Applied Statistics and Econometrics
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Lecture-21
Index Numbers (Part-I)

Hello, friends. Welcome back to the lecture series on Applied Statistics and Econometrics. So, today we are going to start our new topic in the course and that is basically the case of time series data analysis. And in this component of the course, we are going to study two major topics and they are namely index numbers, theory of index numbers and the theory of classical time series.

Although, we are going to cover these topics very briefly, but towards the end of the course, we are again going to come back to the case of time series data analysis, but from the econometric context or perspective. So, now, let us have a look at today's agenda items.

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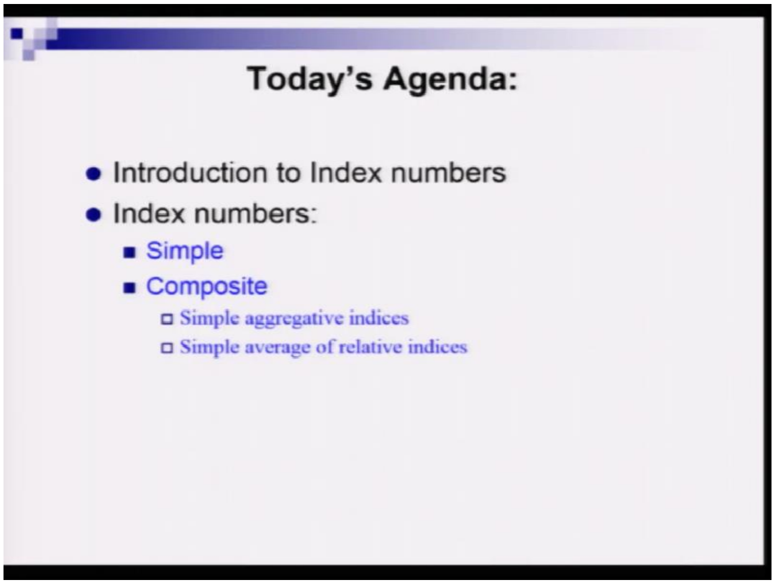

So, now, let us have a look at today's agenda items. So, we will start with a very brief introduction to index numbers which are very useful when you have time series data. And then, we are going to talk about the types of index numbers; broadly speaking, they are of two types simple and composite. And composite index numbers could be of three, four types. But here, in this particular lecture, we are only going to cover two of them, simple aggregative indices and simple average of relative indices.

Data can also come from different periods of time say you are interested to study the per capita income of any country and you have collected data set where you have say 30 or 50 years of data on per capita income or per capita GDP of a country. So, that kind of data set will be called a time series data set.

Now, this time series data has some special features, which are not relevant for a cross sectional data. Of course, we are not going to study all those special features right now, because I have saved some lectures towards the end of this course. So, we will again come back to time series data analyses in the econometrics module of this course.

But as of now, for this week, we are going to study what is known classical time series analysis and we are going to cover some concepts, which are useful to conduct a classical time series analysis. So, you see, with time price changes, it may increase, it may decrease. If it increases or decreases then we have two different terms in economics, they are called inflation and deflation, respectively.

So, the value of money or the value of the unit, monetary unit that is used to measure different economic variables changes and that is why you have to think about a common benchmark, so that you can compare the monetary variable values for different time periods. So, that the in the change in price factor that is the inflation or deflation is duly taken care of. And that is why to start the time series analysis, the best topic is to study index numbers.

So, index numbers is an applied or a mathematical statistical tool or technique that helps us to represent numbers of as of today with respect to some numbers, that is for a reference period. So, what do we mean by that, will be much more clear when we will go through the formal definition of index number and then, we will start the different forms of index number.

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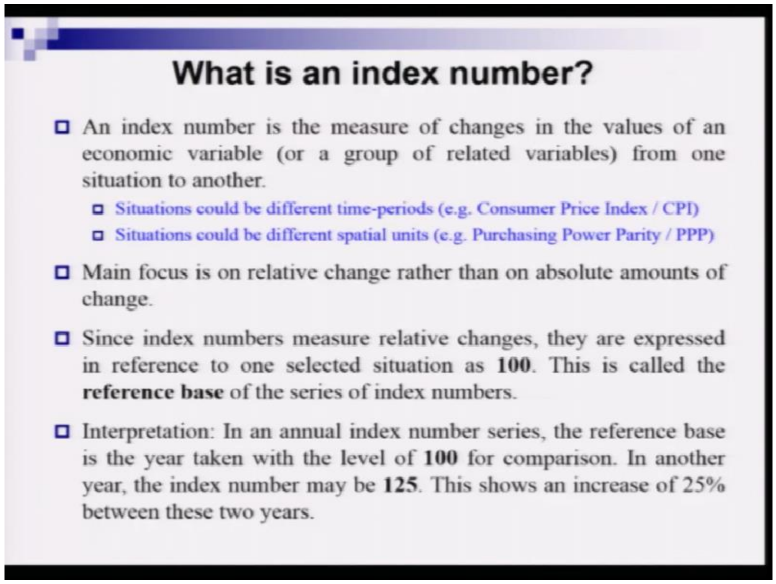

So, formally speaking, this is the way we can define an index number. So, it is a measure of changes in the values of an economic variable or a group of related variables from one situation to another. Well, as I have started the discussion in a time series analysis context, so here, the situation you can say that they could be two or more different time periods and this is what I have started with. But the concept of index number is much broader than time series analysis context.

So here, the situations could also be different special units and here, I am going to give you an example say, have you heard of this concept called purchasing power parity? So, purchasing power parities are basically measuring the value of the currency for one country with respect to the other country.

So, in this context, the situation actually is different country. So, it is not changing with respect to time, well it may change with respect to time, but when we look at the purchasing power parity concept in general then what is changing is basically one country from the other. So, in the purchasing power parity variable we compare the value of currency of one country with respect to the value of currency of another country.

Now, here note that, when we are using an index number, the focus is on relative change rather than on the absolute amount of change. And that is why these index numbers are always expressed in reference to one selected situation and that is called base period or base case and generally that is marked with 100.

Now, why 100, it can be even 1,000? But that is the norm. So, you declare your base situation as 100 and this is called the reference base. Now, how do you interpret when you observe a particular value of some index number series? So, suppose you start with the reference base and the value of the index number for that reference base will be 100 always. And for another year if you observe now the index number value to be 125, then that means that there is an increase of 25 percent between these two years. It implies the year for which you are observing the number 125 and the base year.

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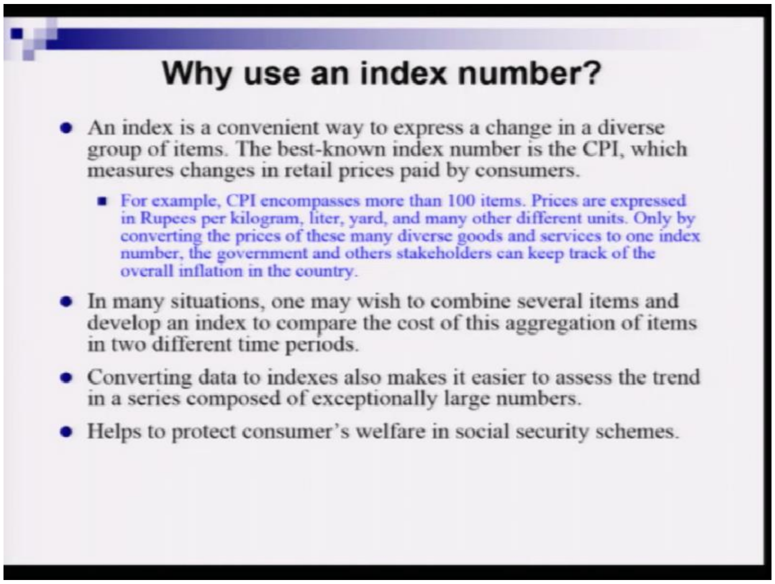

So, now one question can come to your mind that why should we at all study index numbers? Is it just for taking care of inflation? The answer is not really, it of course, the price index numbers which are the mostly used and most commonly referred index numbers, they all are actually taking care of the, in the price changes in the price level or inflation. But this measure is useful in very different contexts, and let us have a look at a least where this index number can be applied.

So, the best known index number is the consumer price index and what does it do? So, it measures the changes in retail prices paid by the consumers. Now, you may ask why, I should bother about the CPI? Well, the CPI actually encompasses more than 100 or 200 items in one single formula.

Suppose, you are interested to calculate the index numbers for all the commodities that you are purchasing from market and you can be interested for an index number for wheat, an index number for oil, an index number for clothes, and what not. But these are all the consumer items that you are consuming.

But what if, you want to come up with an aggregate measure of the price level change in the economy, so that aggregate formula shall encompass all the consumption items that you have purchased in the timeframe. So, the consumer price index number is an aggregate formula, which will actually help you to aggregate these price changes for various commodities.

And note that, here the quantities can have different units also. So, it can be price per kilogram, it can be price per liter, it can be price for year, but although they are related with various units of measurement. But this formula will aggregate all different price indexes into one single formula. And that is why it is very useful in economic analysis.

So, the consumer price index number is an index number, which is referred by the Government and all other stakeholders of national economy, so that they can keep track of overall inflation in the country. Now, in many situations, a person may wish to combine several items and develop an index to compare the cost of this aggregation of items in two different time periods.

So, there could be an example of food. So, if you are interested to measure the consumption expenses over two periods of time then you can maybe interested in different categories of consumption expenses and here, let us assume that we are interested in food.

So, for food, then there could be many types of food. So, there could be like cereals, there could be beverages. Suppose you are interested in cereals. So, then you will collect price data on paddy, wheat, etc., and then you are going to express them in a compact aggregate formula. So again, index number will help you to conduct that kind of analysis.

Now, the third point, so the index number can also convert the data such that it makes it easier to access the trend in a series composed of exceptionally large numbers. Suppose, we are talking about GDP of Indian economy. So, the volume of GDP in Indian economy in 2020 compared to 1950 there is a huge jump in the GDP figure of our country, right and these are a no exceptionally large number.

So, suppose, you want to actually measure a trend in the data and the best way to do that you make use of index number, so that the large number, the GDP numbers can be scaled down to a small number. And then, if you plot then the trend will be much more visible. Then the last but not the least point in this slide, index number calculations actually help to protect consumer's welfare in social security schemes. So, what do I mean by that? So, you may be aware of the pension schemes for the senior citizens, sometimes Government also gives some money in as charity to the poor section of our community. So, these are all monetary payments.

So, now, as inflation is taking place, Government wants to protect the welfare of the receivers of these benefits from the Government. Because as I was telling a couple of minutes back that 500 rupees in 2018 and 500 rupees in 2021, they are not of same value, because inflation already has taken place. So, if the Government wants to keep the beneficiary at the same welfare level as of 2018 in this year 2021 then they can make use of the index number and add just the monetary payment, so, that the consumer remains at the same utility level. So, we are going to discuss this issue in the next lecture.

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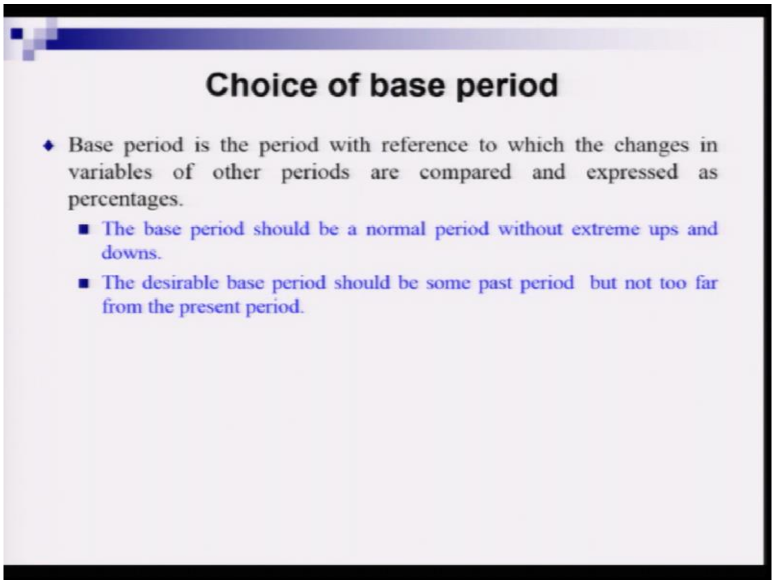

So, I have already referred to this concept called base period or reference point and note that reference point or base selection is a very critical part of index number analysis. So, it is the period with reference to which the changes in variables of other periods are compared and expressed as percentages. So, if you change the base, then of course, these percentages or ratios are going to change. So, where and how you can fix these base period? Because the choice of base is very crucial.

So, statisticians have provided some guidelines to choose a base period and generally, they say that the base period should be a normal period without extreme ups and downs. So, if I talk about an example, I think, the year 2020 should not be taken as a base period because that year was terribly hit by this COVID pandemic and that was an abnormal year.

And the second point that is put forward by the statisticians is the following: so, they suggest that the desirable base period should be some past period, but not too far from the present period. So, if you are interested to have index number for say 2021, if you want to make use of index number for 2021 value then you should not use a series for which the base period is 1990. Why is the case? Because the reference point if it is too far then basically by that time there is a huge change in the consumer psyche or the consumers basket that he or she is consuming and that is why that is going to be reflected in the base year value also.

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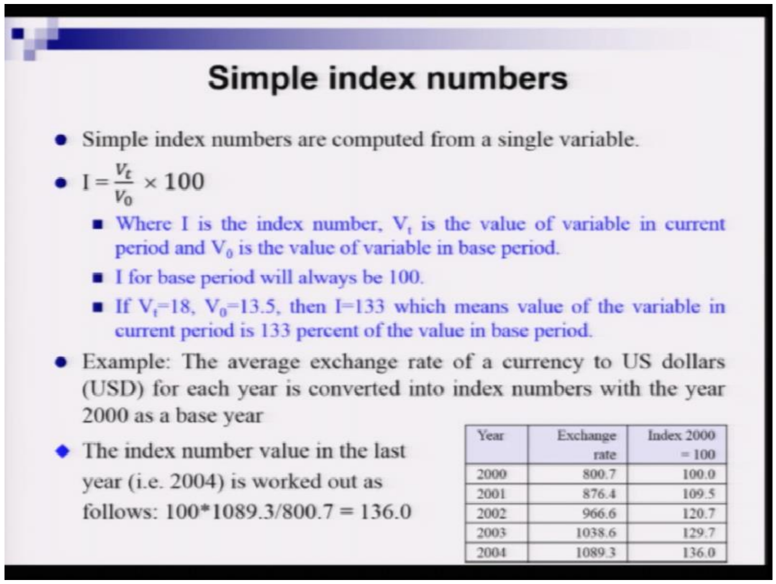

Okay. So, now, we are going to start our discussion on simple index numbers. So, simple index numbers are computed from a single variable. So here, I am showing you a formula. So, suppose I denote my index number by capital I; and there is one variable V, for which I am collecting values for different time periods.

So here, let me tell you again, as we are discussing index number in the context of time series data, I am not talking about special comparison. So, for all comparisons that I am referring to in today's lecture and the next lecture, I am comparing values across time periods, okay, so please remember this. So, the Vt is the value of a variable in current time period and V naught is the value of the variable in the base period. So, base period is denoted by t equal to 0.

Now, I told you that the index number value for the base period will always be 100. So, if we now say that Vt, which is equal to say 18, some arbitrary number and V naught is basically 13.5, then the index number value is 133. So, that means that the value of the variable in the current period is 133 percent of the value in the base period.

So, now let us show you an example through some numbers. So here, I am showing you the average exchange rate of a currency to U. S. dollars for five years. And these are all fictional numbers. So, these numbers are converted into index numbers, let me make 2000 as the base year.

So, you see the table in the southeast corner of the slide, here, I am showing you the values for 5 years from 2000 to 2004. So, exchange rates are given in column number 2. And now, I want to convert these numbers into index numbers. So, index number is taking value 100 for the base year 2000. And then by applying the formula that I have shown you above, I have calculated the values for different cells under the index number column.

So, for an example, if I concentrate on the last year 2004, then the index number value is 136. So how did I get it? So basically, I divide 1089.3 the index number, the exchange rate value by the exchange rate value of the base year that is 800.7 and then, I multiply the ratio by 100. And that is the way I get this 136.0 as the index number value.

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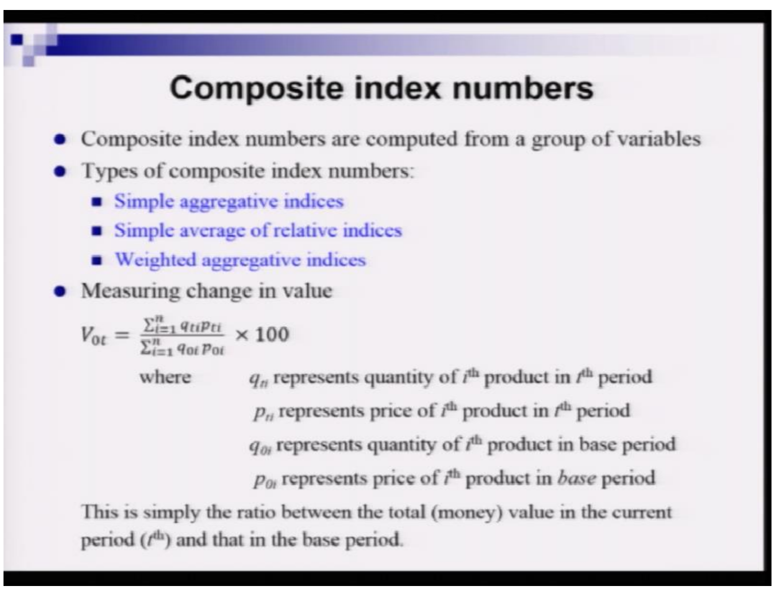

Now, in the next slide, we are going to talk about the composite index numbers. So, the composite index numbers are computed from a group of variables and there are three main types of composite index numbers, namely simple aggregative indices, simple average of relative indices, and the last one is weighted aggregative indices.

Now, the aggregative, weighted aggregating indices can be broken down in many other types of index numbers, but I am saving this for the next class. So, I am going to talk about the composite index numbers through the example of a value index. So, what is a value index? So, here we are going to deal with value as the variable. So, what is value of some commodity or some item? So, value is basically a product of the price of a commodity and quantity of the commodity that you have purchased or consumed or produced, if you are a farm.

So, you see here, within one variable there are actually two variables embedded. So these separate variables say p and qp stands for price and q stands for the quantity of the commodity that we are concerned with. So, for these entities p and q separately there could be movements over time. So, we can have actually index numbers for these two sub-items or the components of the variable value.

So, in this context, actually, the complex or composite index numbers or weighted index numbers come into picture. So here, I am going to show you how to measure changes in the value. So, suppose I want to construct an index number and my base as usual is period 0 and I am interested in the value of the index number or the value in period t.

So, here is the formula. So, you see, I have actually used ratio of two different sums and you see that here, the q, t, I, represents quantity of the product in the th time p, p, t, I represents the price of i-th product in the th period. So, that is basically the components that you find in the numerator. So, you multiply these two variables and then you add for all commodities that you are talking about.

And now, let us focus on the denominator. So, what do we see there? So, we see two items quarter 0 i and p 0 i, what are they? So, quarter 0 i represents quantity of the i-th product in base period and p 0 i represents the price of the i-th product in the base period. So here, you see, we are comparing and number of commodities over two time periods and we are not only comparing the physical quantities or the prices of these n commodities, actually we are comparing the value of consumption or value of production of these n commodities for two different time periods through this complicated formula. So, that is why I am writing here as that this formula is simply the ratio between the total monetary value in the current period, which is the th time period and that in the base period, which is zeroth period.

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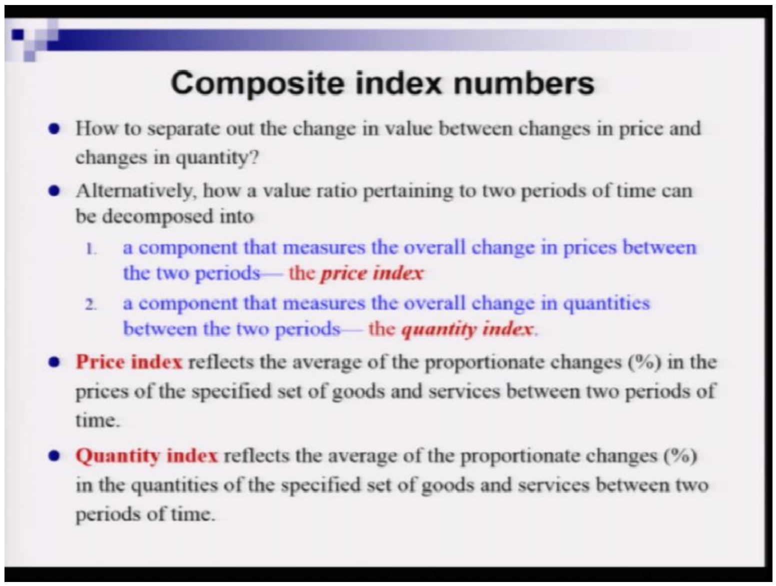

So, now, let us continue the discussion on composite index numbers. So, from that formula, a question emerges, how to separate out the changes in value between changes in price and changes in quantity? So, if you see that there is some percentage change in the value of consumption or value or production, is this entirely contributed by change in prices? Or is it completely coming from the changes in the volume or production or consumption?

So, basically, as I was telling that as V is a product of two variables p and q, so, if we find a change in V, then that change can come from either p or q or both. So, if they are coming from both sources then how do we disentangle these different changes? So, the same question can be rephrased and we can say that, how a value ratio pertaining to two time periods can be decomposed into a component that measures the overall changes in prices between two periods and that is our price index. And another component could be that measures overall change in the quantities between two periods and that is the quantity index.

So, now, we are going to formally define the concept of price index and the concept of quantity index. So, we had already seen the value index in the previous slide. So, let us now have definition in simple layman’s language. So, a price index represents the average of the proportionate changes or you can also say that these are percentage changes in the prices of the specific set of goods and services between two periods of time. And similarly, we can define quantity index, which shows the average of the proportionate or percentage changes in quantities of the specified set of goods and services between two periods of time.

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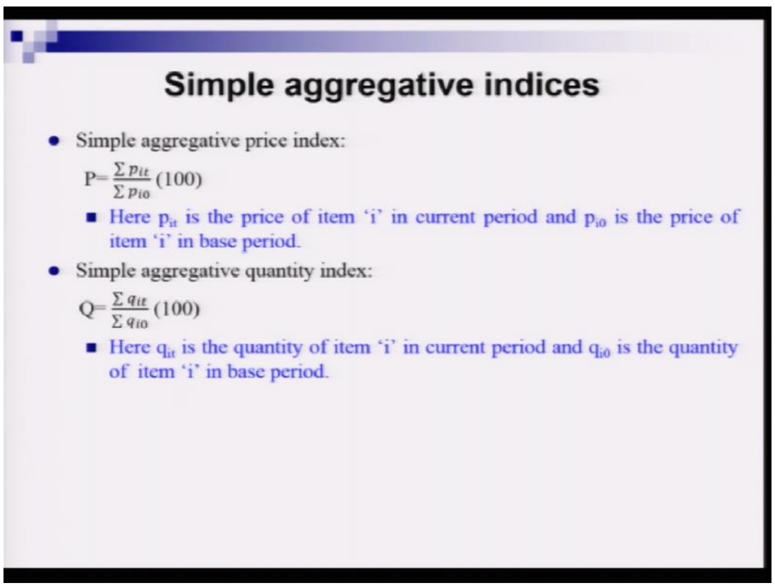

So, now, we are going to talk about the different types of composite indices and in this lecture, we are going to cover only two types of indices. So, we will begin with a simple aggregative price index and here, the formula is given here, and you see that that is given by p equals to a ratio of two sums. And in the numerator, we have some of pit. So, here pit is the price of item I in the current period and then in the denominator we have some of pi naught. So, pi naught is the price of item i in the base period.

And you can surely translate the same formula for the quantity also. So, here you just replace p's by quarter's in the formula that I have shown above and you get the simple aggregative quantity index. And needless to say, that the qit and qi naught abbreviations or notations can be defined in the same way.

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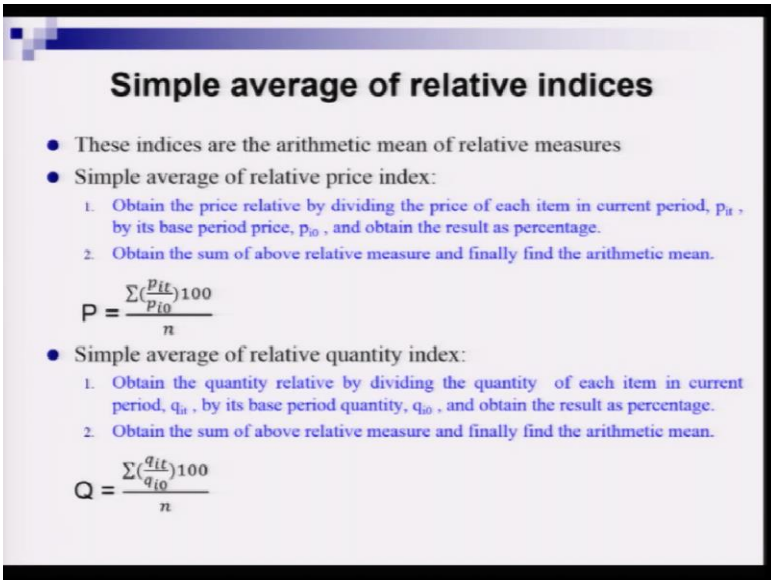

Now, we are going to talk about, briefly about simple average of relative indices. So, these indices are the arithmetic mean of relative measures. So, what do we mean by relative measures? So, when two prices are compared with respect to each other so suppose you have price of a commodity for some time period t and you actually divide that number by price of the same commodity in the base period, then you get a relative measure.

So, the simple average of relative price index can be obtained by following two steps. So, in step one, you obtain the price relative by dividing the price of each item in the current period. So, that is basically pit by its base period price, so that is given by pi naught and then obtain the result as a percentage or ratio and then, obtain the sum of these above relative measures. So, in the second stage, now, you obtain the sum of the above relative measure and finally, find the arithmetic mean by dividing the sum by the number of commodities that you are dealing with.

Now, we move on to the concept of simple average of relative quantity index. Similarly, we can construct the index for quantity or volume of a commodity and again we have to follow two steps. So, first, you obtain the quantity relative by dividing the quantity of each item in current period that is qit by its base period quantity, which is qi naught and then obtain the ratio multiply that ratio with 100, so you will get the percentage. And then, you obtain the sum of the above relative measures for all the commodities and finally, you divide this sum by the number of commodities that you are handling to get the arithmetic mean and that formula is shown here as capital Q.

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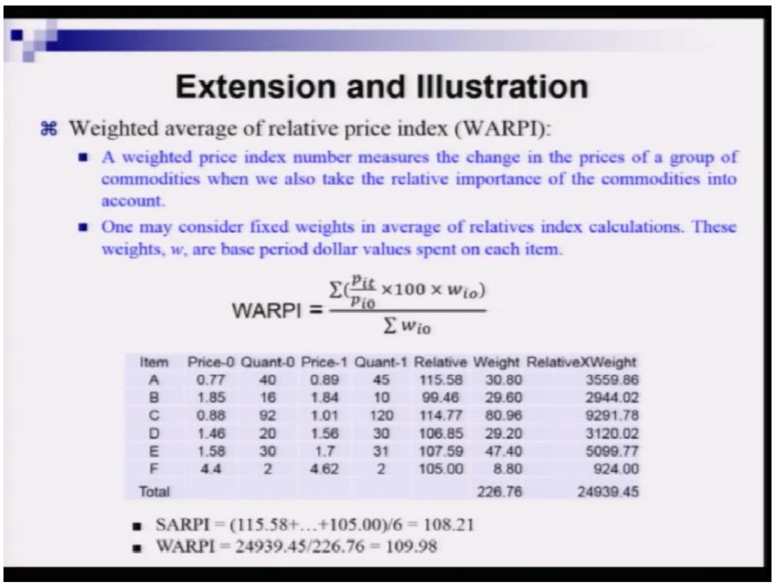

So, let us now look at an extension of the previously learned index number formula and an illustration to end today’s lecture. So, in this slide, I am going to talk about the case of weighted average of relative price index and the abbreviation is WARPI. Now, why do we need WARPI? So, you think about a case or a research problem where a researcher is interested to find out what was the rate of price change or inflation rate for food items.

Now, as I told you that in the food items basket there could be several items like cereals like rice, wheat, there could be milk, butter and bread, there could be sugar, tea, coffee, so, there are several items in the food basket. Now, it is obvious that a consumer is not going to consume all these food items equally. So generally, we can expect that in the food consumption basket, the proportion of the cereals are going to be on the higher side, if not in terms of the value, but definitely in terms of the importance to the consumer.

So, in this discussion, it is evident that if we just simply take an arithmetic mean or simple average of the price relatives to compute the change in price levels, then that may not be a good idea, because the relative importance of different goods in our consumption basket is not same. So, that is why WARPI is proposed.

So, weighted price index number measures the change in the prices of a group of items, where we take the relative importance of those commodities into account. So, while computing WARPI, we consider fixed weights in average of relatives index calculation okay. So, these weights can be denoted by omega or W and these are basically the base period dollar value spent on each item.

If the money currencies not dollar, it can be even rupee. So basically, in some currency units, these weights are calculated. So, here, I am showing you the formula for WARPI. So here, you see we are calculating the price relatives first. So, these are a given by pit divided by pi naught times 100. So, these are basically my price relatives.

Now, I am multiplying each price relative for different commodities with respective weights and they are basically given by wi naught and then I sum over all the items in my consumption basket. And finally, I divide this sum by the sum of the weights. So, this concept is going to be clear, if we look at an illustration.

So, here in this table, I am going to show you a very simple case where I have some items, it can be food, it can be any other major item and then, there are sub items like a, b, c, d, e, f. So, there are six sub-items in that main item for which we are interested to know the price change.

And we are talking about two different time periods one is base period that is denoted by 0 and the current period which is denoted by period one. And for these two periods, I have data on the unit price of these sub-items, six sub-items a, b, c, d, e, f, and then, I also know the quantity purchased and consumed for these six sub-items for both the periods.

So, what I have to do? I have to first calculate the price relative So, to calculate price relative you divide price of period one of a commodity by its price in the base period and then multiply that ratio with 100 and this is the way for sub-item a, I get the price relative value 115.58. And similarly, I can calculate price relatives for the other sub-items also.

And then, I need to calculate the weights. So, here if you remember the weights are basically the value of consumption in the base period. So, here we have to multiply the price of the base period and the quantity purchased or consumed at the base period for each sub-item. So, if I do it for say sub-item a, then I get a weight of 30.8.

Similarly, I can calculate the weights for all other fives sub-items. And that is the way I get my weights and that is given under the weight column of the table. So, I need to take a sum so, I have got the total of 226.76. Now, I have to also multiply the price relative and the weight and that is being done in under the last column of the table and here also, I have to take the sum and the sum is a large number 24,939.45.

So, now, we are done with our calculations in the table. Now, let us look at how we can compute different price index numbers from this table. So first, we will show you the calculation for the simple average of relative price index or SARPI. So, that is basically simple from the previous slides formula. You can remember that this is basically the arithmetic mean of the price relative. So, you actually take 226.76 you divide that by the total number of items in the table, so that is 6 and you get 108.

So, to calculate WARPI, I have to now take the total of the product relative times weight and then divide this by the sum of weights and we get this number 109.98, note that they are different, they are bound to be different, because one is taking care of the relative importance of the items in the consumption basket and the other one is not.

So, let us stop here for the moment. So, in the next lecture, we are going to continue with the discussion on index numbers and there I am going to talk about more popular index numbers, which are called Laspeyres and Paasche index numbers and we are also going to talk about some more applications of index numbers. Thank you.