Angular Kinetics - Part 2

So, let's go. So this is a box. Let's say it is sitting on a surface and force right in the middle, which causes this box to start moving with a certain acceleration. All right, let's see. So the rules tell me that I have to draw the object of interest in minimalist form. Now what does minimalist form actually imply? It means the reduction to the simplest possible form.

Okay, in this case, I am assuming that force is acting parallel to this perpendicular to this plane, right? It is acting in the center of this body. So I can draw a free-body diagram by looking at it from this perspective here. So I can say I see this box, this is L, H, W. I am looking at it from L and H perspective.

Okay, let's do H here. What does it say? Locate the object using a coordinate system. So I have to, right now, this is a random object in random space. If you remember from your kinematics class, that acceleration is the time derivative of velocity, which is the time derivative of position. So I need to be able to define the position of a system.

If I'm measuring the position of this, there is a button on this pen. So if I'm measuring the position of this button, I would say its three centimeters from the tip of the nib or eight centimeters from the back end of the pen. Same way, so I've made this my origin. So same way, let's say I have defined a coordinate system like so, which tells me some important things. Let's say where this object is in space.

I can choose to define a 3D coordinate system as well. For simplicity right now, we are defining it in 2D, let's say. Okay, next is mark the center of mass of the object. So assuming this is rigid and homogeneous box. Rigid means it cannot deform, right? Homogeneous means it has the exact same distribution of material inside it at every location in every direction, right? So if I cut off a chunk of it that has identical properties in terms of distribution of mass, so density is the same everywhere, right? So the mass center would be in the, so if this is the internal structure of the box, the mass center would be somewhere exactly in the center here, right? So we'll call this point G.

In this view, this will be point G. So center of mass is sometimes marked with a circle with an X on it, shade two of the sides. Just a fancier way to do it. Let's say this. And then finally, I have to draw and indicate all external reaction forces and moments of forces.

Okay, let's do that. So assuming all my external forces, one force is acting here. Okay, what else? Is there anything else acting? Well, let's say this is sitting on the floor of a planet like Earth. So there is acceleration due to gravity, which means there's gravity. So there is a gravitational force acting on this, right? Okay, what else? It's not really moving vertically, right? So there's an equal and opposite reaction acting from the floor, Newton's third law, right? So there's an equal and opposite reaction force, what is called the normal reaction force acting on this.

And assuming there is no friction, right? So and let's say there is some friction from the surface, right? So let's define a coefficient of friction. So if the force is acting in this direction, the friction and the body is moving along the negative x-axis, the friction force will act around the positive x-axis. So this is F, this is friction force F_f, which is μ times the normal reaction force where μ is the coefficient of friction. Wonderful. Okay, so I have drawn a very simple free-body diagram by following all the simple rules, right? I took this 3D object, I drew it in the simplest possible form, which gives me all the information, right? I locate, located it in a coordinate system for reference.

I know exactly where its center of mass is. So I know these coordinates at every given instance. And now I have marked all the external forces that are acting on this body, right? Now I can do any sort of analysis on this body, right? And we have right now marked it in a very simple way. If I wanted to use this for just Newton-Euler analysis, right? For just Newton's equation. If I wanted to do the analysis for linear motion, right? So Newton's second law says

$$\sum \vec{F} = m\vec{a}$$

Okay, now please pay attention to this. These are vector quantities, right? Because this thing is sitting on the floor, it is moving in this particular direction in the negative x-axis direction according to this coordinate system, right? So I have to write all my forces as vectors, okay? Now I have said that force is acting along negative x-axis, okay?

$$-F + F_f = ma$$

Now I don't know this (F_f) right now, okay?

$$F_f = \mu N$$

But wait, I can calculate N very easily. So if you notice, we wrote this equation along the x-axis. If I write a similar equation along the y-axis, I will get

$$-W + N = m.0 = 0$$

Which means

$$N = W = mg$$

So now I can substitute this value here and get

$$-F + \mu mg = ma$$

Or I can say

$$a = \mu g - \frac{F}{m}$$

So I have used a free body diagram to calculate the acceleration of this body should a force F act on it in this particular manner. We can do the exact same thing for a system that rotates, right? So let's say this is a box and I have put an axis through it, right? And I am applying a force on it, which is causing it to rotate. Actually, let's make it more realistic.

So this is where it penetrates. Somewhere comes out here, okay? And I am applying a force at a distance d. Okay. Now I would encourage you to pause the video here, take a couple of minutes, and try to draw the free-body diagram of the system. So I want to reduce the system to its simplest form, right? All right.

Let's say it is constrained in the vertical direction, right? Because of this pole that is passing through here, okay. Now from previous experience, you might know that if you applied this force at a certain distance, which is the eccentricity we talked about, you will start rotating this object. Okay. Let's say I'm interested in finding with what angular acceleration this body will start rotating at this given instance. So what is the simplest reduction of this system? If I look at it from the top, right, if I look at it from the top, I will see this.

Okay. I can define my coordinate system here itself. Let's say this is x-axis. Okay. So previously, this was the y-axis coming out of the plane, right? And this is the z-axis. So now we are looking at x-z plane, right? And there is a force acting here at this point.

This point is located at a distance of d. Okay. So I have drawn it in the simplest possible form. I have identified a coordinate system and located this object in space with reference to that coordinate system. I marked the center of mass of this body as well.

Okay. So this is point G. Okay. And then I have marked all the external forces that are acting on it. Is that correct? No. There is one force missing here.

So this is rotating about this point, right? But it's not linearly translating. Right now, there is a net force acting on the system. To be able to move this, however, we know that it is not going to linearly translate. It's only going to move rotationally.

It's only going to rotate. So what is preventing it from linearly moving? That is this pole right here. Okay. When I apply a force in this direction, and I'm applying a force on this platform here, it is trying to move against the pole. So the pole is applying a reaction force equal and opposite to the applied force.

Okay. So this here ends up forming a couple. Right. Okay. So if I was to look at the analysis of the system now, so for the linear motion, I've already said

$$\sum F = -F + F = ma = 0$$

$$a = 0$$

So that means this center of mass is not translating back and forth. There are no other axes to consider here. Okay. Now let's look at the rotational motion.

What is the moment of this force?

$$M = Fd$$

Okay. So moment of the external force, right, about this point, or you can take the moment of the couple, they both will end up being the same, right?

$$M = Fd = I\alpha$$

Now the moment of inertia of this system has to be defined. You have to calculate that or measure that.

And if you have that value, then the angular acceleration α becomes

$$\alpha = \frac{Fd}{I}$$

Okay. Okay. Now, we are not going to go into how to calculate moment of inertia for different kind of geometries, because as we saw, it is the distribution of mass from the axis of rotation, right? So for a simple mass rotating at a radius, at a distance of R, it is MR^2. However, for this, you have to consider infinitesimally small mass segments and their distance from the center of rotation.

So we are not going to go into too much detail on that. You can easily look up the moment of inertia of any geometry, or at least for within the scope of this course, we will provide you with that value. So now we have gone through two separate examples, one for linear motion, right? And one for rotary motion on how to use free-body diagrams as a tool to write down the equations and analyze the system. Okay.

I just want to point out something here. So in this analysis, for example, we were calculating the acceleration, right? Now in the chapter on kinematics, you also learned how to calculate acceleration from displacement values, right? So what is the difference between this approach and that approach? Where do you use acceleration, where do you use position values or velocities to calculate acceleration? And where do you use the forces to calculate acceleration, right? So the answer to that lies in the method of your measurement and what do you have as the outcome of an experiment. So if I have a object moving in space, and I know the position of this object at different instances of time, and that is the only measurement I have, then I can calculate acceleration from that. And I can plug it in into the Newton's equation, right? The second law and I can calculate what is the force, net force acting on the body that is giving it that kind of an acceleration, right? Whereas if I know the force that is acting on a system, I do not have measurements of the kinematics of the body. And I can calculate the acceleration of the body, right? So one is a forward approach, right? Where I know the forces and from that I am calculating the acceleration, this is called forward dynamics. If I have the accelerations, if I have the position data and from that, I have calculated accelerations, and from that I am calculating the forces and the moments, then that is called the inverse dynamics. We will discuss this in more detail when we come to modeling.

So now let us see how we can actually use these principles for human body, in human body. If you recall from the muscular system, how muscles attach to different parts of the bone and by linear contraction of the muscle, right? We are inducing a rotational motion. So we have done very simple, we have looked at it in a very simple way, in a non-analytical way so far, where a linear force is causing a rotary motion of this particular segment. Now let us see if we can analyze this, right? Okay, so when this person is trying to do this bicep curl, they are doing a rotation about a certain point, which is the axis of rotation, right? Let us say this is the axis of rotation, this is the axis of rotation of the joint, okay? Now this by itself is a little complex right now.

So let us see, let us see if we can use the principles we just learned to draw something simpler and analyze the system. So I see, let us assume this person is doing a perfect bicep curl by keeping their upper arm absolutely still, stationary. So I have one body, right? Which is, let us assume fixed to the ground, right? I have, I have a second body, okay. And let us assume this is a warmer ground, that person is not using any dumbbell, right? I see there is a muscle attached, but it is not attached at the joint axis, it is attached a certain distance from it. So the muscle, let us do the muscle in red, so the muscle is attached somewhere here, muscle is attached somewhere here, right? And the muscle contracts, right, which means there is a force acting in this direction.

So this is the point of force application, this is the direction of force, okay? So at any given instance, let us say the force acting is F, right? Now what have we learned that I need to find the perpendicular distance from the axis of rotation. So the perpendicular distance is d, right?

$$\tau_m = F_m d$$

So it is this torque that actually causes this hand to rotate up, okay. If I was to extend this and say, so here is a small exercise for you to try out, right? Let us say this person is strong, okay, this person is lifting up a weight of the weight of this dumbbell, let us say this is 10 kgs, let us say this is 20 kgs, right? Assuming g equals to 10 m/s^2 that means the force that is acting at the end of this person's hand is about 200 Newtons, right? Okay, now if I know the joint, the segment lengths, right? Let us say the forearm weighs a certain amount, right? So the mass of forearm is 2 kgs, 20 Newtons, okay? Assuming this is at a fixed distance, you can assume, let us say this is 0.2 meters, okay? Let us assume this is 0.5 meters, right? So what is the force F_m acting here? Let us say this perpendicular distance to this muscle at this instance is 0.01 meters, 0.02 meters, okay? All right, so I actually sort of have a free-body diagram here, okay? So first of all, but we can draw a free body diagram, but we can draw a free body diagram, okay? So I have the weight of the arm, the weight of the dumbbell, the force acting, okay? And let us say this is my coordinate system. And let us say this is my coordinate system. Now all I have to do is calculate F of m, right? Now there are multiple ways to do it.

I could write, you know, linear equations or I could just go into the moment of each force, right? If I say I am doing isometric exercise where I am holding this dumbbell in place at 90 degrees like that, right? So the net moment acting on this is F_m times 0.02, right? Now this moment is causing it to turn in counterclockwise direction, right? Whereas these moments, the moments of these forces will cause it to turn in the clockwise direction, right? So their sense is opposite.

$$F_m * 0.02 - 20 * 0.2 - 200 - 0.3 = I\alpha$$

And since it is isometric,

$$\alpha = 0$$

So I can say that

$$F_m = \frac{4 + 100}{0.02} = 5200N$$

Okay, so now you can imagine. So now you are equipped with the tools to do some very basic calculations in human anatomy, figure out the kinds of loadings that can be applied in both static or dynamic cases. And calculate the torques acting on the system.