Angular Kinetics - Part 3

So, now that we have looked at a couple of tools for analyzing a dynamic system, let us take a look at an example in real life applications of biomechanics. So we will be defining a problem in two dimensions, so planar movement. And as you recall, this will be movement in the sagittal plane. So the question is, or the problem is, determine the joint reaction forces, the forces that are acting at the individual joints and the moments at the ankle, knee and hip joint, okay, for a situation. The situation is swing phase of walking. So swing phase of walking defines the duration when you have your trailing foot, when you are walking, your trailing foot leaves off of the ground, and then you are in the process of resetting it, bringing it to the front to take the next step.

So the ground reaction forces are zero. And there are a couple of data given, which is the kinematics that has been measured for this particular scenario. So the kinematics are the ankle joint center is located at this particular location, knee here and the hip is at this location. Now to be able to do any sort of computation, you must know all the other details like the mass, the mass moment of inertia, the accelerations of the segment and the angular accelerations of the segments along with the center of mass locations.

So for the foot, for the lower leg and for the thigh segment, these three data are, these data are given. So right now it is a very text problem. Let's convert it into some graphics. Let's try and visualize this. Let's say this is the leg of the person.

This is the, this here is the foot, the lower leg, the knee joint and the thigh, and this is the hip joint. So all I've done is put the individual coordinates that were given in the previous slide on here. And if you'll notice, I've set up a coordinate system. So the measurements, and I'll go back to the previous slide, the measurements are given in meters. So there is this X and Y coordinate axis.

This is a 2D movement. And the person's leading leg is probably somewhere in this vicinity over here. Let's say this is ground level 0, this is (0, 0), and this is right after takeoff, right after the foot has lifted off the ground. So in addition to the joint center locations that have been provided, we also have the center of mass locations. So this is the center of mass location.

This is the center of mass location of the thigh segment. This is the center of mass location of the shank segment or the lower leg. And this here is the center of mass of the foot. Now, why are these defined independently and not just one single center of mass for the entire foot segment? Because we are treating each segment as its own rigid body, which means it, every part of this body has the same properties. It is rigid, there is no deformation, right? So there is a center of mass associated with that and it's more or less linear.

So we are not considering this dimension here right now, but it appears in the moment of inertia calculations. So that data is already provided to you. How do I go about solving a problem like this? And what exactly is the problem that I have to solve? So when I was walking and now my trailing foot is in the air, at each joint, there are certain forces that are acting. So when you say kick in the air, you will feel a certain snap at your knee, a certain force at your hip, to the joint reaction forces. So I want to identify what is the force acting at these points.

Additionally, I have observed that these segments with the center of masses are moving with certain accelerations linearly, right? And have a rotational acceleration component as well. So it could be doing a complex movement, right? Something like this. And we have split it into its individual components. So there's a linear acceleration along the x-axis, there's a linear acceleration along the y-axis, and then there is a rotational component. So I'm interested in solving these joint forces and the joint moments.

Now you might ask why do this to begin with? So this has a lot of sports applications, clinical applications. And the reason is your joints are designed to handle certain amount of loads. So if you were to backpack and carry your body weight, your knee joint might give up halfway. Whereas if you were carrying lower weight, your knee joint might be able to sustain that for a longer hike. Same way for any sports activity, if you are performing a movement and it has a potential to injure your joint, that can be identified by doing calculations like this and calculating, identifying what is the moment, what is the force acting at the interface.

These are the forces that are being taken by the bones, by the cartilage, by the tendons. So how are these forces acting? Then they can be correlated with whether it's likely to cause injury or whether it is a safe movement. So let's take a look at how we go about solving this kind of problem. Now given that all of these are rigid bodies, what have we learned about rigid bodies that we must do first? We must draw a free-body diagram. So I will draw the free-body diagram of the foot first.

You might ask why am I starting with the foot. Why not with the other end? So let's just follow along with this calculation and then we'll be able to make some decisions and make some sort of intuitive understanding about the flow of forces and torques. And this does not just apply to human body. It could be robotics, it could be exoskeletons, whatever area you're working in, you want to apply these principles and it'll be applicable there as well. All right, so here is this segment representing the foot and because I'm only concerned about the two-dimensional nature of the foot, the depth is not present here. So if I was to consider this foot in isolation, this is the center of mass of the foot, which means there is a weight acting down.

This is the weight of the foot that is acting down. Now the foot was attached here to the shank. The foot was attached here, the foot was attached here to the shank, to the lower leg. So there is some force interaction between the two segments. So we will assume that those reactions are positive.

Whenever something is unknown, just assume it in the positive coordinates, and then whatever sign comes out will naturally align with the sign convention. The other thing is there is some rotation of the foot happening. So if we go back, the foot is rotating with an angular acceleration of 5.12 radians/s^2. So there is a moment acting on it, which is also unknown.

We'll call this and I will just quickly change this nomenclature. We'll call this ankle x and y to align with this sort of a nomenclature. So what happens next? We apply the Newton-Euler equations or so

$$\sum F_x = m_f a_x$$

So what are the, all the forces acting on this rigid body in the x-axis? It's only A_x. So I can say

$$A_x = m_f a_f = -5.3N$$

So these values I've already pulled in and I've done the computation, but you can repeat it for a sanity check on your end. So I have the x-axis force. What about the y-axis force? So let us do the computation in the y-axis. A similar equation will apply,

$$\sum F_y = m_f a_y$$

Yes. So what is, what are all the forces acting in the y-axis?

$$A_y - m_f g = m_f a_y$$

And if you do this calculation, you will get something like

$$A_y = 19.9N$$

Now these values don't seem a whole lot big, but regardless, that's the reaction force. So what this means is there is a positive reaction force A_y. So A_y, which was assumed to be positive came out to be 19.9 Newtons.

So there is a force of 19.9 Newtons acting in the positive y direction, whereas the force acting in the x direction is in the negative direction actually. So that is -5.3 acting in this direction actually. So either a force of 5.3 Newtons acting in this direction, or you can say a -5.3 Newton force acting in this direction in the positive x-axis. All right. So we have done force in the x direction, force in the y direction. Now the third is the moment. And for that, we will apply the moment balance equation, which is the

$$\sum M = I_f \alpha_f$$

Now this is written about the center of mass. So what will this be? And I want to spend a little bit of time here. So the moment exerted here, there will be one moment because of A_y. Let me just change to the highlighter.

So there is one moment from A_y. Let's try and see what that is. So this is the segment. And there's a force acting in this direction. So let's see the force acting in this direction like this at this instance. So it's trying to rotate this in the clockwise direction.

So the moments in that are counterclockwise are assumed positive as because of the sign convention where the moments about the z axis, z axis is coming out of the plane. So the moments about this in the counterclockwise direction, and that's the positive direction. So the moments acting from A_y about the center of mass, so the center of mass is here for the segment and I'm applying a force and that means it is a negative moment. So you can calculate the distance between these two points in both these axis.

So in the y direction, it will be 0.189 minus 0.117. So this distance and this distance is what we'll need to calculate.

So this distance is about 0.373 minus 0.303, which is 0.07. So this is about 0.07 meters and in the other direction, it is 0.072 meters. All distances are being measured in meters. So what is the moment applied by this force?

$$-A_y * 0.07 - A_x * 0.07 + M_A = 0.011 * 5.12$$

Now notice that I have not put in the actual value of A_y and A_x that I have already calculated. So this convention here still holds. So I have done this assuming it was a positive force.

So I could have written these equations in the very beginning itself, before even solving each of these. So let's say if I plug the value of

$$A_y = 19.9N$$

And

$$A_x = 5.3N$$

and I solve for this equation, then I get a value of

$$M_A = 1.01Nm$$

Pretty small amount of torque, but it's still there. All right, so I was able to analyze this entire foot segment independently. And I was able to calculate all the values, all the unknowns from the unknown reaction force in the x direction, the reaction force in the y direction, and the moment acting on the ankle. Now all of this is possible because we know the kinematics. So before you do the dynamics, I will emphasize this point, reiterate this point again.

You must know the kinematics of the system. No matter what you are solving, be it biomechanics, be it movement of any complex multi-body system or single-body system, you must know the kinematics. Okay, so let's see. We will now move on to the lower leg. And you might ask me, oh, why the lower leg? Why not the thigh? So again, just follow along and you will gradually realize that there is a pattern to this madness.

Okay. So I have my ankle joint here, my knee joint here, right, and the center of mass of the segment here. Okay. Now, if I was to just say there is an unknown force acting here, there is the weight of the body, right? So that is $M_L g_L$. And the knee joint is where the lower leg attaches with the thigh segment.

So you will have a unknown reaction force here. Great. Now, you might ask, oh, but what about this? Right? Did not we already calculate this? To which I would say yes, we calculated something related to this parameter. Let us take a look at it. So let us first, let me just try to build some intuition here. Let us say you are writing the force balance equations and you are writing the equation in the x-axis, right? So you would have one unknown force, another unknown force, if you started from this segment, right, is equals to the mass times acceleration in the x direction. Again, one unknown force in the y direction, another force in the y direction minus the weight of the body is equals to mass times the acceleration in the y direction.

Okay. So to be able to calculate these, I need to have calculated these, right? Otherwise I will not be able to solve the force equation. Okay. So what is this value? So

$$A_{sx} = -A_x$$

$$A_{sy} = -A_y$$

Now, A_x and A_y are the values we calculated for the foot segment.

So remember Newton's third law, every action, the force acting from the foot on the shank is, has an equal and opposite reaction, which is equal to the force acting from the shank or the lower leg on the foot and opposite, right? That is what this negative sign signifies. Okay. So we will quickly erase this and say that because I have already done the calculation, this is A_y and this is A_x. And now we can get rid of this. So now I can write the force balance in the x-axis,

$$F_x = K_x - A_x = m_L a_x$$

So if I plug in the values, now, if you remember, A_x was what? A_x was minus 5.3 Newtons, right?

$$F_x = K_x - (-5.3) = 2.4 * -4.01$$

$$K_x = -14.9N$$

Okay. Same way, if I do the calculation for the y-axis, then

$$\sum F_y = K_y - A_y - m_L g_L = m_L a_y$$

$$A_y = 19.9N$$

So if I plug this value in, if I plug in the constants that were given earlier, the values that were provided earlier, I will get a value of

$$F_y = 50N$$

And same way as I wrote the moment balance equation for the foot, we will write the moment balance equation,

$$\sum M = I_L \alpha_L$$

And the moment of these, so let's mark these distances.

This is 0.420, so 0.1 meters. This distance here is 0.539 minus 0.437, so 0.102 meters. Okay. Same way, this distance here is 0.437 minus 0.303, which is 0.134 meters. And this distance here is 0.131 meters. Okay. So, moment is a vector quantity, right? So we'll have to write the signs of these values, right? So I will write the equations about the center of mass again. So notice that I have not put the joint moments on here. Okay. So before I do this, I must rectify that problem.

Now I don't know the joint moment that is acting at the knee. So I will assume a counterclockwise positive unknown knee moment, M_K, right? Also there is a moment acting at the ankle. So there was a moment acting at the ankle from the shank, right? With shank as reference, the moment was 1.01 Newton meters. Now an equal and opposite moment will be acting on this segment at this point. So on this segment, the direction of moment that is acting will be opposite to the direction of the moment acting on the ankle.

So now I have all the information to write the left-hand side of this equation. So M_K is unknown, which is easy, right? M_A is assumed clockwise. So

$$M_k - M_A + (K_y * 0.102 - K_x * 0.1) + (A_y * 0.134 - A_x * 0.131) = I_L \alpha_L$$

Okay? And if we do this calculation, you plug in the value of M_A as 1.01, okay? So we substitute these values, 1.01, K_y, K_x, A_y and A_x, you substitute with their signs as you have calculated from the previous equations and you will get a

$$M_k = 9.1Nm$$

Now we have been able to calculate everything about this joint because I knew everything, all the forces and moments at this end of this rigid body. Now I can repeat the same thing for this thigh segment as well, but I will, but you should pause the video here and do the calculations yourself before you look at the answer. So I'll just provide you with the answers for this segment.

So H_y, H_x and M_h, right? So you should have gotten H_x as 24.6 Newtons, H_y as 101.6 Newtons and M_h as minus 11.7 Newton meters. So with this, we have now looked at the application of all the tools that we have learned in linear kinetics and angular kinetics and we are well equipped now to extend these principles to any problem statement that we give. For the sake of this course at least, all problems have been defined in two dimensions here, right? You can see this itself takes a lot of time.

So the same principles can be extended to three dimensions and you can actually practice problems for three dimensions. Of course, that's beyond the scope of this course. So we will conclude this section here and I will see you in the next one.