Petroleum Economics and Management
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Module - 11
Management of Petroleum Wealth
Lecture - 52
Model of Economic Growth-II

Hello I am Dr. Anwesha Aditya, your instructor for the NPTEL course, Petroleum Economics and Management. So, once again a very warm welcome in our course, we are in module 11 of our course, where we are discussing how to manage petroleum wealth. So, this is our lecture number 52 in the course, where we will be studying the Model of Economic Growth.

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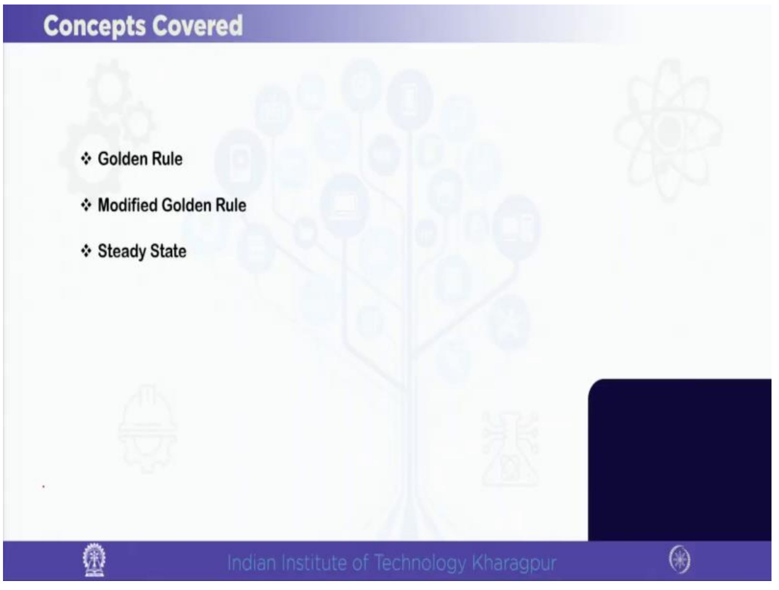

Now, if you remember we have already started developing a model of economic growth, we have discussed the importance of managing the petroleum wealth or overall resource wealth. So, again the theoretical model that we are studying, we are referring or we are giving the example of petroleum, because that is important or relevant for our purpose, but the same model can be applied for any other type of natural resource or mineral resource, which is in fixed supply. So, that means, non-renewable resource.

So, in such a structure if the economy suddenly discovers the resource, but we have discussed with the example and the hypothesis of resource curse and Dutch disease. So, discovering the resource is not the end of the story, you can rather say that it is just the beginning of the story.

So, countries like India face a different problem that we have to import lot of oil. So, you may think that if the countries are countries endowed with huge amount of oil or a very important natural resource. So, everything is good for the country. So, that is not true, ok because we have seen with the experiences of Venezuela and we have also analyzed the country experiences of different Middle-East countries. So, they may not be very good performer in terms of the human development index parameters.

So, how to manage the resource wealth? That becomes very important. It is not about just earning the rents or the export revenue from exporting oil, how to manage the resource prudently, because if we spend too much on current consumption like Venezuela did. So, you deplete the resource or if there is a fall in the price of the resource in the world market like it happened after the shale oil revolution. So, country like Venezuela suffered a lot, because it could not develop the other sectors.

So, we studied the theoretical model of resource discovery and structural change in module 7, where we saw that discovering a resource and getting dependent on the resource can ultimately lead to a shrinkage of all non-resource tradable activity and it finally, it may lead to emergence of a mono-economy and extreme dualism. And that sometimes has happened in many of the Middle-East countries.

So, how to manage the resource? Because we have also the other hand, we have the experiences of developed countries like Norway and Netherlands, who could manage the resource very good and they are actually very good performance. They are not only rich countries, but they are also in the category of developed countries and their human development index ranks are also very good.

So, we see that the country, if the country's level of economic development and growth is already good and then resource is discovered, such countries are able to manage the resource very prudently. Whereas, if resource is discovered in an underdeveloped economy in a poor country, the country may lack the proper human capital to manage the resource wealth and if the resource wealth is already is enjoyed by very few. So, there will be increased inequality.

So, we have already discussed that in the previous module and if you remember, we just introduce the structure of our theoretical model, which is an infinite time horizon model using a micro foundation. So, we wrote down our discounted sum of utility function and we just introduce the maximization principle using the Hamiltonian.

So, in today's lecture, this lecture number 52, we are going to continue with this model. In the next lecture, we will be introducing the impact of resource discovery. So, let us now concentrate on to this particular lecture, where we will be completing the model of economic growth before resource discovery. So, in this context, we will be covering up some very important concepts of golden rule, modified golden rule and steady state with reference to our particular model.

So, as I mentioned in the previous lecture. So, this model we are discussing is mainly from the book of Petroleum Economics by Hansen, but those who want to know the background, because we do not have time to discuss, then you have to study a proper growth a course on economic growth or macroeconomics.

So, you can study, we have referred to the books of Barro and Sala-i-Martin and Abel Bernanke. So, you can study these books and for the solution of the Hamiltonian for dynamic optimization and optimal control theory, you should refer to the book of Alpha Chiang, it is a Dynamic Optimization.

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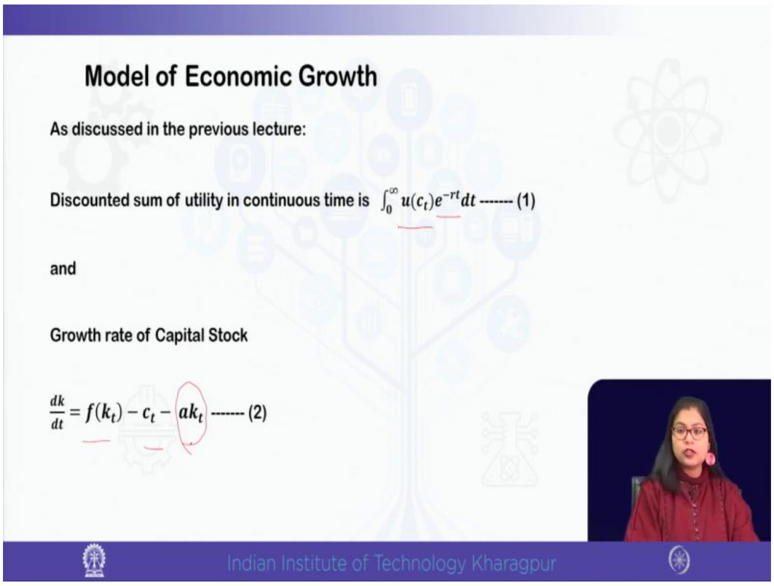

So, if we now just recapitulate our previous lecture. So, we wrote down the discounted sum of utility in continuous time, where uc is the utility function, utility is the function of consumption, but now it is over time. So, we are using the time subscript t and this we are discounting using the social time preference that is r.

So, the problem of the policy maker is to maximize this discounted sum of utility in continuous time. So, you see our integration is from 0 to infinity. So, 0 is the initial period up to infinity, because infinite horizon model makes more sense in the, in its more realistic right. And then we also define that consumption is the gap between production and the capital required for maintaining and increasing the production capital.

So, in with reference to the definition of consumption, we can also write down the growth rate of capital stock. So, capital stock is the gap between see production, consumption and the depreciation. So, we are assuming that capital is getting depreciated at the constant rate a. So, a is constant, a is positive constant.

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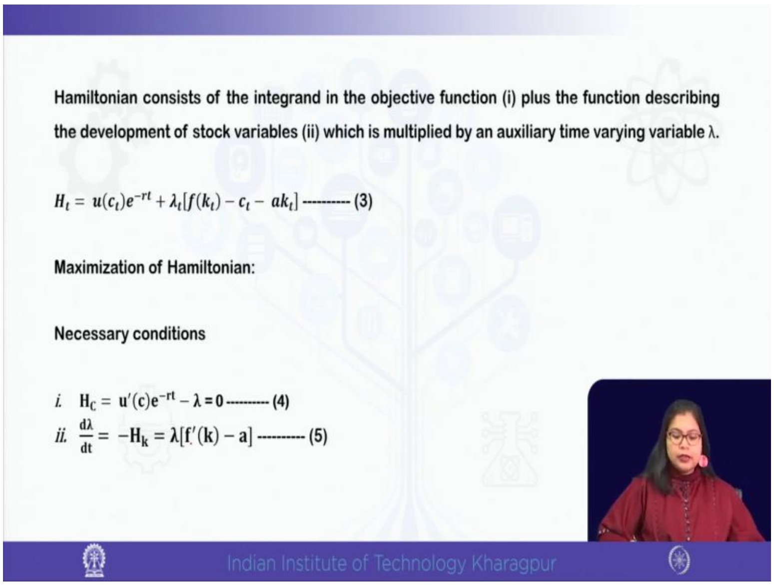

Now, Hamiltonian, so, with this, under this context, we wrote down the Hamiltonian. So, Hamiltonian consist of the integrant in the objective function. So, that means, this ucte to the power minus rt plus we introduce the auxiliary time varying variable lambda. So, this is now time varying. So, we are using it with the subscript lambda t and we multiply this lambda t with the growth rate of capital stock. So, this is our stock variable capital is the stock variable now. So, this is one is the growth rate of capital stock.

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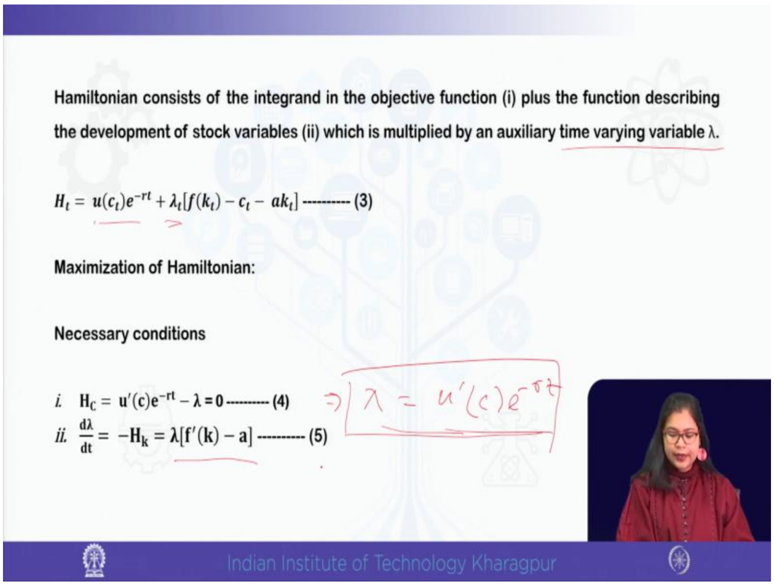

So, we already discussed about the construction of Hamiltonian in the previous lecture. So, just like we use the Lagrange method to solve the problem of utility maximization or profit maximization in a static framework, here we are using the Hamiltonian in the dynamic framework. So, again we have an auxiliary variable lambda, but now this time it is not like the Lagrange multiplier because this is not time varying.

So, we are using the subscript t. Now, the problem of the policy maker is to maximize the Hamiltonian. So, we have some necessary conditions. So, I am not elaborating, we do not have time to solve the Hamiltonian step by step, because we have only very few lectures left. I would have very much preferred to do that, but we cannot do.

So, we are focusing mainly on the results and the implications and I am just briefly telling about the solution procedure and how we finally, get the capital stock optimal capital stock which will maximize the consumption. So, that is our purpose and then we will be introducing resource discovery.

So, what are the necessary condition, first necessary condition is we differentiate the Hamiltonian with respect to our consumption. So, del H del c we are writing it as H c. So, H subscript c is basically del H del c is equal to what we can see it is equal to u prime c to the power minus rt minus lambda, ok because here you have lambda tct. So, this is the second term becomes minus lambda. So, this is equal to 0, this is the first necessary condition. The second necessary condition is when we differentiate the auxiliary variable lambda with respect to time.

So, d lambda dt is equal to minus H k. So, minus del H del k basically. So, del H del k you are writing as H k subscript k H subscript k means del H del k. So, this is equal to lambda f prime k minus a ok. And so, that means, now you see from this 4 what we can write we can get an expression of lambda. So, lambda is basically u prime c to the power minus rt ok; so, now, coming to this using this equation 5.

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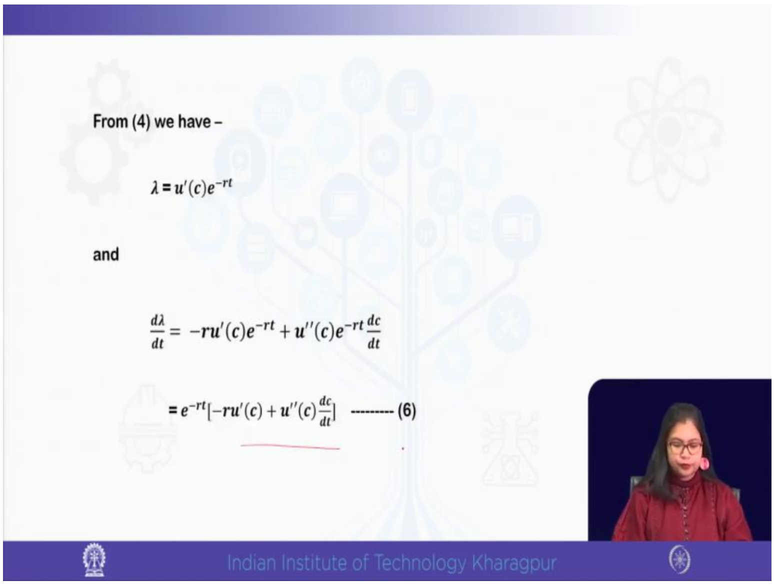

So, d lambda dt is equal to what is equal to lambda f prime k minus a. So, in place of lambda we can write this expression ok. So, this is what we are doing and then finally, you get this expression that we are writing in 6 ok. And we can finally, combine equation 6 and equation 5 because d lambda dt.

So, lambda as lambda you have got this and in the right-hand side you have in right hand side of 5 you have this equation where we are, we already have the expression. So, you see here what we have done we have taken d lambda dt and we have used this condition 4 what we have got the expression of lambda from condition 4. So, from here d lambda dt is this ok.

So, if you said d lambda dt you the get the expression is minus r u prime c to the power minus rt plus u double prime c dc dt into e to the power minus rt right. So, you are taking e to the power minus rt out and then in the bracket you have minus r u prime c plus u double prime c dc dt right. Now, in the what you will be doing in the next step.

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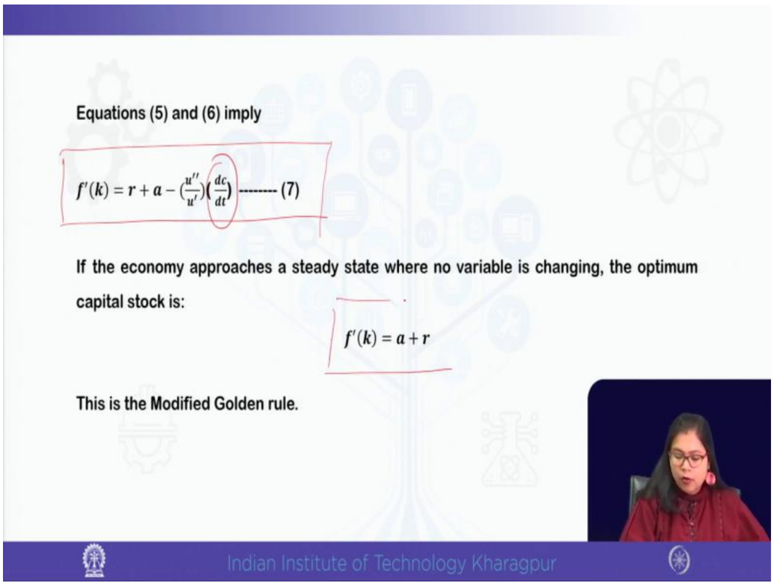

You combine equation 5 and 6 to get some final expression. So, you combine equation 5 and 6 because you see what happens in if you combine equation 5 and 6 here you also had you can find out this expression of Hk and then equate these 2 ok, because you have this here. So, use condition 3 also then finally, if you just do-little bit of manipulation and you divide everything by u prime c.

So, finally, you get this condition in 7 ok it is very simple you can derive that. So, f prime k is equal to r plus a minus u double prime divided by u prime into dc dt ok. So, this just you combine equation 5 and 6 to derive this expression ok. So, you see you actually have divided the 2 sides after you combine 5 and 6 you have divided the 2 sides by u prime c and you finally, get this. Now, what is happening now we introduce a very important concept called steady state.

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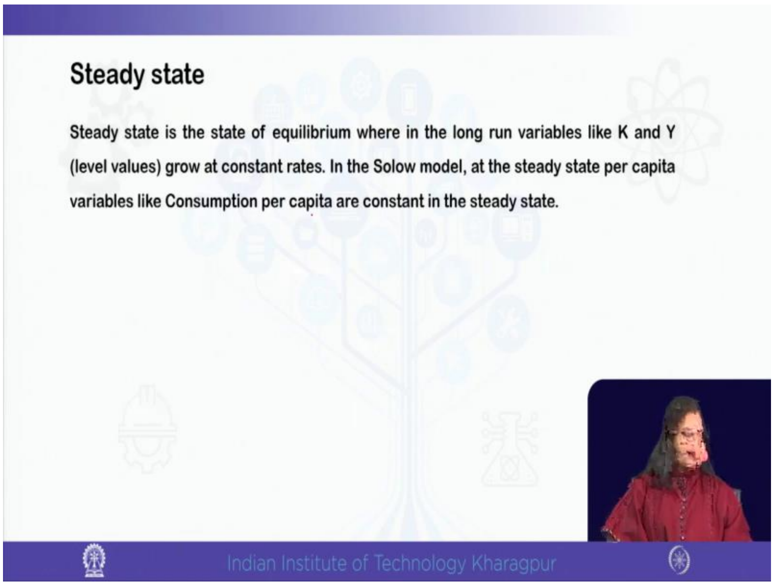

What is steady state? Steady state in the Solow model. So, it is often used in macroeconomics in the models of economic growth it is a situation where it is a state of equilibrium in the long run. So, the level variables like say for example, capital stock or consumption.

So, the level variables they grow at constant rate. So, capital stock population consumption they grow at the constant rate. So, that means, what the per capita variables do not grow. So, consumption per capita do not grow. So, if consumption is growing at the same rate of labour stock. So, what will happen? I mean the labour supply. So, what will happen? The per capita consumption will remain unchanged. So, the ratios do not change ok.

So, in the Solow model at the steady state per capita variables like consumption per capita, capital stock per capita. So, these are constants. So, that means, what this consumption change of consumption over time dcdt will be equal to 0. So, this condition we are using in 7 ok. So, what will become the condition 7 will become f prime k is equal to a plus r. This is very important and this is called the modified golden rule.

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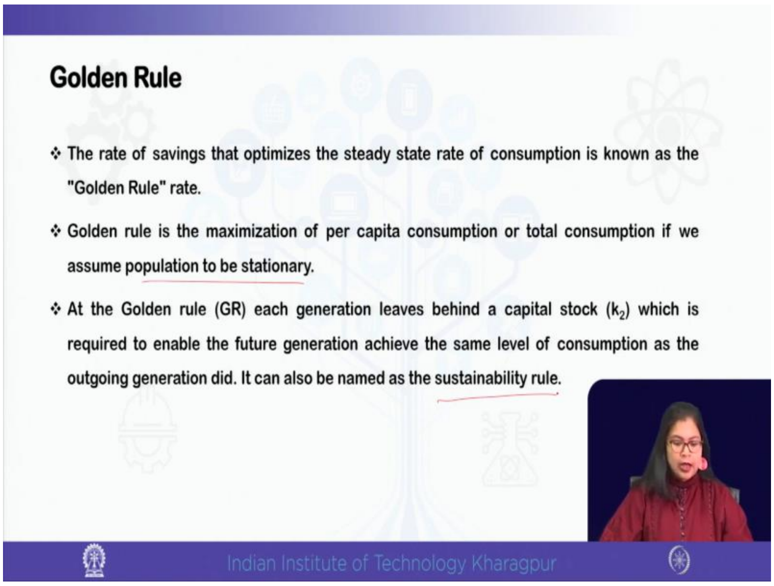

So, before we proceed to the modified golden rule, we need to know what is golden rule overall. So, first we need to know this important concept of golden rule. So, what is golden rule? This is the rate of savings which maximizes or optimizes the steady state rate of consumption. So, golden rule savings rate is that level of savings that leads to maximization of consumption, highest possible consumption.

So, golden rule is the maximization of per capita consumption or we can also interpret it as golden rule is the maximization of total consumption if we assume population to be stationary. So, at the golden rule, we can interpret the golden rule as a sustainability rule. What does it mean? That means, each generation should keep a stock for the future generation.

So, that means, each generation should leave behind a capital stock which is required to enable the future generation achieving the same level of consumption as the outgoing generation did. That means what? That means the present generation has to consume that much. So, that it is investing the rest in I mean getting in capital stock so, that the next generation can also achieve the same level of consumption like the previous generation.

So, the level of consumption should remain unchanged generation after generation. So, that means, the present generation has to decide how much capital stock should be kept for future generations consumption purpose. So, that the consumption over generation remains unchanged. So, this is obviously, the sustainability rule. So, that means, you should not be using more for present consumption. So, that you are keeping a less capital stock.

So, the future generations consumption is less than you that you are not doing in golden rule. In golden rule, you are thinking about the future generation. So, you are consuming that much. So, that the rest is invested as capital stock in order to maintain the same level of consumption for the future generation. So, it becomes sustainable. So, you can consider it as the sustainability rule. So, in simple word it is the maximization of per capita consumption ok.

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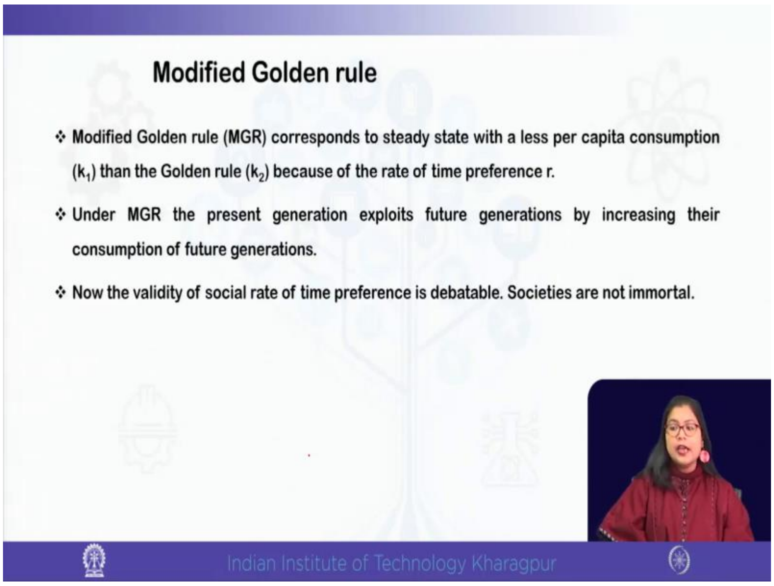

Now, here I have discussed about the modified golden rule. So, what is the difference between golden rule and modified golden rule? In modified golden rule, we are bringing the social rate of time preference. So, you see f prime k is equal to r is f prime k is equal to a plus r this the capital stock which leads to this condition is basically the modified golden rule ok.

Because we have the discount rate r is the social time preference rate. So, when you introduce the discount rate so, we have already interpreted the rate of discount in our previous module. So, we generally do not put equal weightage in present and future. Mainly we put greater weightage on the present than on future right. So, we are discounting the future using the social rate of time preference r.

So, modified golden rule corresponds to the steady state with a less per capita consumption because you are now putting more weightage on a present consumption. So, you see in golden rule you put equal weightage on present and future consumption.

So, f prime k is just equal to a, but in modified golden rule you introduce the rate of time preference r. So, Modified Golden Rule (MGR) corresponds to the steady state with a less per capita consumption than the golden rule because of inclusion of the rate of time preference.

So, under modified golden rule the present generation explores the future generation by increasing their consumption of future generation right. So, now the validity of the social rate of time preference is of course, debatable because societies are also not immortal. So, now if you plot this, if we now plot the capital stock corresponding to the golden rule and the modified golden rule.

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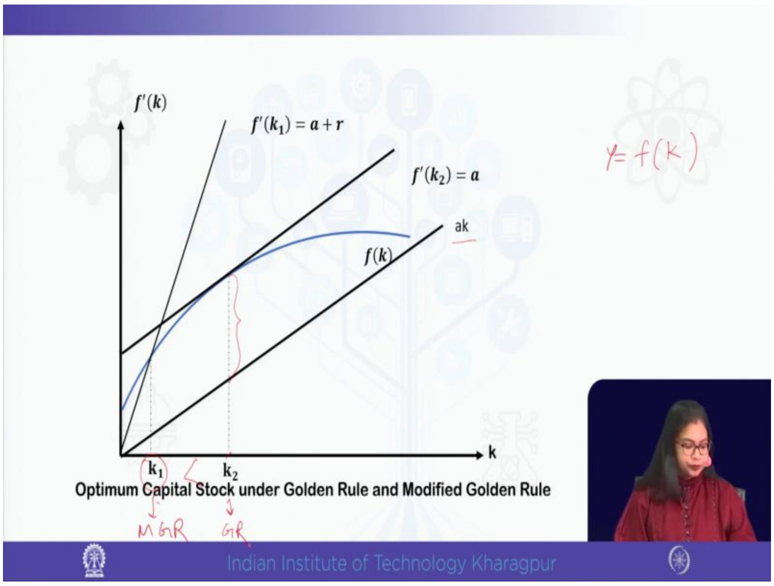

So, what we see over here. So, suppose we are plotting marginal productivity. So, f prime k is what? We are writing in in the previous class we have written the production function y is equal to fk. So, what is your f prime k? F prime k is basically del y del k; that means, your marginal productivity of capital C.

We have already interpreted marginal productivity. So, if you change if you increase one more unit of capital how your output will change. So, we are plotting the marginal productivity of capital against a capital stock k, ok k on the vertical on the horizontal axis sorry and f prime k on the vertical axis.

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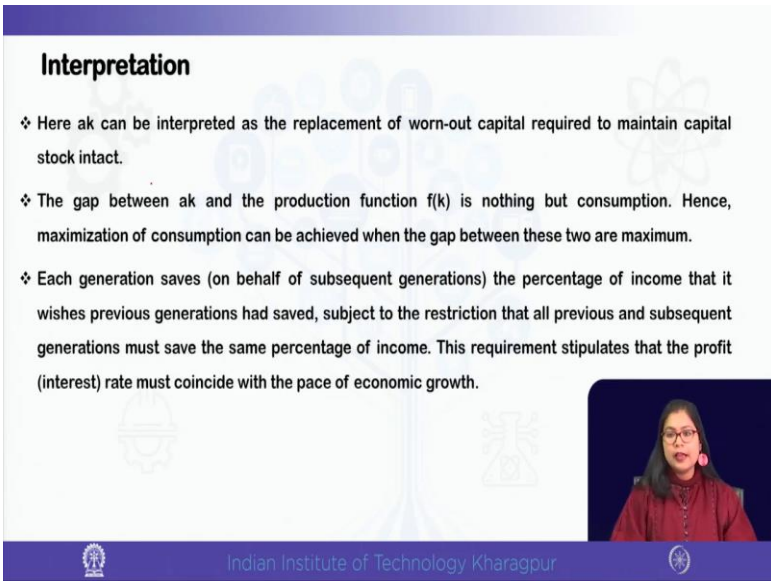

Now, you see a k if you remember a is the rate of depreciation. So, a and that is constant we have assumed the rate of depreciation to be constant. So, ak can be interpreted as the replacement of depreciated capital or own out capital which is required to maintain the capital stock intact ok. Since you have assumed ak is a constant. So, we draw a straight line a 45 from the origin this is your, ak line ok.

Now, what you are doing you are plotting your production function fk ok. Now, so, what is the gap between your output and the depreciation of capital? We know that we have a in the first lecture of this module where we started the developing the theoretical model, we saw that this is nothing but the consumption. Consumption is the gap between production and the amount of capital required to maintain and increase the capital stock.

So, the gap between ak and the production function fk is nothing but the consumption. Therefore, the objective of the policy maker as you know is to maximize the consumption so, that means, consumption will be maximized when the gap between the two is maximized. So, if you draw this fk function ok earlier also if you remember we have we have drawn the production function.

So, the production possibility set we know that production function is the upper boundary of the production possibility set. So, with given amount of input and given level of technology there is a maximum limit of producing. So, production function is basically it is the upper limit of the production possibility set. So, you have plotted the production function and ak. So, the gap between these two lines is basically your consumption.

So, the policy maker will of course, choose the capital stock where the gap is maximum. So, you see and that occurs at a point where the two slopes are same right ak is equal to this f prime k2. So, that means, the capital stock which correspond to this maximum gap between a k and f k is what? This is your golden rule capital stock ok.

So, at the golden rule we already know each it is a sustainability rule the present generation will leave the capital stock k2 which will be required to enable the future generation achieve the same level of consumption. So, k2 corresponds to the golden rule level of capital stock, but we know that we have derived the condition which is which correspond to the modified golden rule because we are considering the discount factor.

So, if you now consider the discount factor, we already have defined we have seen that with the incorporating the discount factor the modified golden rule corresponds to the steady state with a less per capita consumption. So, now k1 you see k1 is less than k2. Yeah. So, k1 is less than k2. So, k1 basically corresponds to the modified golden rule and k 2 corresponds to the golden rule. So, because of the inclusion of the rate of time preference so, you are putting more weightage on present consumption.

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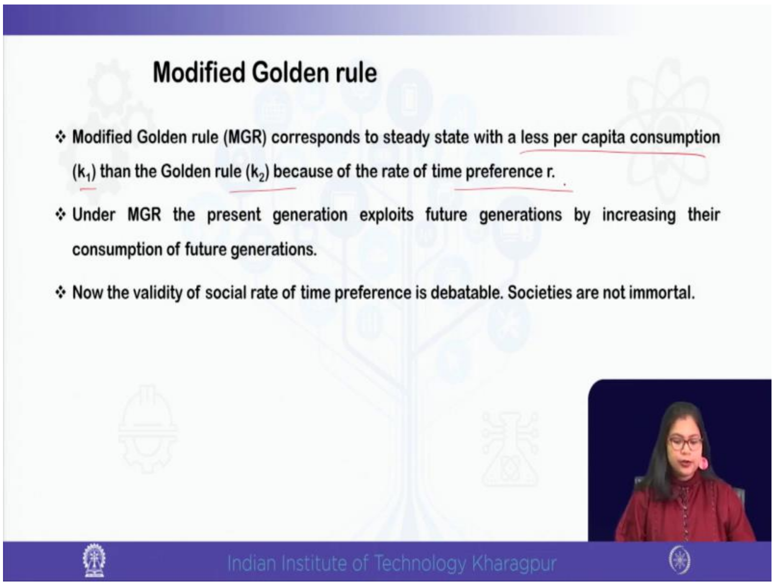

In the modified golden rule, the present generation actually exploits the future generation by increasing their consumption right. So, we have already seen that. So, it corresponds to a less per capita consumption k1 than the golden rule consumption k2 because of the social rate of time preference r.

So, in this way it derived the golden rule capital stock and the modified golden rule capital stock without the discovery or before the discovery of the natural resource or petroleum. Now, in the next module we will be bringing in the discovery of natural resource how our model will change ok. So, we will be writing the rewriting this Hamiltonian with the resource discovery or the discovery of the petroleum or other type of mineral resource.

So, how the now in the next module that means, we will be having another stock variable. In this framework you see you have the only stock variable is capital stock, but then introduce another stock variable. So, how your model will change? So, that is what we will be discussing in the next class ok.

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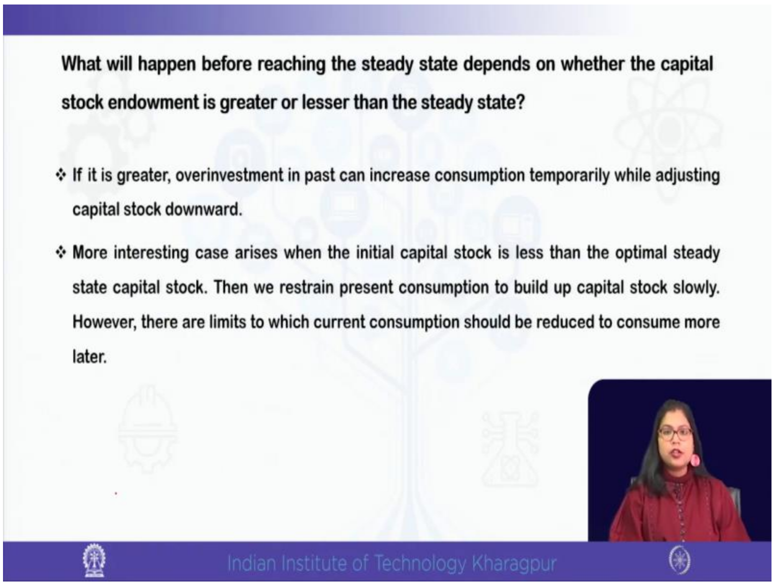

Now, the question is what will happen before reaching the steady state? It depends on whether the capital stock endowment is greater or less than the steady state ok. So, what will happen if before reaching the steady state? So, it is not that the economy starts from the steady state.

The steady state you can interpret it as the means the long run in which the economy will converge in the long run. It is a level; it is the state of the economy which the economy will converge in the long run ok. Now, if you are not starting with the steady state. So, you try to converge to the steady state.

So, during this process what will happen? So, it depends on whether your capital stock endowment is higher or lower than the steady state. Now, if already you are at the beginning if the capital stock endowment is greater, which is very rare for most of the cases it is rarely seen because, in most of the cases your initial capital stock is less than the optimal steady state capital stock.

So, if for example, suppose the initial capital stock is greater than the steady state capital stock. So, what will happen? So, over investment in past can increase the consumption temporarily by adjusting the capital stock downward. That means, if you go back to this figure if it happens that your capital current capital stock the initial one is greater than the steady state. So, obviously, you try to adjust it downward right because you want to come back to the steady state in the long run.

So, over investment in past can increase the consumption temporarily by adjusting the capital stock downwards to leftward. However, more interesting situation arises when the capital stock is less than the optimal steady state capital stock. So, if we consider any capital stock less than k2. So, then what will happen to the left of this? So, then what will happen? Then you see we restrain the present consumption to build up capital stock gradually.

However, of course, you see it is there is a limit to reduce current consumption for increasing the consumption future. So, if your initial capital stock is less than the steady state one the one you want to achieve in the long run. So, what you want to do? Then you restrict the current consumption because you want to invest more for creating the future capital stock.

But you see is it possible to reduce the consumption very much because there should be a limit by which consumption can be reduced because you have to maintain a standard of living the government has to spend on the social welfare scheme. So, there are limits to which current consumption should be reduced to consume more later.

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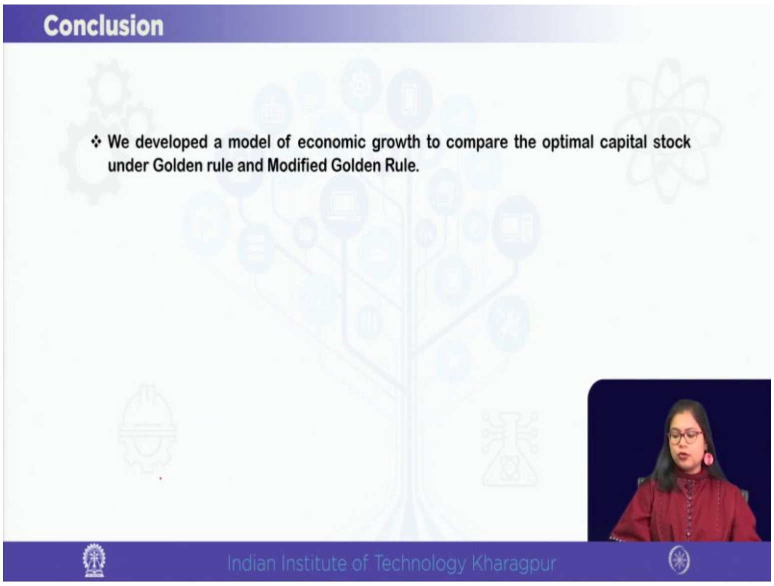

So, you see what we did in today's class we just solve the model before resource discovery. So, in the previous lecture; that means, the first lecture of this module we introduce the model with a micro foundation where the planner this is remember this is a planned economy. So, the planner is deciding how much to consume now and how much to be invested so, that a future consumption can be maintained.

So, we continued the model in this lecture and we completed the model we found out the steady state and the golden rule and the modified golden rule. Without the resource discovery and we found we compare the optimal capital stock under golden rule and modified golden rule.

So, we saw that since we use the discount factor the optimal capital stock under the modified golden rule is less than the capital stock under the golden rule because by discounting future you are putting greater weightage on current consumption. Then we also discussed what will happen if your initial capital stock is less than or greater than the steady state level.

So, we saw that if it is already greater than it is fine, but if it is less than you have means if your initial capital stock is already greater than you have to reduce the capital stock and you can increase the consumption. But if your initial capital stock is less than the optimal steady state means the golden rule capital stock.

So, in that case you have to restrict restraint present consumption to increase the future consumption or to increase to invest more capital for future consumption. However, there should be limit of how much we can reduce of the current consumption. So, see these are some advanced models. So, because of time constraint we are not able to do a step by step solution.

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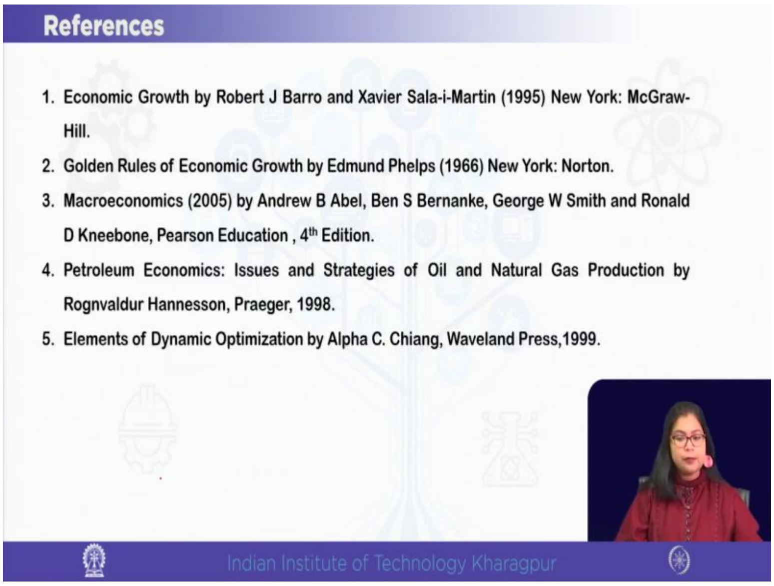

But I have already mentioned you the references though those who are interested should definitely first start with the how to solve the Hamiltonian. So, you can read the book of Alpha Chiang on Dynamic Optimization and then of course, you should go through the books of some very famous books on Economic Growth like the book of Barro and Sala-i-Martin and the book of Macroeconomics by Abel Bernanke.

And of course, the Golden Rule of Economic Growth by Phelps. So, these are very important and famous books on Macroeconomics and Economic Growth. And of course, for the model as I mentioned already, we are mainly following the Petroleum book, Economics book of Hansen. So, in the next lecture I am going to introduce the how the model of economic growth will be changed once we introduce the discovery of the natural resource.

So, thank you very much. See you in the next class.