Petroleum Economics and Management
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Module - 10
Theories of Price Formation of Petroleum
Lecture - 47
Optimum Choice of a Consumer: Problem of Utility Maximisation

Hi everyone, welcome to the NPTEL course Petroleum Economics and Management. And I am your instructor Dr. Anwesha Aditya. So, we are now in module 8 of our course where we are discussing the Theories of Price Formation of Petrol. So, in today's lecture is lecture number 47 in the course where in today's class we will be discussing the Optimum Choice made by the Consumer. So, we will be discussing the Problem of Utility Maximisation in today's class.

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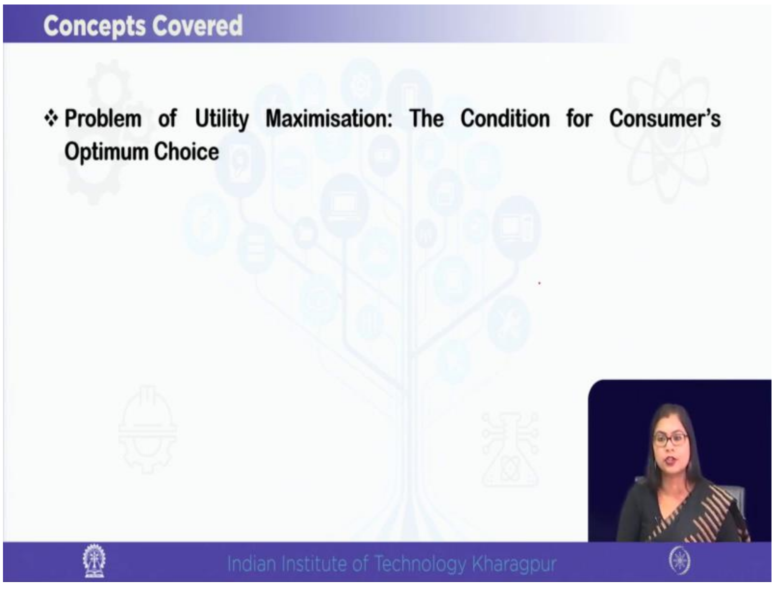

Now, you see what we are doing in module 10. In module 10 we are studying how pricing of a mineral resource or any type of non-renewable or exhaustible resource, which is in limited supply, how the pricing is done, because we have a given endowment of that particular resource in a particular point of time. So, we have to use the resource prudently so, that we have to optimally allocate the resource over time.

We should not be running out of the resource; we should not exhaust the resource and at the same time we should not also be using the resource in such a way that we do not use the resource currently and we keep it for our future purpose. And in future we can have a technological breakthrough whereas, in which some very good substitute replaces the resource. So, the resource can become useless also.

So, we have to use we have to manage this resource very prudently because you see we have also earlier also in when we studied the oil price and how during the lockdown oil price the WTI future price became negative. We have pointed out that it is not about just being endowed with the resource, we have to use the resource prudently, we have to also create a good storage and transport facility. If there is some demand side shock, we should be able to store the resource prudently.

And we have also studied the experiences of the countries that many times the endowment of natural resource may not also be good for a country's economic growth and development. So, we have already studied experiences from Middle East countries which are very much disturbed politically and they are not also very good in terms of the development parameters like education, health and we have discussed that. So, we have discussed the phenomena of Dutch, disease and resource curse.

So, using the resource prudently is very important rather than just having the resource because we have also seen the examples or case studies of Norway or Denmark or even see Netherlands for that matter. So, Netherlands for it is due to the discovery of natural resource in Netherlands and there was a stagnation and recession. So, this name Dutch disease came up, but afterwards Netherland recovered from Dutch disease.

Even there was a balance of payment crisis in 1986- 87 that time in Norway after discovery of petroleum, but Norway was also finally, Norway did better also we know that these countries they perform very good in terms of the human development index also.

So, therefore, when resources discovered in a country which is already developed these countries can manage the resource prudently because they are already diversified. So, they use the rent or the wealth enjoyed from that resource much prudently because if that oil rent or the wealth enjoyed or gained from exporting the resource if that is not used properly if that income or that profit is just enjoyed by the very few people in the country.

So, there will be increased dualism, there will be increased inequality and leading to more corruption and political instability and more increased rate of crime, violence. So, it is not about getting endowed or discovering a resource managing the resource prudently is also very important. And we have also seen with some countries that the countries may also deplete the resource if the country is becomes too much dependent of the resource we have studied the experience of Venezuela.

How Venezuela is a typical case study of the resource curse and Dutch diseases because Venezuela economy was very much dependent on oil rents, the export revenue and from oil and it did not develop the other sector. So, the economy went into a huge recession and the currency depreciated a lot in 2018.

So, that means, using the resource too much will not be good for the country. So, that means, we need to know how to allocate the resource and what should be the optimal pricing because the pricing can dictate the use of the resource if the price is too low in the present period in at present.

So, that may lead to over optimal over exhaustion of the resource. So, high price can also discourage the consumers to use the resource too much because we know that by law of demand there is negative relationship between price and quantity demanded. So, in module 10 we are starting a theoretical framework in which we are finding out what should be the optimal allocation of the mineral resource.

So, not specific to petroleum this model theoretical model can be applied for other resources also. So, how it should be optimally allocated between now and then; that means, present and future? So, for our purpose we are studying a very simple theoretical model involving two periods and then what should be the pricing.

So, if you remember in the first class of this module in that means, in the last lecture we have plotted the marginal utilities of consumption of the resource in period 1 and period 2 and we saw that if we put equal weightage on current consumption and the future consumption, then the resource will be almost equally be allocated between the two periods and the price will also be the same.

However, that will not be the case because I mentioned already that it is not the final allocation because often, we put greater weightage on the present as compared to the future. So, we will be bringing the discount factor and we see that the marginal utilities of the two periods will not be same. So, we will be doing that, but before proceeding further in the theoretical model, we need some conditions to study the theoretical model successfully.

So, we need to know how the consumers the individual consumer of the household maximizes utility. So, in today's class we will be deriving the consumers condition for optimum choice when the consumer or the household is making a purchase decision.

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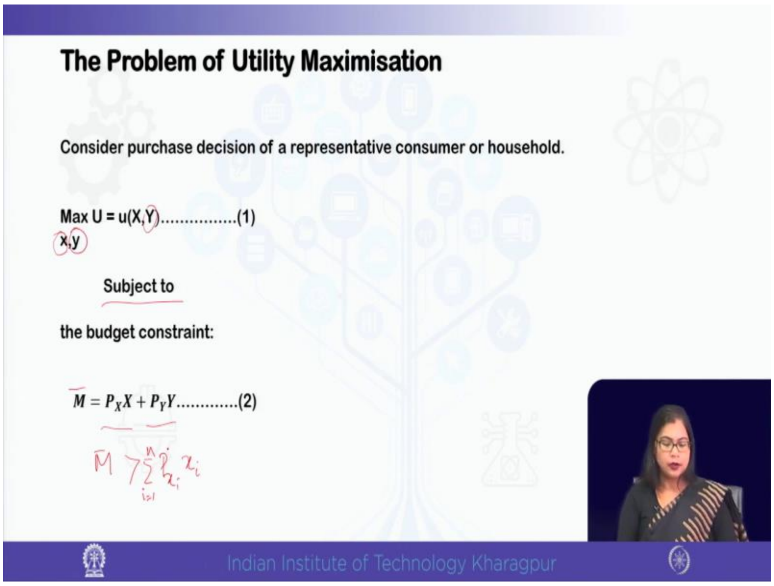

So, let us consider a representative consumer or household. So, when we say household, we refer to a single unit, ok the household as one unit not how many individuals consist of the household. So, the household is considered as a single unit. So, when we make purchase decision what do we want? See we often see that lot of discounts are going on the festive seasons are coming. So, we may want to buy a lot.

See, we want the best of things in our life, but can we buy everything that we want? No, why? Because often we are constrained by our purchasing power, we do not have unlimited income. So, we have already studied that economics deals with scarcity and choices because how if when if economy is also very rich there may not be unlimited resources.

So, just like the economy the individuals are also we are also constrained by our purchasing power. If you even if you consider a very rich person, but see income is or the purchasing power is not unlimited. So, when we are making our purchase decision how much we want to buy which will give us the satisfaction or utility.

So, that decision is subject to the budget constraint because we may like to buy the goods very the best of the goods branded products and in many quantities, but that may not be affordable because all of us our we are constrained by our budget. So, we can state the problem of the consumer or the household. As the consumers objective is to maximize his or her utility. Ok and utility we have already defined that utility is the level of satisfaction received from consuming the goods and services.

So, often we use our two good framework to keep our life simple and we can also represent the two goods in a two dimensional figure. So, that is easy convenient way and it also makes sense because what we can do? We can think of one of the good as a particular good let us say your laptop or mobile and you can also consider say all other goods as a composite good.

Suppose in your daily life you are consuming n number of goods. So, the first good you can consider it is your laptop or your cell phone and from good two to good n you can consider as a composite good say good y. So, x is suppose your laptop or mobile and good y is of the set of all other goods. So, the objective of the consumer is to maximize utility by choosing the amount of consumption of the goods. Hence, we are writing this choice variables are amount of consumption of the two goods.

So, maximize utility which is a function of the quantity of consumption of the two goods and the consumer or the household maximizes utility by choosing the amount of the consumption of the two goods, but the household or the consumer is subject to the budget constraints.

So, what is the budget? Budget constraints means see we do not have our unlimited income right. So, suppose your income at a particular point of time or purchasing power is fixed by say $\bar{M}$ over time it can change, but at a given point of time your income is given say $\bar{M}$.

So, M is your total income or your purchasing power and you see what is your total expenditure? Expenditure is the expenditure you are incurring on the two goods. Suppose you are consider considering that you are only consuming these two goods where you can interpret good y as the composite good as I was mentioning. So, what will be the total expenditure?

So, total expenditure will be the sum of expenditure on good x and good y. And suppose P_x is the per unit price of good x. So, what will be your expenditure on good x? So, if you are consuming x unit of good x. So, this will be $P_x X$ and what will be the expenditure on good y? If P_y is the per unit price of good y and you are consuming y units of good y. So, $P_y Y$ will be your total expenditure on good y.

So, your total expenditure on the two goods will be $P_x X + P_y Y$ and your total expenditure cannot exceed your income right So, if your income is greater than the total expenditure suppose if your total income $\overline{M}$ is greater than your total expenditure say you can also write i running from 1 to n ok summation or you can also write it as $\sum_{i=1}^{n} P_i x_i$.

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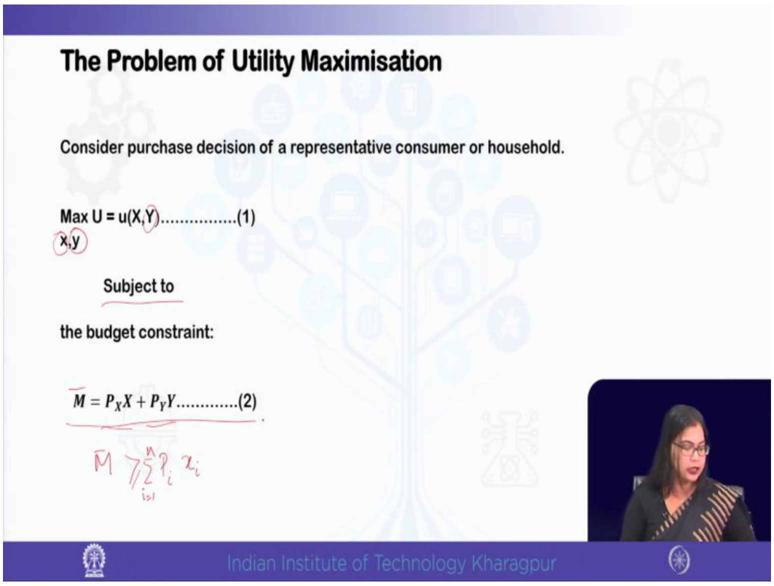

So, if your so, what does this inequality mean? So, the left hand side is your total income and the right hand side your total spending or total expenditure. So, if your income is greater than your expenditure so; that means, you are saving one part. But if this condition holds with equality; that means, your total expenditure is equal to your total income. But of course, your total expenditure cannot exceed your total income right.

So, you can have the condition the budget constraint as with the greater than equal to sign your income is greater than or equal to expenditure and extreme case you are spending all your money you are not saving. So, income is equal to your expenditure and that is how you are writing the condition the budget constraint in condition 2. So, in this simple framework actually the consumer does not save anything at the optimum. So, total income is spent on the consumption of the two goods.

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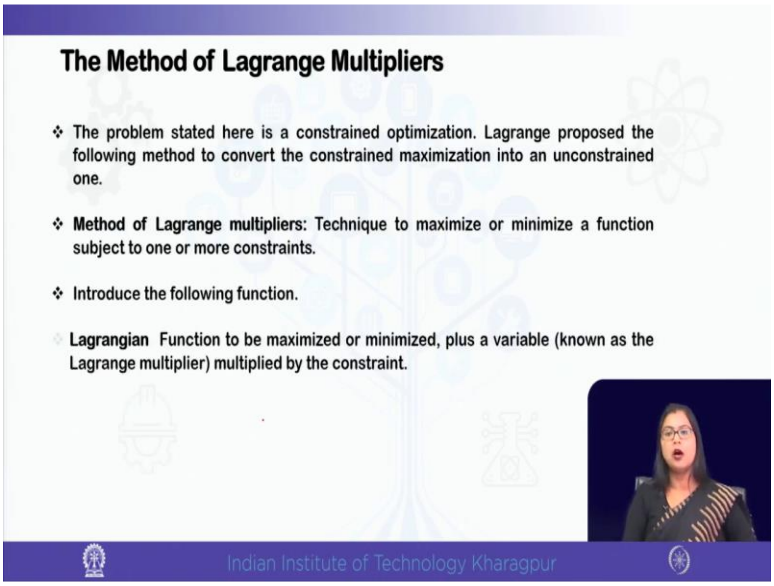

Now, we have to solve the problem of the consumers optimum, we have to find out how the consumer decides how to how much of the two goods to be consumed. Because you see here the choice variables are x and y because the consumer cannot influence the prices of the goods. See we are assuming a perfectly competitive market.

See in most of the cases we as the buyers we cannot influence the market price. We have to buy the goods and services that we need for our consumption at the ongoing market price because as consumers of buyers we individually are very minuscule amount of the entire market.

So, the consumer can maximize his or her utility by choosing only the amounts of the quantities of consumption of the two goods. He or she cannot choose the prices ok. Now, we use a very famous method called the method of Lagrange multiplier in which we convert this constraint maximisation into an unconstrained one.

You see this maximizing utility subject to the budget constraint this is of course, a constraint of optimization right. The consumer is the economic agent is maximizing the objective function the utility function with respect to the constraint. So, it is we have seen that its more convenient to solve an unconstrained optimization rather than a constraint one.

So, hence Lagrange proposed a method in which we can convert the constraint optimization into an unconstrained optimization ok. So, this is the method of Lagrange multiplier which is a technique to maximize or minimize a function subject to one or more constraint. So, this is a utility maximisation problem similar type of problems can be analyzed like say for example, the cost minimization.

See optimization can be both maximisation or minimization. So, we have studied with say in the case of firms we have written down the profit function. So, the firm can maximize profit by minimizing the cost also. So, you can also use the Lagrange multiplier method for minimizing the cost of a production by a individual firm.

But here we are considering the problem of utility maximisation using the Lagrangian method for our purpose. So, it is just for convenience for ease of solving the problem. So, we introduce the Lagrangian function which is which constitute. Now, in the Lagrange function we incorporate both the objective function and the constraint in the same function.

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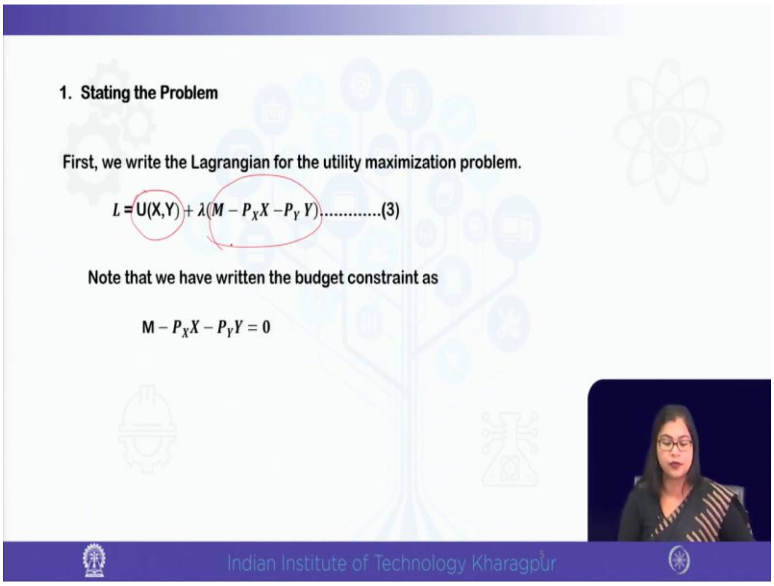

How? So, that means, we define this function say L ok. Earlier we are writing this problem as maximize utility subject to the budget constraint. So, we had one objective function and one budget constraint. Now, we are constructing the Lagrange where we are incorporating both the objective function that is the utility function here and the constraint also in the same function.

And for doing that we are introducing a new variable λ which is called the Lagrange multiplier ok. There are interpretation of the Lagrange multiplier, but unfortunately we do not have time to go into that detail. So, those who are interested you can go for some advanced courses on microeconomics or even mathematical economics for interpretation of Lagrange multiplier ok.

In this problem this is the marginal utility of money income, but I cannot devote more time because of our time in constraint. So, we write the Lagrange for the problem of utility maximisation. So, we are incorporating both the objective function and the constraint in the same function by introducing the Lagrange multiplier λ ok.

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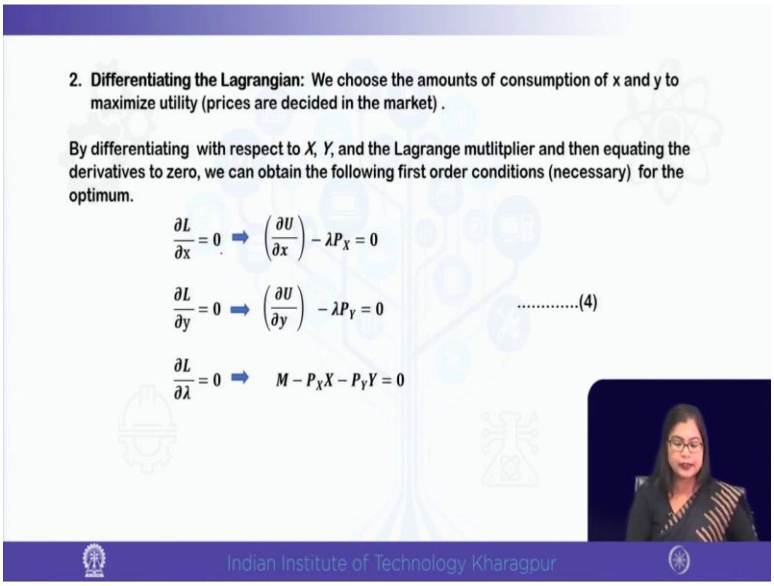

So, now how do you solve the problem? So, use the first order condition. So, as I told that there are two choice variables the amount of consumption of good x and good y. Now, in the Lagrange the way we are writing the Lagrange introducing the Lagrange multiplier. Now, we can also use this Lagrange was another choice variable.

So, there are three first order conditions. So, the consumer uses the three first order condition to find out the optimal level of consumption of good x and good y. So, what do we do? We differentiate the Lagrange with respect to x, y and λ and we set the differentiation. So, $\frac{\partial L}{\partial x} = 0$, $\frac{\partial L}{\partial y} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$. And hence we get these three conditions.

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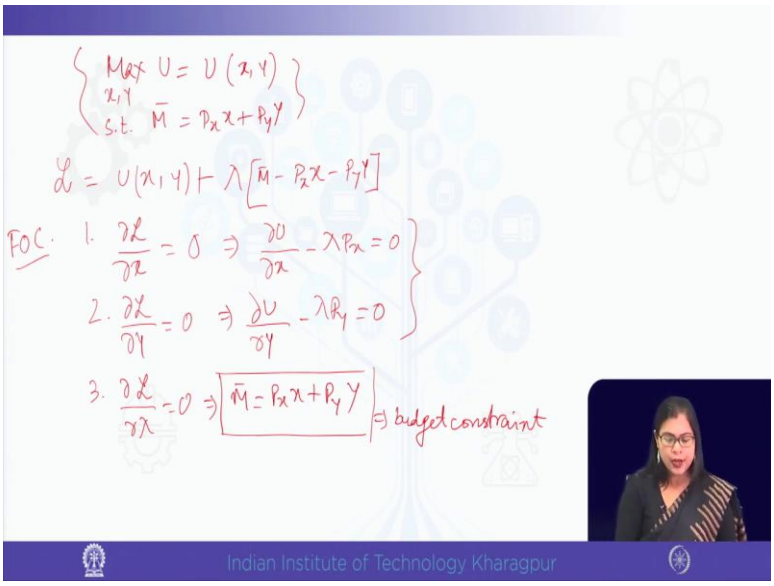

So, I thought that I will be writing. So, instead of just depicting it in the PPT let me write quickly. So, maximize utility this is the problem of the consumer. So, we can either write x_1, x_2 or we can also go by our example where we write the two goods x and y ok. Maximize utility, where utility is a function of amount of consumption of good x and y.

So, the choice variables are x and y subject to the budget constraint $P_x X + P_y Y = \bar{M}$. So, what we do? We convert this constraint maximisation into an unconstrained one by using the Lagrange where we incorporate both the utility function that is the objective function and the budget constraint in the same Lagrangian function.

What we do? We introduce the Lagrange multiplier λ. Now, we use the first order condition. So, the first there are three first order condition as just now I mentioned. First first order condition is to set the Lagrange L with differentiate Lagrange with respect to the first choice variable x and set this $\frac{\partial L}{\partial x} = 0$. So, what do we get over here? So, this is

$\frac{\partial U}{\partial x} - \lambda P_x = 0$ This is the first first order condition which is the second first order condition? It is $\frac{\partial U}{\partial y} = 0$. So, that makes this $\frac{\partial U}{\partial y} - \lambda P_y = 0$

and the third first order condition is $\frac{\partial L}{\partial \lambda} = 0$ which makes $P_x X + P_y Y = \bar{M}$. --- (B)

So, you can easily see that the third first order condition is nothing but the budget constraint itself. So, that means, at the optimum the budget constraint is to be satisfied. So, that means, at the optimum in this framework the consumer is consuming both the goods.

So, the consumer is not saving in this simply simple framework, but of course, we do have saving in reality, but in this framework, we assume that the consumer is consuming on the budget line. So, that means, is exhausting all the income on consumption of these two goods. Now, what you do in the next step? We combine the first two first order condition.

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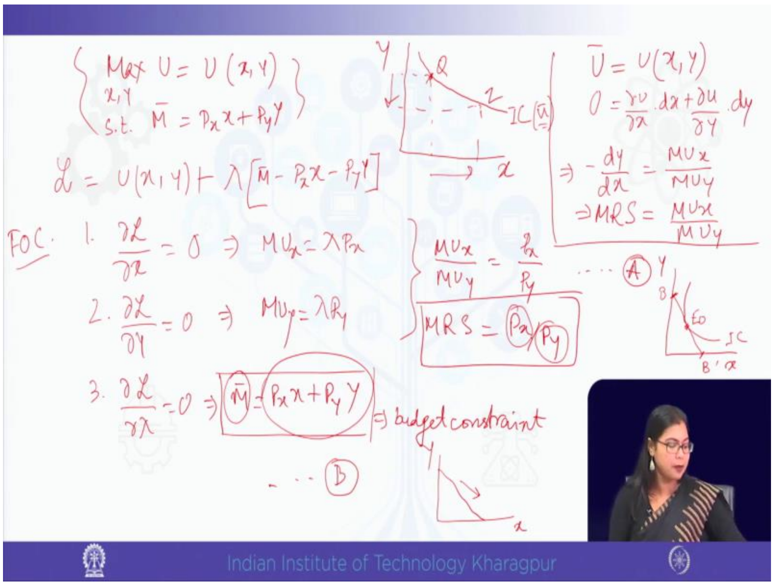

So, if we divide 1 by 2 what we get? You see we can just manipulate the first two first order condition and we can write $\frac{\partial U}{\partial x} = \lambda P_x$ and similarly $\frac{\partial U}{\partial y} = \lambda P_y$ right. Now, what is $\frac{\partial U}{\partial x} = 0$? $\frac{\partial U}{\partial x} = 0$ is nothing but marginal utility of good x we have in the previous lecture we have defined what is marginal utility.

So, this is the change in utility as the consumer is consuming more of one good. So, and $\frac{\partial U}{\partial y}$ is MU_y. So, if we now divide 1 by 2 what we get? $\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$ -- (A)

because λ is getting cancelled out ok. So, the ratio this is the condition for the optimum.

So, combination of this you can say this is condition A and this is condition B. So, that means, the consumer is consuming at the on the budget line and the point where the ratio of marginal utility is equal to the relative price or that means, the ratio of the prices of the two goods.

Now, you see there is some interpretation of the ratio of the marginal utility. So, if you remember in the last class, I also discussed the concept of indifference curve. So, indifference curve is basically the locus of the utility function. If we consider a particular level of utility and what are the combinations of the two goods that yield that same level of utility?

So, suppose if we plot these two goods x and y. So, there are you see in this figure you take any point. So, any point will show you this on any point correspond to what? Any point correspond to this combination of x and y how much satisfaction this combination yields? So, suppose this is point Q you can consider other points also point Z. So, these are the different combinations. So, if it so, happens that the utility obtained from this different combinations of the two goods are same.

So, if we draw if we join all such points we get a locus which we call the indifference curve. So, along the indifference curve your utility is same. So, suppose this correspond to this level of utility $\overline{U}$. So, this is your utility function $U = xy$. So, along the indifference curve your U level of utility is same ok.

So, if we now find out the slope of the indifference curve. So, the see in this indifference curve if we move from point Q to Z what does it mean? So, you are increasing the consumption of good x and you are reducing the consumption of good y so, that your level of satisfaction obtained is same ok.

So, you are sacrificing the opportunity to consume good y to consume more of good x ok. So, that means, the level of utility is same. So, if we find out the slope of the indifference curve what we do? If we totally differentiate see in the left hand side your utility is same along a particular indifference curve. So, the left hand side becomes 0. What about the right hand side? The right hand side it is $\frac{\partial U}{\partial x} * dx + \frac{\partial U}{\partial y} * dy$ So, now what will be the slope of the indifference curve? This will be $-\frac{dy}{dx}$ is equal to what? See $\frac{\partial U/\partial x}{\partial U/\partial y}$. That is nothing but marginal utility of good x divided by marginal utility of good y. So, the ratio of marginal utility is nothing but the absolute value of the slope of the indifference curve.

So, there is a trade-off between consumption of the two goods because you see we have already said that we are considering the products, which have positive marginal utilities; that means, the good commodities. So, for the good commodities you see there is a trade off because from point Q you see if you increase the amount of consumption of good x without reducing the amount of consumption of good y.

So, your utility will increase you will not be stuck to the indifference curve $\overline{U}$ right. So, that means, if you add more unit of consumption of good x you have to reduce the consumption of good y in order to maintain your level of satisfaction ok. So, that is why you see there is a trade-off. So, this is your the indifference curve is negatively sloped and the absolute value of the slope of the indifference curve there is a particular name to this is called the marginal rate of substitution ok.

The marginal rate of substitution this is the rate at which the consumer wants to substitute one good for another so, as to maintain his or her level of satisfaction. So, we can see from here that the ratio of marginal utility is nothing but the marginal rate of substitution. So, this is like you can interpret it as the opportunity cost because we have earlier also defined opportunity cost, I am not going into that detail we have discussed it in detail when we discussed the isoquant and even, we discussed the shape of PPF.

So, if you are moving from point Q to Z you are actually sacrificing the opportunity to consume more of y because in order to consume more of x you have to sacrifice the opportunity to consume y. So, you can interpret it as the opportunity cost. So, here coming back to our context of consumers utility maximisation. So, that means, we can write condition A as MRS = $\frac{P_x}{P_y}$.

So, this is the final condition for our purpose which we need. So, this is the condition for consumer optimum and that should be occurred at the means on the budget line because in the budget line you see if you calculate the slope of the budget line you can easily see that the absolute value of the slope of budget line will be the relative price that is $\frac{P_x}{P_y}$.

So, that means, under some conditions like the oil behavior preference. So, I am not going into the detail under the condition of strict convexity. So, we can show that the tangency between this indifference curve and budget line actually correspond to the unique and interior optimum. So, we do not have time to discuss all this, but this is the optimum in the case of oil behavior preference ok.

So, this is suppose the budget line say BB'. So, this is the point E_0 is the optimum for the consumer ok and this is the condition for that optimum MRS = $\frac{P_x}{P_y}$. So, this is our condition which we need to study the intertemporal allocation of oil or the mineral resource over time.

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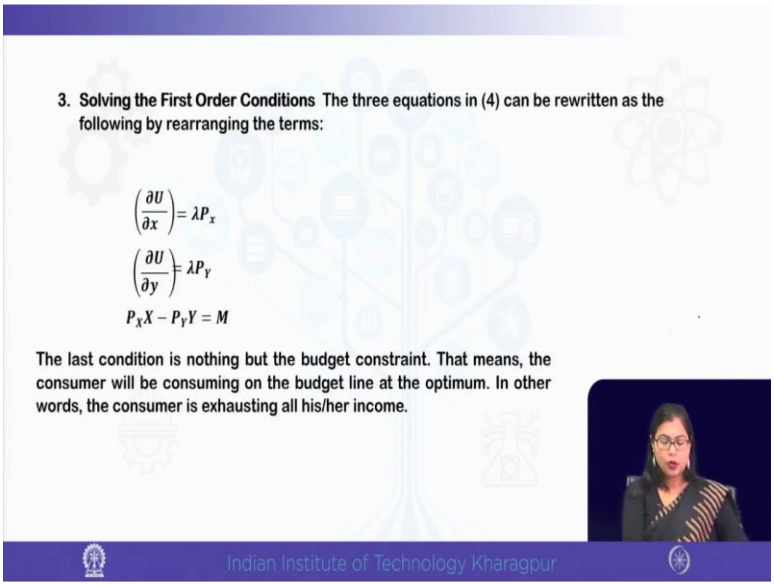

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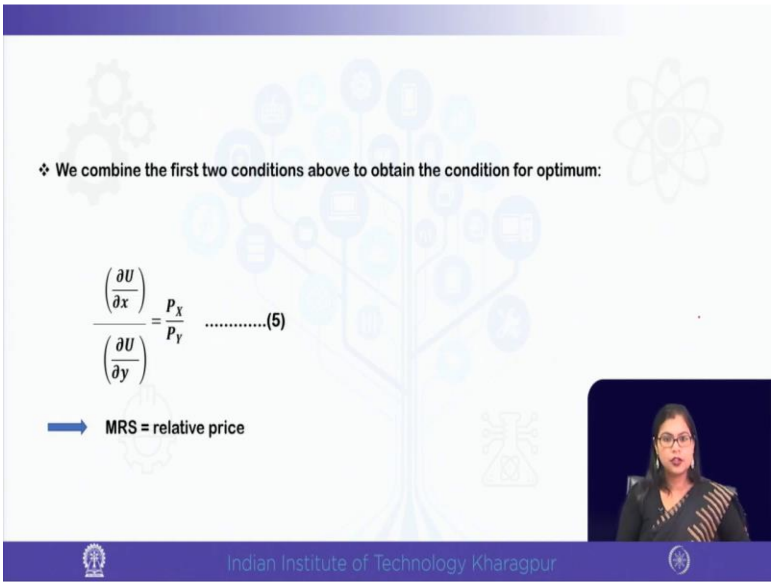

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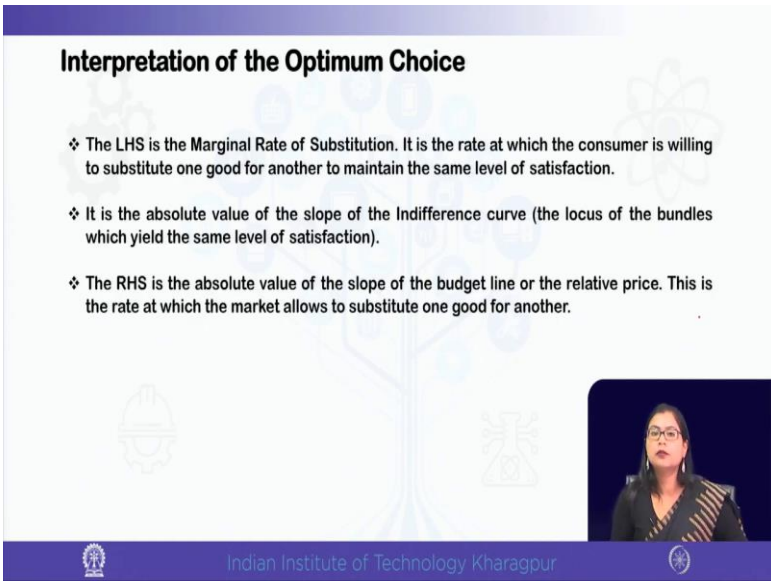

So, we have already done we have solved this I have written all this. So, I am skipping these slides because we have already obtained the equation the condition for the optimum and under certain conditions.

So, those who are interested you can go for some advanced course on microeconomics to study the this condition. So, MRS = $\frac{P_1}{P_2}$ that correspond to the point of tangency between indifference curve and the budget line and the optimum occurs on the budget line itself.

So, how do we interpret this condition A that is MRS = $\frac{P_1}{P_2}$ or $\frac{P_x}{P_y}$. So, the left hand side or the MRS as I defined it is the rate at which the consumer is willing to substitute one good for another to maintain the level of satisfaction. So, it is the absolute value of the slope of the indifference curve ok and you can also interpret it as the opportunity cost. And what about the right hand side? The right hand side this is the absolute value of the slope of the budget line or the relative price.

So, what is that? This is the slope of the budget line means, this is the rate at which the market allows the consumer to substitute one good for another ok. And means, that is also an opportunity cost because in the budget line is also negatively slope. If you plot the budget line you see this is the rate means because you are constrained by your budget along the budget line your total income or purchasing power is same. So, if you want to consume more of one good you have to reduce the consumption of the other good.

Hence, the budget line is also negatively slope because if you increase the consumption if you move down along the budget line let us say and if you want to increase the consumption of one good you have to also reduce the consumption of the other good because if you increase the consumption of both the goods your budget constraint will not be satisfied with equality you need more income right.

So, if you want to consume more of good x you have to reduce the consumption of good y you have to sacrifice the opportunity to consume good y. So, this the slope of the budget line is $\frac{P_x}{P_y}$ the absolute value or this is the relative price ok. So, what is the difference? You see MRS is the ratio of marginal utility. So, this is the rate at which the consumer wants to substitute one good for another ok. And there are goods which cannot be substituted.

So, depending on that we have different types of shapes of the indifference curve and what about the budget line? The budget line slope is given as the relative price the absolute value of the slope of the budget line is the relative price. So, these what about these prices? As I already mentioned that these prices P_x and P_y these are not decided by the consumer. So, these are decided by the market. So, you see the relative price or the absolute value of the slope of the budget line.

So, this is the rate at which the market allows the consumer to substitute one good for another whereas, the left hand side in the consumers optimum condition A. So, the MRS is the rate at which the consumer wants to substitute one good for another. So, the optimum is the one where these two rates are just same.

So, the rate at which the consumer wants to substitute one good for another is same as the rate at which the market allows the consumer to substitute one good for another. Otherwise, what will happen? If suppose for the timing suppose this is not satisfied with equality.

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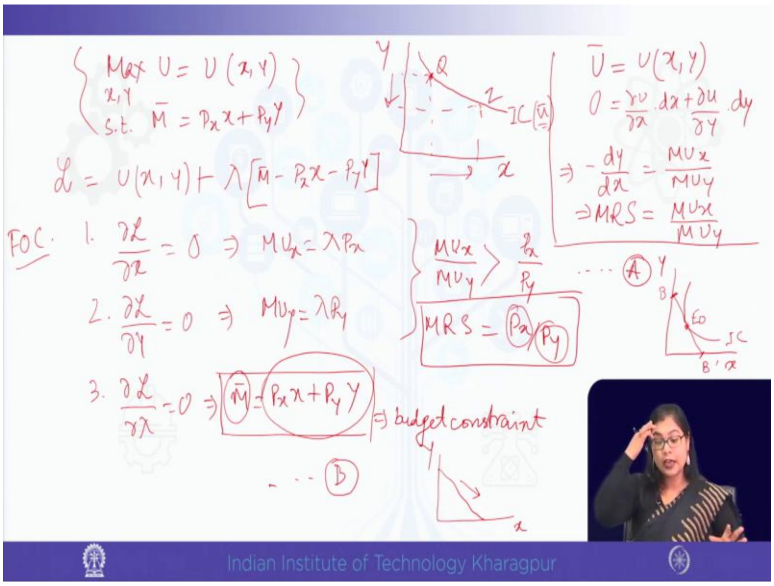

So, $\frac{MU_x}{MU_y} > \frac{P_x}{P_y}$. So, this will not be the optimum why? Because the consumer if the consumer increases the amount of consumption of good x the consumer will be better up so, a consumer will not stop at this point at this situation where $\frac{MU_x}{MU_y} > \frac{P_x}{P_y}$. But you see if the consumer then increases the consumption of amount of good x what will happen? We have law of diminishing marginal utility in play.

So, by law of diminishing marginal utility after consumer increases the amount of consumption of good x. So, marginal utility of good x will fall. So, eventually you will be coming back or returning to the equality. So, the optimum in this case will be M = $\frac{P_1}{P_2}$ or $\frac{P_x}{P_y}$; that means, the ratio of marginal utility is equal to the relative price.

So, this is the optimum choice for the consumer which will be using to study the inter temporal allocation of petroleum or other type of mineral resources and how pricing is also done based on the inter temporal allocation and for that we need this condition which we derived into this class.

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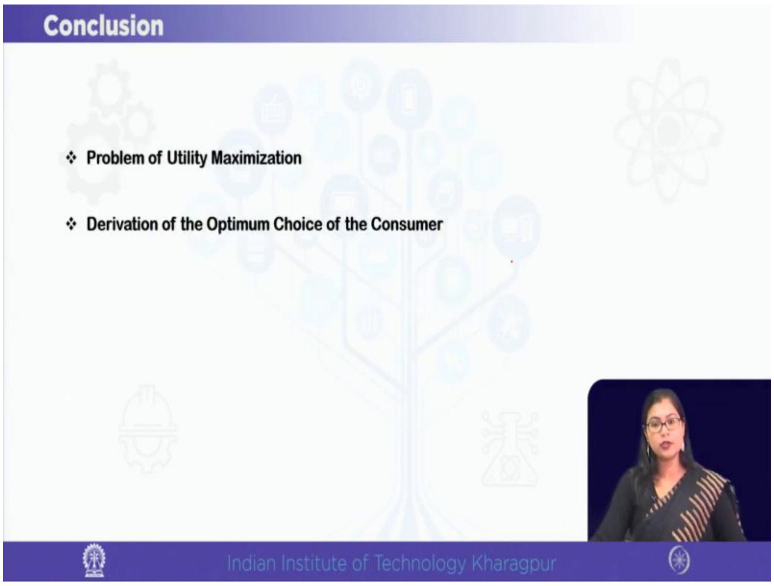

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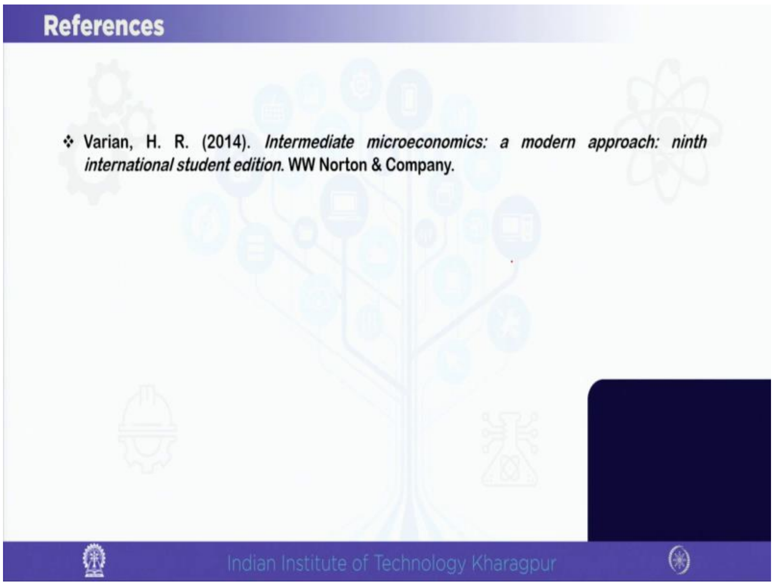

So, for today's class we mainly followed the Intermediate Microeconomics book by Hal Varian.

So, thank you very much look forward to discuss with you the Theoretical Model on Inter Temporal Allocation and Pricing of Petroleum.