either true or false, it has only 2 values and it is based on the material implication.

The material implication  $A \text{ plus } B$  is defined as  $\text{not } A \text{ or } B$  it is introduced by. So, all the classical logic are on the material implication. So, now, classical logic is not appropriate for formalizing human reasoning. We will be seeing some examples where cannot be applied and all, where does not make to apply make sense to apply this classical logic. Classical logic fails to provide satisfactory account of the following things, one is conditionals statements. If all the conditional statements are simply expressed, as if  $p$  then  $q$  then, there will be some kind of problems that, we come across. We will talk about paradox of material implication etcetera. And then, we will see that not all conditions can be expressed in terms of simple.

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- ❶ Classical logic fails to provide satisfactory account of the following: conditionals, arguments involving possibility, necessity, logic of knowledge and belief, vagueness etc.
- ❷ Non-classical logics are developed to overcome several defects of classical logic.
- ❸ Classical logics obeys transitivity, property of bivalence, and monotonic. But common sense reasoning is non-monotonic.

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So, before that we have presented in formal logic and fallacies and then Aristotle and logic, of knowledge belief vagueness etcetera. They all present numerous problems the classical logic; that means, we need to move beyond the classical logics and need to talk about some other kinds of logic. So, non classical needs are basically developed to overcome these defects in the classical logic; that means, what essentially happens here is that, classical logic is based on some fundamental laws of logic. You have to withdraw at least 1 of these fundamental laws of logic.

So, usually classical logic obeys transitivity property of bivalence; that means, it has own 2 values and it is considered to be more atonics in a sense that, even additional of new premises will not lead to the withdrawal of conclusion that you have drawn earlier. And the problem here is that, but mostly the common sense reasoning that we usually employ in day to day discos, is considered to be Non-monotonic; that means, additional of new information, you need to withdraw the conclusion that you have drawn earlier.

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Two reasons for doing non-classical Logic

- 1 Classical logic is of no help to represent **intensional concepts** like modality and time. But, these notions are pervasive in common sense reasoning.
- 2 Human knowledge may be **incomplete**, and **inconsistent**. Classical Logic cannot express incomplete and inconsistent information.
- 3 It Fails to explain **vagueness**, which is part and parcel of our life.

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A classical logic; obviously, will be having no help to represent intentional concepts like, modality and time, these are considered to be intentional, but in this course we have focused our attention only on extensional kind of concepts; that means, the truth value of a sentence is solely determined by truth value of this individual constitutes. Then we say that, the logical operator is considered to be extensive in nature. If the truth value of a compound formula is not solely determined by as a truth value of its individual constitutes, then it is called as intention.

Human knowledge is usually a the incomplete and; obviously, it can be inconsistent withdraw lot of inferences based on, incomplete information did not wait for the perfect information to come by, we base our decisions on incomplete and inconsistent kind of information and classical logic cannot express this incomplete and inconsistent kind of information. We are not charging this classical logic, but basically classical logic is a starting point and it is used for capturing the mathematical reasoning. And it also fails to explain, the concepts which are related to vagueness and as we know, vagueness is considered to be important parcel of our life.

Most of the time, we will be using expressions English languages are obviously, considered to be vague, which consists of vague predicates such as tall, rich, poor etcetera all. These things judgment about these things, inferences based on these things will be purely based on our observations rather than, the principles logic etcetera.

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The slide is titled "Non-classical Logic:" and is divided into two main sections. The first section, "Complementary Logics", lists two categories: 1. Modal Logics: Tesnse Logic, Epistemic Logic, doxastic logic, Deontic logic, Dynamic Logic, Conditional Logic, Intensional Logic. 2. Modal Logic: Normal and non normal Modal Logic. The second section, "Deviant Logic", lists Intuitionistic logic, para-consistent Logic, and Many-valued Logic (Fuzzy Logic). At the bottom of the slide, there is a footer with the text: "A. V. Ravishankar Sarma (IIT K. Limitations 5 / 30 December 14, 2013 5 / 30".

Now, Non-classical logics have emerged as the response to this particular kind of defects in the classical logic. Their number of logics which are, which exists there these are Modal logics Deontic logics, it talks about time epistemic logic is logic of knowledge and belief doxastic logic logic of belief deontic logic talks about logic of obligation forbidden etcetera and all. Dynamic logic, conditional logic, intentional logic and other lots of names which are given to these logics and all.

So, depending upon the usage to capture some of the interesting inferences will be making use of these different kinds of logics. And we have; on the other hand, these are all extensions of classical logic. It is not, enough that only classical logic explains all kinds of situations and all, but logic has to be applied to our day to day discos as well. It just did not have to apply to devils machines etcetera and all or computers etcetera well it has to applied to our day to day discos.

So, in that process we will be either extending the classical logic or we will be deviating from the classical logic. Deviating from a classical logic in a sense that, we have 3 fundamental laws of logic. Law of identity, Law of excluded middle, Law of non-contradiction. If at least you deviate from 1 of these things; that means, you draw 1 of this fundamental law of logic and you are doing deviant kinds of logic. So, intuitionistic logic where law of excluded middle did not bein your, in that logical system then it is intuitionistic logic.

If you drop the law of non-contradiction, you are dealing with para-consistent logic and you are allowing for many values rather than T and F we are talking about, many value logics and 1 instance of many value logic is the fuzzy logic. The father the founding father of fuzzy logic is, of the view that doing classical logic is like coming to the party with formal dress, shoes, tie, hat everything and all. So, that is what is considered to be doing formal logic. And doing fuzzy logics or non-classic logics is like, going to the party with simply T-shirt, jeans slippers some kind of comfortable dress etcetera and all or short hair something like that.

So, that is what we prefer mostly. So, this way nicely puts this analogy into the context and where we distinguish classical and non-classical logics; that means, non classical logics is fuzzy logics.

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Basic Principles of Logic

- 1 Law of Identity:  $P$  is  $P$
- 2 Law of Excluded Middle, Principle of Bivalence:  $P \vee \neg P$ .
- 3 Law of non- contradiction.  $\neg(P \wedge \neg P)$ .  $P$  cannot be both  $P$  and  $\neg P$  at the same time and the same sense.  
A contradiction occurs when one statement excludes the possibility of another and yet both are claimed to be true. Truth is not self-contradictory

3 Three laws: Foundation for mathematical, physical, and rational thinking

Are all these laws complete?  
A variety of arguments can easily be produced to show that these laws are incomplete: i.e., they do not specify all reality, for parts of reality can be shown to contradict one or more of Aristotle's laws.

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So, we will be talking about, some of the basic principles of logic they are like this. Law of identity, which is stated as P is P law of excluded middle, either P is a case or not P is a case. And law of contradiction it is not the case that, something is simultaneously true and false, that is law of non-contradiction a contradiction occurs when 1 statement excludes the possibility of another, but yet both are claimed to be true. So, truth is not self contradictory.

So, these are the 3 fundamental laws which define this classical logic. Now, this is the the 3 laws are considered to be foundation for mathematical physical and rational

thinking. Now, 1 question that we can ask is this thing or all this laws completely describes all kind of phenomenon or not. Now, a variety of arguments can easily be produced to show that, these laws are considered to be incomplete; that means, it applies to only static case when, it come comes to dynamic case it may not apply. They do not specify all kinds of reality, for parts of reality can be shown to contradict 1 or more of Aristotle laws.

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Heraclitus(500BC): The problem of change

Heraclitus pointed out that, for a thing to change, it must turn into something else, and then asked how a thing could be something other than itself?

You cannot step in to the same river twice.

- 1 if Aristotle's laws are taken to be all the fundamental laws of logic, then logically there can be no change whatsoever, because change negates all three laws. I.e., either change does not exist or it is totally illogical.
- 2 Since all measurements, detections, thoughts, and perceptions are simply changes, then it follows that these operations logically cannot exist.

<http://www.cheniere.org/books/aids/appendixIII.htm>

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Heraclitus in 500 BC is come up with this problem of change, is of the view that you cannot step into the same water twice, if you step into the water twice water goes away and then, when next time when you step into it will not the same water. So, everything a change is what, Buddha also talks about. So, Heraclitus pointed out that, for a thing to change it must stand out to be something else for example, if this is a duster, it looks like same even now or day after tomorrow also, it looks like let to be the case like that, but something is changing in this particular kind of duster.

So, if this duster changes means it must done into something else and then he asked how a thing could be something other than itself. If this duster changes to some other thing of course, this duster only with some other properties then, how can whatever is changed duster is same as the old duster see. It is in this context, you cannot step into the same water twice. So, now if Aristotle laws are taken to be all the fundamental laws of logic, then logically they can be no change what is. So, ever because the change negates all

these 3 laws for example, if; that means, either change does not exist or it is totally illogic. Illogic in the sense that, it is a it is violating all the 3 fundamental laws of logic, since all measurements detections thoughts, perceptions are simply considered to be changes that dynamic in nature whereas, Aristotle logics are simply appraised to in this static case, it follows that these operations logically cannot exist and then, in that context logicians they introduced.

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Modal Sentences

Examples

- 1 If it is necessarily the case that 2 is the smallest prime number, then 2 is the smallest prime number.
- 2 It is known that Ravi is richer than Ramesh, then Ravi is richer than Ramesh.
- 3 If it is morally obligatory that you love your neighbour, then you love love your neighbour. **Flase.**

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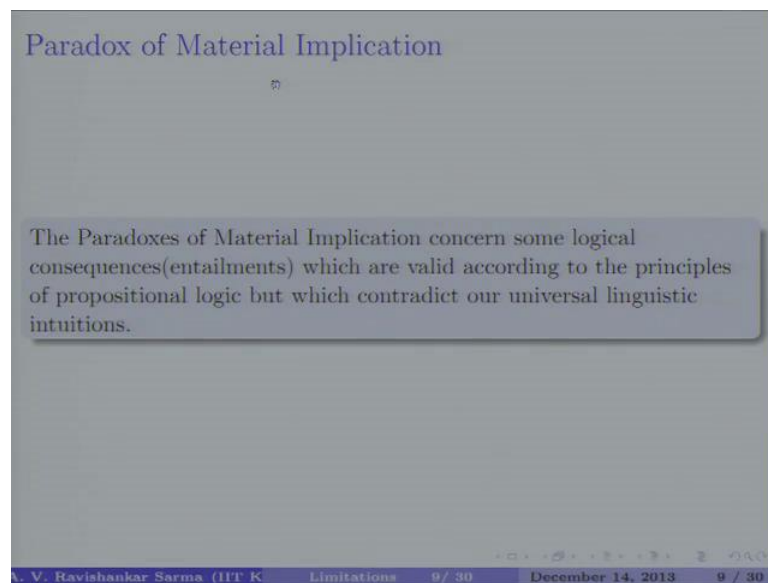
The forth fundamental laws of logic, if you aloof, if you accept the forth law then you need to deny the 3 laws of logic or if you accept the third law of logic then, it is explicit implicitly you will be the forth law will become implicit etcetera. So, what happens is that when, something changes and all E is not equal A, that is law of fails if that, fails then law of excluded middle also changes. And then of course, it has its consequence on law of non contradiction; that sense, how the 3 laws fails.

This is 1 problem with respect to listening and the second problem is when you are referring to modal sentences heat move away from the classical logics. 1 example could be, if it is necessary in the case that 2 is the smallest prime number then, 2 is the smallest number. You need to have some kind of modal operators to make it distinct from the actual operators. So, we need to maintain the difference between something is actually true, something is possibly true, something is necessarily true. This is what, we all the time use in the day to day discos.



We can say it is possible that, water exists on the moon; obviously, it is not necessary that it exists on the earth. In other hand, you might you will be saying that, it is necessary that 2 plus 2 is equal to 4. So, what do you mean by saying that, it is necessary that something is a case that it is possible that something is a case. And it is actually is a case that P is a case; actually the P is the case is referring to some kind of actual sentences. So, that is what we are interested in this particular kind of course.

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Now, we are said that this is based on first order logics, classical logic is based on paradox of first order logic is based on material implication. If we accept material implication in to consideration, it leads to some kind of problems which we called as paradox of material implication. It concerns some logical consequences which are; obviously, considered to be valid principles of prepositional logic, but they contradict to our universal linguistic intuitions. All these things which I have presented here are considered to be an instance of paradox of material implication.

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Instances of Paradox of Material Implication:

- 1  $p \vdash q \rightarrow p.$
- 2  $\neg q \models p \rightarrow q$
- 3  $p \rightarrow s \models (p \wedge q) \rightarrow s$
- 4  $(p \wedge \neg p) \rightarrow q$
- 5  $\models p \rightarrow (q \vee q)$
- 6  $\models p \rightarrow (q \rightarrow p)$
- 7  $p \wedge q \models p \rightarrow q$
- 8  $\models (p \rightarrow q) \vee (q \rightarrow p)$

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So, from p introduced q implies p and not q pm plus q etcetera all. This is list of implications which are for the classical logic, but when you make use of this kinds of inferences in the day to day discos then we generate some kind of a counterintuitive results.

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Examples:

Example  
 $\neg p$ : There is no oil in my coffee.  
 $q$ : I like it.

Example

- 1  $p$ : I will play football tomorrow.
- 2  $q$ : I break my leg today.

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Instances of Paradox of Material Implication:

- 1  $p \vdash q \rightarrow p.$
- 2  $\neg q \models p \rightarrow q$
- 3  $p \rightarrow s \models (p \wedge q) \rightarrow s$
- 4  $(p \wedge \neg p) \rightarrow q$
- 5  $\models p \rightarrow (q \vee q)$
- 6  $\models p \rightarrow (q \rightarrow p)$
- 7  $p \wedge q \models p \rightarrow q$
- 8  $\models (p \rightarrow q) \vee (q \rightarrow p)$

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Examples:

Example  
 $\neg p$ : There is no oil in my coffee.  
 $q$ : I like it.

Example

- 1  $p$ : I will play football tomorrow.
- 2  $q$ : I break my leg today.

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One simple example that, you can take into consideration let us assume that knot p stands for, there is no oil in my coffee, if there is oil in my coffee that is represent as p and not p represent there is no oil in my coffee and q means that I like it. So, now, you substituted into the inferences that we have here, not p plus q plus p then, this will become like this. There is no oil in my coffee implies that, something like there is there is oil in my coffee then I like it.

So, that is seems to be going against our intuition. So, in the same way if suppose p stands for I will play football tomorrow, q is I will break my leg today. So, for example, q implies p implies q; in that case, I will play football tomorrow from that, what follows is this thing. I will play tomorrow, if I break my leg today.

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Paradox of Material Implication

$$P \rightarrow Q \stackrel{\text{Def}}{=} \neg P \vee Q$$

- ❶  $P \models Q \rightarrow P$ : I am alive to if I am dead, I am alive
- ❷  $P \models \neg P \rightarrow Q$ . I'm alive to if I'm not alive, Im famous.
- ❸  $(P \wedge Q) \rightarrow R \models (P \rightarrow R) \wedge (Q \rightarrow R)$ .

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It goes completely against our intuition it is like p implies q implies p; suppose p stands for I am alive, q stands for I am dead. So, I am alive from that q implies p will come as an outcome as an instance of that 1, logical consequence of p. I am alive to, I am dead then, I am alive and all. So, all these things when you put it into the day to day discourses, it presents numerous kind of promise and what needs to be done and all. Then we need to move away from classical logic, as we need to talk about fixing this connective or you need to talk about, you need to bring in relevance into picture and then talk about relevance logic etcetera and all.

It lead to different kinds of logics in particular and there are some other kinds of examples like, what about those sentences which are referring to future. So, these are called as future contingent sentences. So, we can only talk about truth value of sentences which occur in the past, are there are certain things which of actually true and all. And what about the future, the sentences which are referring to future for example, if you say, that I will be in my native place on December 25<sup>th</sup> and all. So, it booked my ticket

etcetera. As well and good then, how do we fetch the truth value of that particular kind of statement now?

So, if I say that sentence is true, then I have to be in my native place, and I have no choice that, I can drop my plan etcetera and all. That makes it necessary to, suppose that is false and it pushes us to another extreme that it makes me impossible to go to my native place; suppose if I cancel it now, I can still plan for my trip on. so, and so, date and all. If that sentence is false, that I will be in my native place is false, then it pushes us to another extreme that is it is impossible for me to go to my native place and all.

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Aristotle's Sea battle argument

A general is contemplating whether or not to give an order to attack. The general reasons as follows:

- 1 If I give the order to attack, then, necessarily, there will be a sea battle tomorrow
- 2 If not, then, necessarily, there will not be one.
- 3 Now, I give the order or I do not.
- 4 Hence, either it is necessary that there is a sea battle tomorrow or it is necessary that none occurs.

The conclusion is that either it is inevitable that there is a sea battle tomorrow or it is inevitable that there is no battle. So, why should the general bother giving the order?

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So, both extremes leads to some kind of position which is called as fatalism. And what do we do with these future contingent sentences. So, how do we deal with this kinds do we need to dismiss those things which are contingent sentence or do we need to allow them all, classical logics have no answer. So, we need to move to many well and logics where, sentence is needed to true nor false can be represented as 1 by 2. So, I do not go into the details of this particular kind of thing, another interesting important thing are what we call it as paradox are; obviously, valid arguments, but if they are considered to be counter intuit.

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**Sorites Paradox**

Consider a heap of grains of sand. Take away one solitary grain from the heap, you still have a heap, for one grain of sand is not enough to make the transition from heap to non-heap. So, if a pile of 10,000 grains of sand makes a heap (call this statement  $h_{10,000}$ ) then a pile of 9,999 grains also makes a heap.

- 1 A heap of sand is comprised of a large collection of grains. (Premise 1)
- 2 A heap of sand minus one grain is still a heap. (Premise 2)

The premises seem true.  
The argument seems valid.  
The conclusion seems false

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So, now, let us consider simple example, why classical logic fails to explain this particular kind of phenomenon. So, consider a heap of grains of sand. And let us assume that, you have a heap of sand and all and we started removing 1 by 1 grain after another now. So, take away 1 solitary grain from that particular kind of heap, you still have a heap and all. Nobody will be in a position to say that it is not a heap and all still it retains it is heapness. Heapness is considered to be property a predicate, which is considered to be a vague predicate.

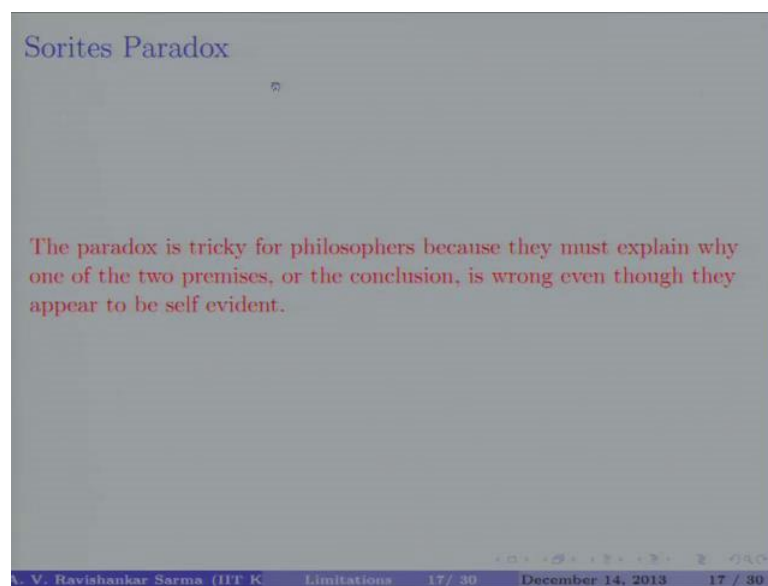
So, for 1 grain of sand is not enough to make that transition from heap to non-heap and all. So, in that sense if a pile of 10,000 grams of sand makes a heap then call the statement as  $h_{10,000}$  then a pile of 9,999 grains also makes a heap. So, now, a heap of sand is comprised of a large collection of grains this is a definition that we as we beginning with we began with, an heap of sand minus 1 grain is still considered to be heap. So, these 2 premises are unquestionably true and all; you just remove 1 grain it is not going to lose.

The same way, this problem can be extended to any different context where we can simply understand this thing. A person with full of hair, is considered not to be bold; if he removes 1 hair is not going to become, there is not going to change much. If person with some let us say, he has 1million kind of hair and all he is not considered to be bold a person with 1 million minus 1 is not considered to be bold. So, like that you will be using

Modasponance n number of times and then, at the end of the day you will say that, even if you do not have any hair and all. You will still consider to be not bald and all that is little bit surprising for us and it is counterintuitive to us.

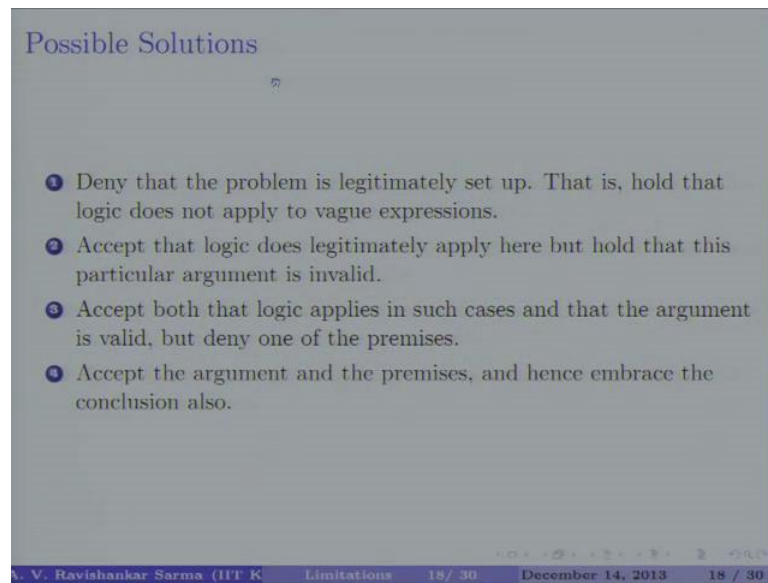
So, here the problem is that, you have used Modasponance which is considered to be; obviously, valid principle of reasoning. The premises are considered to be true the argument is valid. So, the conclusion is counterintuitive, it is not at least we are not in a position to accept that it is true.

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So, this paradox is tricky for the philosophers because they must explain why, 1 of the 2 premises, or the conclusion is wrong even though they are they appear to be self evident. So, each time when you come to the next step, it is application of which is; obviously, valid kind of influence.

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Possible Solutions

- 1 Deny that the problem is legitimately set up. That is, hold that logic does not apply to vague expressions.
- 2 Accept that logic does legitimately apply here but hold that this particular argument is invalid.
- 3 Accept both that logic applies in such cases and that the argument is valid, but deny one of the premises.
- 4 Accept the argument and the premises, and hence embrace the conclusion also.

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So, what is happening here is this you have a heap and you started removing that particular kind of thing, 1 after another and all even if you remove all the grains and all still you will have a property such as it still retain its heapness, that is little bit counterintuitive to us. So, what you do with the classical logic and all. Do we dismiss this particular kind of predicates and then, totally dismiss it that is deny the problem that it is 1 legitimately set up and all or you hold that logic does not apply to vague expressions logic in the sense, classical logic or accept that logic does not legitimately apply here, but hold that this particular argument is invalid; that means, your premises are true and conclusion is false somehow using degree theoretic account etcetera and all.

You can show that, you have conclusions are probably true, but premises are probably true and the conclusions can still be false. If you invoke the degree of truth and then, accept both logic apply that means etcetera. All apply in such cases and the argument is considered to be valid, but deny the premises you say that 1 of the premises is considered to be wrong. Like this n number of solutions, which are provided and which led to different kinds of logics which we usually call it as many valued logics. It led to fuzzy logic as well; the same example is used to motivate used as a motivation for doing fuzzy logics as well.



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**Liars Paradox**

The liar paradox is an ancient conundrum of logic. It was originally cast in the form of a fable.

In ancient times, all the inhabitants of Crete were incapable of making a true statement. Epimenides, who lived in Crete, made the following statement: **All Cretans are liars.** Is Epimenides lying?

**Essence of Liar's Paradox**

**This sentence is False**

Any attempt to assign a truth value to the statement in (1) leads to a vicious cycle of contradiction (sometimes called a vicious circle. If the statement is true, then it is false. But if it is false, then it must be true. Thus we have a vicious cycle of contradictory statements

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Another important paradox, which I like to bring to your attention which has not found any solution in the classical logic, that is the Liars Paradox; the Liars Paradox is considered to be an ancient conundrum, it was there right from the period of Greeks, it was originally cast in the form of a fable. The fable goes like this, in the ancient times, all the inhabitants of Crete were capable of making true statements. Epimenides is considered to be 1 Crete; hence he belongs to that particular country, who lived in Crete and made the following statements. All he said in 1 country all Cretans are Liars Epiminides is also considered to be belonging to Cretan now, the question is he a Liar if he is a liar he can he tell the truths.

So, now, if he says that all Cretans are liars now, the question is Epimenides lying or not. So, now, slightly different kind of problem, which emerges with respect to self referential kind of sentences such as this thing this sentence, is false. If you say that particular kind of thing now if you ask your selfish this sentence true or false. So, now, in the case of classical logic we have said that a sentence has to be either true or false it only takes 2 values now what about these particular kind of sentences, that this sentence is false is neither true nor false. How do we incorporate these particular kinds of sentence and all. Do we dismiss this particular kind of sentence or be salient about these things or you incorporate these things and then talk about extending your logics and all. These things presents problem to that classical logic.

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Liars Paradox:

- 1 The liar paradox says to consider the statement: **this statement is false**. If **this statement is false** is true, then it is false, which would in turn mean that it is actually true, but this would mean that it is false.
- 2 if **this sentence is false** is false, then it is true, which would in turn mean that it is actually false, but this would mean that it is true, and so on ad infinitum. Thus the statement **can be neither true nor false**.

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So, I do not want, to go into the details of Liars Paradox just I'll end it end this thing by saying that liars paradox is to consider the statement this statement is false if this statement is false is true then whatever it says is whatever indeed stating is true then; obviously, the statement is considered to be false, which would turn out to mean that, it is actually true, but this would mean that this is actually false. So, the problem here is that when, it is true is a problem it leads to contradiction if false also it leads to contradiction and all.

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Historical Attempts

Hierarchy levels of truth

- 1 distinction between object and meta-language.
- 2 This statement is (false)<sup>1</sup>.
- 3 The level for the word false in (1) is level 1, while the level of truth values for the entire statement is level 2.
- 4 We should then attempt to determine if it is (true)<sup>2</sup> or (false)<sup>2</sup>. This method is unsatisfactory precisely because it introduces a hierarchy of truth values.
- 5 Such a hierarchy seems entirely ad-hoc and is not referred to in statement.

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So, that is leads to a tricky kind of situation paradoxical kind of situation again, there are different attempts which are made to resolve this particular kind of paradox. If you maintain the distinction between object and meta language, may be you can come out of this particular kind of paradox for example, truth of a particular kind of sentence we can talk about, in the higher language and all you cannot talk about truths of particular kind of sentence, which is the object language to maintain a distinction between object and meta language is a way to resolve this particular kind of problem.

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Three Valued Logic

- 1 This sentence is false.
- 2 This sentence is **not true**.

With a three-valued logic, (2) is paradoxical. The statement in (2) is called the **strengthened liar paradox**.

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There are some other attempts in the form of by taking into consideration 3 valued. Logics such as a you represent this sentence is false, is neither true nor false neither true nor false is represented as something called as paradoxical or nonsense or something like that. It takes a value 1 by 2 which is different from 1 and 0. So, there are other kinds of paradoxes which sets limit to the logic that is 1 particular kind of paradox which is important. In the context of that is in the Russell's paradox.

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Russell's Paradox

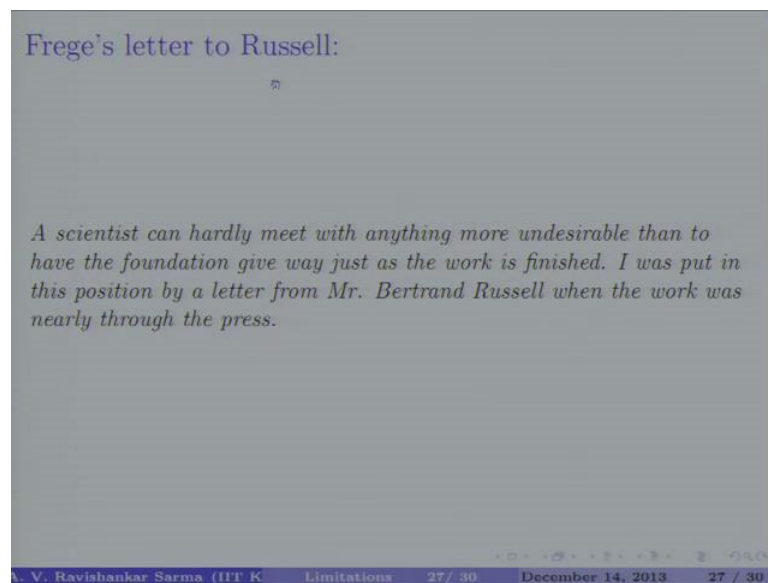
Consider set of all those sets which are not members of itself.  
 $r = \{x: x \notin x\}$

- 1 **Case 1:** If  $r$  is a member of itself, then it is one of the sets that is not not a member of itself, so  $r$  is not a member of itself.
- 2 **Case 2:** If  $r$  is **not** a member of itself, then it is one of the sets in  $r$ , and hence it is a member of itself.
- 3  $r \in r$  iff  $r \notin r$

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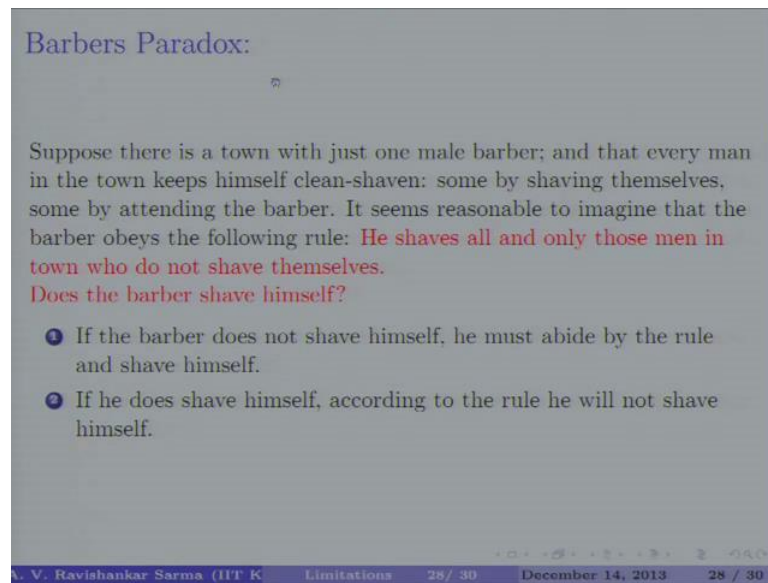
So, now, consider a set of all sets which are not members of itself. So, that is represented by  $r$ . So, is that  $x$  belong to  $x$  now, the question is case 1 is that if  $r$  is a member of itself, then it is 1 of the sets that is not a member of itself. So,  $r$  is not a member of itself; in the case 2, if  $r$  is not a member of itself then, it is 1 set, 1 of the sets in  $r$  and hence it is not a, it is a member of itself. It is not a member of itself it has to be a member of itself; that means,  $r$  belongs to if and only if or does not belong to  $r$ .

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So, whether sets of all sets belongs to itself or not is a question? Which is posed by Russell and now in a letter to? Frege he writes like this; a scientist can hardly meet with anything more undesirable than to have foundation to give away just as the work is finished. He has finished a grand book and all and then after that this result has come to him as a surprise it shakes the foundations of the logic itself. So, it was put because set theory logic is vested on set theory is considered to be shaky, in the sense that it led to this famous paradoxes that is 1 paradox is the Russell's paradox. So, now, he is of the view that his position by letter from Bertand Russell when the work was almost nearly through the press when this result has come and shake in the foundations and all.

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Barbers Paradox:

Suppose there is a town with just one male barber; and that every man in the town keeps himself clean-shaven: some by shaving themselves, some by attending the barber. It seems reasonable to imagine that the barber obeys the following rule: **He shaves all and only those men in town who do not shave themselves.**

Does the barber shave himself?

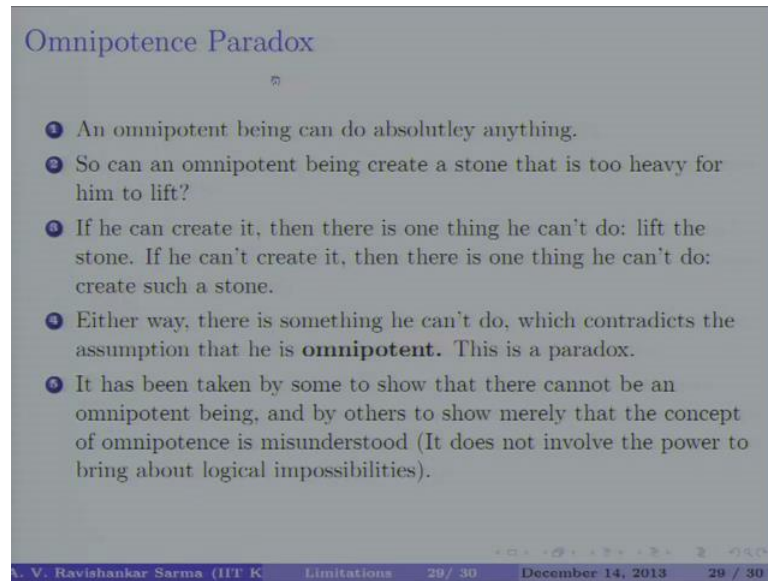
- 1 If the barber does not shave himself, he must abide by the rule and shave himself.
- 2 If he does shave himself, according to the rule he will not shave himself.

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So, so this is the famous paradox, which Russell's paradox which is expressed in terms of barbers paradox goes like this. Suppose, there is a town in which there is just only 1 male barber was there, and man in this the town keeps himself cleanly shaven; that means, they have to go to the barber to shave themselves. Some by shaving themselves, some by attending to the barber, and all it seems reasonable to imagine that barber obeys the following rule. He sets it in the notice board he says like this. He shaves all and only those men in the town, who do not shaves themselves; only those people he shaves that do not shave themselves, they will go to the barber for shaving and all.

So, now, the question is does the barber shave himself? So, the problem here is that if the barber does not shave himself, then he must abide by the rule and he has to shave himself. If he shaves himself, he violets whatever he has said shaves a only those men in the town who does not shave themselves. And if he does shaves himself, then according to the rule, which is in the red color he will not shave himself. In both cases there will be problem, there on way in which he can shave himself he will be growing his bred like anything. So, these are some of the counterintuitive results that we come across especially in the context of classical logic.

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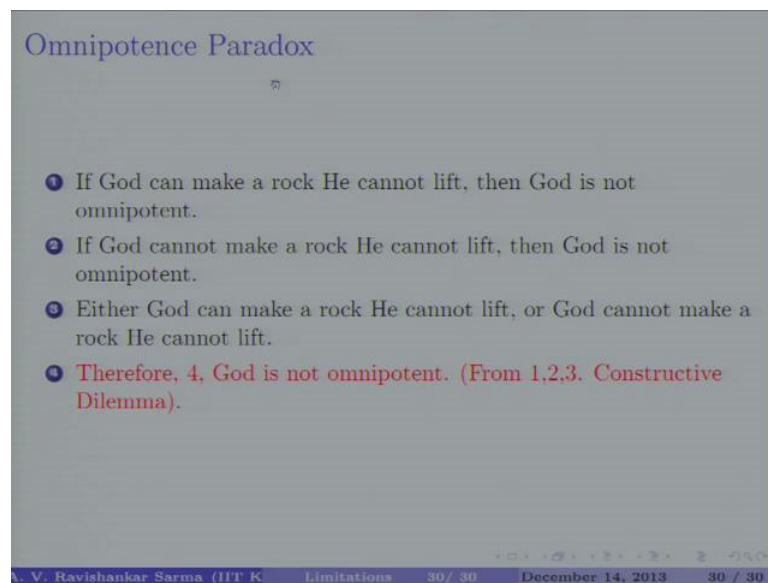


Omnipotence Paradox

- 1 An omnipotent being can do absolutely anything.
- 2 So can an omnipotent being create a stone that is too heavy for him to lift?
- 3 If he can create it, then there is one thing he can't do: lift the stone. If he can't create it, then there is one thing he can't do: create such a stone.
- 4 Either way, there is something he can't do, which contradicts the assumption that he is **omnipotent**. This is a paradox.
- 5 It has been taken by some to show that there cannot be an omnipotent being, and by others to show merely that the concept of omnipotence is misunderstood (It does not involve the power to bring about logical impossibilities).

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Omnipotence Paradox

- 1 If God can make a rock He cannot lift, then God is not omnipotent.
- 2 If God cannot make a rock He cannot lift, then God is not omnipotent.
- 3 Either God can make a rock He cannot lift, or God cannot make a rock He cannot lift.
- 4 **Therefore, 4, God is not omnipotent. (From 1,2,3. Constructive Dilemma).**

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There are some other important paradoxes such as, Omnipotence paradox with this, I will end this lecture. So, suppose if you say that God can make a rock which he cannot lift, then God is not omnipotent. So, usually we say that omnipotent means, he is capable of doing anything, omnipresent means he is present everywhere. So, now, the question is can God create a stone which cannot lift it. So, if God indeed can make a rock that he cannot lift then there is; obviously, God is not considered to be omnipotent. If god cannot make a rock, then there is something which he could not do; that means, he cannot lift then God is; obviously, not considered to be omnipotent.

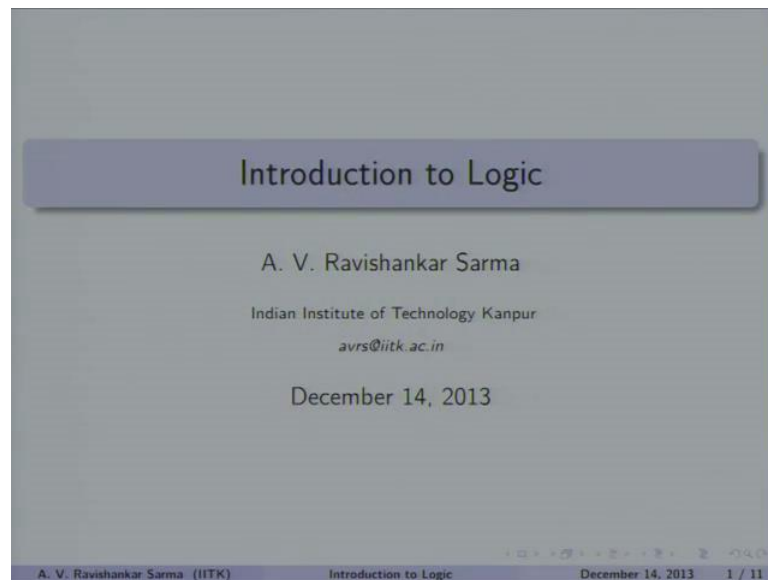
So, either God can make a rock or he cannot if he makes around he cannot lift, there is a problem or God cannot make a rock; that means, there is something which you cannot do, that is he cannot lift and all. In both cases, there is a problem therefore; you can say that God is not omnipotent. So, in this lecture what we have seen is this that, we have presented some kind of limitations to the classical logic. You should not be under the impression that, all kinds of reasoning we are trying to cover in terms of first order logic. There is lots of things, which are considered to be, which come under the category of common sense reasoning, which cannot be captured in terms of first order logic.

So; that means, we need to extend the first order logic or we need to deviate from the first order logic and talk about deviant logic by dropping 1 of the fundamental principles of logic. Such as, Law of entity, Law of excluded middle, Law of non-contradiction etcetera to drop. The law of non-contradiction you are doing Para consistent logic, if you are dropping Law of excluded middle; you are talking about many valued logics or fuzzy logics etcetera. So, as far as possible what we understood from this course is that, this first order logic in particular, is a starting point is a basic beginning point for doing all other kinds of logics and all.

So, it obeys all the nice properties and all it has wonderful properties, it has soundness consistence completeness etcetera and all. So, the moment you drop some of these fundamental laws of logic, then it will be at the cost of this nice features which has consistency completeness, soundness, etcetera and all. So, this first order logics basically tries to capture mathematical reasoning; this mathematical reasoning is based on mostly is based on the material implication. So, what we have done in this class is that, in the course we discussed about first order logic, in this lecture we presented some of the problems or challenges to the first order logic.



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So, welcome and then we will be talking about, introduction to logic I am the course instructor for this course my name is Ravishankar. So, I will be dealing with, I am the course instructor for this course introduction to logic. So, as you all might be wondering why, this logic course is start in the humanities department. So, if you take the history into consideration, logic has began the discipline of philosophy and then, it has moved to mathematics and then now, it has taken shelter in the department of computer science. So, I will be talking about, what I am going to discuss in this particular kind of course, then what are the topics that, I am going to cover in this particular kind of course, before I begin. So, I will start with an important quote. So, it says like this.

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The slide is titled "Proverbs" and contains the following text:

- 1 He who knows not and knows not he knows not: he is a fool - shun him.
- 2 He who knows not and knows he knows not: he is simple - teach him.
- 3 He who knows and knows not he knows: he is asleep - wake him.
- 4 He who knows and knows he knows: he is wise - follow him.... Arabian Proverb.

Lao Tzu  
To attain knowledge add things every day.  
To attain wisdom delete things every day.  
..... Lao Tzu (604 BC - 531 BC) Chinese Taoist Philosopher

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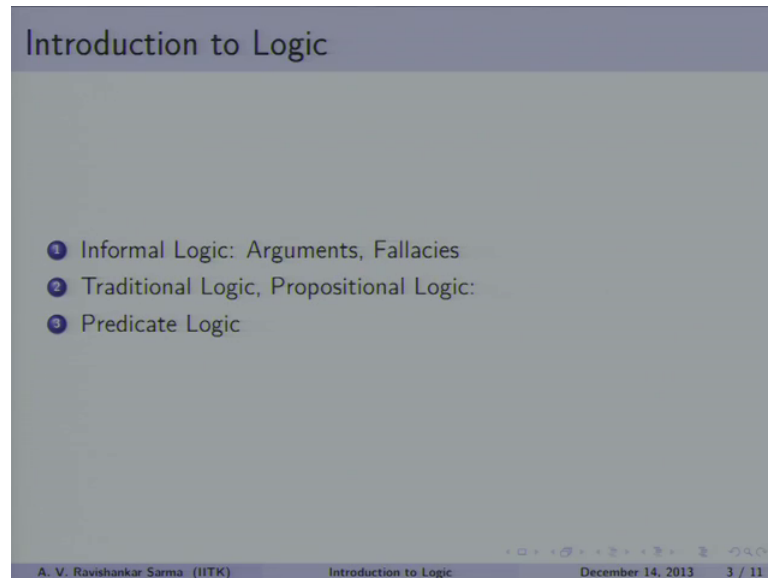
He who knows not and knows not that he knows not, is considered to be a fool he to shun him. And he who knows not and knows not that he knows not, is considered to be a simple person and we need to teach him. And a person who knows that, that he knows his that he does not know is considered to be he is falling asleep. So, we need to awake him and then finally, we have people like, those who knows that he knows that, he knows that something is the case, is considered to be wise and we need to follow him. So, this is a famous Arabian proverb and all it tells us that, there are 4 kinds of people which exists in the world.

So, the forth 1 is the 1 which we will be following, they have they are considered to be having some kind of wisdom. Mostly teachers will be having this kind of thing and I in another context, other quotation which I like to bring to your attention is to attain knowledge things everyday that is what, we have been doing all the time will be accumulating lot of knowledge etcetera; day by day, but if you go to the science etcetera and all. The first thing that, they will tell you is to empty your mind see to attain wisdom, we have to delete things every day.

So, what do you mean by saying that deleting this everyday and all. We might have accumulated knowledge out of our prejudices biases etcetera and all; we need to give up those things which we have accepted out of our prejudices, biases or some other things which might be just some kind of opinion etcetera and all. This is a famous quotation by

Lao Tzu. So, now coming back to this course, what is that we have trying to do in this interesting and exciting course.

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So, first we will be starting with some of the basic concepts of logic, where we will be introducing what you mean by, an argument and what kind of arguments exists and if once you identify that these are the arguments then what kind of argument it is. So, in this context we introduce inductive and deductive kind of arguments as 2 different kinds of arguments that you come across in logic then, we will discuss about some of the important properties of this particular kind of arguments. And then we say we will move on to another interesting and very exciting topic that is considered to be fallacies.

So, fallacies are considered to be mistakes in the argumentation and both deductive and inductive arguments can be fallacies and all these fallacies can be used as some kind of strategies and all. they are used as some kind of persuasive strategies, which we commonly come across we usually come across day to day discos even you see that many politicians etcetera in order to owe the voters they will be making use of this fallacies and all; they will be making some empty promises etcetera and all they play with the emotions etcetera and all they will be making use of many fallacies.

So, then we will move on to traditional logics which are due to Aristotle, which has dominated from more than 21000 years; then, we will be taking up the theory of which more or less serves as some kind of predicate logic only, but Aristotle has no did not

have this kind of formal equipment, but yet it he had discussed on the important inferences of this categorical positions and all; how 2 categorical prepositions leads to another kind of categorical preposition. So, that is where we introduce theory of syllogisms; so, theory of syllogisms has some kind of limitation then we move on to the propositional logic where we discuss about logic of prepositions.

So, a preposition is a sentence which can be simply spoken to be a as a true or false, then we discuss preposition logic is all about logic of 5 connectives that we are trying to use that is and or implies if and only if and negation, it will basically discussing about the properties of this connectives. Since this is a starting point of our representation of knowledge, this is the minimal tools that with which we can represent our knowledge then preposition logics are not sufficient enough, they are not rich enough. Especially know, we do not have relations predicates etcetera and all quantifies etcetera and all; which are missing in the preposition logics in order to make the language richer, then we will be introducing 2 more quantifies, 2 quantifies that for all x and there exists some x. You argument the propositional logic with these 2 quantifiers then we will be talking about the predicate logic.

So, in the context of preposition and predicate logic will be talking about, some of the important decision procedure methods with which you can judge, whether a given well formed formula in this preposition and predicate logic is considered to be valid or when we say that 2 statements are considered to be consistent to each other or this was some of the important logical properties that we will be discussing with respect to this decision procedure methods.

So, at least 4 or 5 decisions based procedure methods that we have used in this course, to start with the most simplistic kind of method that is the truth table method, which we have used in 2 different senses direct and indirect kind of truth table. And then what is what occupies a central position for this course is, the decision procedure that occupies the central position for this course is the semantic tap locks method. So, we will making use of this semantic tap locks method in both in the context of predicate logic and propositional logic and we will talk about validity consistency etcetera. So, what is the learning outcome of this course? So, why should we do this particular kind of course...

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**Learning Outcome of the Course**

- 1 Learning to distinguish good from bad arguments and make the process of making argumentation- effective.
- 2 To be able to represent various knowledge claims with in the language of first order Logic (Propositional and Predicate Logic)
- 3 To learn more about the decision procedure methods in the context of propositional and predicate logic
- 4 To be able to solve some logic puzzles while making use of basic concepts of this course.
- 5 To learn underlying techniques of some of the important decision procedures.

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So, we will be able to learn to distinguish good from bad arguments that are occupies the first part of this course. So, ultimately logic is all about study of argumentation as well. So, we need to identify what is considered to be good argument or effective argument compared to bad argument etcetera, and make the process of making argumentation effective. So, in this context we introduced 1 important moral of argumentation which is due to Stephen toolbin. So, toolbin has introduced a very nice moral of argumentation we discuss it in later details, about that particular kind of thing. And 1 should be in a position to 1 should be able to represent various kinds of knowledge claims within the language of first order logic given in an English language sentence. You should be able to translate it into the language of preposition logic or predicate logic depending upon this phrases exists and all.

Example: if sentence begins with for all x etcetera and all; some nun etcetera and all. You will be using predicate logic and if you, if it is enough that you express it in terms of prepositional just simple prepositions and all prepositional logic would surface. And 1 should be in a position 1 should be will be able to learn more about, some of the decision procedure methods to start with truth table, semantic tap, locks method and 1 of the truth procedure methods such as natural deduction method and resolution defeatation method etcetera. These are the decision procedure methods that we will be using.

In this particular kind of course, but what occupies central position is the semantic tableaux method. And you will be 1 of the unique feature of this course is that, we studied the basic principles of logic, by making use of by solving some kind of logical puzzles. So, these logical puzzles are due to famous logician Raymond Smullyan. So, we have used different kinds of puzzles in this course knights and naves puzzles etcetera tiger lady and tiger etcetera, interesting puzzles are that 1 can make use of to make these basic concepts clear to students and also to learn some kind of underline techniques of some of the important methods which we have discussed just now.

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**What is Logic?**

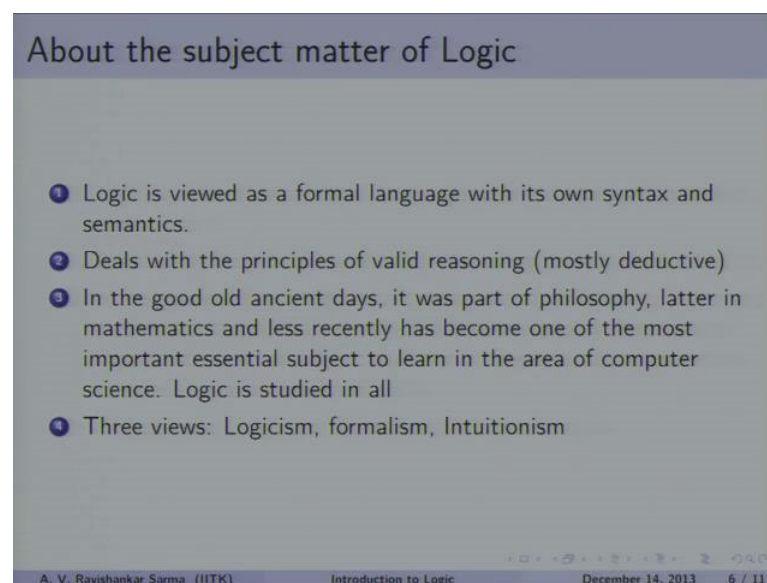
- 1 Logic is the study of the principles of valid demonstration and inference.
- 2 Logic is a branch of philosophy, and the word derives from Greek (logos), which means word, thought, idea, argument, account, reason, or principle..
- 3 Logic concerns the structure of statements and arguments, in formal systems of inference and natural language.
- 4 It also deals with topics like validity, fallacies and paradoxes, reasoning using probability and arguments involving causality.

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So, 1 important question that come that might come to your mind is this is that, what is considered to be logic why it is studied in humanities discipline etcetera. So, logic is usually considered as the study of the principles of valid demonstration or principles of valid inference. It is not enough that, something follows from something, but it has to be valid and it has to be sound as well. So, logic is considered to be a branch of philosophy and the word or logos derives from the word logic derives from the word logos, which means; word thought idea argument account reason principle etcetera. We make use of reason to be the most important thing out of these things logic also concerns with the structure of statements and arguments in formal system of inference and in the natural language.

So, it also deals with topics such as validity, fallacies, paradoxes etcetera; that means, reasoning using probability and arguments involving causality is mostly taken care by inductive kind of logics. This is not what occupies the attention and all; we will be dealing with validity fallacies paradox etcetera. Mostly in this course we will be focusing our attention on deductive reasoning. So, what about the subject matter of logic; logic is used as a formal language which is on syntax and semantics and we discuss relation between these 2 things syntax and semantics syntax will be taken care by provability and semantics is taken care by logical consequence.

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- 1 Logic is viewed as a formal language with its own syntax and semantics.
- 2 Deals with the principles of valid reasoning (mostly deductive)
- 3 In the good old ancient days, it was part of philosophy, latter in mathematics and less recently has become one of the most important essential subject to learn in the area of computer science. Logic is studied in all
- 4 Three views: Logicism, formalism, Intuitionism

So, whatever provable is true and whatever true is proven and system is considered to be complete. It deals with the principles of valid reasoning that is what we have discussed till here and in good olden ancient days it was considered to be part and parcel of the discipline philosophy and it is still widely studied in the area of philosophy. There are many problems philosophical problems which are raised in the Greek period still considered to be problems in the contemporary literature on logic it occupies the attention of logicians in the contemporary literature of logic.

So, just say it has a part of mathematics in the sense they shift from since the fall of Aristotle in syllogistic logics it moved to mathematics in particular, where the attempt was made to reduce mathematics logic the program is called as Logicism. And rest is recently it has become 1 the important and most essential subjects to learn in the area of

computer science. So, logic is studied in all these disciplines even till now also it is a kind of interdisciplinary kind of a subject. So, the 3 views which are dominating in the logic or Logicism formalism and intuitionism I am not going to the details of it of course, these things will become explicit.

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**Different kinds of Logic**

- 1 **Formal logic** is the study of inference with purely formal content, where that content is made explicit. An inference possesses a purely formal content if it can be expressed as a particular application of a wholly abstract rule, that is, a rule that is not about any particular thing or property.
- 2 **Informal logic** is the study of natural language arguments. The study of fallacies is an especially important branch of informal logic. The dialogues of Plato are a good example of informal logic.
- 3 **Symbolic logic** is the study of symbolic abstractions that capture the formal features of logical inference. Symbolic logic is often divided into two branches, **propositional logic** and **predicate logic**.
- 4 **Mathematical logic** is an extension of symbolic logic into other areas, in particular to the study of model theory, proof theory, set theory, and recursion theory.

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So, there are different kinds of logic which we can talk about; first is formal logic, formal logic means; it is a study of inference with purely formal content, where that content can be made explicit. We are not worried about the content of the argument and all where we are only worried about the form, if  $p$  then  $q$  and  $p$  that is why  $q$  follows, but there are certain kinds of arguments which require the analysis of the content. Such as, this room is made up of atoms which are invisible; that means, this room is invisible these kinds of arguments requires you to analyze the content of the argument and all.

So, there you have used shift in the meaning of the usage of the word atoms, in the premises that is why that kind of argument is called as a fallacy and that fallacy is called as informal fallacy. There are certain kinds of arguments which requires the analysis of content, but mostly you will be dealing with formal logics where what matters to us is only form of an argument. An informal logic is considered to be study of natural language or arguments that occupies the first part of our course, study of fallacies is an especially an important branch of informal logic.



So, that is the reason why we look into the informal logic; in the beginning of this course especially for example, the dialogues of Plato are a wonderful example of informal logic. And there is other way which you can define logics with the name symbolic logic symbolic logic is a study of symbolic abstractions that capture the formal feature of logical inference. Symbolic logic is often divided into 2 branches that is what we have done in this course. We are going to do in this course preposition logic and then the predicate logic.

So, what occupies the second part of this course is the propositional and the predicate logic. And there is another thing which is important, that is mathematical logic it is an extension of symbolic logic into other areas such as, in particular to the study of model theory, proof theory set theory and recursion theory we are not going to study all these things, but we will be focusing our attention partly on model theory and partly on proof theory. In the context proof theory we introduce rasal wh ited aximated system and then system and then we also talk about some of the important proof procedure method such as natural reduction and how do we reduce theorems from the given axiomatic system these are things which will be studied.

So, if see the content of this course it is a mixture of all these things formal logic formal logic to certain extent we are taking into Aristotle logic also considered to be formal logic partly. We have taken into consideration that and in informal logic we will be studying various kinds of fallacies and in the symbolic logic that is the core of this particular kind of course, that is preposition and predicate logic. And as far as possible we introduce the concepts of preposition and predicate logics with the help of some kind of puzzles solving some kind of puzzles; we get familiarize our self with some of the important decision procedure methods such as semantic tableau method. And in the mathematical logic we focused our attention on proof theory it is a mixture of all this things.

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The slide is titled "Nature and Scope of Logic". It contains two numbered bullet points:

- 1 This course doesn't study all kinds of reasoning, but basically captures mathematical reasoning.
- 2 We restrict ourselves to two valued logic, which obey the fundamental laws of Logic: Law of identity, law of excluded middle, law of non-contradiction, monotonicity.

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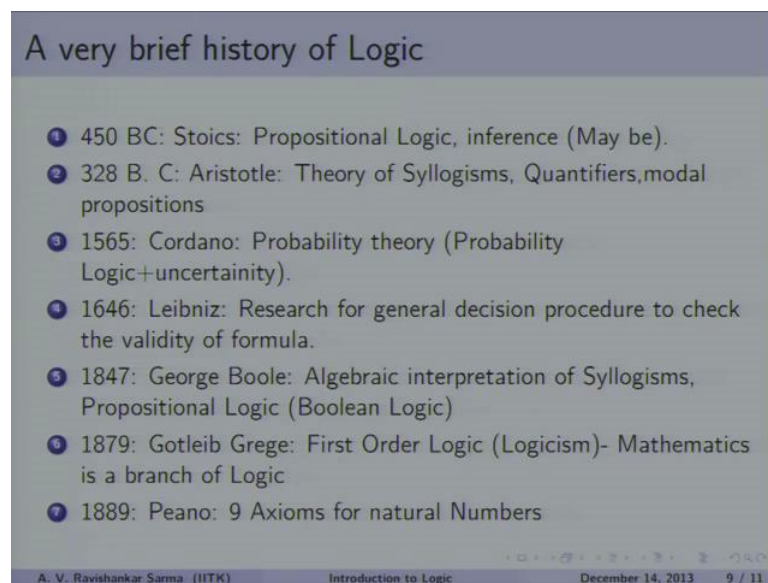
So, nature and scope of this logic it should not be under the impression that all kinds of reason that we are trying to cover, in this particular kind of course, and there are other kinds of reasoning which we employ in day to day discourses is common sense reasoning, which is considered to be non monotonic for example, if you say all birds fly Tweety is a bird and Tweety flies and if somebody comes to you and tells you that Tweety of course, it is bird, but it will not come under category of penguins and penguins does not fly. So, now, what you will going to do. So, are you suppose to withdraw the conclusion that you are drawn that Tweety flies or what exactly you are going to do at this stage.

So, accepting the information will need to the withdrawal of the conclusion that you have drawn earlier. So, this is not what is permitted in the classical logics because classical logics are considered to be more atonic the deductive and monotonic in nature. So, this course does not study all kinds of reasoning all though it tries to capture some of the things of course, logic has to update to day to day discourses to certain extent in solving puzzles etcetera and all. We make use of the basic principles of logic when we actually come to the day to day discourses we will talk about some of the limitations of these first order logic. Especially when it is referring to wague predicates when, it is referring to sentences such as this sentence, is false whether it is true or false and that kind of questions liars sentences.

For example, etcetera all these things presents some kind of challenges to the classical logic. In the same way when you use material implication in the day to day discourses you have counter intuitive results.

So, in that sense we are not going to talk about all kinds of reasoning, but we are restricting ourselves to just 2 valued logics it is predicate and preposition logic takes care of that 1; which obeys the fundamental laws of logic. That is law of identity law of excluded middle and the law of non contradiction, then of course, the monotonicity, as far as possible that we are trying to capture the mathematical reasoning, is a most minimal kind of ways to represent our knowledge claims its used as a language for the for the representing the knowledge. So, now, what is important here is that, just we need to consider a brief history of logic is not considered to be the complete kind of all the complete details are not there in this 1.

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A very brief history of Logic

- 1 450 BC: Stoics: Propositional Logic, inference (May be).
- 2 328 B. C: Aristotle: Theory of Syllogisms, Quantifiers, modal propositions
- 3 1565: Cordano: Probability theory (Probability Logic+uncertainty).
- 4 1646: Leibniz: Research for general decision procedure to check the validity of formula.
- 5 1847: George Boole: Algebraic interpretation of Syllogisms, Propositional Logic (Boolean Logic)
- 6 1879: Gottleib Grege: First Order Logic (Logicism)- Mathematics is a branch of Logic
- 7 1889: Peano: 9 Axioms for natural Numbers

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So, but it is very difficult to go through a brief history of logic it started with there are many things which might be missing in this list, but of the most important things that usually find it in the history logic are these things. I will go through it quickly to start with we have stoics they are before Aristotle they seems to be the major proponents of these propositional logic. And they also propose some rules of inference which needs to be studied in greater detail, some research is still going on this direction to what extent they have come up with the rules etcetera.

So, then we have an important work by Aristotle his works *Organon* which consist of a set of books and all like *Para et cetera*. Where he has introduced in 1 of these things he introduced theory of syllogisms; in a way more or less he has introduced quantifiers because categorical proposition starts with all some none etc and all. They are all quantifiers only more or less, but formal interpretation is missing in that particular kind of thing. And then, he also talked about modal propositions *et cetera* and all; although did not deal with these things in much more greater detail these are things which are already there and followed by that we jumped to the medieval period 1565 Cardano he has come up with probability theory probabilistic logics *et cetera* uncertainty in 1646; Leibniz has come up with a research a grand program that is research for general decision procedure to check the validity of a given formula.

So, this is considered to be the origin of the computer computers *et cetera* and all. Usually treated as the origin of the computers, in 1847 this is considered to be the most important work in the first order logic, this is starting point where the logic has taken the shape of mathematics in particular logic there is a turn there is a mathematical turn in logic. So, that is this thing George Boole is come up with algebraic interpretation of syllogisms and he has also come up with the propositional logic.

What he has done is he has given algebraic interpretation of syllogisms and 1 of the most important works in first order logic is this thing Gottlob Frege is spelling mistake here Frege. First order logic is considered to be the father of first order logic, where he tries to reduce mathematics to the branch of logic and 1889 you have Peano's 9 axioms for the natural numbers they are considered to be the important thing. And in the twentieth century in the 19<sup>th</sup> century with mid 19<sup>th</sup> century the monumental work is due to Bertrand Russell and Whitehead *Principia Mathematica* and Hilbert's program has emerged after that 1 he provided decision procedures for mathematical theories and he also presented 23 challenging open problems they are still considered to be open problems and Wittgenstein is said to be attributed to the development of the truth tables and proofs based on the truth tables and the celebrated result of Gödel's completeness theorem one of the lectures we will be discussing about, that is related to first order logic.

So, her brand has come up with an important theorem which is called as deduction theorem which we will be dealing with in 1 of this lectures a proof procedure for it is considered as a proof procedure for first order logic for based on propositionalization and

Kurt Gödel's very important result in 1931 is a path-breaking result which sets a limit to the program of Russell, Whitehead, and then Hilbert's Academics. It is the incompleteness theorem in the context of consistency of Peano's axioms.

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20th Century

- 1 1903: Bertrand Russell and Whitehead: Principia Mathematica.
- 2 Hilbert's Program: To formalize all existing theories to a finite, complete and consistent set of axioms. Provide decision procedures for all mathematical theories. Also presented 23 challenging **open** problems.
- 3 Wittgenstein: Proof by truth tables.
- 4 1927: Kurt Gödel: Completeness theorem of First order logic
- 5 Herbrand (1930): Deduction theorem, A proof procedure for FOL based on propositionalization.
- 6 Kurt Gödel: 1931: Incompleteness theorems for the consistency of Peano's axioms
- 7 1936: Gentzen: A proof for the consistency of Peano's axioms in set theory.
- 8 1936: Church, Turing: Undecidability of FOL.

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In 1936 Gentzen has come up with method of natural deduction he shows a proof for the consistency of Peano's axioms in the set theory. So, we it is not the case that we will be dealing with the all these topics and all, but as far as possible will be dealing with some of the important interesting topics out of this. So, in 1936 Church and Turing has come up with the undecidability of first order logic. And after 1950's most of the work is done in the area of computer science and the mathematics in particular of course, I choose our philosophical in nature only.

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The slide, titled "After 1950", lists ten significant milestones in the history of logic:

- 1954: Davis Putnam: First Machine generate Proof.
- 1955: Beth and Hintikka Semantic tableau Method
- 1957: Newel and Simon: First machine generated proof in Loical calculus.
- 1957: Kanger, prawitz Lazy substitution by free and dummy variables.
- 1958: Prawitz: First provers for FOL
- 1958: Kurt Godel: A method for proving consistency of axioms with type theory (Is type theory consistent?)
- 1959: Gilmore, Wang Developed more provers
- 1960: Davis-Putnam/Longman: Davis Putnam Procede
- 1963: Robinson- Unification and Resolution.
- Age of Non-classical Logics: like fuzzy, default, Modal logics.

At the bottom of the slide, the text reads: "A. V. Ravishankar Sarma (IITK) Introduction to Logic December 14, 2013 11 / 11".

So, philosophers mathematicians computer scientist I mean; all work together mostly logicians work together on these particular kinds of problems. So, 1954 Davis Putnam has come up with first machine generated proof automated kind of proof and then 1955 Beth and hintikka has come up with semantic tableau method. And semantic tableau method is considered to be occupying the central position for this particular kind of course, and newel and Simon has come up with first machine generated proof in the context of logical caculas 1957 Kanger and Prawitz come up with the some interesting method and which is lazy substitution by free and dummy variables.

So, Prawitz has come up with first proverbs for first order logic and then, 1958 Kurt Gödel is come up with the method of proving consistency of axioms with the type theory. And these are some of the important developments after 1959; and after 1963 of course, we will be making use of Robinson's unification resolution refutation method; in 1 of these lectures. So, in the context of propositional logic and then, after that after 1967 ties and all it was a turn towards non classical logic there was a lot of dissatisfaction with respect to classical logic which fails to explain many interesting phenomenon.

So, there is a shift towards fuzzy default and model logics. So, there are things some of if you take the history into consideration and we will try to do justice to some of the important content that that arises out of this history and we will be not dealing with all this works and all, but just to present the continuity and all just to know the subject

matter of logic, I have presented a brief history or time line of logic. These are some of the important developments there are many things which might be missing and all.

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**Books**

- Patrick Hurley, *Concise introduction to Logic*
- Mendelson, *Introduction to Mathematical Logic*
- Shawn Hedman, *A first course in Mathematical Logic*
- Russell and Whitehead: Bertrand Russell and A N Whitehead, *Principia Mathematica*, 1910
- Raymond Smullyan, *Forever Undecided: A Puzzle Guide to Gödel*, 1987
- Raymond Smullyan, *Logical Labyrinths*, A. K. Peters Ltd, 2009

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So, as far as course is concerned I will be using this particular kind of text book Patrick Hurley concise introduction to logic, there are many good books on introduction to logic. Mendelssohn introduction to mathematical logic Shawn Hedman a first course in mathematical logic and then, we will be making use of the original work by Russell and whitehead that is principia mathematica. We will be taking we will making use of a portion of it then, we will be talking about various kinds of proofs based on this axiomatic system .Then what is interesting in this course the paradoxes and the puzzles; for the puzzles we refer to Raymond Smullyan's book, there is lot of books which are written by Raymond Smullyan. What is the name of the book? Logical labyrinths etcetera these are the books which will be referring to. So, these are the online differences which will be following.

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Futher references to get in touch with logic:

- <http://home.iitk.ac.in/~avrs/avrs/PH142/>
- <http://groups.google.com/group/sci.logic/topics>
- <http://www.cs.nyu.edu/pipermail/fom/>
- <http://world.logic.at/>
- <http://philosophy.lander.edu/logic/links.html>
- <http://groups.google.com/group/sci.math>
- <http://sakharov.net/foundation.html> [best site]

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So, this is considered to be an interesting and exciting course in the sense that, it has all the flavors and all philosophical logical and mathematical flavors philosophical. In the sense that, we will be dealing with some of the philosophical issues such as liars paradox, Russell's paradox etcetera and all. And then it has mathematical flavor deals with the foundation of mathematics or the problems related to the problems of mathematics. And it also deals with computation flavor in the sense that, many methods that we will we making use of decision procedure methods, which we are trying to make use of. We will have some kind of implications for the computer science, as well it is in that sense it is considered to be an interesting and exciting course.