Angular Kinematics

Hello everyone, Welcome to Module 4, introduction to the concepts of Biomechanics. In this section, we will continue our discussion on the kinematics. In this section, we will look at the angular kinematics, look at the various methods to measure the angles, and then we will look at the relationship between angular and linear kinematics, followed by an example where we will calculate the joint angle for a movement. So, what is angular kinematics? So angular kinematics is nothing but study of motion in terms of angles or angular displacement, angular velocity, and angular acceleration. As in the previous section, we discussed about the linear displacement, velocity, and acceleration. So, the first thing that comes to our mind is why we need to study the angular kinematics.

It is particularly very relevant when analyzing movements involving rotation about an axis such as joint movements in the human body or rotational motion in sports equipment. Understanding angular kinematics is crucial in analyzing and optimizing movements involving rotation. It also provides valuable insights into the mechanics of joint movements, sports technique, and the design of equipment that involve rotational motion. For example, in this example, we can see an equipment is being rotated about an axis over here.

And similarly, at different joints, the athlete is also making a rotational motion. So, let's start with an example to understand what angular kinematics is. So let us assume a weight or an object is suspended by a rope at a position which is noted by number 0, or it is known as mean position, and then it moves to position 1 which is at an angle θ_1 to the original position in anti-clockwise manner. And after certain amount of time, that object moves to position 2 in clockwise manner like first, it moves to anti-clockwise manner by an angle θ and then come back to mean position and then goes to position 2, which is in clockwise direction by an angle θ_2 from the mean position. So let us define the position, displacement and distance for these three positions from 0 to 1 and 1 to 2 or 0 to 2 itself.

So, angular positions will be θ_1 and θ_2 from the mean position. If we look at angular distance from position 0 to position 2, so the position 0 to 1 it moves by an angle θ_1, and from mean position to position 2 it is θ_2. So, the total distance traveled, the angular distance, becomes the sum of two angular positions. Now, let us look at the angular displacements. When we are looking from position initial position or mean position at 0 to position 2, the displacement is θ_2 only.

However, when we look from position 0 to position 1 that displacement is θ_1. So here we see how we can define angular position, distance, and displacement. It is in a similar manner how we discussed for a linear kinematics. The only difference here is over there we were looking the angular, sorry the linear distance in meters, centimeters, kilometers, or any standard unit of length. However, the standard, like international standard unit for length, is meters.

Over here, the international standard unit for angle is radians. So now let us look at the various units involved in the measurement of angular motion. So the angular motion can be measured either in degrees which we are very much familiar or in radians. So degrees is nothing but 360th part of a full circle. So for example we draw a circle and then start dividing into 360 equal segments like here shown and then you keep on drawing those lines and divide the whole circle into 360 equal parts.

So one such part and the angle between the lines is known as 1 degree. Whereas radian is nothing but is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of circle. For example, if we take this as the radius and then another position in the circle or divide the circle in a such a way where the length of this and this. So this is radius and this is arc. So arc length when equal to the radius the angle over here is known as 1 radian.

So the next thing is how we can measure angles since we talked about what angle is in day-to-day life what are the different methods how we can measure the angle. So for example if you are talking about human movement science in this particular course so the angles various angles which we can link to our mechanical analysis are the joint angles. So for example, if I am flexing my arm so the angle between my forearm and the upper arm. So this angle we are interested in this angle or this segment my forearm what angle it is making with the horizontal plane itself. So there are two ways how we can measure angles.

The first one is known as relative angle and the second one is known as absolute angles. In relative angles, it is the angle between the longitudinal axis of two adjacent segments. For example in this case adjacent segments are forearm and upper arm and the angle between these two is known as the relative angle between forearm and upper arm. Whereas absolute angle is nothing but angle between a segment and the right horizontal of the distal end. Over here I mentioned right horizontal but it can be you know generally defined as a horizontal plane or a vertical axis. Horizontal axis or vertical axis itself.

So relative angle are also known as joint angles whereas absolute angles are known as segment angles. So it should be measured consistently on the same side of the joint. For joint angles this thing we should keep in mind to have a consistent results. Whereas when an angle between two segments is 0 degrees and it is defined as the fully extended position. For example, if I am fully extending like the angle between these two segments is 0 degree then when it is defined as 0 degree.

But mathematically if you see if you are moving this angle from here or here that becomes 180 degree. So in absolute angles, they should be consistently measured in same direction as I discussed earlier from a single reference either horizontal or vertical. Over here in this example, I just showed the horizontal but you can measure the segment angles with respect to vertical axis also. So now let us look into more detail how we can calculate the absolute angles or the joint angles. So for example there is a segment let me draw a segment for example something like this.

So maybe this is your foot, lower leg or simply leg and upper leg or we call it thigh segment also. So we just isolate this component over here from our knowledge from right angle triangle what we do is we draw horizontal and vertical lines which meet each other at 90 degrees and then using the right angle triangles and trigonometric identities we can clearly find the angle between these two segments. So absolute angles can be calculated from the end point coordinates using the arc tangent or inverse tangent function. So this is helpful when you know the end points. For example, in this equation or in this example if we know the x and y values for our in two-dimensional example x and y values for our foot segment knee segment sorry not segments joints foot like ankle joint, knee joint, and the hip joint then what we can do is we can use the trigonometric identities to calculate the angle.

For example,

$$Opposite = y_2 - y_1$$

So upper value is y_2 and lower value is y_1. So difference will give us perpendicular or opposite side.

Similarly,

$$adjacent = x_2 - x_1$$

And then we can use the tangent function to calculate the tanθ

$$\tan\theta = \frac{Opposite}{adjacent}$$

And then find the inverse of this function to calculate the angle θ over here

$$\theta = \tan^{-1}\left(\frac{Opposite}{adjacent}\right)$$

So when we use standard units like if we are using the units for these quantities as meters the angle which we get is in radians.

To convert radians to degrees what we need to do is

$$180^{\circ} = \pi\, rad$$

So,

$$1^{\circ} = \frac{\pi}{180}\, rad$$

Or in other way

$$\pi\, rad = 180^{\circ}$$

Then

$$1\, rad = \left(\frac{180}{\pi}\right)^{\circ}$$

So this value will help us to convert radians into degrees and degrees into radians. Now, let us look at how we can calculate the relative angles. So relative angles can be calculated in two different ways.

First one is by using the law of cosines, and this method is helpful only when you know the segment length. For example, as we were discussing in previous example, using our absolute angles. So in this what we have, we have endpoints for ankle, knee, and hip given, but this particular method law of cosines is useful only when you know the segment lengths like A, B, and C and that will help us to calculate the angle θ. How we calculate the relationship between A, B, and C and angle θ is given by this equation using law of cosines

$$c^2 = a^2 + b^2 - 2ab(\cos\theta)$$

$$c = \sqrt{a^2 + b^2 - 2ab(\cos\theta)}$$

And the value of A and B can be calculated using the similar right angle triangle principles.

$$a = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

$$b = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Now look at the second method to calculate the relative angle and that is also when you find two absolute angles and then this process is useful when you can calculate absolute angles easily.

So let us take at the same example, but for absolute angles, we need the angle between the segment and either horizontal or vertical. So here the dotted lines represents the horizontal axis. So what you do first is you calculate the angle between the leg and the horizontal axis similarly angle between the thigh and the horizontal axis and then using this information we can calculate the relative angle or the knee angle. So what we do is it is given by this equation here

$$\theta_{knee} = \theta_{leg} + (180 - \theta_{thigh})$$

Knee angle is nothing but angle subtended by the leg segment with the horizontal we add 180 degrees and then subtract the thigh angle from it. So let us look what does it mean.

So if we extend this over here so let me draw this again so what we will do is this is our segment so if we extend it. It will go like this and if we go like this. So what is happening so this angle it is θ_{leg} and this whole angle is nothing but θ_{thigh}. So we are interested in this angle over shaded here. So from where this equation came so this θ_{leg} is nothing but so we want to find out this angle. So what we do is we add this angle since this is 180 degrees and we subtract 180 this and this angle is θ_{leg} because these are opposite angles so they are equal.

So let me draw one more time in a clean manner so over here, if we have these vertical and horizontal axis this is our lower leg and this is our upper leg so what is happening we are interested in finding this angle. Over here, if we extend this so angle between horizontal and the leg segment is this, and similarly, this is angle between horizontal and the leg, so this is again θ_{leg}. However, we know with this horizontal this is making θ_{thigh}, and let me change the color so that and this whole angle over here it is 180 degrees. So to find the angle this angle which is shaded over here that is nothing but 180 degrees minus θ_{thigh}. So, this angle plus θ_{leg} will give us the angle at the mean joint.

I hope this is clear otherwise you can practice or try to draw yourself then you will understand how this angle can be calculated. So now let us move to the next concept that is angular velocity. So as we discussed in the linear kinematics about the linear velocity here also angular velocity (ω) is nothing but

$$\omega = \frac{change\ in\ angular\ position}{change\ in\ time}$$

And the direction of angular velocity is given by right-hand thumb rule. For example if some like in our earlier example when an athlete is throwing a shot put so the direction in which the implement or the equipment is moving for example the shot put whether it is moving clockwise or anti-clockwise depending upon that what you will do is you place your hand over the axis and then try to rotate the or curl your fingers then the direction of the thumb will give you the direction of the angular velocity. So angular velocity this concept is known as right hand-thumb rule.

The SI units for angular velocity is radians per second. Similarly, angular acceleration is when the implement or the object in motion about an axis changes its direction. For example, initially when if you see the whole movement for a shot put, the athlete will start when the shot put is on the ground or below the height of his hand, then start rotating, and then eventually it changes its direction. So the circle which that equipment sports equipment or sports implement is making changes its direction or the plane of rotation, then what you will have is with that your direction of angular velocity also changes. For example, initially, if it is in the horizontal plane, then we can see ω_i is the initial angular velocity, and after some time, if it is changes its direction angular it moves from this position to this position.

So this is ω_i and this is ω_f like after certain amount of time initial time and final time and then the change in angular velocity is $\Delta\omega$. So angular acceleration is donated by the Greek alphabet α it is nothing but

$$\alpha = \frac{change\ in\ angular\ velocity}{change\ in\ time}$$

Or in other words

$$\alpha = \frac{\omega_f - \omega_i}{t_f - t_i}\ rad/s^2$$

So now another thing since we are discussing both linear and angular kinematics and in our previous discussion we already know in a general motion or the motion which happens in day-to-day life we have both linear and angular motion. So, general motion is always a combination of linear and angular motion. So now let us see the relationship between linear and angular kinematic parameters.

So let us assume s denotes the linear displacement in meters, θ is the angular displacement in radians, v is the linear velocity in meter per seconds, ω is the angular velocity in radian per seconds, small a is the linear acceleration sorry over here it should be meter per second square and angular acceleration is in radian per second square and r is the radius of rotation in meter. So linear displacement is nothing but

$$s = r\theta$$

Similarly linear velocity is

$$v = r\omega$$

And linear acceleration is

$$a = r\alpha$$

So these are the scalar components which I represented over here. However, since these are vector quantities which is beyond the scope of this course at the moment but since these are vector quantities they are governed by the principles of vector algebra.

So now let us see the example which we discussed earlier in our linear kinematic analysis or draw vertical jump where we looked at three specific positions. First is the touchdown, second is the maximum knee flexion, and third is the toe or feet off when the athlete is ready to take a vertical jump. Now what we will do is we will take all these three moments in the drop vertical jump and try to calculate the knee angle using the information which we have so far. So now let us look at the first phase of the drop vertical jump, which is touchdown phase. In this, we were given the location of ankle, knee, and hip joints.

So, we are interested to find the knee angle. So, let me draw over here. So, this is our thigh segment and this is our leg segment. So, this is leg, this is thigh. So, we are interested in this angle. Since we do not know the length of leg and the thigh segments, we will first calculate the absolute angles and then find the relative angle using the absolute angle.

So in this case what we will do is we will draw a horizontal line at the ankle and knee and we are then going to calculate the θ_{leg} and θ_{thigh}. So let me draw the thigh segment again, and then the horizontal, and then we drop a vertical line over here. So this is 90-degree angle. Since we are given the coordinates for hip and knee joint as given in the image. So let me call this triangle as BCD, where the distance CD is nothing but the difference between the y coordinates given here

$$CD = 87-52$$

And BD is nothing but that this difference between the x coordinates of hip and knee joints.

$$BD = 54-26$$

So we will calculate this angle as

$$\tan\theta = \frac{CD}{BD}$$

When we plug in these values we will get a value of θ as

$$\theta = 54.34°$$

Since we are interested in finding this angle over here, the shaded portion so that is nothing but

$$\theta_{thigh} = 180° - 51.34° = 128.66°$$

Similarly, if I draw the leg segment over here again we will draw a vertical axis as well as a horizontal axis, and we are interested over here in the angle known as θ_{leg}. Let me call it A, B, and E. So, in this, we will calculate

$$\tan\theta_{leg} = \frac{BE}{EA} = \frac{52.15}{28.26}$$

So here, I interchangeably used the knee and ankle x coordinates, the reason being here we are interested in the magnitude only we are not concerned about the direction as of now.

So with this we will calculate θ_{leg} as

$$\theta_{leg} = 86.91°$$

And finally, once we know the absolute angles we can calculate the relative angle between leg and thigh segment which is given by

$$\theta_{knee} = \theta_{leg} + 180° - \theta_{thigh}$$

When we plug in those values, we will get

$$\theta_{knee} - 138.25°$$

which seems about right looking at the picture. Now let us look at the next phase, that is the maximum knee flexion phase. So over here if we draw similar ways so this point is 28 and 14, this is 8 and 45, and 52 and 58. So using similar calculations which I would encourage all of you to try yourself then your

$$\theta_{thigh} = 163.54°$$

$$\theta_{leg} = 57.17°$$

$$\theta_{knee} = 73.63°$$

Here also, what we will do is just to give you a hint similarly; we will divide the thigh segment and then your leg segment into right angle triangle. So, this is your leg, and this is your thigh. So, here we are interested in calculating this is your θ_{thigh}, and this angle would be your θ_{leg}.

So, this way you can calculate the knee angle. Now let us look at the final segment or the takeoff instant of the draw vertical jump. Here also we will do the similar calculations. What we will do is here you can clearly see your thigh segment is something like this and your leg segment is something like this. What you will do is you will again draw a vertical line over here and create a right angle triangle, and similarly, at the thigh segment, you will draw a. So, let me separate it for you for better clarity. So, here and then, when we extend we are interested in θ_{thigh}, and similarly, for your leg segment, we are sorry for the not-very-straight line.

So, that will give us θ_{leg}. So, in this case, why after you calculate plug in the values, you can calculate the values and match that

$$\theta_{thigh} = 92.29°$$

θ_{knee} would be around, oh sorry, θ_{knee} would be for the θ_{knee} we need θ_{leg} first

$$\theta_{leg} = 85.71°$$

$$\theta_{knee} = 173.41°$$

So, in this example, we learn how we can calculate the angular kinematic parameters specifically in this case, we just looked at angular displacement using the methods which we described earlier in this section.

So, similarly, if you know the angular displacement like these three events happen over a span of time, if we know the difference from touchdown to maximum knee flexion, then if that time is t_1, then your angular velocity average angular velocity to reach that position would be

$$\omega = \frac{\theta_{knee_{MF}} - \theta_{knee_{TD}}}{t_{knee_{MF}} - t_{knee_{TD}}}$$

So, that will give you the value of angular velocity and similarly, if you know the angular velocity at the touchdown and maximum knee flexion and the time interval, then you can calculate the angular acceleration. So, in summary, what we have looked in this particular section, we looked at the linear and angular kinematics and the various methods to find them, followed by the biomechanical considerations while calculate performing a kinematic analysis like how you will look at the different time points while a movement or activity is being performed and then how we can calculate linear and angular parameters for them and then we discuss this linear as well as angular kinematics using a example of draw vertical jump. Thank you.