Energy Resources, Economics, and Sustainability

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Week – 08

Lecture – 03

Lecture 39 - LCA Computational Structure

Hello everyone. Welcome to the course Energy Resources, Economics and Sustainability. The past few classes we have been trying to understand the different basics of life cycle assessment. In the last class particularly we have been discussing the different impact categories, their sources, their likely midpoint and endpoint impacts, what are the different chemicals which are responsible for these kinds of emissions. Further, in today's class we will try to get a brief understanding of the computational structure of an LCA. So as we have tried to understand in the previous classes that LCA is basically an accounting feature which you can do simply with the help of a pen and paper. But that is true only when the number of emissions as well as number of individual process units in a complete process are quite small. What would happen if there are thousands of emissions as well as a couple of thousands of initial unit processes in a particular process. This entails a use of some mathematical structures which we will try to understand in today's class. So let us move to a white board. So for using different mathematical structures for LCA we first need to understand the concept of linear spaces. We will try to put in the different inputs and outputs from a unit process in the form of a single vector. So let us now try to understand this computational structure with the help of a simple example.



Let us consider a unit process which is for the production of electricity. So the output from this process could be electricity production. And this electricity production let it be 10 kilowatt hour or 10 units of electricity that is getting produced. For production of this electricity you would need some kind of fuel that would have to go in. So let us consider it to be a hypothetical fuel which is input at the rate of 2 liters. And further in this conversion process you can also expect some kind of emissions to happen. And again let us try to estimate these emissions could be that of CO2 and SO2. So let us assume that this process leads to production of 1 kg of CO2 and maybe 0.1 kg of SO2. So again this is a hypothetical process. So we have 2 liters of fuel going through some process which will lead to the production of electricity and in the process of doing so it is producing 1 kg of CO2 and 0.1 kg of SO2. So if I am talking about linear space I would have to define a vector in which I can define all these inputs and outputs. So let me define a vector such that it takes into account the liters of fuel. Then kilowatt hour of electricity, kgs of CO2 and kgs of SO2 which are the two major outputs. So let me define this in terms of a matrix and maybe I can denote this matrix with the name P and the values can go in here. The fuel that is getting used is 2 and since it is an input I would put a minus sign to it. Then the electricity that is produced is 10 units. So it would be 10. Then we have 1 kgs of CO2 and 0.1 kgs of SO2. So this could be one kind of notation for a process like this. Now let us also try to extend this. Let us let there be another process for which the output is the production of fuel which is used for electricity production in here.



So let us think of another hypothetical process again and let me define it here. So the output of this process would be the fuel production that we are later on using for production of electricity. Just a minute, yeah. So it is fuel and let us assume that this process would be producing fuel at the rate of 100 liters. The input to this particular process could be crude or the crude oil or crude fuel. So whatever the source, again this is a hypothetical process and let me assume the input is 50 liters. So again I am using a lesser input than the output just to show that like in this mathematical structure it does not

like it is not the concern with the maths whether it does make common sense or not. It is just doing mathematics and nothing else. The emissions from this process again let me assume it is emitting some amount of CO2. This could be 10 kgs of CO2 and maybe 2 kgs of SO2. And now I would like to use the same vector notation that I have used earlier for representing this fuel. But you would note that in the earlier notation I did not had the element of crude fuel whereas in this we have and I do not have an element of electricity here. So I would have to extend the vector that I have defined earlier to also include another element and so I would have to extend this and this would then include the liters of crude. And so I can represent the new process maybe by another vector which is P2 which goes in as. So the first element that I had was the liters of fuel so we would have 100 liters of fuel that is getting reduced in here. Then the second is the kilowatt of electricity so I do not have any electricity production in here. This goes to 0. The kgs of CO2 that you find in is 10 kgs of CO2. SO2 would be 2 kgs and then we have an addition in here that is of crude fuel that is a new element that we have added and that is an input at the rate of 50 liters. So we would have 50 liters as a negative. And in a similar way the earlier vector that I would have defined for the process P1 maybe denoted with P1 would have another element 0 here because this does not include any crude in here.



So now let us try to define these processes together with a combined notation a vector with a capital P which basically includes both the unit processes P1 and P2 and I can define this as a joint of the two vectors which would be -2, 10, 1, 0.10, 0 and then 100 which is the liters of fuel that is produced 0, 10, 2 and -50. So the two processes taken together could be denoted with the help of a combined matrix which is denoted by P the alphabet P and further this matrix can be broken down into two different matrices which is A and B. So what do I mean by A matrix? When I say about the A matrix it is basically

the economic matrix which consists of the economic flows. What are the economic flows? These are the flows that are coming from the human activities and in particular process or this process this would be the fuel which is denoted by the human intervention as well as the electricity that is denoted by the human intervention. What is the B matrix? B matrix is basically the environmental matrix which are the flows in and out of the environment which basically pertains to the CO2 emission, SO2 emissions as well as the crude input. So if I talk about this matrix the above one is the A matrix which is also called the economic matrix or the economic flows and the below matrix would be called the B matrix and this is what is called the environmental flows or the environmental matrix. So basically the combined matrix of the two processes could be given in this form. The above matrix is normally a square matrix and the below matrix could have any dimensions. It doesn't make much of a difference. So the above matrix is basically do with the flows input and output that are coming from the human intervention, the different processes and the B matrix particularly pertains to the different kind of emissions into the environment or the flows that we are taking directly from the environment. And particularly you can also see the rows basically pertain to the different kinds of products whereas the different columns would pertain to the different kinds of processes. So if I am talking about the rows this basically pertain to the products which you can see -2 litres of fuel getting consumed or 100 litres of fuel getting produced in here or 10 kilowatt hour of electricity produced or 0 kilowatt hours of electricity being consumed. And when I am talking about the columns this basically pertain to the different processes. So in this we have two unit processes. In the first process we would have the fuel being consumed leading to the production of electricity and in the second process we would have the crude fuel going in leading to the production of refined fuel. So this is with respect to the input. Now as we have understood in the previous classes that in an LCA I would want the results specific to a functional unit. Now I would want all the emissions to be normalised with respect to a functional unit. So in this case let us assume that I would want the emissions that pertain to a functional unit of 1000 kilowatt hour of electricity produced. So I would want the processes to be adjusted such that at the end I would have 1000 units or 1000 kilowatt hour of electricity produced. So my functional unit in terms of the matrix notation that I would have would be 0 and 1000. So it is just similar to the A matrix in which the first row basically refers to the production of fuel and the second refers to the electricity. So this is basically referring to fuel and we are not concerned about what the fuel is produced and what we are concerned about is 1000 unit of electricity needs to be produced. So in the matrix notation we would have 1000 kilowatt hour of electricity that is what my output should be. And corresponding to this output I would want to know the different emissions that could come in the form of SO2, CO2 and the crude that I am using. So these are the three basic emission that I have considered in this calculation and this emission could be notated with the help of another vector G which is giving us the three emissions G1, G2 and G3. These elements basically

pertain to CO2, NO2 and crude that would come up for the production of 1000 kilowatt hour of electricity. The combined matrix that I am more interested would be the Q matrix which is basically a combination of the F and G matrix. The F matrix basically pertains to the function unit which is 0 and 1000 and the G matrix refers to the three different emissions that I would want to know so that I can take these emissions to the different impact categories and calculate or evaluate the impact that is coming from this different kinds of emissions. The first thing is let us try to do this analytically. If I would want to produce 1000 units of electricity and if I consider this unit process possibly I would multiply this process by 100.



So P1 multiplies by 100 gives me 1000 units of electricity and for this 1000 units of kilowatt hours of electricity I would need 200 liters of fuel. So for 200 liters of fuel I would have to multiply the second unit process by 2 because it is already producing 100 units of fuel and I just multiply that with 2. So let me go here and P2 should be multiplied by 2 and this should be the two scaling factors. So if I multiply the first process by 100 and the second process by 2 I should be getting the functional unit of 1000 kilowatt hour of electricity that is what the aim is.



And further if I would want to quantify the total CO2 emission so we have 1 kg of CO2 from the process 1 multiply that with 100 plus I have 10 kg of CO2 that is coming from the process 2 and multiply with the scaling factor of 2 and this is what my total CO2 emissions are going to be 120 kg. If I talk about the SO2 emissions this would be in a similar fashion 14 kgs you can do the calculation and since the crude is just getting used up in the second process so the crude would be consumed at a rate of minus 100 liters because it was minus 50 liters getting produced and we have multiplied the process by 2. So far you would think that the processes the calculation can be done analytically using a pen and paper but think of when you have many unit processes in series or in parallel with recycles involved and you would have emission that could be maybe 1000 or more. In that case analytical solution might not be possible and that is where we would need to make use of different scaling factors and these scaling factors would basically be analogous to the factors of 100 and 2 that we have calculated. So what is the scaling factor?



So this is basically a matrix that consists of the scaling factors for the different economic flows or different processes that we would have to multiply it to get the final emissions. And how do we estimate that? So we are having an understanding that the matrix A which basically pertains to the economic flows multiplied by the scaling factor should yield to the functional unit which is 1000 units of electricity production.



Or I can say that we have the scale matrix in terms of A or if I write in terms of equation we can have the elements A11 multiplied by S1 plus A12 multiplied by the factor S2 gives us F. So in this the first case this would be -2 litres of fuel that is getting consumed multiplied by S1 plus 100 litres of fuel that is getting produced in the second process multiplied by S2 leading to a total of 0 because we do not want any fuel to be produced in the functional unit. Another equation that we can write is A21 S1 plus A22 S2 leading to the factor F2 which is F1 here. So this is basically 10 kilowatt hour of electricity produced multiplied by the scaling factor of 1 plus the second process does not produce or consume any electricity so it is 0 multiplied by S2 leading to a total of 1000. We solve these 2 equations and the scaling factor come out to be 102 that we have seen in the previous we have just done it analytically and we can also define this with the help of matrix.



Which goes in the A matrix which has the 4 elements and is a scale matrix multiply that with the scaling factor S1 and S2 and finally leading to the functional unit F1 and F2. For the current problem we would have just F1 is equal to 0 and F2 is equal to 1000 and the notation that we have put in is the A matrix multiplied by the scaling matrix leading to the functional unit. So this is the underlying maths that would be used for coming up for the functional unit and normally you would be aware of this A matrix and the functional unit. We would have to estimate the scaling factors and the scaling factors would in that case would be equal to an inverse of A matrix multiplied by the functional unit. So this is how you would come up with the scaling factors. So when you have just 2 processes you can calculate the scaling factor analytically but if you have many processes you would need this matrix algebra.



So as we have estimated in the previous example this A matrix is nothing but a scale matrix with -2, 10, 100 and 0. So basically the rows in this the first row basically signifies the fuel so in the first process we have -2 liters of fuel used and in the second process we have 100 liters of fuel getting produced whereas the second row basically is the electricity production. So in the first process we have 10 units of electricity getting produced and in the second process we do not have any electricity coming into picture. And you can just get an inverse of this matrix so if I do an inverse so I am not going into the calculation you can do it yourself. So the inverse of matrix A would be 0, 0.1, 0.01and 0.002 so the matrix need to be necessarily square if you would want an inverse to be there and if you would have to calculate the scaling factor this would be A inverse into the functional unit. So in this case we would have the same matrix 0, 0.1, 0.01 and 0.002 multiply that with the scaling factor oh sorry the functional unit which is 0 and 1000 you multiply the two and what you get is 100 and 2. So 100 was the factor that was used to multiply to the first unit process and 2 was the factor that was used to multiply the second process so as to arrive at the final output of 1000 kilowatt hour of electricity produced. So this is a very simple example portraying how would you calculate the different scaling factors so as to arrive at a particular functional unit. So once you have determined the scaling factor the end in still remains which is quantifying of the total emissions of CO2, SO2 and the crude that would be used.



And for that we would be using the notation which is the emissions which are given by the G matrix would be the multiplication of the B matrix into the scaling factor. So G was basically a different emission the total emissions from the complete process B was basically the environmental matrix which was giving in the individual emissions to the environment from the different processes and S is the scaling factor. So if I put in these values of this matrix so the B matrix is known to us is 1, 0.1 and 0 so the first process has 1 kg of CO2 emission 0.1 kg of SO2 emissions and 0 liters of fuel getting used. The second process had 10 kgs of CO2 emissions 2 kgs of SO2 emissions and it used – 50 liters of fuel. I multiply that with the scaling factor which is 100 and 2 and this multiplication would lead to the final emissions which comes out in the terms of 120 kgs of CO2, 14 kgs of SO2 and – 100 liters of the crude and this is something that we have also calculated analytically. So this is basically the CO2, the SO2 and the crude that is getting produced.



Further we can also simplify this methodology or this pathway by using the combined matrix where the total emissions G could be given by multiplication of B into A inverse F. So I have just replaced the S matrix by A inverse F and then the B multiplied by A inverse matrix I can sum it together multiply that F and this could be further denoted by lambda capital lambda into F and where this lambda basically represent B into A inverse and I can also directly have a value attributed or calculated for this lambda which in this case would be 0.1, 0.02, -0.5. So I am giving you the example so that you can try it on your own if you want 0.12, 0.014 and -0.1. This is the kind of maths that would normally be used by any LCA software so normally when you are doing a concrete LCA or an LCA of a pretty extensive process you will be using some kind of software. Typical examples could be SimaPro, Gabi, OpenLCA, Umberto and some of you also might want to use your own excel sheets and this is the type of calculation that would normally be going in the background in the software or you can do similar things in an excel. We will just show you an example how this kind of calculation can be done in excel as well. So this was with respect to a simple process what if I need to make some updates. So let us

try to update the functional unit. Now in the initial case I have assumed that the functional unit would be 1000 units of electricity.



What if I now want to update the functional unit such that it leads to 1000 kilowatt hour of electricity sorry 100 kilowatts of electricity and 100 litres of fuel. So I want two outputs which leads not just the electricity but it also takes into account some amount of fuel production as well. So I do not need to worry I just need to update my matrix and do the calculation. So the F matrix would be updated and I do the multiplication and the final S matrix that you should be getting. So you can try this out on your own. Just update the F matrix, the multiplication of A inverse into F and the final S matrix that you should be getting should be 10 and 1.2. So the first unit process should be multiplied by a factor of 1.2 and the ultimate result would be the CO2 that you are getting produced would be 22 kgs. The SO2 that is getting produced would be 3.4 kgs and crude that would be consumed for this particular function unit would be -60 litres.



Further we can also rearrange or we can also update the process. Suppose that the process 2 I want to update that, let me go back. So this was the process I am talking about. So suppose this is the process for the production of fuel also has another input in terms of

electricity. So in the first process we have electricity getting produced and in the second process it is getting consumed and maybe it is taking in 10 kilowatt hour of electricity as well. So in that case you would need to update your A matrix. Again I would not go to piecewise calculations which you can try it on your own.

$$S = \left(\begin{array}{c} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ \end{array} \right)$$

$$S = \left(\begin{array}{c} 1 & 0 \\ 2 & 0 \\ 2 & 0 \\ \end{array} \right)$$

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So the A matrix will get updated and let us consider the original functional unit of 1000 units of electricity produced. So functional unit again gets updated to 1000 kilowatt hour of electricity but in this case the second process has now been updated to also take in 10 kilowatt hour of electricity for the production of fuel from the crude fuel. So in that case you have an updated A matrix, you multiply that with a functional unit of 0 and 1000 and finally the scaling factor that you should be getting should be something like 1 or 2.41 and 2.041. So this particular functional unit takes care of the electricity that is consumed by the second process as well. And finally the emissions that you should be getting would be CO2 emissions of around 122.45, SO2 emissions of 14.29 and a crude intake of around -102.04. So when you have these complexities involved that the processes are using the inputs and outputs interchangeably, this is where this matrix algebra structure is

going to come in very useful to you. So let me also try to show you the application of this particular matrix algebra calculation with the help of a simple excel sheet. Let me go to MS Excel file.



So this is a typical excel file for the same process that we have just sold in together. So what you have on the y axis or the different, the row A is the different fuels or different products that are being formed in terms of liters of fuel and electricity.

So you have these values -2 and 10 and then you also on the different columns you would have the different processes like electricity production and fuel and then on the B matrix which is given in greenish tone is having the different emissions or the environmental flows in terms of CO2, SO2 and crude. And I have written the formulas for this and what I just need to update in here is the function unit. So if I put a function unit of 1000, it gives me the scaling factor. So what you see here is simple matrix algebra, multiplication of the A inverse matrix with the F and finally it is normalizing the different processes. I can also update this in here like in the same way we have done it before.

So in case I would want the function unit to change to 100 and 100, so 100 units of fuel and 100 units of electricity, the scaling factor changes and so does the individual processes can be seen in here. Further I can also update the process in here. The first process now also takes in some amount of electricity, maybe 10 units of electricity in here. That was the process, the function unit again remains the same. 0 and you get the scaling factor which I have just shown you and then you can also quantify the different emissions. So this is how you can do an LCA using Excel sheets. Let me give an example of a real life LCA that we have done. So this is what a particular LCA sheet would look like. So this is a process for production of ammonia from biomass. You can see there could be different emissions that are coming in. I wouldn't go into details, just to show you like what are the typical emissions and if you see the inventories of emissions, this could be pretty

You can see here the emission inventors that can go in here, this could be pretty large and in this particular LCA what we have taken into account is almost 1500 or more emissions which you can see in here and this is where it becomes a bit difficult to quantify all these things with a pen and paper and you would need some kind of tool, you can use Excel and what you see in here is the same mathematical tool being used for normalizing and calculating the final emissions. With this we have given you a brief understanding of the computational structure of an LCA. Again there are many complexities involved, specifically when you are dealing with the cut off of the different flows, what if they are multifunctional unit processes or flows loop recycling. So you would have to take in these different considerations and maybe adopt different kinds of methodology for when you have these kind of problems. But this is just to give an understanding how things are done for a typical life cycle assessment.

And in the next class we will try to portray you the utilization of a commercial LCA software which is SimaPro and to show you the functionality of SimaPro with the help of an example to help you understand how an LCA can be conducted with a commercial software. With this we end today's class. Thank you.