Angular Kinetics - Part 1

Welcome back to another module in Human Movement Science, Kinetics. Today, we will be studying Rotational Motion. So let us get into it. Rotational motion or rotational dynamics is, so let us get into it. Rotational dynamics or kinetics is the branch of classical mechanics that deals with objects that rotate about an axis. So this top is rotating about its axis and then about another axis.

So all of these rotational motions, the cause of this as you remember, we studied when we studied linear kinetics, we studied the cause of motion. Here again, we will be studying the cause of rotational motion. Now, you might wonder where these kinds of movements are critical for this course or for sports in general and the answer is pretty much everywhere. There is one simple example, cricket bowling action, right? The person is moving, is running up, is moving their shoulder, their arm about their shoulder joint doing rotary motion like this and completing with a throw.

All this time, they were also running and rotating their legs about different joints, right? So we do need to understand what is the nature of forcing required to move these joint segments and that is what we will focus on interpreting, understanding, right, in rotational dynamics. Now, before we get to the dynamics, it is of utmost important that you remember displacement, velocity, and acceleration. So in linear kinematics, you have studied the displacement, velocity, and acceleration, same way you have studied it for angular quantities and how to measure them. So this will be required for doing any computation in this field. Okay.

Now, another question is why even study? So building on top of rotational kinetics, as I showed you just now for bowling action, let's just take a simple running action. The subject will be moving their individual joint segments, right? They'll be swinging their arms back and forth, right? So the forcing required to move your individual segments, the rotary forcing is intuitive. If you think about it physiologically, and we'll see why it is intuitive when we look at muscle action, right? And so we would like to understand in addition to the linear force between joints, we would also like to understand joint moments. So I use the term here moment. You might remember similar term momentum, but it's not the same thing.

We'll talk about what is a moment. In physics, moment is also an instance in time. So we'll use these terms, several terms like this. And so I'll just lay them out briefly here. Moment or torque or couple.

We'll also look at leverage, right? And how there are so many levers in our body. We'll understand what is a lever, right? And how we can use the principles of levers to study the joint movements. Let's dive into it. Let's look at what is moment. So moment is typically used in terms of a linear force when it can cause a rotary motion.

Moment and torque are sometimes used interchangeably. And even mathematically, they end up being the same. At this point, it's more semantics than anything else. So if I was to say the moment of a force is the measure of the force's tendency to cause a body to rotate about a specific point. And this is measured as

$$Moment = Force * Perpendicular\ Distance$$

So if you remember the concept of line of force, let's say I have this disc. And I'm applying a force on it at the corner. And this disc is free to rotate about this axis. Now the interesting thing is I will extend back the line of action. And I will drop a perpendicular from this axis on this line of action of the force.

Remember that all of this is in one single plane. So this force is acting in this plane. And this axis of rotation is perpendicular to this plane. So this is this distance d and this is the force F. So the moment of this force is

$$M = F * d$$

Another way to look at it is I can directly, another way to look at it is I can measure the distance to the point of action. Let's call that D, capital D. So the moment of the force would be

$$M = F * D\ Sin\theta$$

Right now we are dealing with planar kinetics. And we'll keep our scope limited to that.

Now we have also used the term torque. So what is torque? So let's go back to linear kinetics again and try to recall what was force. Force was an external agent that caused a movement in the body. It was the external agent that would produce a change in momentum. And that's how we define force, change in momentum.

Derivative of mv, where mv is the momentum. So force was the derivative, time derivative of momentum. This quantity is momentum. One way, torque is the rotational analog of force. What does that mean? So analog, GUE, so analog means that it is the rotational equivalent of force in the linear system.

So what that means is if I am applying, so if you remember, force was a linear vector. Right? I can also apply, can apply a force in a twisting direction, like this is what this arrow is supposed to indicate. So force in the twisting direction, and I'm using that term very loosely right now. So force in the twisting direction cannot really be represented by a linear vector. So for that, we have what is called torque, right? Where torque tau is defined as mathematically

$$Torque\ (\tau) = \frac{d(Angular\ momentum)}{dt}$$

Now I'm going to say that angular momentum, how do I define angular momentum? So for linear momentum, we said that it is the tendency of a body to move, right? We measured it by the mass of the body and the velocity with which it was moving. But when we come to rotational systems, there's a slight difference. And so let's understand these analogs or the equivalences between linear systems and rotational systems. Okay. So let's try to understand these analogs or equivalences between the linear systems and the rotational systems.

So in linear system, we had mass of a body as m. Okay. Now if the same mass m was moving in a straight line, it's a linear system. If it was rotating about an axis, right? If it was rotating, let's say about an axis, like the earth rotates around the sun, right? So it's rotating about an axis. Let's say this is a circular trajectory.

Now we'll define a quantity called moment of inertia. So inertia was measured by the mass of the body. So inertia was quantified by the mass of the body. The moment of the mass inertia is quantified by

$$I = mR^2$$

where R is the distance to this mass m. Same way we have a force in the linear system.

So I have this mass, I apply an external force on it, right? And it starts moving with an acceleration a or at an instance, it has an acceleration. Same way we have moment or torque, which is

$$M = F * d$$

And just like we have linear momentum, we have

$$p = mv$$

Now a rotating object, let's say this is rotating with the speed of ω. So angular momentum would be replace the mass with the analog of analog in the rotation system, which is the moment of inertia and the linear velocity with the angular velocity.

$$L = I\omega$$

So that means that if I, if a mass can only rotate about an axis and I am applying a torque on this, so this could also have the torque. Then

$$\tau = \frac{d(I\omega)}{dt}$$

Let's go back. Let's go back here and take a look at the effect of the force. So this force, the moment of this force is equal to the torque that is being applied on this disc right now.

Now this doesn't need to be a disc. It could be anybody, any object rotating about any axis. The only things that matter right now is the perpendicular distance to the line of action of that force that is acting on the body. That's all that matters for our calculations or to understand this quantity as a fundamental quantity. Now I want you to do a sort of a mental exercise.

So you might have experienced this. We all remember merry-go-rounds hopefully in parks. If you start, if you, if your friend is standing on a merry-go-round and you're trying to rotate it, right? If your friend goes closer to the axis, right? It's much easier to rotate him as opposed to when he is outside, right? That means it was the same mass, but I was changing the radius at which by moving the friend in or out, you are changing the inertia of the system, right? So if I look at this quantity here, right? So this is for a point mass. For a point mass system

$$\tau = \frac{d(mR^2\omega)}{dt}$$

Okay. So mass being constant, the torque that has to be applied to achieve a certain velocity will become proportional to the radius at which the mass is located. So if your friend is rotating and is on the outside, right? It will be much more difficult to attain a certain velocity or you'll have to apply a larger force resulting in a larger torque acting on the system, right? Another thing we commonly do is take a string, tie something to the end of it, right? And then rotate it. Now depending on how far this object is, right? I will have to generate additional torque. So right now I'm applying more torque to rotate it at a certain speed. If I was trying to maintain the same speed at a shorter radius, it's much easier, right? So these are some practical experiences that you might have had.

And this is the quantitative reasoning behind that experience. Okay. So one other quantity I want to talk about is couple of force. So if I take a system which is rotating about its axis and two forces equal and opposite are acting on this system, right? Two forces equal and opposite at the same distance.

Okay. So now if I was to look at the system and these are the two forces and these are the two forces, the sum of the forces is equal to plus F, right? This is in the coordinate system. So in this coordinate system, let's say this is X and Y.

$$\sum F = +F - F = 0$$

But even though the net force acting on this thing is 0, which means it's not going to translate, right? By the laws of, by Newton's second law, right? Net force acting on the system is 0. But it is free to rotate about this center point. What will happen? This force will apply a moment about this, which is F into this little d.

This force will apply a moment about this, which is again F into little d. And if you notice, both are causing the rotation in the same direction, same sense, right? So in this case, both are causing it to rotate. Both are causing it to rotate in a clockwise direction, right? So it's causing it to rotate like this, like so. So resultant force is 0, but the resultant moment is

$$\sum M = Fd + Fd = 2Fd = FD$$

Should quantify that as perpendicular distance between those two forces, right? So now let's, so here we developed a sort of an intuitive understanding as well, that if a force is acting in equal, that if equal and opposite forces are acting, even if they are not causing linear motion, they can still cause rotational motion.

Now let's see what other, another important characteristic of the force that is being applied for rotation is where it is being applied, right? So like we said about your friend on the merry-go-round, the distance of the person from the center change the, okay. So like we said, your friend is standing on this merry-go-round, let's say, right? And they ask you to rotate them. They ask you to, and they ask you to rotate the merry-go-round, okay? Now what would you do? Very intuitively you would go to this point, to the very edge, and try to push it, right? What would happen if I try to push here at, what would happen if I try to push here at the very center, if I tried to apply all the force on that axle, on the shaft on which the merry-go-round is centered? I mean, you can go out and try it, but I'm pretty sure you realize that it won't rotate, right? The best rotation, the easiest rotation will happen is if you apply the force, if you apply the force at the very edge, right? So the eccentricity of the force from the axis of rotation is essential to cause rotation, right? Is required to cause rotation in the absence of which rotary motion is not going to happen.

Okay. Okay. So we have so far defined a couple of important things that are required for understanding the dynamics of systems or defining the dynamics of systems in doing rotational analysis. Before we move forward, I want to equip you with a tool for you to be able to analyze any system, okay, with relative ease. And these are the free-body diagrams, okay? They are a visualization tool. They are, you know, they help you write your equations as well whenever you actually do the analysis, right? They're very useful in any system, wherever you're trying to do any sort of kinetic analysis, be it linear translation, be it rotary motion, or a combination of both, okay? So we'll follow a couple of rules or we'll follow a couple of steps to draw the free body diagram and then we'll see how we can actually use it, okay? So to do this, let's say there's an object and I have to draw the, so to draw the free body diagram of something, we have to define that something, okay?