Population and Society

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Lecture No. #11
Demographic Models

So, students today we are going to start a new module, and that is devoted to demographic models. For students of sociology who do not have any background in algebra, not even in statistics, it may be difficult to comprehend some of the advance concepts or advance material in the modeling. But I think it is important that all of you, whether you do quantitative methods or qualitative methods all those who are doing sociology of population must at least understand, what is modeling? What is the purpose of modeling? And in what context statistical or mathematical models have been applied to study demographic process or what kind of questions can be answered, if you apply statistical or mathematical models.

I will try to be as simple as possible, and I will assume that you have only a basic understanding of mathematics may be all of you have done mathematics up to high school level, some of you have done mathematics up to intermediately those who have done mathematics up to intermediate level for them it is not difficult, but those who have done mathematics only up to high school level for them some lectures may be difficult.

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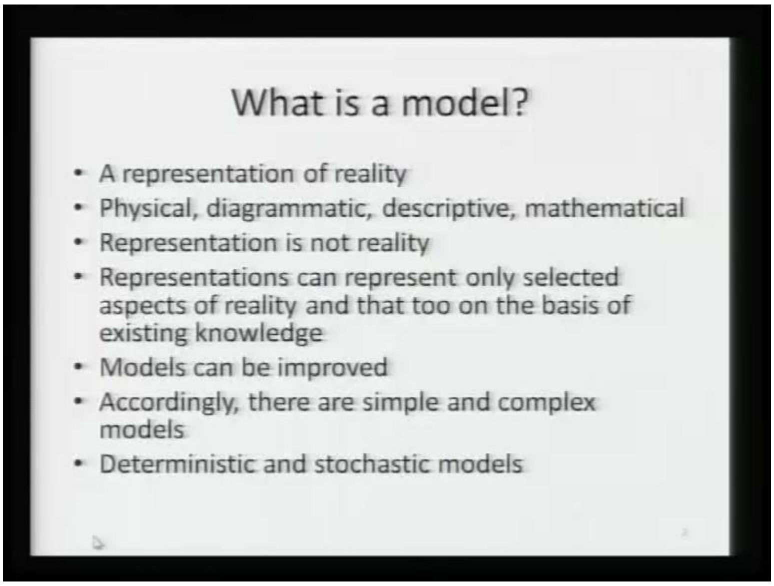

So, now this series of three lectures is devoted to modeling, demographic models in particular. First, before we talk about demographic models, the first question is what is a model? You must have seen various types of models that you or your friends made in your primary classes, model is a representation of reality.

So, you make model of an aero plane, you make model of a house, the models can be physical models can be written descriptions in the form of paragraphs, models can be diagrammatic you make a diagrammatic representation of something, that can also be a model or you can express certain relationships in algebraic equations or you write probability density function of a random variable these are all models.

Models can be static models can be dynamic models can be descriptive, exploratory, mathematical, statistical there are simulation models and then these physical, diagrammatic, descriptive, mathematical models represent one particular aspect of reality.

So, if I say that the population of India is growing with a certain decade rate of growth. I am assuming that the population of India is growing linearly and; that means, if I know the population in the base year or population in one census say the population of the last census 2001 and I know there linear rate of growth or decadal rate of growth and I assume that the population is growing linearly then I can project what would be the population of India in 2011 census. And I can compare that figure with the figure that I will get in 2011 census, if my model is correct then the predicted figure should be very close to the figure that we are going to get in 2011 census.

If the figure is not same then it means, either the assumption of linearity is wrong or that the rate of growth that I assumed for decadal growth during this decade 90, 2001 to 2011 that growth rate is wrong. Now, this representation does not tell us anything about what has happened to fertility, what has happened to mortality, what happened to age composition what happen to say international migration.

It is possible to build models which will include some more aspect of reality in addition to decadal rate of growth like, you can include migration rate or you can develop models which consider everything fertility, mortality, age composition migration and in that way you can develop a more complicated more complex simulation model, but model is still a model cannot consider all aspects of realty or the complete reality.

Representations of models can represent only selected aspects of reality, and that to on the basis of existing knowledge. If then existing knowledge shows that population can grow in a linear fashion then we assume that our growth model is linear, if the existing knowledge show that no population grows in some other fashion say geometric fashion or exponential fashion we assume that our model should be geometric or exponential.

If the current state of knowledge tells me that no if some other definite curves say logistic growth model which represent the growth of population better than simple linear or geometric or exponential models then I assume a logistic growth model. So, what kind of model I develop that depends on the state of knowledge existing in a field at a given point of time. And as knowledge improves model can also be improve so; that means, you can consider more aspects of reality and that way you move from simple to complex model.

Starting with say model of decadal growth assuming that population grows in a linear fashion. You have a simple model you can go to more complex model which considers growth rate, age distribution, sex distribution, migration, fertility, mortality and may be many more things.

And models are also divided into two category deterministic model and stochastic model. Deterministic models are models which assume that it is possible to completely determine a process means, you can have a complete mathematical understanding of the processes or relationships involved.

If write y equal to a plus Bx and say that population is growing in a linear fashion like this y standing for population x standing for time then I have a deterministic model. But if I say that growth of population is a random event and we cannot hundred percent predict or we cannot exactly predict what is going to be population of India at a future date.

We can only say in probabilistic terms that there is this much of chance like I can say that there is 95 percent chance that the population of India in 2011 census would be a close to say something between one point three billion to one point four billion something like that a statement I make a statement like this. Then I am using a stochastic model, stochastic models deal with uncertain events errors or random events or events which can be described only with the help of probability models and exact equations simultaneous equation model or exact equations cannot help in developing those models.

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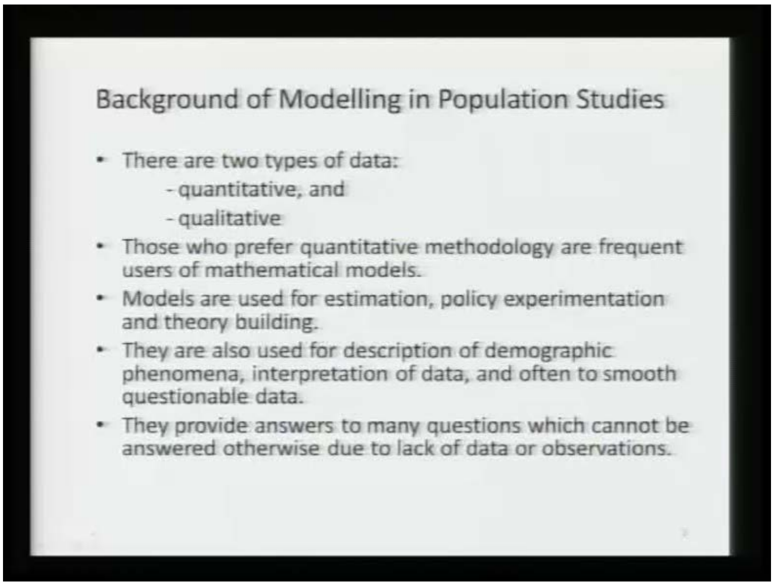

Now, background of modeling in population studies is as follows there are two types of data that we have seen quantitative and qualitative. Those who prefer quantitative methodology it is these people who go more frequently for mathematical models. Those who do not use quantitative methods those who use say qualitative methods for them mathematical modeling is impossible.

Mathematical and statistical models are used only by those who depend on quantitative data who collect quantitative data who develop measurements of attitudes, beliefs, values, behavior and try to explain them in terms of socioeconomic demographic characteristics. Models are used for estimation policy experimentation and theory building for three things.

When I the examples which I gave you that I want to estimate population of the next census date what will be the population of India in 2011. I am using a model for estimation policy experimentation means; I want to study what are the likely consequences of having different experimentations or interventions in dealing with certain social problem.

Like there are there can be many ways of a reducing poverty. You can focus on industrial development, you can focus on rural development, you can focus on big industry, you can focus on small industry. And by policy experimentation by modeling it should be possible for us to know whether in terms of reduction in poverty rate which policy is going to be more beneficial.

To answer this question whether we should go for big industry or we should go for small industry medium industry or household industry or we should promote informal sector more or formal sector more we need a linkage between investments and productivities in different sectors and the level of poverty percentage people living below the poverty line.

Now, once that kind of model is available with us we can decide that if the purpose is to reduce poverty from the existing level of say twenty seven percent to twenty eight percent to fourteen percent what should we do given the constants of time and resources and theory building you will see that many types of models have also been used for building theories.

So, models can also help in explaining certain phenomena you will see this in the case of migration how certain models have been used to explain migration phenomena causes of migration, consequences of migration, process of migration, rapidity of migration, different aspects of migration process. They are also used for description of demographic phenomena interpretation of data many times models are used for interpretation of data and often to smooth questionable data

Now, the other day when I was talking to you about census I said that census data suffer from various types of errors and I gave you the example of age. In data on age there is the problem of age heaping resulting from the fact that in reporting ages people prefer figures ending in certain digit like 0 and 5 and avoid certain other digits like 9 or 7.

So, if I am to use say age distribution singular age distribution of population of India from a census for projection purposes and I find that initially, in the age distribution there are errors and I make projections based on those data; obviously, my future projections of age distribution will also be erroneous. So, before using age data for modeling or for projections or predictions I must correct these data we must identify the kind of errors we have in the data remove them and then use for prediction purposes.

So, when we have data of these kind data of questionable reliability then models often help and models will tell me that normally a population like India with certain level of fertility mortality what kind of smooth age distribution it will likely to have. And using certain techniques of estimation of validation you know I would like to select that kind of model which is closest to empirical data in certain respects, and then in place of using the empirical data which is full of errors I would rather use the simulated data or the data that I get from model building.

Now, these models provide answers to many questions which cannot be answered otherwise due to lack of data or observations. You will see I will give you some examples, of how there are certain questions related to populations which cannot be answered on the basis of observations or hard facts.

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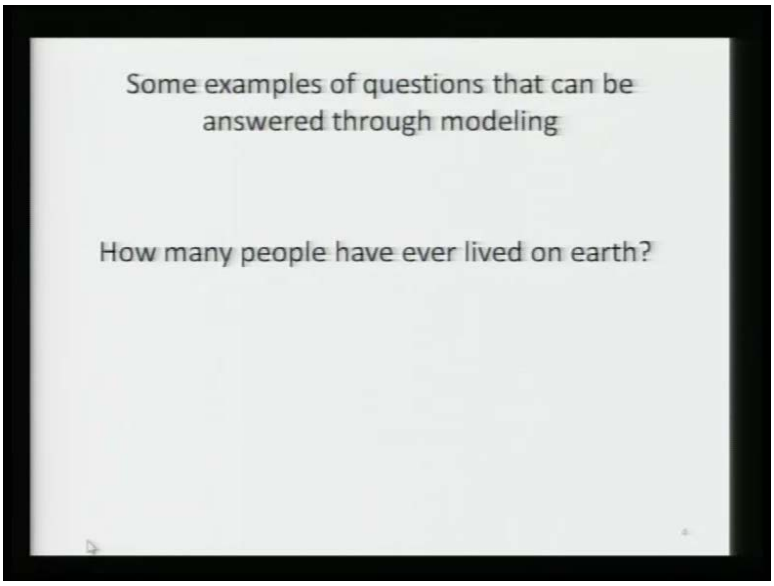

For example, this is a question how many people have ever lived on earth. Then the kind of method that we have discussed so far census, focus group discussion or surveys sample registration scheme or interviews of key respondents or experimental data. These data cannot help, we cannot collect data to answer this question, how many people have ever lived on earth, but if I understand the process, and if I can make some assumptions regarding the growth curves of population growth, then it is possible to provide estimate of how many people have ever lived on earth.

Let me give you an example, suppose it is possible for me to identify certain periods during which number of births has being increasing at a fix rate we do not exactly know when did man first appeared on this planet earth,

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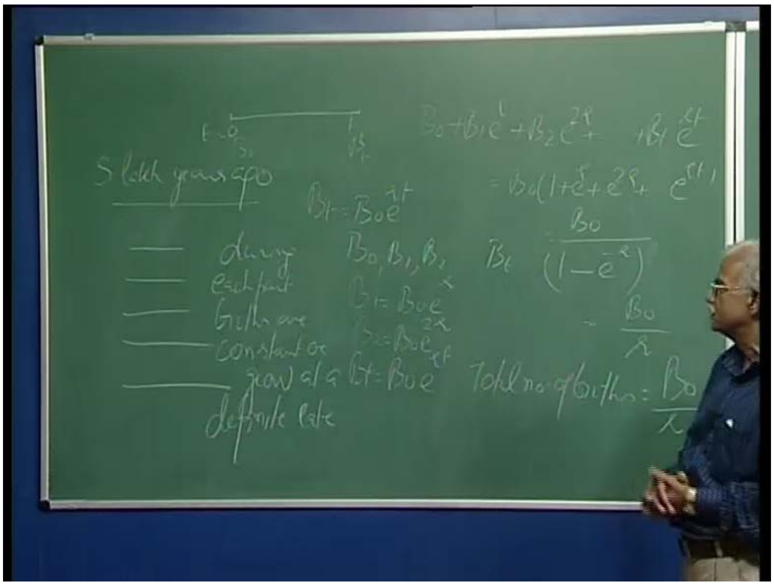

but suppose we say that it was around 5 lakh years ago that man first appeared on earth. And suppose this history of 5 lakh years can be divided into a number of part. During which during each part births are constant or grow at a definite rate let it be exponential rate.

So, if I take one particular time period, say 0 t equal to 0 to t. And I know that number of births here is B0 and number of births here is B t, and we may assume that Bt is B0e raise to power r t where t stand and r is rate of growth.

I know that some of our students may have difficulty in understanding the logic of continuous variable. So, let me give the example of discreet variable let the number of births be B0 B1 B2 up to time t. And let every time say after a gap of one year say B1 is B0 e raise to power r where r is the rate of growth rate of growth of births or B2 is B0 e raise to power 2r.

In general Bt equal to B0 e raise to power rt. Now, the total number of births during this time 0 to t will then be B0 plus B1 e raise to power r plus B2 plus e raise to power 2r etcetera and Bt e raise to power rt. Which can be written as B0 1 plus e raise to power r plus 2r plus e raise to power rt.

Now, approximate value of this it is possible to solve this kind of equation approximate value of this function would be B0 divided by 1 minus e raise to power minus r or roughly B0 divided by r.

Now; that means, if we know the rate of growth of births during certain time period during which birth births number of births has been increasing exponentially. Then the total number of births total number of births can be known as B0 divided by r births in the beginning of that time period divided by the rate of growth very simple other and Nathan key fits.

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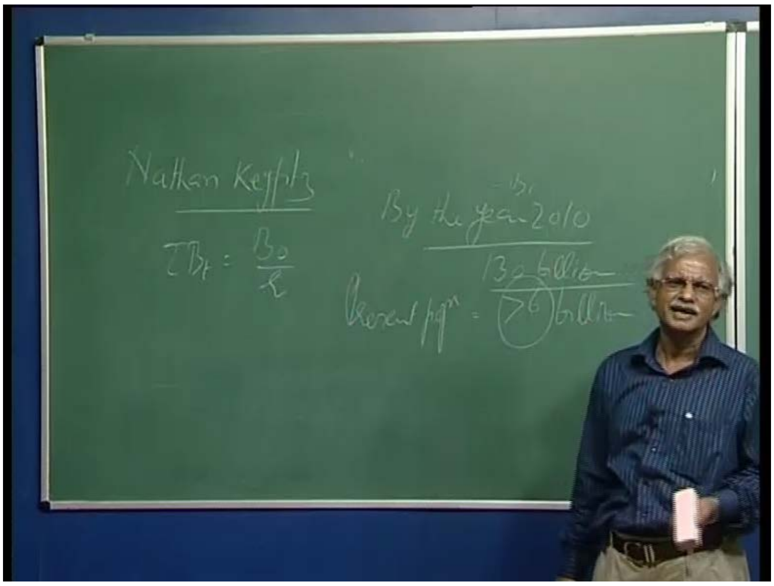

A noted English demographer Nathan Key fits divided the history of mankind into a number of periods during which growth of births was relatively constant. Later on when I discuss theory of demographic transition, you will see that it is possible to divide the history of mankind into number of such periods. Because you will also see that for most of the history say ninety percent history or more than ninety percent history of mankind growth rate of population has almost been 0, the number of births has been same as number of deaths and the population did not grow.

So, using that formula that sigma BT is B0 divided by r you can calculate how many humans were born during a given time period. And if you add all these numbers you can know how many people have ever been born on this planet earth. When Key fits made this estimate about three decades ago he estimated a of about eighty billion.

Today if you do this exercise I did this exercise for two thousand ten. I can say that roughly I arrive at the figure of hundred thirty billion of course, it depends on what your assumptions regarding different time periods are what your assumptions regarding rates of growth of births during different historical times are.

But overall I can say then that. So, far by the year two thousand ten means starting from the emergence of human population starting from the time first man appeared on this planet earth nearly 130 billion people have been born 130 billion people have ever lived on this planet earth. And you know that population of the world today its more than 6 billion present population in more than 6 billion; that means, of all the people ever born on this planet earth 6 by 130 nearly 5 percent are present today.

Is it not an interesting figure, that on the basis of some simple mathematics you can say that nearly 130 billion people have ever lived on this earth. And you can you can find out from your national and international data that of these hundred thirty billion people 6 billion people or slightly more than 6 billion people are living; that means, nearly 5 percent people 5 percent of all those people who have ever lived on this planet earth are alive.

So, this kind of interesting answer can be given to question how many people ever lived on earth by using a simple mathematical idea. A concentrate of growth of births the idea of dividing history of mankind into a number of historical periods during which births were either constant or they grew at a certain fix rate. It is also possible to take into account changing births rate or changing rates of growth of births and for that you will require a more sophisticated model. So, I said that we go from simple to complex.

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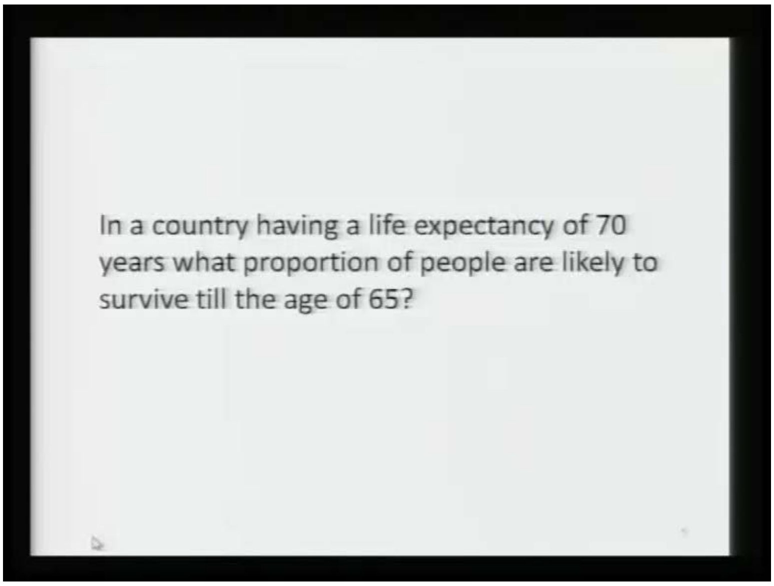

Another interesting question which a model can answer, that in a country having a life expectancy of 70 years what proportion of people are likely to survive till the age of 65. Substantive part or the conceptual problem is this, that today while average age of life is constantly increasing everywhere in all countries. You know what were the average age of life in India 100 years ago it was 22 years; that means, a child born in year 2000 a child born in year 1901 could expect to live on the average for 22 years. And that was because mortality was high infant mortality was high there were epidemics food shortage malaria, kala azar, influenza status of women was low fertility was high lots of women died during child birth considering everything it is possible to explain why life expectancy was so low as 22 years.

There had been years where life expectancy was even less than 22 years it was only 20 years or even less. Today in India life expectancy has gone up to 64 years. So, in 100 years time see the big change that life today a child born in India child may male or female we are taking of an average child combining both males and females if a child is born today then that child is expected to live for 65 years 100 years ago a new born child was expected to live for 22 years only. So, there is a big change big change of nearly 38 years.

So, because of this change because of improvement in life expectancy and also because of change in birth rate, in 1901 where life expectancy were 22 years a woman on the average produce 7 or 8 children. Today where life expectancy is 64 years an average woman in India produces 2.7 children less than three.

Now, because of the twin effect of a these factors average age of population is going up less number of children are born, but more of them survive. So, age of population is going up age composition is changing while talking about composition of population I said that population is ageing and higher is the decline in birth rate and more is the improvement in life expectancy more is the aging. And this aging is causing all kinds of problems economic political social.

Aging has become a big issue in developed countries it is not such big issue in developing countries like ours today, but in years to come if birth rate continues to decline today average fertility, is as I said two point seven, but ah if we follow the trends of developed countries it is possible that a time will come when we will have average number of children as 1.5 less than two you may have them. So, called second demographic transition and the number of children may be less than two; that means, a couple a husband and wife may not be replace even by two children.

So, on the fertility side this is going to happen on the mortality side life expectancy is increasing. So, more and more proportion of population is becoming aged for policy purposes therefore, for planning purposes for next 5 year plan government of India may like to know that if life expectancy increases from present 64 to 70 what will be the proportion of people who will survive till 65. Or if life expectancy increases from 64 to 75 what will be its impact on proportion of people surviving to 65 you see that in some of the developed countries like Japan or Germany or sweeten life expectancy has gone up to more than 80 we cannot expect it to be more than 80 during next 5 or 10 years, but our life expectancy may also improve significantly and therefore, for policy planning purposes we may like to know.

Now, this is a question again we see the existing methods and methodologies of sociology will not be able to answer, you cannot conduct a survey to answer this question its dealing with future population, you cannot have experimental design it will be unethical immoral to have two groups of people.

One group were like life expectancy is 64 only and another group life expectancy is raised to 70 by suitable health intervention programs and therefore, you cannot compare we are average ages or age distribution of population, but if you know the relationship between age distribution of population on the one hand and fertility and mortality on the other. We will see a little later how this can be achieved then it is possible ah to predict on the basis of mathematical formula that if life expectancy goes up to 70 what will be the corresponding change in proportion of people surviving to 65.

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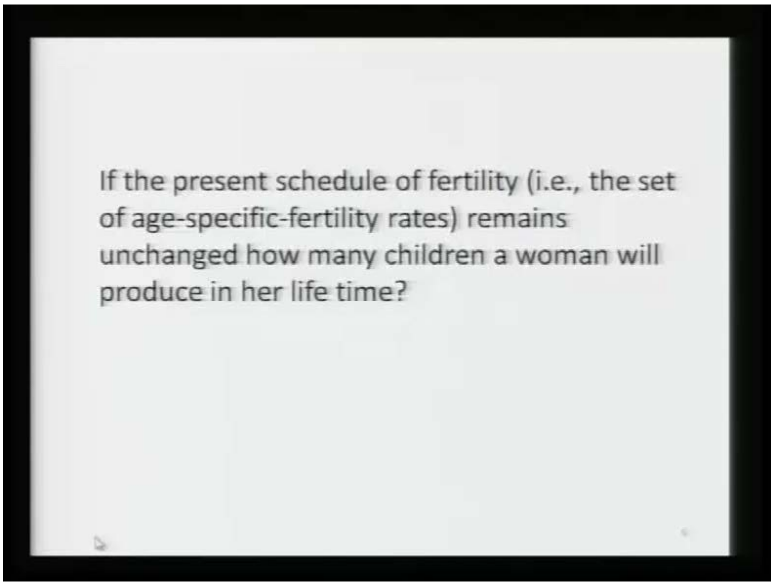

So, this is another question for which we require demographic or mathematical modeling another question, if the present schedule of fertility schedule of fertility means age sets of age specific fertility rates that is I define age specific fertility rate as number of children born to women in certain age group divided by number of women in that age group average number of women in that age group and some total of all that is total fertility rate.

So, if the present schedule of fertility means age specific fertility rates at different ages remain unchanged how many children a woman will produce in a life time according to n f h s 3 it is about 2.7 actually, it is also a model what you have is inputs you have age specific fertility rates or probabilities of producing babies at different ages in the reproductive period and by using certain mathematical logic.

You are saying that if group of women enters reproductive period say age of 15 or age of 17 at the same point of time and throughout their life they experience age specific fertility rates as shown by your schedule of fertility then by the time they cross the reproductive period say 45 or 49 on the average they will produce. So, many children

So, when I say that according to NFHS 3 total fertility rate is 2.7 I am only adding age specific fertility rates at various ages. It is a model total fertility rate are derived in this way is based on modeling this is also a model it is based on certain assumptions and one assumption is that age specific fertility rates will not change this is an assumption this is not reality.

Women who are experience experiencing certain age specific fertility rate at the age of 20. 10 years later when they are 30 they will not necessarily experience the same fertility rate which women of age 30 are experiencing today because over the years fertility rates are changing, but this is an assumption and with this assumption add certain mathematical work I can say that if age specific fertility schedule remains unchanged then on the average when women are crossing their reproductive period they have produced 2.7 children on the average this is based on that model we will see what kind of model is this.

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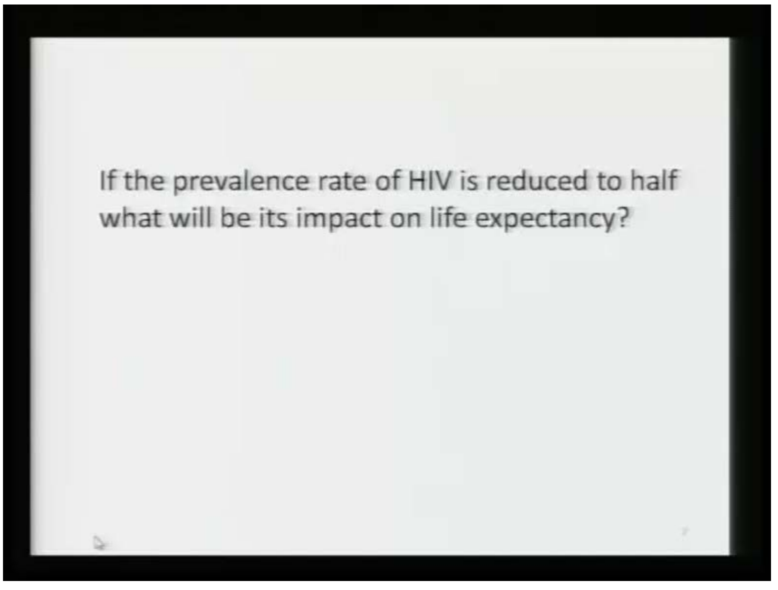

Another question which would be of interest to health planners, that if the prevalence rate of HIV is reduced to half what will be its impact on life expectancy. You see this question is specially significant in the context of those African countries where prevalence of HIV is quite high there are countries where prevalence of HIV is as high as

20 percent or 30 percent means, 20 to 30 percent of all the adult men or women are carrying HIV human immune deficiency virus which is producing a very high mortality at subsequent ages which is also making children orphan and therefore, all health planners everywhere they are interested in reducing HIV prevalence rate or if HIV has not acquired that epidemic proportion as in African countries.

In India according to 2006 figures 0.63 percent of all women in the reproductive period were HIV positive. So, India does not have that serious a problem of HIV, but we have to be careful only 20 years ago 25 years ago African countries also has the same level of prevalence and in just 25 years time their prevalence increase from this small level of 1 percent or less than 1 percent to 20 percent to 25 percent. So, we have to be careful that is another issue.

Now, these African countries in order to tackle this problem of HIV are making health interventions. So, that people go for safe sex or counseling pre testing post testing all efforts are being made to reduce prevalence of HIV and some countries are successful also our country also quite successful in reducing HIV rate among certain communities in the country where earlier HIV prevalence was much higher.

Now, for planning purposes then a question would arise what would be the gain of reducing HIV for the life expectancy that if prevalence rate of HIV is reduce to half I said that in 2006 it is 0.36 suppose it is reduced to half of its present level means it becomes 0.18 due to health intervention due to all efforts made by unique and Naco suppose after 5 years. HIV prevalence rate in India reduces from 0.36 to 1 8 what will be its impact on life expectancy; obviously, life expectancy will increase from the present level of 64 to something what will that something be.

Other factors remaining same if life expectancy is not improving due to any other factor it is improving only because of the factor that HIV prevalence rate has been halved then what will be the corresponding gale in life expectancy. For this you have to have a mathematical model that links life expectancy to probabilities of dying due to different causes, and then by eliminating HIV prevalence by half you can see what its impact on life expectancy will be.

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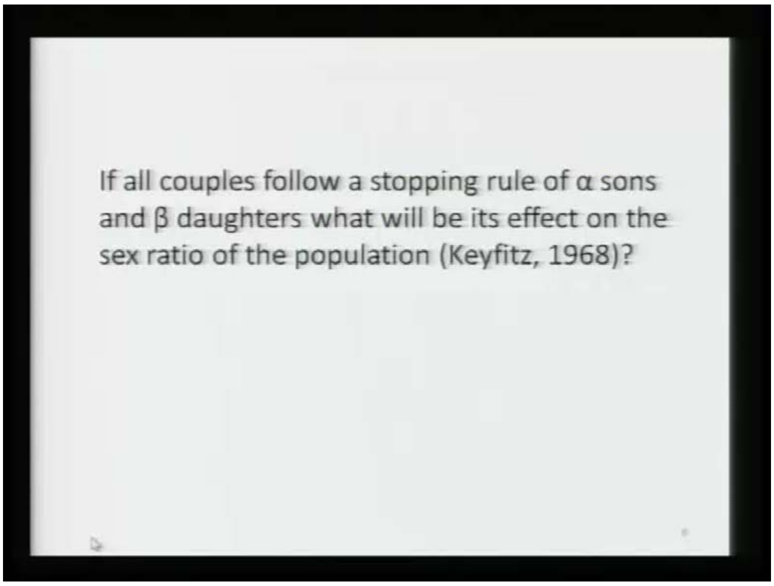

Another question another interesting question I discuss this question even in elementary statistic course because I find that students with 11th standard mathematics of algebra can understand this and it is interesting. The question is that if all couples follow a stopping rule of alpha sons and beta daughters what will be its effect on the sex ratio of the population.

You see all countries have some or other stopping rule. Stopping rule means after how many children couples stop producing children when in 60 KAP surveys started knowledge attitude practice service they showed that Indians want two sons and one daughter, but diverse populations in India may diverse different number of sons and different numbers of daughter some may want three sons some may want two sons some may want one son some may want no daughter or one daughter or two daughters.

In general if the stopping rule in notations is alpha sons' alpha sons and beta daughters people want alpha sons and beta daughters, what will be its effect on the sex ratio population or on the total number of children. You will say how can we answer this using the existing tools and techniques or methodologies or methods it is not possible to answer this question, but I will say that if I have an elementary understanding of probability density function.

Actually, in my class I start with much lower concept and concept of additive and multiplicative probabilities. When you are mutually exclusive and then the probability that one of two or more things will happen is some of their probabilities then you have independent probabilities then the probability that two or three things will happen simultaneously is multiplication of respective probabilities.

Now, using this additive and multiplicative probabilities no advance probability distribution it is possible to show what will be the average family size if everybody wants to produce alpha sons and beta daughters.

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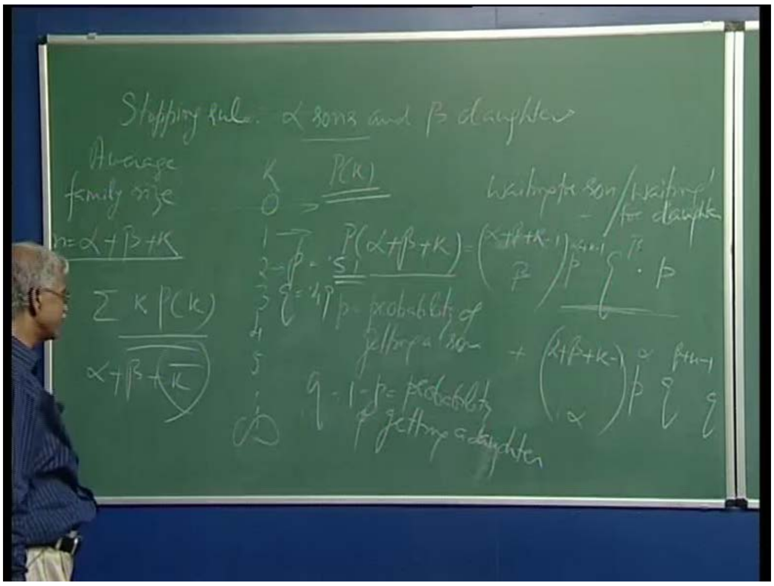

So, I am telling that let the stopping rule be alpha sons and beta daughters. That a couple will stop producing children once they have had alpha sons and beta daughters and I want to know that with this stopping rule what will be the average family size.

It is simple, but for this I have to know the probabilities of different couples stopping at 0 1 2 3 4. If I know what are the probabilities of couples stopping at 0 1 2 3 4 children I can calculate the expected value for that probability distribution or average of that distribution that will be the average family size under this stopping rule.

Now, it is obvious that in under this stopping rule everybody will produce alpha plus beta plus K children. And K will take values 0 1 2 3 4 5 in theory we can go up to infinity nobody can produce infinite number of children, but in theory it does not matter actually writing infinity you will see that as the number is increasing as the value of K is increasing then the probabilities are declining at a very fast rate and beyond a certain numbers eight or nine probability will be almost zero.

So, when K is 0; that means, couples stop after alpha plus beta children. What will be the probabilities then how many; that means, how many couples what is the probability of K how many couples will stop after K may be 0 nobody will stop after 0 because everyone has to have alpha sons and beta. So, K can be 0, but number of children n cannot be n if n is this then n cannot be 0 n has to be more than or equal to alpha plus beta. So, K can be 0 you want to know what is the probability of K equal to 0 probability of K equal to one and. So, on in general let me write probability of alpha plus beta plus K children K 0 1 you just put values of K is in this expression and you will get these probabilities.

If there are two types of couples one type of couples that they were waiting for son or another type of couples who were waiting for daughter. When they stop at alpha plus beta plus K then either they were waiting for one more son or they were waiting for one more daughter. And the probability that couples will stop after alpha plus beta plus K would be this probability plus this probability because, these are mutually exclusive ones one cannot be waiting for both son and daughter a couple is either waiting for son or is waiting for daughter.

What is the probability that the couple is waiting for son let p be the probability of getting a son probability of getting a son and q or 1 minus p probability of getting a daughter. Usually a usually P is 0.51 it is more than 0.5 and thus q is around 4. Around 0.49 if P is 0.51 then q is 1 minus p or point four nine p plus q is 1. So, a child is either male or female.

Now, the probability of is probability of this kind that out of alpha plus beta plus K children. So, this means if they are waiting for the next child to be son they have already had n of daughter's beta if they are waiting for son; that means, they have already achieved their target with respect to daughter.

So, they have had beta and it does not matter what is the order of getting beta it may be first daughter third daughter fifth daughter eleven daughter, but if they are' stopping at K then in first alpha plus beta plus K minus 1 this combination beta out of first alpha plus beta plus K minus 1. They have had beta daughters and they are waiting for a son only.

So, out of these q I can write q raise to power beta and p the remaining ones are alpha plus K minus 1 out of alpha plus K plus beta minus 1 children they have beta daughters and the remaining ones are on sons.

Now, they have a son into P P is the probability of getting a son. So, now, they have a son similarly if they are waiting for a daughter; that means, they already have completed their target with respect to sons they have alpha sons out of alpha plus beta plus K minus one out of one less than alpha plus beta plus K they already had alpha sons. So, p raise to power alpha and the remaining ones means beta plus K minus one or daughters and now they have a daughter.

So, q for students of sociology this may be sometime a bit complex probability express let me repeat it again there is a stopping rule that in a culture everybody wants alpha sons and beta daughters. Now, the question is that with this stopping rule what will be the average family size in order to know average family size we must know what are the probabilities of stopping after 0 child 1 child 2 children 3 children 4 children etcetera. Now, in this kind of combination nobody will stop before alpha plus beta everybody stops after alpha plus beta plus K, but K can take any value 0 1 2 3 4.

For all these values of K if you know the probability of K then simply the expected value of K which can be written as sigma K probability of K. This will be the expected value of this probability distribution. So, and once you know the expected value of this probability distribution; that means, alpha plus beta plus mean of K which will be obtain from this probability distribution you can know after how many children they will stop. in order to get this probability of alpha plus beta plus K imagine that there are two types of couples there are some couples who were waiting for a son and there are other couples who are waiting for a daughter.

Now, these couples were waiting for a son; that means, they have already had the required number of daughters. So, beta out of one less than alpha plus beta plus K they had beta daughters. So, this combination why combination because we are not insisting that first three should be daughters or second third seven should be any out of alpha plus beta plus K minus one any beta daughters will do and probability of getting a son is this.

So, p raise to power alpha plus K minus one and q rise to power beta q rise to power beta means q into q into q into q beta times and now finally, in alpha plus beta plus K the case we get a son. So, this probability gets multiplied by p. Since these are independent events in every trial means every time a child is born probability of getting a son or a daughter is independent of what the probability of getting a son or daughter earlier was. So, I am simply multiplying p s and q s you see that q is multiplied beta times and p is multiplied alpha plus K minus one time all remaining ones are son.

Or in this case you people are waiting for a daughter; that means, they have already had required number of sons which is alpha. So, alpha plus beta plus K minus one out of. So, many children you have alpha sons’ p raise to power alpha probability of son is p. So, p raise to power alpha and all the rest are q or daughters. So, q rises to power the difference between this number and alpha and now they have a daughter q and they stop.

So, the expression for that probability will be this now these two are mutually exclusive events. So, the probability of stopping after alpha plus beta plus K would be this you put different values of K into this expression and you will get probability corresponding to K equal to 0 K equal to one K equal to two K equal to three and. So, on and you can obtain the mean of this probability distribution the mean of this probability distribution is K bar. So, the average number of children in that population would be alpha plus beta plus K bar.

Now, using this kind of logic you can also calculate what will be the sex ratio of this population. For each combination you calculate what is the sex ratio and then you can tell those overalls if stopping rule is this what. So, you can know that if everybody wants a son means stopping rule is alpha equal to one everybody wants one son, but no daughter beta equal to 0.

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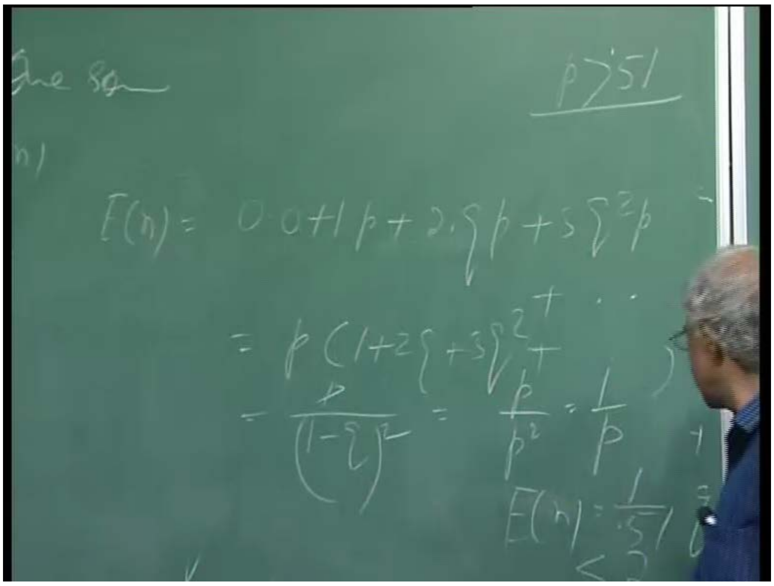

If everybody wants a son that expression will be much simpler and you can see what will happen if everybody wants a son what will happen whether proportion of sons will increase or proportion of daughters will increase stopping rule one son can you guess if I write n number of children after which people will stop signal 0 1 2 3 4 probability of n what will be the probability of stopping after 0 because one son is must.

Probability of stopping after one those will stop after one who is first child is a son. So, p who will stop after two those who had first daughter and then a son. So, that probability will be q p who will stop after three children those who had daughter and then a son. So, q square p and. So, expected value of n for this simple probability distribution would be 0 into 0 plus 1 into p plus 2 into q p plus 3 into q square p etcetera, or p can be taken out say 1 plus 2 q plus three q square and. So, on or p upon 1 minus q square or P P upon 1 minus q square or p upon 1 minus q is p.

So, or 1 upon p in this that means, if p is exactly 0.5 if p is exactly 0.5 means probability of getting a son and probability of getting a daughter is same then the expected value of n would be two in a population in which everybody wants a son on the average people will have two children if probability of P P is more than 0.5 I said that it may be around 0.51 then probability of NAP is more than 0.51 then expected value of NEN expected value of N which is one upon p is 1 upon 0.51 it is less than 2.

So, a probability of getting a son is more than 0.5 then the expected number of children will be less than two. Using this kind of logic you can also calculate what will be the sex ratio of this population and again depending upon what kind of mathematical assumptions you make sometime you can get varying values of sex ratio for this population.

Nathan keyfitz was the first demographer to present this kind of mathematical modeling, but it is not interesting you can has to see that if stopping rule is one son then what will be the average family size if it is one son one daughter what will be the expected family size and. So, on now this question that and this is stopping rule basically refers to gender preferences.

So, the action to which a society is gendered determines its fertility level this is a outcome of this kind of modeling and what is exactly the relationship between degree of genderness and overall fertility levels provided a society is rational family planning methods exist, they are widely known if all these things happen then we can estimate what will be the total number of children what will be this is something equal to total fertility rate.

So, I have made a collection using Nathan key fits formula, I have made a connection between stopping rule and total fertility rate. This logic can be extended further and more complex mathematics can be worked out for answering other types of questions some other questions are like.

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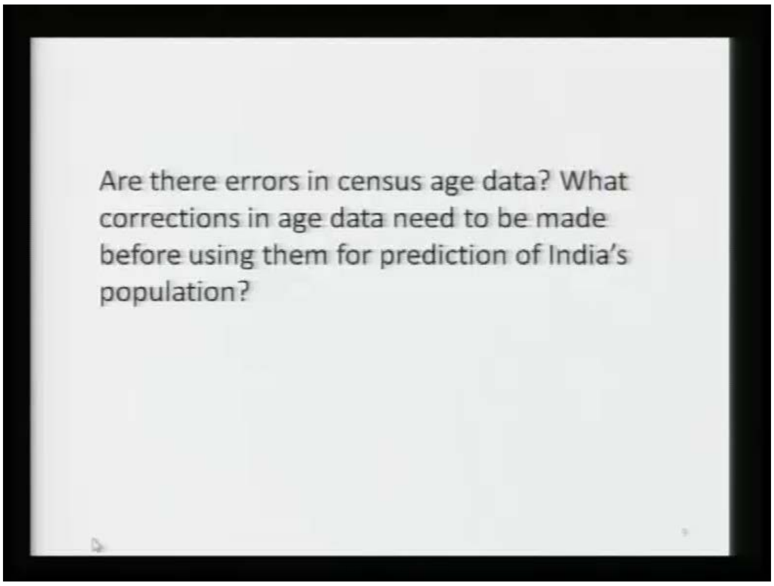

Are there errors in census age data how do we know whether census age data suffers from errors or not you must have some theoretical understanding of depending on the levels of birth and death rates how should age distribution or smooth age distribution look like what corrections in age data need to be made before using them for prediction of India's population.

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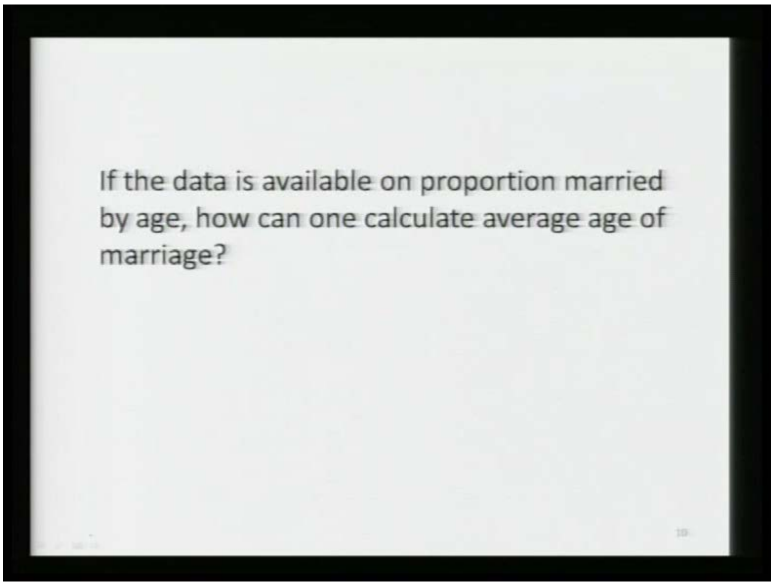

Another question if the data is available on proportion married by age how can one calculate average age of marriage this is this question has been of great significance to

Indian demographers. We required for policy making for studies of health and family welfare for studying success of family planning program we require calculation of age of marriage, but in the censuses we did not have a direct question of age of marriage we had a question on marital status whether a person is married or unmarried. So, we had data on proportion married according to age, if we have some kind of modeling if we can develop a relationship between proportion married and average age of marriage then we can estimate this average from that.

So, first there are some questions in population studies which cannot be answered simply on the basis of survey or census or observations or interviews or focus grow, but they can simply be answered if you know little bit of mathematics and little bit of probability distributions. I will give you one example of how an interesting question like how many people have ever been born on this planet earth can be answered on the basis of simple mathematics.

Now, this is a question how many people are ever born which cannot be answered on the basis of survey or anthropological investigation or anything, but just look suppose the history of mankind can be divided into a number of parts. So, that it in each part you can assume that the rate of growth of birth has been constant.

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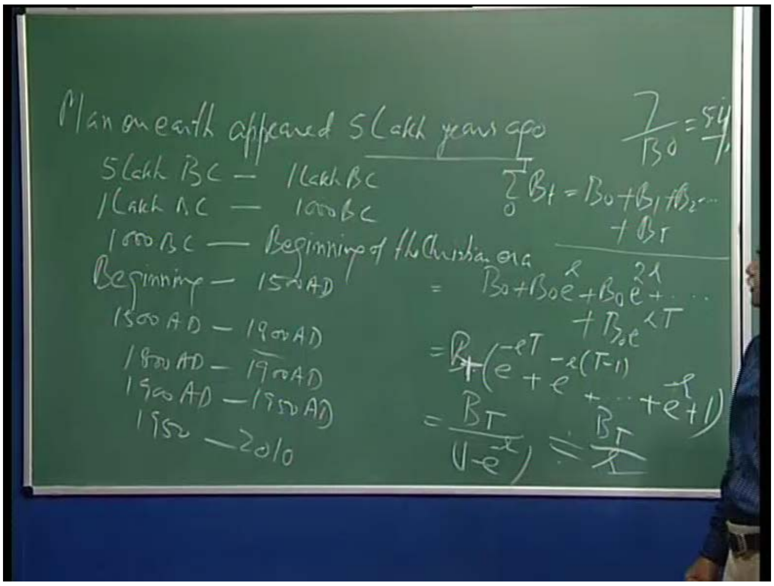

Say nobody knows actually when did man first appear on this planet earth, but let me assume as the that man appeared man on earth appeared 5 lakh years ago. Here this 4 or

4 3 that is not important what I want to show is how can we calculate how many people have ever been on this planet earth.

Now, this 5 lakh years ago we know that for a very long period of time population of the world of different region different countries remain more or less constant number of births and number of deaths were more or less equal and more or less constant. So, if I can broadly divide history into say 5 lakh BC to 1 lakh BC 1 lakh BC to 1000 BC then thousand BC to beginning of the Christian era. Then from beginning of this Christian era to say 1500 AD because up to 1500 a d the growth of world population was more or less constant then 1500 AD to 1800 AD the date after which world population started growing though at very slow pace then 1800 AD to say 1900 AD 1900 AD to 1950 AD and then 1950 to 2010 the present date.

We find that more or less these periods births were constant now if births are constant or they are growing at a slow pace say rate of growth r during a certain time period then I can write that total number of births during some period 0 to capital T. Bt is number of births at time t and I am calculating how many children or how many people are ever born between time 0 and capital T. This can be written as B0 number of births in the initial year plus B1 number of births in the second year plus. So, on going up to Bt for small t equal to capital T and if I assume that these births are growing declining at a constant rate r then I can write that this is B0 plus B0 is e raise power r plus B0 e raise power two r and so on finally, B0 e raise power r capital T.

Now, B0 can be taken outside and it can be written as B0 or if take say B0 e raise power r t this same expression can also be written in the form that Bt e raise power minus r t. If I write B capital T here e raise power minus rt plus e raise power minus rt minus one and this will go up to B this will go up to e raise power minus r plus one.

So, what I have written that here I have written births in a particular year. As multiple of two things birth in the initial year multiplied by the proportion by which births are grown. Here I have taken B capital T there is a reason for that the reason is that now, it is possible for me to use the logic of geometric series and write some of this expression from some of geometric series. Here these e r e 2r etcetera, are all positive terms and it is difficult for me to write some of this thing a using a simple formula that I will use.

While in this case because these are all ratios. So, it is possible for me to simplify the matter and this becomes B capital T divided by one minus e raise power minus r or approximately equal to B capital T divided by r. How simple this is we want to know how many how many people are born in a particular time period say 5 lakh BC to 1 lakh BC. We know that number of births were constant, but if the number of births is changing even then there is not much of problem number of birth may be changing increasing or decreasing using this series assuming that the births are increasing at rate exponential rate are per year I can write like this or I can start from B capital t number of births in the final year and writer an expression like this.

Now, this is a geometric series and in this geometric series the ratio between two consecutive terms is less than one. So, it can be written like this and once I write like this then the approximate value of this is simply, be in the final year of the time period divided by rate of growth. By using this method we calculate how many people are born in this period how many people are born in this period and. So, on and you add them you will arrive at a god approximation of how many people have ever lived on planet earth.

I was doing this exercise and I found that. So, far nearly hundred thirty billion people have lived on this earth hundred thirty billion people have ever been born on the planet earth. And of this hundred thirty billion people close to seven billion are present today the world population today is around 7 billion.

So, you can see that out of 137 are present nearly seven by 13 nearly 60 percent 7 by 130. So, nearly 5.4 percent you can say that nearly 5.4 percent of all those people who have ever been born on this planet earth are present today the world population is 7 billion and hundred thirty billion are ever born.

So, it is an interesting I do not know whether there is any practical use of this for planning or anything, but it is an interesting question and science deals with human curiosity how many people have ever lived on this earth and using this simple geometric series this logic I can say that about 130 billion people have ever lived on this earth and of them 7 billion are present. So, nearly 5.5 percent of all those born are living today.

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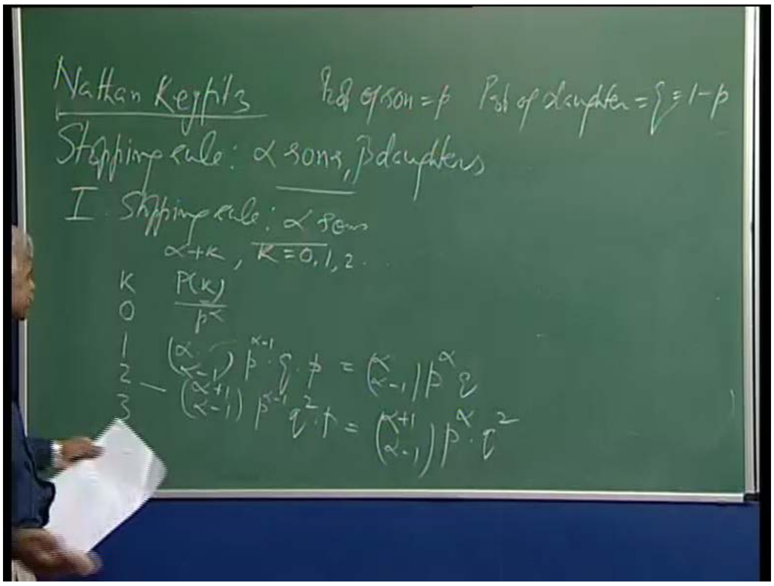

Is it Nathan key fits a well known demographer in his book introduction to population mathematics dealt with the question of that if in a culture there is a stopping rule that people want to have alpha sons and beta daughter then how many children will be born.

You know that in all society there is a gender bias and people do not want equal number of sons and daughters and; that means, that the number of children in the population will be more than alpha plus beta because everybody will not be. So, fortunate as to have alpha sons and beta daughters in first alpha plus beta children. It is possible if you have just the basic idea of probability distributions it is possible to calculate for you what will be the average number of children in such a population and let me show this exercise first with the stopping rule alpha sons.

Let first stopping rule, is alpha sons. I have to write to calculate how many children will be born in this population per women or per person I must know the probability distribution of number of children under this stopping rule very simple.

Now, if stopping rule is alpha sons then; obviously, kapok's are going to produce alpha plus K children K goes from 0 2 theoretically infinite, but it cannot be infinite may be four five six something values of K 0 1 2 3 4 and probability of K. What is the probability let somebody will stop at alpha or alpha plus one, or alpha plus two, by using simple rules of additive and multiplicative rules of probability you can write down such distributions simply. Now, in case somebody stops at K equal to 0 it means that all first alpha children are sons.

Let the probability of son probability of son be p and probability of daughter be q or 1 minus p students of population know that p is slightly more than 0.5. That is beside the point p if p is 0.51 then q is 0.45 now of all the alpha children all must be sons then only couples will stop at K equal to 0.

So; that means, p into p into p into p alpha times and the probability will be p raise to power alpha. When we come here what is the chance that couples will stop at children alpha plus 1 under this stopping rule that everybody wants to have alpha sons it means if they have come to alpha plus 1 then it means that out of first alpha children there was a shortage of one child one son one; that means, out of alpha children first alpha children they had alpha minus 1 sons and 1 daughter and they were waiting for 1 more son to be born in their family.

So, out of first alpha children there are alpha minus 1 son. So, alpha minus 1 combination out of alpha children there are alpha minus 1 sons and the probability of getting alpha minus 1 son is p raise to power alpha minus 1 and there is one daughter. So, probability of daughter is q into now if they are stopping at alpha plus one. So, last child is son. So, multiplied by p or alpha combination alpha minus 1 p raise to power alpha and q.

Similarly, who will stop after alpha plus 2 those will stop after alpha plus 2 who among first alpha plus 1 child had only alpha minus 1 sons and they were waiting for 1 more son. So, alpha plus one combination alpha minus 1 or they are stopping at alpha plus 2. So, out of first alpha plus one they had only alpha minus 1 sons which order in which order that is not important, but they have only alpha minus 1 son. So, p raise to power alpha minus 1 and the remaining the difference between the 2 is of daughters. So, they have 2 daughters and now they have a son and they stop.

So, alpha plus 1 alpha minus 1 p raise to power alpha and 2 square like this you can see that probability of stopping at alpha plus three will be alpha plus 2 c or combination alpha minus 1 p raise to power alpha and q raise to power three like this you can derive this is only for your for illustrative purpose.

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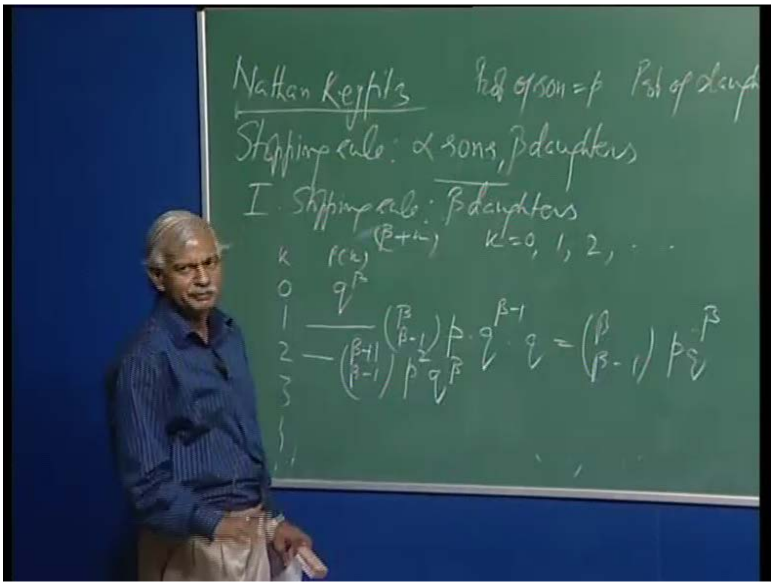

Similarly, if the stopping rule is beta daughter nobody wants a son everybody wants a daughter and minimum number of daughters is beta, then again you can imagine that people are going to stop only after beta plus K. K may be 0 if they are fortunate enough to have beta daughters consecutively without having a son they will stop at beta otherwise if there is a shortage of 1 they will go for beta plus 1 or beta plus 2 and. So, on and you can likewise write their probabilities like a K 0 1 2 3 4 and so on.

Theoretically we go up to infinity, but infinity does not make any sense here and probability of K the probability of stopping at beta plus 0; that means, all the beta children consecutively are daughters so; that means, q raise to power beta the probability that a couple is going to stop at beta children means K equal to 0 will be q raise to power beta first child is daughter second child is daughter third child is daughter all first beta children are daughter. Who will stop after beta plus 1 those who will who out of beta children first beta children had only beta minus 1 daughters; that means, there is 1 son and; that means, there is 1 son and beta minus 1 daughters and now the next child to be born is daughter again. So, beta minus 1 combination p and q rise to power beta this will be the probability.

Similarly, who will stop after having beta plus 2 children that will be beta plus 1 combination beta minus 1 and p now it will be square and q beta. In case of third K equal to 3 it will become beta plus 2 combination beta minus 1 p cube powers of p will keep on increasing as the number of children is increasing powers of p will increasing will increase and q power will remain same beta as soon as they achieve beta daughters they will stop.

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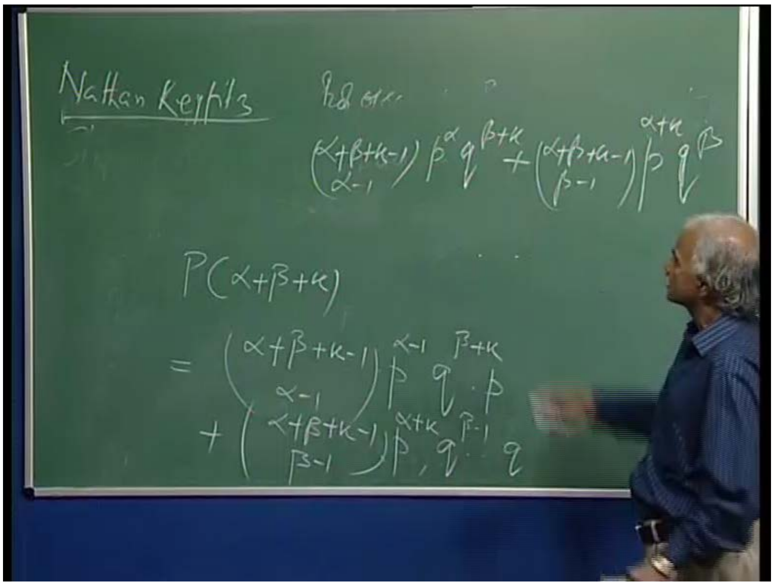

In general if I have to write an expression for alpha sons and beta daughter the original question it is simple, under the stopping rule that people stop after having alpha sons and beta daughters they will have alpha plus beta plus K children here K may be 0 may be 1 may be 2 and so on.

Let me write just 1 you are not going to be students of mathematics the purpose is only illustrative I want to show that this an understanding of simple rules of probability how certain fascinating questions like this, that may culture if there are stopping rules like alpha sons and beta daughters what will be the average family size.

You can also calculate what will be the sex ratio of population under this stopping rule or how different stopping rules changes in alphas and betas can produce changes in sex ratio. So, the probability of producing alpha plus beta plus K children.

So, this time there are two types of couples either the couple is waiting for a son or the couple is waiting for a daughter; that means, out of alpha plus beta plus K minus 1. If they are waiting for a son; that means, they have one son less than their target alpha minus 1 in which order that is not important.

Out of first alpha plus beta plus K minus one child they have alpha minus one sons since the probability of getting a son is p. So, p raise to power alpha minus one and the difference between the two is number of daughters. So, q raise to power this whole thing minus alpha minus 1 beta plus K this is the number of daughter they have and since the probability of having a daughter is q. So, this and now they have a son and they stop.

Another possibility is that out of alpha plus beta plus K minus 1 they had beta minus 1 daughters only suppose, or there is another category of couples who are waiting for a daughter; that means, out of first alpha plus beta plus K minus 1 if they are stopping at alpha plus beta plus K then it means that out of alpha plus beta plus K minus 1 which means in the last alpha plus beta plus K minus 1 children they had only beta minus 1 daughters and they were waiting for 1 daughter the probability of daughter being q. So, beta q beta minus 1 and the difference between the two consist of sons. So, the difference this alpha plus beta plus K minus 1 minus beta minus 1 that is alpha plus K alpha. So, alpha plus beta plus K minus one combination beta minus one p rise to power alpha plus K into q into q rise to power beta minus 1 and now they have a daughter. So, this is the expression.

The probability that they will stop after having alpha plus beta plus K children is this since these two are mutually exclusive events either the couple for waiting for a son or they were waiting for a daughter. So, these two probabilities can be added or 1 can write to simplify this further I can write that alpha plus beta plus K minus 1 combination alpha minus 1 p raise to power alpha and q raise to power beta plus K plus alpha plus beta plus K minus 1 combination beta minus 1 and p raise to power alpha plus K and q raise to power beta.

This is a probability that under the stopping rule of alpha sons and beta daughters people will stop here, just too since all of you do not have mathematical background. So, let me show you what will happen if the stopping rule is very simple.

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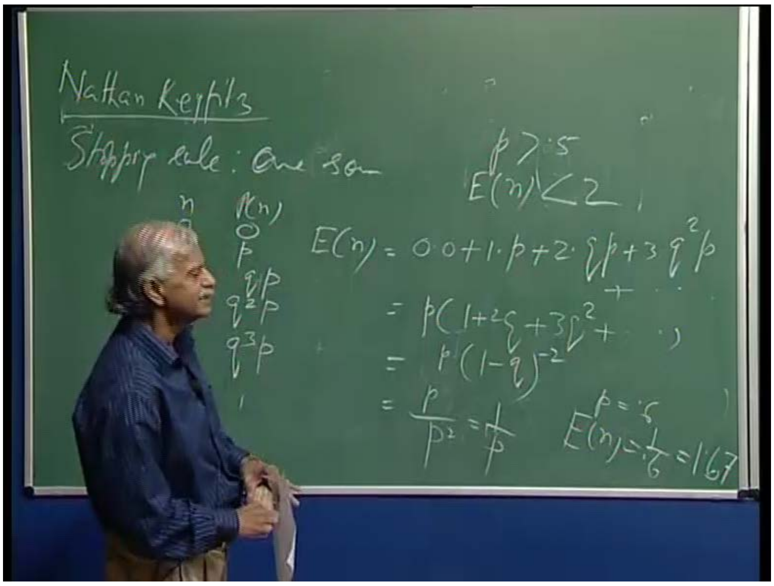

If the stopping rule stopping rule is one son then the probability of stopping after n children 0 1 2 3 4 5 probability of n nobody will saw after 0 because everybody wants to have a son and who will stop after one those who have a son probability is p who will stop after two those who have ah out of for whom first child is daughter q and second child is son. Who will stop after three who have first two daughters and then son who will stop after four who have three daughters and then son.

The average number of children for this distribution would be expected value of this probability distribution you can check that sum of all these probabilities will be 1. So, expected value of n in this would be 0 into 0 plus one into p plus 2 into q p plus 3 into q square p and. So, on and this will be p one plus two q plus 3 q square plus. So, on or p 1 minus q minus 2 this is this is a series which you get from the binomial expansion of this. So, p upon one minus q square and 1 minus q is p. So, p square of 1 upon p. If p is 0.5 probability of getting a son is 0.5 then average number of children will be two there will be one daughter and one son.

If p is more than 0.51 or p is more than 0.5, then expected value of n will be if p is more than 0.5, then expected value of n which is one upon p would be greater than less than 2. So, they will have less than two children, like if p is 0.6 then expected value of n is one upon 0.6 or 1.66666 or 1.67. If the chance of getting a son is 0.6, then on the average a couple will produce 1.67. I am not interested in exactly whether, what is the weather probability of getting a son is 0.5 or 0.6 or 0.51, the point is that there are some questions like this, which can be answered on the basis of knowledge of simple rules of probability and simple algebra, they cannot be answered by using other methodologies or methods of social sciences. .

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