

# Energy Resources, Economics, and Sustainability

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Week – 02

Lecture – 05

## Lecture 10 - Fossil Fuel Resources-II

Hello everyone, welcome back to the course Energy Resources, Economics and Sustainable Sustainability. In the last lecture, we have been discussing about the depletion of the fossil fuel resources, how fast or how slowly that can happen. We have studied two different approaches, one was the static R by P ratio and the other was an exponential rise of the consumption of or the production of the fossil fuels. Both of them give us extreme cases which are unlikely to happen. What would likely to happen is that we would have an exponential increase in the production of resources, then the resource production is going to be stagnant for a few years and then this might be accompanied by an exponential decay. And let us try to understand this particular aspect in this class.

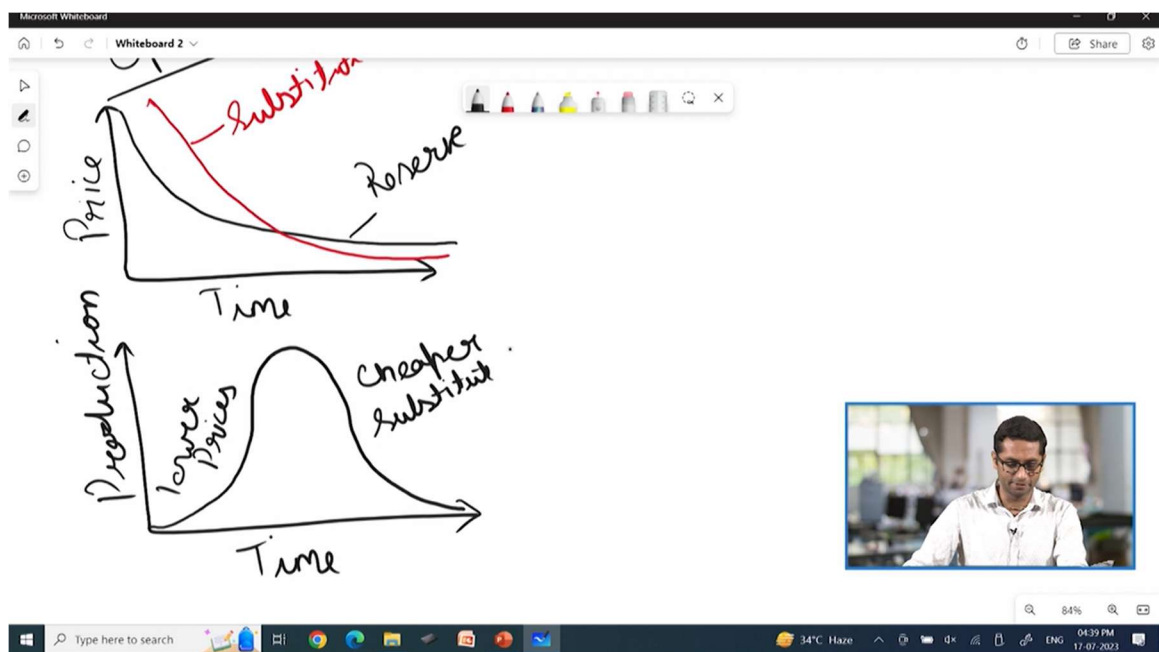
### Life Cycle of fossil fuels

- All mineral resources, including fossil fuels, are finite.
- In the last three centuries, the rate of consumption of the fossil fuel resources by far outpaced their rate of formation.
- It is apparent that if we continue mining and using fossil fuels at their current levels, these resources will be exhausted in the future.
- The questions “when will the fossil fuels be depleted?” or “when will the petroleum be depleted?” have very high political, social, and economic significance and may only be answered by detailed mathematical models that include all the factors, which influence the global demand of the several types of fossil fuels.



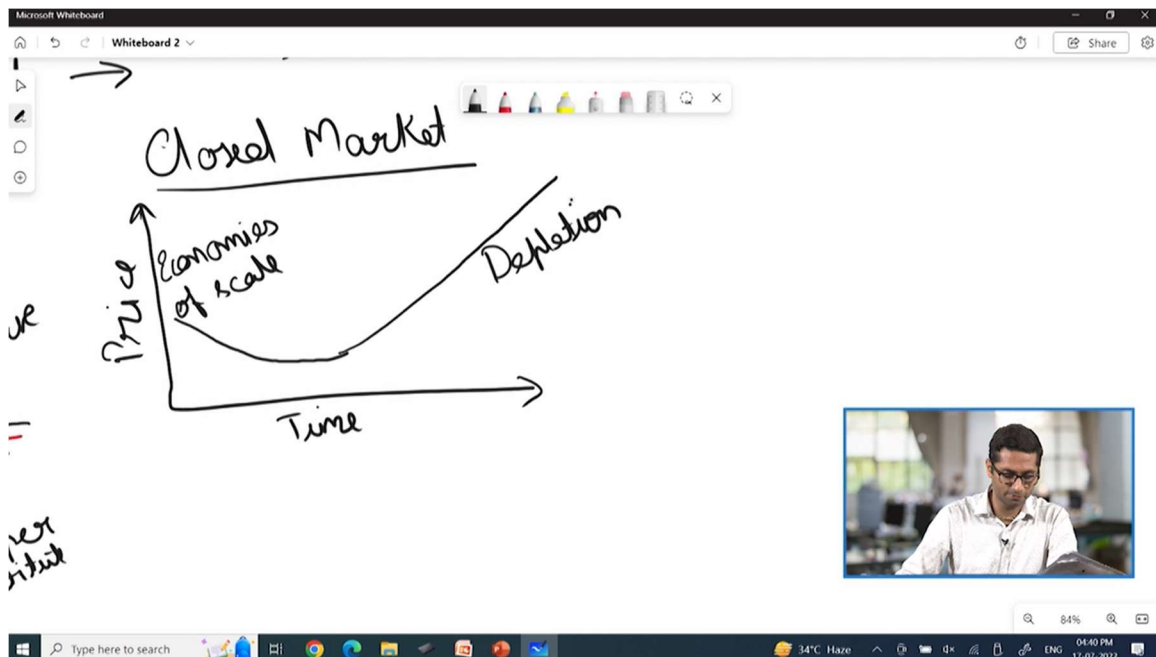
So if I talk about the life cycle of fossil fuels, we are well aware that fossil fuels as resources are finite. In the past few centuries, the consumption rate has by far outpaced the production. So we have been increasing the consumption at an exponential rate. And it is apparent that someday it is going to happen, but like the prediction of when that is going to happen has not been very good in the past few years or so.

So, for example, like M. King Hubbert have been able to predict the oil peak for the US and that came out to be pretty accurate. But if we see the situation today, the production in the US even after peaking in the 1970s has increased beyond that. So the phenomena of the production of consumption of the global resources in terms of fossil fuels is quite complicated. So let's go back to the whiteboard and try to make a model that make use of some of these relationships and try to see that if we can come up with a simple model to enhance our understanding.

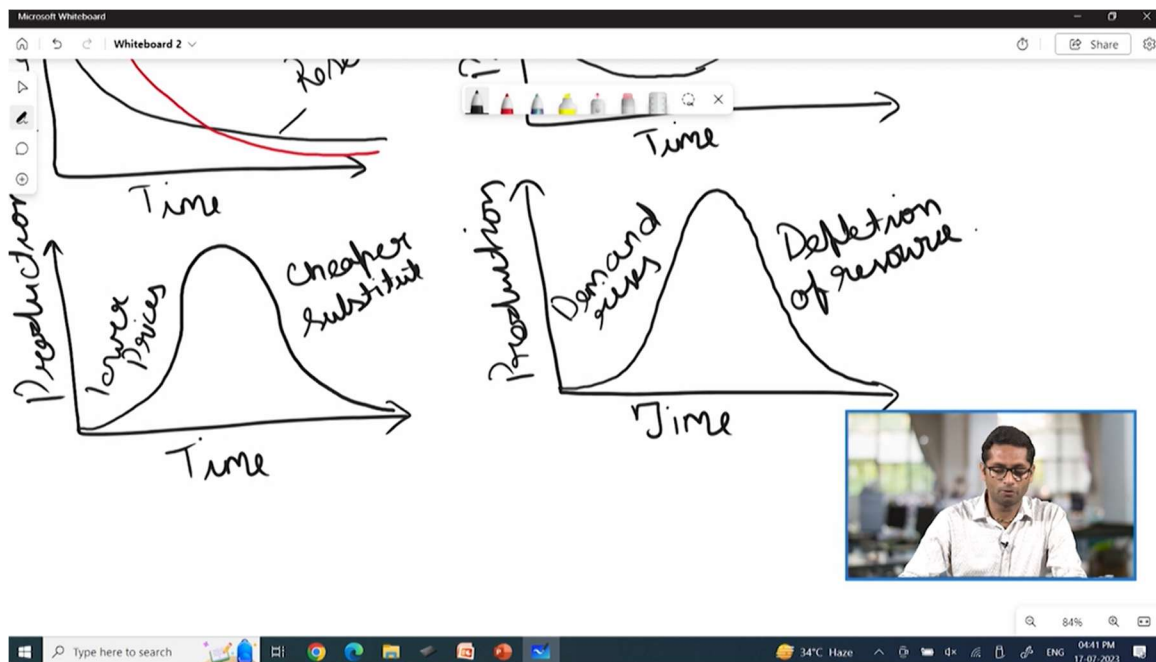


So what has been predicted in the past is like depending upon whether we are going for an open market or the closed market, the production rate of finite resource is going to follow almost a similar trend. So let us try to understand that. So let me start in the case of an open market. So in the case of an open market, if I have a graph like this where the y-axis gives me the price for resource and on the x-axis I have the time frame. This is how the price of a resource is going to change in the future.

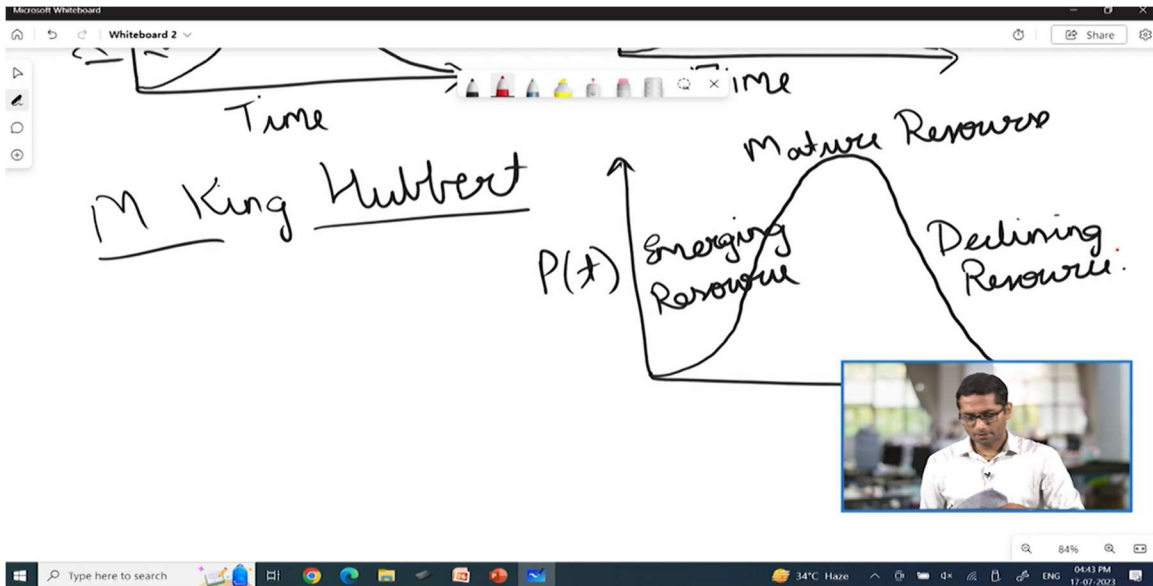
There is going to be a decrease and then stagnate. So the price is reducing as more and more resources getting produced or getting consumed. And something else happens in open market that there would be substitutes available and slowly with time as one resource is getting used and the substitute might become much more economical and might take up a good amount of market share. So this is what I am talking about the substitute. And sorry for that. And by this graph I am taking over the reserve of fossil fuel. And if I see the production rate that is coupled with this kind of production rate, so again I have the production rate on the y-axis and time on the x-axis. So we would have an increase in the production of a resource. It peaks and then as the substitute take over there is a decline. So the production remains constant at a place which is also called the peak oil or peak coal and as the substitute takes over in terms of the price we would have a decline in the consumption of that particular resource. It does not mean that the particular resource will get exhausted but its price is going to stay stagnant or might increase slightly because lesser amount of resource is available in the market. So it is basically the lower prices that would generate the exponential growth but we will also have eventually a cheaper substitute which will bring down the consumption of a resource. So this was with respect to an open market. Something similar could also or like phenomena could be seen in the closed market as well but of course for different reasons.



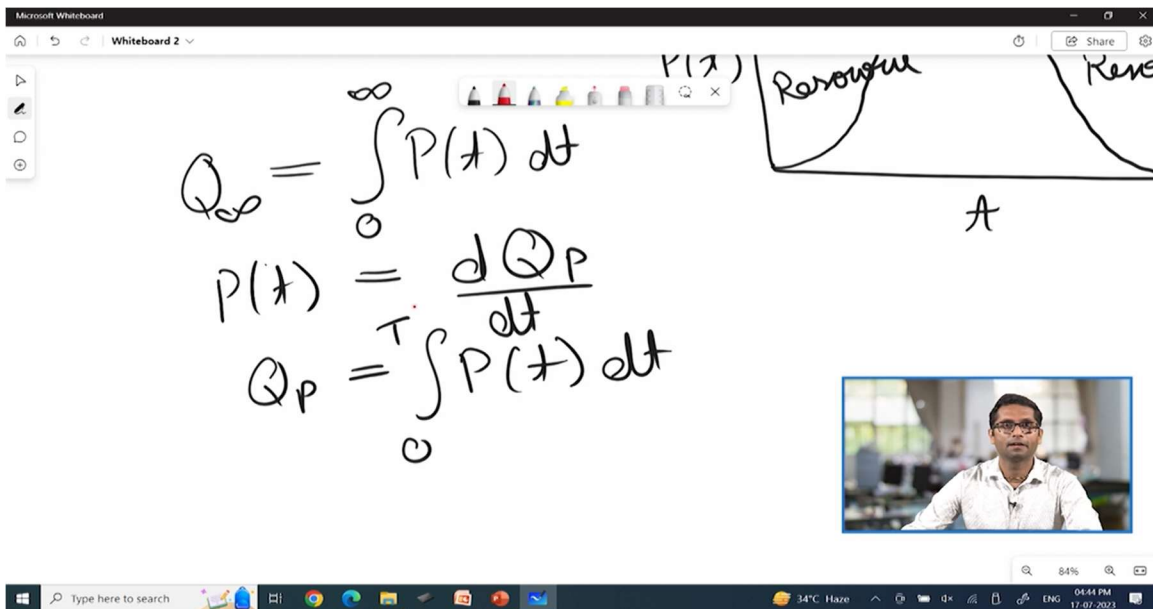
So if I am talking about a closed market what is going to happen is if we consider the same graph on the y-axis I have the price and on the x-axis I have the time. So there would be a decrease in the price of the resource. It becomes to a minimum and then there is going to be an increase. So first we would have the economies of scale. We would have bigger and bigger plants or refineries that would basically make a particular resource cheaper but as the depletion starts to kick in people would want to say this is all for the future when it could fetch a higher price. So that is why the price would increase and even in this case if I consider the total production versus time.



The production in here and we would have the time on the x-axis. We would have a graph like this where the peaking is going to occur and here we have an increase in the production because the demand rises and then we have the depletion of the resource leading to a decay in the production amount. So irrespective of whether we are going for an open market or a closed market this is what the prediction of the different people have been like this is the likely trend that is going to be followed for a finite resource. So let us try to understand if something similar was to be followed by a resource like coal in India like how do we make an equation for it and how do we do a prediction for the same.



So this kind of curves were initially predicted by M King Hubbert and he was a geologist with the geological survey of the US and it says in his own order that these kinds of curves were also called as Hubbert's curve and so the basic assumption and that lies in this is that we would have a curve and on the y-axis we would have the production that is taking place the time on the x-axis and the production would increase reach a maxima and come down. So when I am in the increasing phase the resource is called as the emerging resource on the top it is called the mature resource and finally it is called a declining resource. So let us try to derive an equation for this consumption.



So for continuing this we also need an estimate of the total amount of a reserve or the resource that is available. So for this calculation let us try to make a guess that this number is known to us and that goes by  $Q_{\infty}$  which is the total amount of the resource that is available and that would basically be integration of the production for a particular year and integrating that from time 0 till whatever is known to us till almost infinity. And if I say the production at an year for that case would be  $dQ_p$  and  $dt$ , by  $Q_p$  I mean the cumulative production till that particular year. So  $Q_p$  is basically production integrated from the time  $t$  is equal to 0 till a time  $t$  which is present. So let me introduce you three terms one is  $Q_{\infty}$  which is the total amount of reserves that is available.  $P_t$  is basically the production for a particular year and  $Q_p$  is the cumulative amount of production till that particular year.

The screenshot shows a Microsoft Whiteboard interface with the following content:

- Handwritten equations:
 
$$\frac{dQ_p}{dt} \propto Q_p$$

$$\frac{dQ_p}{dt} \propto (Q_{\infty} - Q_p)$$

$$\frac{dQ_p}{dt} = B Q_p (Q_{\infty} - Q_p)$$
- A video feed in the bottom right corner showing a man with glasses and a white shirt.
- Whiteboard toolbar with various drawing tools.
- Windows taskbar at the bottom showing the search bar, task icons, system tray (34°C, 84% battery), and date/time (04:46 PM, 17-07-2023).

Now the rate of change of  $Q_p$  with respect to time which is  $dQ_p$  with respect to time would be proportional to the amount of resource that we have already consumed. And this rate of change of  $Q_p$  with respect to time would also be proportional to the amount of resource that is left in the earth's crust which is  $Q_{\infty} - Q_p$ . Let me repeat again now the  $Q_p$  which is the cumulative amount of production and the rate of change of that cumulative amount would be related to the amount of  $Q_p$  till that particular year and it would also be proportional to the amount of the all reserves that is left in the earth's crust. So of course it would like if the consumption has been increasing there would be an increase further if the amount that is left in the earth's crust tends to be reducing there

would be a reduction in that as well. So if I add both the terms the rate of change of  $Q_p$  with respect to time could be given by the equation  $B$  is a constant in here I am multiplying with  $Q_p$  and  $Q_{\infty} - Q_p$ . So this is basically dependent upon what the consumption has been in the past and how much oil or a particular resource is left in the earth's crust.

Microsoft Whiteboard

Whiteboard 2

$$\frac{dQ_p}{Q_p(Q_{\infty} - Q_p)} = B dt$$

$$Q_p = \frac{Q_{\infty}}{1 + A e^{-B Q_{\infty} T}}$$

S shaped Curve  
Logistic curve  
Pearl curve:

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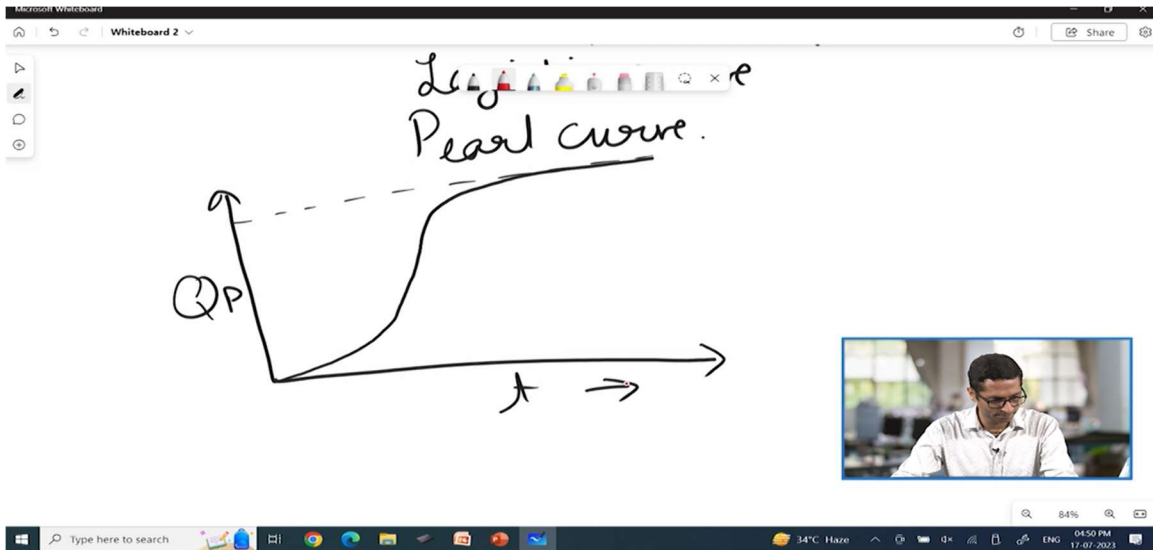
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So this is a simple equation in which I can try to derive a solution. So I can have  $Q_p$  so we can bring some terms from the right hand side to the left hand side denominator which would be basically  $Q_p$  multiplied by  $Q_{\infty} - Q_p$  and what remains on the right hand side is the constant  $B$  multiplied with the change in time and I could do the simple integration and the solution that I would be getting would be  $Q_p$  is equal to  $Q_{\infty}$  which is total amount of reserves  $1 + A$  which is another constant  $e$  raise to power  $-B Q_{\infty} T$ . So this kind of relationship could be derived this which basically gives me the logistic curves or which is giving us the peak concentration or the peak consumption of the different oil resources and it depends upon two constants which could be empirical derived based upon the past data which are  $A$  and  $B$ .  $Q_{\infty}$  is again we have to make an assumption with respect to the reserves that are available to us and  $T$  is basically the time and there are different names to this particular relationship one is the S shaped curve or there could be another name which is called the logistic curve. People also refer to this as the pearl curve and this particular relationship has been used in the scientific community for different kinds of predictions as well. So one particular application could

be the inclusion of any new technology into the market sometimes follow the S shaped curve and this might not be the exact relationship there are variation to this logistic curve there could be like cos h or different kinds of elements that might be brought into the relationship but the typical shape of this curve remains more or less the same.



So if I look towards the cumulative production or the consumption of a resource using this relationship this would go as like there would be a picking up of the consumption then there is an exponential rise and then it reduces asymptotically to a max and this is how the QP would look like and this is the time in here and this is because of the S being formed in here that it is also called the S shaped curve.

$$QP = \frac{Q_{\infty}}{1 + A e^{-B Q_{\infty} t}}$$

$$1 + A e^{-B Q_{\infty} t} = \frac{Q_{\infty}}{QP}$$





And finally the relationship again let me put it down it is  $Q_P$  is equal to  $Q_\infty$   $1 - A e^{-B Q_\infty T}$  so as I told like  $Q_\infty$  would basically be coming by the different reports of a particular country so if you would want to look towards the reserves of the coal you can use it the government of India websites and you will get the updated numbers for that and for the numbers  $A$  and  $B$  let me try to help you out in this respect so we can rearrange the terms in this equation and they can go as  $1 - A e^{-B Q_\infty T}$  and this would be equal to  $Q_\infty$  by  $Q_P$  production I can further rearrange the terms wherein I can take the  $Q_\infty$  by  $Q_P$  production on the left hand side and minus 1 and this would be equal to  $A e^{-B Q_\infty T}$  I can take log on both the sides so  $\log(Q_\infty / Q_P - 1)$  equal to  $\log A - B Q_\infty T$  I can take  $\log$  on both the sides so  $\log(Q_\infty / Q_P - 1)$  equal to  $\log A - B Q_\infty T$  and if I see like how this equation looks like this can be applied to linear regression where I can have the term  $Y$  as in here and then this could be  $C_1$  which is which is and the like extract on the  $Y$  axis plus  $C_2$  which is a slope multiplied by time so given the past data I can very well come up with the constants  $B$  and  $\log A$  given that we would have the quantity of the reserves that is available is known to us and the production capacity in the past known to us I can easily derive the values of this constants  $A$  and  $B$  and come up with the dates when the oil or the coal or natural gas is going to peak for a particular country or not.

Microsoft Whiteboard

Whiteboard 2

$\log\left(\frac{Q_\infty}{Q_P} - 1\right) = \log A - B Q_\infty T$

$B Q_\infty T = \log A - \log\left(\frac{Q_\infty}{Q_P} - 1\right)$

$Y = C_1 + C_2 T$

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$$Y = C_1 + C_2 t$$

$$P = \frac{dQ_P}{dt} \quad \frac{dP}{dt} = 0$$

$$\frac{d^2 Q_P}{dt^2} = 0$$

Further like this gives me the total how the production is going to change over the years but when it is going to peak I can again use simple like maths to do that so if you remember earlier like the production for a particular year was the differential of QP by DT and a differential of DP by DT and equating that to 0 would give me the time at which the production is getting to a maximum or for short what I would have to do is a double differential of QP by DT square I need to equate that to 0. So I know what QP is so the production at any year would be B into QP Q infinity minus QP so taking a differential of this equation which would be DP by DT and equating that to 0 should give us B and DQP by DT and this should be Q infinity minus QP and then I also need another term which would be B QP and minus of DQP by DT. We know that the constant B is not equal to 0 and also we are aware that the rate of change of the production capacity cumulative production capacity with time is also not equal to 0.

$$\frac{d^2 Q_P}{dt^2} = 0$$

$$P = B Q_P (Q_\infty - Q_P)$$

$$\frac{dP}{dt} = 0 = B \frac{dQ_P}{dt} (Q_\infty - Q_P) + B Q_P \left( -\frac{dQ_P}{dt} \right)$$

So you can divide the above equation by these two terms and what we get is a very simple relationship which is  $Q_{\infty} - Q_P - Q_P = 0$  and  $Q_P$  is equal to  $Q_{\infty} / 2$  so this basically gives us that the peaking is going to occur when we are exactly at the half of the total reserves of a particular resource.

Handwritten equations on the whiteboard:

$$\frac{dQ}{dt} = \frac{A}{1+BQ} - Q$$

$$B \neq 0, \frac{dQ}{dt} = 0$$

$$Q_{\infty} - Q_P - Q_P = 0$$

$$Q_P = \frac{Q_{\infty}}{2}$$

So this gives like this curve is symmetric around the center which means the peaking is going to occur just at the point when the reserves are equal to half. And how do I get the time of peaking so I can replace and this value of  $Q_P$  in the earlier equation which would be  $Q_{\infty} / 2$  is equal to  $Q_{\infty} / (1 + Ae^{-BQ_{\infty}t})$ . So these two terms cancel out each other and we have  $2 = 1 + Ae^{-BQ_{\infty}t}$  or  $1 = Ae^{-BQ_{\infty}t}$  and the  $t$  here where I am referring to is the peaking time of  $t_m$ . So the value of the time at which the peaking is going to occur would be if I take a log of  $A$  divide that with  $BQ_{\infty}$ .

Handwritten equations on the whiteboard:

$$\frac{Q_{\infty}}{2} = \frac{Q_{\infty}}{1 + Ae^{-BQ_{\infty}t}}$$

$$2 = 1 + Ae^{-BQ_{\infty}t}$$

$$1 = Ae^{-BQ_{\infty}t}$$

$$t_m = \frac{\log A}{BQ_{\infty}}$$

So the time of peaking is basically a function of A, B and the total reserves that we are having. So this is the time at which a resource is going to peak and the variables A and B we can determine from the past consumption of a particular resource. And finally we also would have the total amount of resource that it would be coming from a particular country. Using this we can calculate what is the time at which a particular resource would peak and in a similar fashion we can also use the above relationships to come up with like what is the time in which 90% of the resource would get utilized or 99% of the resource would get utilized. In this particular relationship it is not going to happen that the resource is going to be 100% used up because it is asymptotic towards the end.

But we can very well try to estimate like at what particular time this particular resource is going to get utilized. And this is one of the simplistic model for like for giving the predicting of the peak oil or peak coal and you could use a similar model for different countries. But as we are like in present world we also have much complex models where these terms have been replaced by much complex terms and one of the things or one of the simplicity in this model was like the decline is also symmetric to the increase. But in the real world that is most probably not the case and then there have been different models. One particular example would be the Gompert's model which have come up with other kind of relationships. The trend more or less remains the same but it becomes more and more complex. So let us also try to do a simple exercise where we try to estimate like using this particular model how the coal is going to peak in India.

The screenshot shows a Microsoft Whiteboard interface with the following content:

- Handwritten equation:  $Q_p = \frac{187105}{1 + 600e^{-0.06t}}$
- Handwritten note:  $t = 0$  is 1960
- A video inset in the bottom right corner showing a man with glasses speaking.
- Windows taskbar at the bottom showing the date 17-07-2023 and time 05:00 PM.

So let us try to have or come up with a simple model where the  $Q_p$  which is the cumulative production capacity for the case of India is given by a simple relationship which is 187105 which is basically the  $Q$  infinity. So I am picking up that as a static quantity and divide that with 600 which basically refers to the term  $B$  in the above equation. So if I go with the above equation sorry the term  $A$  so  $A$  and then  $e$  raise to power minus  $0.06t$  which is basically a multiplication of  $B$  into  $Q$  infinity. And for this relationship I am assuming that  $t$  is equal to 0 is the year 1960 and this is the place when the coal production in India picked up.

Microsoft Whiteboard

Whiteboard 2

$t = 0$  is 1960

$$t_{\text{peak}} = \frac{\log A}{B Q_{\infty}} = \frac{\log 600}{0.06} \approx 106.6 \text{ years}$$

$t = 0, 1960$   
 $t_{\text{peak}} = 2067$

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So if I would have to go for the time for the peaking of the coal or the  $t$  peaking this would be given by the relationship  $\log$  of  $A$  divided by  $B$  into  $Q$  infinity and if I use in the above logarithmic equation this would be nothing but a  $\log$  of 600 divide that with the multiplication of  $B$  into  $Q$  infinity which comes out to 0.06 and the answer for this particular case would be roughly around 106.6 years. So given that the start of this equation  $t$  is equal to 0 is considered as 1960s and the time for the peaking of the coal will be somewhere around 2067. So this is the time when the coal that is available in the country is likely to peak if we carry on with the consumption rate and this is again based on a simplistic model that we have just hypothesized. In reality they would be much more complex models but this is just for our understanding. Using the same equation we can also try to come up like what would be the time span when we have used 90% of the available coal.

Microsoft Whiteboard

Whiteboard 2

Share


$t = 0, 1960$

$A_{max} = 200,000$

$Q_p = 0.9 \times 187105 = \frac{187105}{1 + 600 e^{-0.06t}}$

$1 + 600 e^{-0.06 \times 90} = \frac{1}{0.9}$

$T_{90} = 143 \text{ yrs}$



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So in that case the P value would be or the Qp would be 0.9 into and the total production capacity that I assume is 187105 and this I am equating with the term 187105 which is the equation and I divide that with 1 plus 600 e raise to power 0.06 into t. So we would have these two terms cancel out each other and what remains is 1 plus 600 e raise to power 0.06 and this is basically t 90 when 90% of this particular resource reserve has been utilized and what we have on the RHS 1 divided by 0.9 I can do this calculation and the time for the t 90 would come around to be around 143 years and if 1960 was the start year this t 90 is going to happen around 2103 AD.


Microsoft Whiteboard

Whiteboard 2

Share

$T_{90} = 2103 \text{ A.D.}$

Stadium R/P (209 yrs)  $\rightarrow$  Logistic Curve (-80 yrs)  $\rightarrow$  Exponential Rise (+50 yrs)



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So this is a much more realistic time frame when we are going to run out of all the coal reserves that are available in the country roughly 80 years from now. And we also understand like this prediction that we get is somewhere around the middle of the earlier predictions. So if I go with the static R by P ratio this gave me a prediction of around 209 years from now. Then we had the logistic curve which we have done in the present class and this gives us like from today it will be roughly 80 years and this of course is greater than the exponential rise which gave us our prediction of roughly 50 years or 49 years. So with this we have tried to understand three different types of models that could be used for the prediction of the use of a particular reserve. We have taken a simple case of the coal reserves in India and given up on the different scenarios how fast or how slowly they are going to get consumed and if we are taking about the case of logistic curve when it is probable that the coal consumption in India is going to peak. So let us go back to our discussion in the slides.

## Some influencing factors

- The total amount of the resource
- Technological advances
- Environmental regulations
- Political and military events
- Substitution effects
- The state of global trade

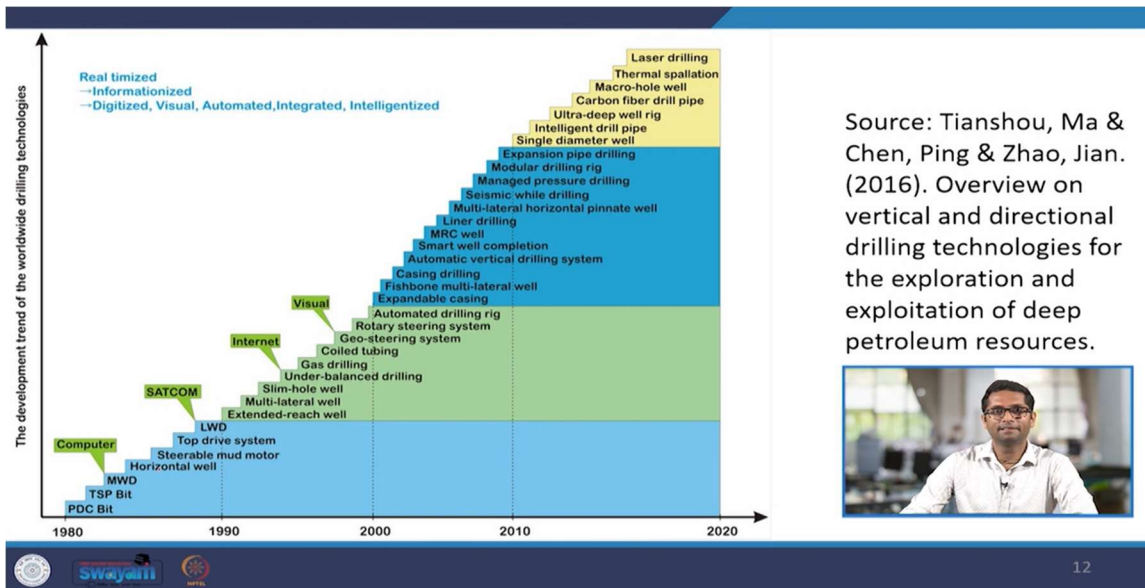


So we have been discussing the life cycle of the fossil fuels and we have been discussing like the most appropriate would be the logistic curve but there are few other influencing factors which are absent in this particular relationship and they being like the technological advances. If there are some breakthrough technological advances that would convert a lot of resources into reserves or the  $Q$  infinity increases with time the logistic curve doesn't take that into account and that is something that has been the case for the past many years. Further the environmental regulations so as we move in the future there have been more and more treaties or the protocols that have been signed by

different countries which basically aims to put a cap on their CO<sub>2</sub> emissions and indirectly a cap on the type of resources that they would want to use.

So in short they are trying to cap the types of resources and amount of those resources that they would want to use in the future and this affects how the resources are going to be available in the future. Even in case there are lot of resources available in the future but there is a cap that you cannot use those because of the CO<sub>2</sub> emissions possibly the mathematical relationship that we have derived fail at this point of amount of time. Further there could be political and military events that happen in different parts of the world that would affect the geopolitics and the relationship between the countries there have been treaties that would be forming because of the relationship and it would affect the particular resource in a particular country. You can take the example of the US so we have taken how M King Hubbert when he discovered the logistic curve with fairly simple tool he was able to predict the oil peak in the US in the 1970s and that was much appreciated and we also saw that there was an oil reduction after that but comes the year 2010 or so there have been a drastic increase in the oil production again. And what has been attributed to the reduction in the 1970s or the 80s could be the cheap oil that was available from the Middle Eastern countries and countries like Venezuela. So the companies in the US found it to be more profitable to import the oil than go for domestic production. So there are like political and military scenarios or the profit scenarios or that are available that might affect the use of this particular resource. Further there could be substitution effects we would have other sources of energy that would become much more economical. A typical case could be like much of the developed world being going for natural gas based power as compared to coal based power and one of the reasons that is attributed is like natural gas is more economical to use and much more efficient to use as compared to coal. So there is a natural substitution that is occurring and further there is a state of global trade that keeps on altering as we move in time. So there are different alliances that are formed or that get dissolved with time and that affects how the different sources of energy which are coal, oil and natural gas are traded. So these are some of the factors that affect the consumption and production of a particular finite resource like the fossil fuels and it is very difficult to take into account these kinds of resources in the models that we develop.





For this like just to give you an example of the different technologies that have been coming up so like since the last 40 years or so there have been significant improvement in the drilling technologies some of those are shown in front of you like just to give you an example would be the fracking for the shale oil and shale gas and also horizontal drilling that came up. So people have been coming up with new and new techniques that are converting resources into reserves. And also like we will end the class with another model that might give you some perspective.

## A new model

- The initial part of the production curve follows an exponential growth.
- The second part of the curve follows a linear growth. At this stage, the resource is characterized as a mature resource. Most of the fossil fuel resource is produced and consumed during this period of linear production growth.
- There are no empirical data on the trends and shape of the production curve when the energy resource becomes a depleting resource and production continuously decreases on a global scale. In the history of civilization, there has not been an energy resource that was thoroughly depleted on a global scale by human consumption. Because of the currently high rates of consumption, fossil fuels may become the first such energy resource that will be depleted on a global scale. Given the lack of data, the new model makes the assumption that the area under the depleting resource (the total amount of the energy consumed during this period) is equal to the area under the emerging resource curve.

Source: Michaelides, E.E. (2018). Energy, the Environment, and Sustainability (1st ed.). CRC Press. <https://doi.org/10.1201/b22169>

So this is another model that was proposed and it comes up like for the initial part we have an exponential growth then for the mature resource the resource increases slowly but linearly and finally we would have a decay for the resource. One of the reasons why we have not been able to come up with a peak prediction nicely is that have been like the world has never seen any major resource being consumed till the end till date. So there have been an exponential rise as well as a stagnation phase but we have not we are yet to witness a particular resource to get declined because of the human consumption and that is one of the reasons why people have been coming up with the different theories how the production and the consumption are going to change with time. So what this particular model does was like it has three different equations coming up or trying to predict the exponential increase and the stagnation phase as well as the decay phase which are given in front of you.

- For the time period of emerging resource

$$P(t) = \frac{P(T_1)}{\cosh^2\left(\frac{\omega_1(T_1-t)}{2}\right)}, \quad t \leq T_1$$

where  $\omega_1$  is the inverse of the timescale of the emerging resource and is obtained from the empirical production data during the emerging resource stage.

- For the time period of a mature resource, where the linear growth occurs,

$$P(t) = P(T_1) + \frac{dP}{dt}t, \quad T_1 \leq t \leq T_2$$

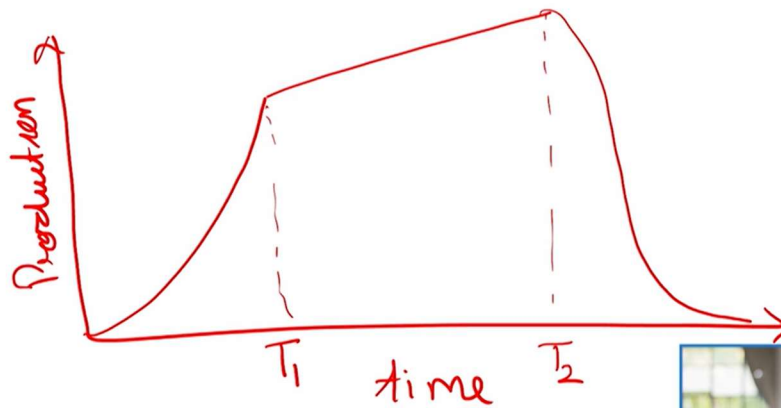
- For the time period of a depleting resource,

$$P(t) = \frac{P(T_2)}{\cosh^2\left(\frac{\omega_2(T_2-t)}{2}\right)}, \quad T_2 \leq t$$

Source: Michaelides, E.E. (2018). Energy, the Environment, and Sustainability (1st ed.). CRC Press. <https://doi.org/10.1201/b22169>



I would not go into the detail. Given the references if you are interested you can check those references. But what this model does essentially is like come up with a prediction. So it come up with a model which can go something like this. So we have the production on the y axis and of course we have the time phase on the x axis. So there would be an exponential rise and this is till a period t1 and from the period t1 till t2 there is a linear rise and then there is again a decay as the resource tends to decay.



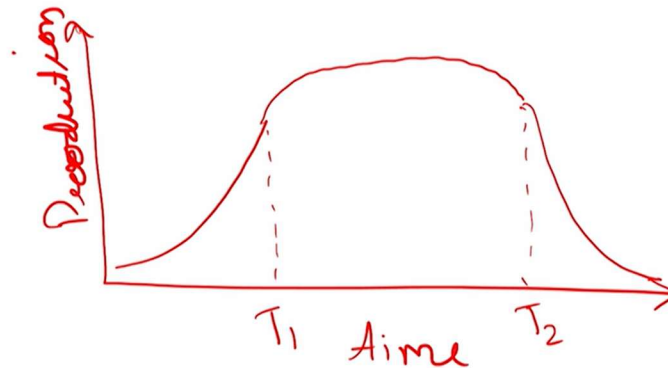
So this is another type of model that has been predicted to come up with a trajectory of the use of finite resource. Then another variation to this particular type of model could be a model that is symmetric around the center. So it also has the concept of P equal and would be given with a sort of quadratic equation like this which would have two real roots one positive and one negative.

$$\frac{1}{2} \left[ \frac{dP}{dt} \right] [T_2 - T_1]^2 + P_{T_1} [T_2 - T_1] + \frac{4P_{T_1}}{\omega_1} - Q_\infty = 0$$

$$\frac{1}{6} \left[ \frac{dP}{dt} \right] [T_2 - T_1]^2 + P_{T_1} [T_2 - T_1] + \left[ \frac{4P_{T_1}}{\omega_1} - Q_\infty \right] = 0$$



So essentially it would have just one answer and this kind of model would look something similar to this diagram that I am drawing in front of you. So we would have again the production and we would have the time.



So we would have the production increasing. So this happens up till the year  $t_1$  and then there is a quadratic equation and then we would have  $t_2$  and then again an exponential decay. So this is another type of prediction that is made for how a finite resource would end up being depleted. But nonetheless there have been shortcomings in all the models and so I like to end up this class with a quote by Edelman which is one of the pioneers in this field. He said that to know ultimate reserves we must have ultimate knowledge.

Nobody knows this and nobody should pretend to know. So again this is something that is very difficult to estimate but of course there have been different models to help us do this calculation but not all of them could be to be exactly right. So with this we end today's class. Thank you.