Linear Kinematics

So, now coming back to our original discussion how to define motion. So, it can be defined with respect to change in position which is happening over time. So, what are the mathematical descriptors like how you can describe a motion in terms of mathematics. So, the various descriptors or mathematical variables which can define motion include position, distance, displacement, speed, velocity and acceleration. We will look at these one by one. So now let us understand the concept of position, distance and displacement.

So, over here in this image let us assume this shaded area like blue shaded area is non paved area like there might be grass or like an open field in these areas and this white region between these positions is a paved road like there is a pathway or walkway over here. So, what we will do is we will look the distance that this these squares have sides of 10 meter and the distance between squares is 10 meter. So, first let us define position. So, position is nothing but location of an object with respect to a specified reference frame.

Similarly, we can define distance is length of the path travelled like the total path travelled is known as distance whereas displacement is the shortest distance between the initial and final position and all these quantities are measured in the SI unit for these quantities is measured, but however you can use kilometers, feet. There are other units which are being used, but thus SI unit for these quantities is meters. So, let us understand the concept. Let us say a person is standing right here at position A and he has to move to position B. So, how you will define position? So, our reference let us put a reference frame over here. So, at this position the location of the person is (0, 0). He has not moved.

So, let me draw our reference system like this reference system is like this and it is present over here this red dot. So, what is happening over here? So, this is the starting position. So, at starting position the value in or the movement in x and y direction is 0 and 0 and when the person reaches B what would be the position? But there is another condition. So, the person has to move a specific path. For example, let me change the color over here.

So, he has to move like this following the green path to reach position B. So, what is happening over here? He is travelling 20 meters this way, 20 meters this, 10 meters over here, 10 here, 20 here, 20 here, 10 again, 10 again, 20, and 20. So, the total distance from position A to B which is being covered by this person would be 20, 40, 50, 60, 80, 100, 10, 20, 100 and. So, distance would be 160 meters whereas, the displacement would be let me change the color again. So, in this position let us look at the displacement also.

So, displacement is the shortest distance from A to B. So, here also what we can do is we can apply the Pythagoras theorem to calculate the distance between A and B. What we will do is we will let us say this point this edge is C we will calculate the distance between A and C and B and C and then find the value of AB. So, from here A and distance between A and C is 80 meters and similarly over here also 80 meters. So, then what would be the distance between A and B is? This which will be approximately 113.14 meters.

So, you can see how we can define distance and displacement with respect to the position of the points which we are given. For example, we can solve another example where an athlete which is bit more complicated than the previous one. So, over here an athlete is moving from position A to position B then coming back from position B to C and then again from C to D. So, the position like let us place our reference system over here x and y.

Since the we are just looking at the horizontal motion in x. So, this is known as one dimensional where only one dimension is involved. So, in this the distance between A and B is. So, let me put A and B over here and then this is distance and this is displacement. So, between A and B the distance is this is the starting point and this is the end point is 40 meters. Between A and C is athlete goes to 40 B and then move back 20 meters back.

So, that will add to 60 meters and finally, A to D is again going to this position. So, that would be 100 meters. Similarly, if we look the displacement between the motion from A to B that would again 40 meters, but between A and C starting from here 0 to 20. And finally, between A and D displacement is 60 meters. Now, since we look started our example with a drop vertical jump.

So, over here also we can see the position of. So, the knee joint over here, here and here which is different in all three time points or the critical points which we want to observe. Similarly, the location of elbow joint is different in all three position. So, these type of analysis or the knowledge of position distance and displacement will help us to understand the mechanics of the movement. So, now let us look at individual phases of this movement to understand more about how distance and displacements are being discussed.

For example, in frontal view we look at this. So, in order to get the location of these points I considered an axis or reference frame over here which is in horizontal it is x and in vertical it is y. So, in this notation when we want to record the 2D coordinates what we do is we separate x, first value is x and second value is y which is being separated by comma in the in the brackets. So, over here if I consider this reference frame. So, the location of the ankle joint, knee joint and let me change it to red again.

So, the location of the hip joint with respect to this reference point or the origin which is at (0, 0) the locations are shown in the figure. I did intentionally on the sagittal view. So, that you can try to observe or analyze this position in the frontal view. So, if you place a reference system over here you have to use the another calibration thing. Particularly in this view this is the distance between these two is 48 centimeter. If you do this you will get these coordinates. Similarly, at maximum knee flexion these are the values which we can get from 2D video analysis and finally, in the feet of position you can clearly see there is a variation in the x and y coordinates.

So, which will help us to understand the concept of position, distance, and displacement. So, in this example so let us look at all these three positions. So, you can clearly see the position at the touchdown phase for ankle is different from maximum knee flexion and finally, at the toe off. Similarly, the same situation it is the three different joints like ankle, hip and knee. So, I would recommend the participants to calculate the position, distance and displacement.

For example, if this is activity location A, this is B, and this is C. So, you can try to find out the distance and displacement between A to B, B to C and A to C using these coordinates. So, that will give you a good exercise to understand how these concepts can be applied in a sports setting also. So, now let us understand the concept of speed and velocity. We will start with the same example which we use to discuss our position, distance, and displacement.

So, speed is nothing but defines how fast something is moving and it is measured as the distance moved in a specific amount of time. So, mathematically it is nothing but

$$Speed = \frac{Distance}{Time}$$

So, distance covered in a specific amount of time. In a velocity it is nothing but speed in a given direction. So, like you are in your vehicles when you move you see the speed on the odometer like 40 kilometer per hour not with the direction like whether you are moving towards east, west, north or south.

So, when you include the information on the direction, so that will result in velocity. So, velocity is nothing but

$$Velocity = \frac{Displacement}{Time}$$

So, distance is a scalar quantity, displacement is a vector quantity and then scalar divided by a scalar results in scalar and scalar divided by sorry vector divided by a scalar since time is a scalar quantity results in a vector. So, velocity is a vector quantity. So, SI unit for both speed and velocity is meter per second.

So, now let us solve this problem for speed and velocity. So, in this what happens is from our previous knowledge we know the distance from this point to this point was 80 sorry it was 160. So, let me erase it. So, the distance between these two points A and B following the specific path. So you will move following these arrows to reach point B the distance was 160 and the displacement was 113.14.

So, let us assume when the person walks using this path it took him 80 seconds to reach from point A to B then our speed becomes

$$Speed = \frac{160}{80} = 2\ m/s$$

However, for displacement when we calculated the displacement following this double line over here or let me change the color for better visualization. So, following this green line the displacement was 113.14 and it took for our person under consideration 60 seconds to complete this path then our velocity will become

$$Velocity = \frac{113.14}{60} = 1.89\ m/s$$

So, this is how you can calculate the velocity for a very simple case.

Now, let us look at another example which we discussed earlier also. So, now let us try to find out the average speed and average velocity between A to B, A to C and A to D. So, let me draw these let me put these points over here A to B, A to C and A to D. So, now we had to define the time also like how much time because we need distance, displacement and time to calculate speed and velocity. So, let us assume from A to B the person took approximately 8 seconds it is in seconds. A to C it took same because he goes from A to B first then B to C took 4 more seconds and then again from C to D.

So, the total time this takes 8, 12 and then another 8 over here. So, the total time from A to D is 20. Distance is we have already discussed. The distance between A to B was 40 meters and the displacement which is also measured in meters was 40. From between A to C is 60 displacement and 20 distance and between A to D it is 100 and 60. Now, let us calculate the speed and velocity for these points. So, between A to B is distance divided by so, it is distance divided by time whereas, for velocity also it remains same.

Between A to C it is whereas, it is 1.67 and finally, between A to D and over here. So, we can see clearly during these movements from A to B or A to C or A to D the average speed remains same whereas, the average velocity varies between A to B, A to C and A to D. So, this analysis or the what we learn from this example is with the knowledge of speed and velocity we can look into more specific details about the movement. So, next let us look at the concept of acceleration.

So, acceleration is nothing but rate of change of the velocity of an object with respect to time. It is being measured in meter per second square. So, to understand this let us take a similar example which we discussed for distance, displacement, velocity and speed. So, here there is an athlete who starts from position A and reaches position B. We need to calculate the average acceleration between A to B.

We have given the athletes or the person under consideration starts with the zero velocity from A and reaches with the velocity of 10 meter per second at point B and it took him 5 seconds to reach from point A to B. So, what would be the acceleration? Let us denote acceleration by small a that would be

$$a = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{10 - 0}{5 - 0} = 2\ m/s^2$$

Since now we have covered about the discussed and covered some examples on displacement, velocity and acceleration.

So, let us see how they are related with each other. So, once we have a displacement and we divide it by that time taken to cover that displacement or to reach to that displacement that results in velocity. As we know velocity was given by displacement divided by time. Similarly, when we divide the change in velocity by the interval of time it took to change from initial velocity to the final velocity that results in acceleration.

Now, let us look other way around. When you multiply the average acceleration with the time interval during which it happened you will get the velocity and similarly when the average velocity is being multiplied by the time interval during which it happened we will get the displacement. So, this is a very important concept which we will be using in our examples or in bio mechanical analysis very often. Another thing to understand over here is what is the effect of direction of motion on direction of acceleration. So, this concept will be very helpful when you are observing a movement without looking at the numbers. So, what we will do is to understand this table it seems too much information over here.

Let me break it down for you to understand it in more optimized way. For example, first we decided conditions which we change specifically there are three possible scenarios in terms of conditions. For example, whether an object or a person is speeding up, whether their speed or motion is not changing or whether they are slowing down. What we will do is we have these three condition and then repeat these three again. Since we are looking at the direction of motion, so whether the direction of motion is positive.

So, how you will define positive and negative? Depending upon the position of your origin for your reference system. So, if you are moving in opposite direction that will become your negative x and over here that will become your negative y. So, that is what it means by positive and negative. So, speeding up will have two conditions when the direction of motion is positive and when the direction of motion is negative. So, for better understanding I club positive into 1 and negative into 1.

So, you do not have to change back and forth between these two columns. So, let us consider when the direction of motion is positive and we will look at these three variations when the person is speeding up. So, when the person is speeding up that means the velocity or the change in motion which we call velocity is positive. In that case our direction of acceleration will be positive. In the second scenario when the direction of motion is positive but there is no change in the motion then our acceleration will also become 0 because change in velocity we know change in velocity divided by change in time give us acceleration.

So, in the first condition if that number is positive, time is always positive we will get positive. If in second condition we do not have any change, so in the second condition we do not have any change divided by positive value that will become 0 again. In third condition when the object or person is slowing down then change in velocity would be negative. What will have when we divide negative divided by positive we will get the negative value. So, that is why the direction acceleration is negative. Now, let us look from other way around when let me change the color over here.

So, now we will take again these three conditions and now all the motion or the direction of motion is negative. For example, you are speeding up although you are speeding up in the negative direction but change in velocity would be positive and as we know time is always positive. So, we will so we know change in direction is positive like change in sorry when the change in motion is positive but you are moving in negative direction. So, that will result in so let me clean it up first. So, when you are moving in the negative direction, so what is going to happen is final position minus initial position if this is 0 let us say this is minus 1 this is minus 2.

So, you are speeding up means your value is increasing for velocity. So, final position would be minus 2 and you are subtracting it from minus 1. So, the final answer is negative 1. So, what you will the condition for acceleration will become. So, change in motion is positive but you are going in the negative direction.

So, that will give you the negative acceleration. It is sometime also known as deceleration also. So, negative divided by positive will give us negative. Similarly, when there is no change then the direction of there is no acceleration. So, there is no direction of motion and finally, when the object or the person is slowing down in the negative direction itself. So, what will happen is so initial value is higher than so initial is higher than the final value.

What will happen? Change would be positive divided by positive. So, you will get a positive acceleration. So, I hope this will be helpful and can be utilized in your observational analysis. Another concept which is very important in kinematic studies is equations of motion. So, I do not want you to think like we are going more into mathematics but it is just I am trying to connect whatever we have studied so far so that you will have a better understanding why we spend so much time in understanding the concept of position, distance, displacement, speed, velocity, and acceleration.

So, in this case the equations of motion have certain assumptions. So, first one is acceleration is constant. So, the change in motion is constant. So, you do not have you know varying values of acceleration whether going in positive direction that is known as acceleration going in negative direction or slowing down is deceleration. So, over here acceleration is constant means no change in acceleration.

So, let us assume the displacement is being denoted by small s, small u is the initial velocity in meter per second, v is the final velocity, a is the acceleration in meter per second, then t is the time in seconds. We will use these notations to define our equations of motion. So, the first equation of motion is between the relationship between the first equation of motion is the relationship between initial velocity, final velocity, acceleration and time.

$$v = u + at$$

The second is similarly, the second is again a relationship between displacement, initial and final velocity and time.

$$s = \left(\frac{u + v}{2}\right)t$$

Third equation of motion is again relationship between initial and final velocities along with acceleration and displacement, but no time.

$$v^2 = u^2 + 2as$$

The fourth one would be in displacement, initial velocity, time, acceleration.

$$s = ut + \frac{1}{2}at^2$$

And the last one would be the displacement, final velocity, time and the acceleration involved.

$$s = vt - \frac{1}{2}at^2$$

So, the derivation of these equations is beyond the scope of this course, but however, I would recommend to go over any standard text to see how these equation of motions can be derived. We will see how these can be applied in a sporting activity now, so that you will have a better understanding why we need to study this. So, before moving to the application part of the equation of motion, I would like to emphasize another type of motion which is like very common in sports that is known as projectile motion. And what are the assumptions in projectile motion? That is after the initial force, the objective is just under the action of gravity.

For example, when you kick the ball, once the ball is airborne; there is no other force acting on it. For simplicity here, we are ignoring the effect of air resistance or other environmental factors because that will be covered in an advanced course on biomechanics. So, here what will happen is, the object is just under the effect of, so let us see if like since this is airborne phase already, so it might have started from here and then landed at a certain point. So, let me complete this. This might be the original position of the ball and this is the final position of the ball, this horizontal line represents ground.

So, we are not going to make it difficult. So, what I am going to try to do over here is try to explain the projectile motion and how to calculate various parameters involved in a projectile motion using the concepts which we have already studied. So, over here what we will do is, so we will draw our reference system over here. So, this direction is positive x and this direction is positive y and the opposite direction is over here is negative y and over here would be your negative x. So, since this motion is happening in this direction, so what we will do is, we will divide the this complex motion into simpler horizontal and vertical motions. So, what will happen when ball is moving? So, ball has again x and y components over here at this position also, at this position also.

So, what we are going to do is, we are going to look at all these positions separately. So, what is the meaning of these arrows? So, if there is a force, since force is a vector quantity or velocity is a vector quantity, so since it is moving up, so up means it is going in positive direction and moving forward, moving forward in the horizontal x direction, so that depicts the direction of the motion. And if this is the maximum point and after the maximum point, the ball started coming down, so that is why I put a arrow which is downwards. And we will simply use equation of motion which we discussed recently, those equation, simpler equation of motion which just involve quantities like displacement, initial velocity, final velocity, oh sorry, so which involves initial velocity, displacement denoted by small s, initial velocity by small u, final velocity by small v, I just curl it over here, acceleration by a and time by t. Since in this case, we assume there is no other force than a gravity, then there is only vertical acceleration which is acceleration due to gravity and there is no horizontal acceleration.

To make things easier, what we will do is for horizontal motion, we will use a subscript as x and for vertical motion, we will use subscript as y. Similarly, u_x, u_y, v_x, v_y, a_x and a_y and time is scalar quantity, so it is independent of the direction. So, now let us analyze a soccer kick using these principles which we discussed in this module so far. So, in this example, what is happening? An athlete is kicking a ball like if this is the starting point and then ball moved from here, so it is moving, so this is the velocity vector, the initial velocity is small u. So, velocity is 20 meter per second and it is making an angle, launch angle is or your angle with which the ball started moving from the ground is 45 degrees and the acceleration due to gravity is 9.81 meter per Second Square.

So, in this example, which is a very common example you might have seen in a soccer game like a player comes, hits the ball and then it lands covering certain distance, reaching at a certain height and takes certain amount of time. So, this is the goal of this example. So, we will be interested in finding maximum height of the ball, total time of flight and horizontal distance the ball travels which is also known as range. So, let us break this problem into each sub like smaller sub problems, for example, for maximum height of the ball. So, from our knowledge from equation of motion we know, sorry, from our knowledge from equation of motion we know initial velocity and final velocity along with acceleration and displacement are related by this equation.

$$v^2 = u^2 + 2as$$

Since over here we are interested in the. So, if I draw a smaller version of this like ball is starting from here and then landing over here. So, this is the I will denote small i and small f the position and this angle of velocity is 45 degrees and this is 20 meter per second. So, maximum height.

So, maximum height is this distance. So, to understand this problem. So, what we will do is we will since we are interested in a vertical distance. So, we will use the vertical components only. So, let us look at this point again, where our velocity is given. So, velocity is a vector.

So, it is making an angle of 45 degrees with the horizontal. So, using the concepts from trigonometry, we can calculate the base and perpendicular distance. So, we will look at the components of velocity. So, this is initial velocity u is given as 20 meter per second. So, the velocity component of velocity in horizontal direction would be

$$v_x = 20 \cos 45 = 20 * 0.71 = 14.14\, m/s$$

Similarly velocity in y direction would be

$$v_y = 20 \sin 45 = 20 * 0.71 = 14.14\, m/s$$

So, that is again. So, here we see our initial velocity components in horizontal and vertical direction has same magnitude. So, when we look at this point the final point where we had to calculate the maximum height. So, when the ball is travelling since at this instant, since earlier the velocity vertical velocity component was on upper side, the going in positive direction. So, over here it is negative side. So, at that particular instant, so what is happening it is moving at that particular instant, it is not going any more further in the y direction.

So, what happens is your if there is no displacement in the y direction that results in 0 velocity in the y direction. So, what will happen is the velocity in y direction final velocity in the y direction would be 0. So, when we plug these values in this equation what it will give us is

$$0^2 = 14.14^2 + 2 * (-9.81)s_{max}$$

After solving these equations you will get

$$s_{max} = 1019\, m$$

So, you see we did not use and we do not need to cram any formulas or look into complicated mathematics. It is just the uses of trigonometric principle basic trigonometry where we can use those identities in the resolution of vectors or looking at the components of vectors and then using those components as a very simple one dimensional problems like along the horizontal or x axis or in vertical or y axis.

Now, let us look at the second point over here where we are interested in the total time of the flight of the ball. So, let me draw this again for a better understanding. So, the ball starts from here and then lands over here. So, we are interested how much time the ball will take to reach from this point to this point sorry over here this point to this point. So, what information do we have? We have velocity information at the initial point and we have partial information about the velocity at the final position also.

And we know initial velocity final sorry final velocity is related to initial velocity and acceleration.

$$v = u + at$$

So, if we look at the motion in vertical direction what will happen is so initially we have u_x going in positive and u_y going vertically upwards, but at this position what is going on you have a vertical velocity here and final velocity over here. Another thing to notice over here is

$$u_y = -v_y$$

It is just negative in directions. Why? Because once the ball is in the air. So, it was under the influence of gravity only to reach this position, but what happened from there it starts again and then reaches to its initial velocity considering there are no effect of air resistance or any environmental factors. So, what we have over here is so in terms of y direction or vertical motion what we can see is

$$v_y = u_y + a_y t$$

$$-u_y = u_y + (-g)t$$

So, what this will give us is if we move this equation to this side that will become

$$-2u_y = gt$$

or rearranging this equation will give us this

$$t = \frac{2u_y}{g}$$

And when we plug in the values we already calculated we will get time as

$$t = \frac{2 * 14.14}{9.8} = 2.88\ s$$

So, now we see how these equation of motion can simplify the analysis by breaking down the motion into horizontal and vertical direction. How this is possible over here we have utilized all the mathematical skills which we discussed during this lecture.

For example, we used right angle triangle to find the components of a vector. We also used sine and cosine theta or the trigonometric identities to find the values of horizontal and vertical components and also we are using these equations of motion to solve a complex problem into simpler thing. Now, look at the third point where we want to calculate the horizontal distance which the ball has travelled. So, in this again we are interested to calculate the distance s between initial and final position. So, what you will use is we will use this equation to find the displacement in the x direction.

$$s = ut + \frac{1}{2}at^2$$

So, let us plug in the values. Since in horizontal direction there is no acceleration. So, we can simply put a_x as 0. So, we want to find out s_x. So, that is the horizontal distance.

$$s_x = u_x t$$

In the previous slide we calculated the time as 2.88 s. So, that will result in a distance of

$$s_x = 14.14 * 2.88 = 40.72\ m$$

So, here we have solved a complex problem which seems complex like kicking a soccer ball during a match and then try to calculate the time it was in the air, the total horizontal distance it cover and the maximum height it reached using very basic knowledge of mathematics or I would say just the high school level knowledge of mathematics using very simple equations. How we were able to achieve this by breaking down the problem into a very simple and smaller problems. So, with this I would like to thank you for your attention and recommend to go over these slides again to understand it in a better way. Thank you.