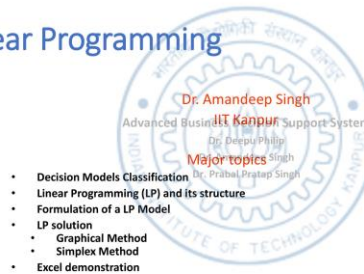


Advanced Business Decision Support Systems
Professor Deepu Philip
Department of Industrial Engineering and Management Engineering
Indian Institute of Technology, Kanpur
Professor Amandeep Singh
Imagineering Laboratory
Dr. Prabal Pratap Singh
Indian Institute of Technology, Kanpur
Lecture 16
Linear Programming Introduction

Welcome back to the NPTEL course on Advanced Business Decision Support Systems. In the last 4 weeks, Professor Deepu Phillip has shed some light on what are the models? What are Decision Support System models? What are Systems? And, the classification of the models was given based upon the Decision Tree, the kind of the decisions that we take. MS Excel demonstration on the decision tree was also taken.

Linear Programming

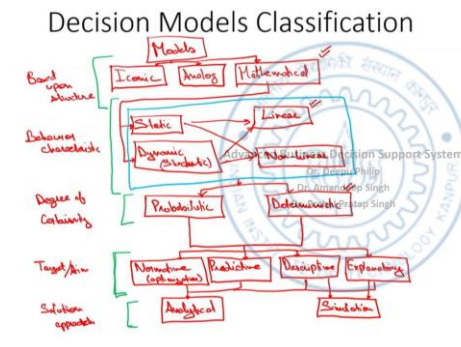


NPTEL Course: Advanced Business Decision Support Systems

I am Dr. Amandeep Singh, I will focus this week on Linear Programming, which is Deterministic Modelling for taking decision when theta is available. Theta is in available in a structured form and been more determined we are more close to what realistic decision could be.

But there are certain assumptions in linear programming, linearity, additivity, divisibility. Those also we will discuss and we will try to see how do we formulate the problem, how do we try to solve the problem, if it is set in the linear program objective. So, this week we will cover Decision Model Classifications. I will give you a separate set of the classification based upon the structure, function, certainty, generality, quantification and so on.

Then, we will see what is Linear Programming by definition and what are its characteristics and its structure. Here also we will try to see its assumptions. Then, we will try to formulate a linear programming model based upon the statement that is given, the data that is given, how could we formulate a maximization and a minimization problem? How can we convert a maximization into minimization and vice versa. Then, we will try to see solutions to the linear programming problem using graphical and one of the most common methods known as Simplex methods. These two methods will also be taken in MS Excel demonstration.



Decision model classifications, we have seen the classification of the models based upon the systems, based upon the decisions that we take. Now, let us try to see another view for the classifications of the models for taking decisions. It could be based upon the validity, usability, the value, the time reference, if I say time reference, the model could be Static or Dynamic. If I say structure, it could be iconic, analog or symbolic. Let me try to put it in a block diagram here.

I would say models based upon the structure of the model. It could be Iconic, it could be Analog or it could be Mathematical. What is Iconic model? When I say the word Iconic, it means, an icon on your screen you have icon for MS Excel. Icon means it could be scaled up or scaled down model of the original, so that is Iconic model. So, such models retain, some of the physical characteristics of the system that they represent.

So, the examples could be, we take decisions while reading map, we take decisions while looking at the photographs, we take decisions while even watching some small videos. So these are all iconic representing the situation and while looking at the drawings, looking at the icons, representing the overall system or maybe type of the engine, the car has or the drawing that is representing the structure of the building, these all models could be used to take small decisions. Then comes analog. Analog in itself means, when we are trying to compare the system with the previous one or a similar system. For example, in the case of the output, maybe the speedometer of your car or maybe the

watch that you have nowadays. The watch has digital time it is 2 p.m. or it is 2:02, 2 p.m. 2 minutes.

When you try to see the hand of the clock, that is analog clock that is comparative to the previous time, but digital is the specific output 2 p.m. 2 minutes. Similarly, in the speedometer if the speed is given 70 miles per hour or 71 miles per hour, this is a continuous system it is comparative. So, analog is something that is compared to the physical system that is present there already.

So, it resembles to the previous system and it retains a set of characteristics of the system. So, such models are more general available. The examples which I have given to you like the speedometer, like your watch, all these are the kinds of the analog models. Then, comes the major type that is the focus of this course the Mathematical models. Mathematical models means, when a model is represented in form of an equation.

Any of the equations that you have learnt like the Pythagoras equation, is a mathematical model. The hypotenuse, the base, the perpendicular is compared to each other that is mathematical model. If it is represented in a mathematical form or an empirical mathematical equation, that is known as the Mathematical model this could be further classified into various forms. Mathematical model based upon the behavior characteristics, could be classified into two types that is based upon their time frame. It could be Static, it could be Dynamic.

So, depending upon the time reference, the model has is it changing with time or is it not. If it is not changing with time, the model is known as Static model. But if it changes with time that is known as the Dynamic model. So, a contrary to the mathematical model could be also verbal models, maybe where only the words, maybe the spoken language or written sentences those do come those are also called of one of the models. So, in Mathematical models, the Static model gives us a status of a system at a particular point and it does not take into account the changes over the time.

For example, if I am trying to study an inventory model, the Economic Order Quantity (EOQ) models those are the static models for the specific time. Yes, there are EOQ models with price breaks as well. With price breaks, could also be static model specific time that we are covering. But if the Dynamic system is involved into it, that becomes a Stochastic model. So, Dynamic model takes into account the change over the time.

So, time is one of the variables which is deriving the optimal solution. So, Dynamic programming is used in a Dynamic model. So, it is also known as Stochastic model. Static and Dynamic, which are the types of the Mathematical model. Now, these models

depending upon the data or the information that is available or the kind of the complexity that you are trying to build the model could be linear or could be non-linear.

Because the title of this week is linear programming, the models what we will be covering in this week would be linear models. That is, the variables that we are taking would have the degree of the order 1 only. They would not be x^2 , they could not be $1/x$ or so. So, the model would be majorly linear discussed in this course. Linear is when the variables attains degree of the order one, if it is degree other than order 1, that is less than 1 and more than 1, the models becomes non-linear.

Both the Static and Dynamic models could be linear or could be non-linear. It could be like this or it could also be like this. Now, how certain the models are? How certain are you with the data that you have? How many assumptions are you taking? how well is the data structured that determines the degree of probability? So, based upon the degree of certainty, the models could be further classified into Probabilistic and Deterministic.

Probabilistic models, where we use probability theory, wherever we use the exponential distribution, wherever we use may sometimes the binomial distribution for example, input, CPM, those are all Probabilistic models. Deterministic, like the Linear Programming, when the data is available, we only need to formulate the problem depending upon what we have in hand.

The data is more closely available. So, that is Deterministic model. These Deterministic and Probabilistic models, could be further classified into various forms depending upon what is our aim to work upon the model. These come from both if I put this into a bigger box, the Probabilistic and Deterministic models could come from any of these. Now, depending upon what is your target, what is your aim or what is the scope of the work that you are trying to see the models, could attain various forms.

I would just list them here Normative, Predictive, Descriptive and Explanatory. Now, what are these kinds? The Normative models when is as the word says, it talks about the normalization of the data or these are also known as the Optimization models. That is, we try to find the best or the optimal solution to the problem that we have formulated based upon certain limitations, certain assumptions, those always would be there. We use mathematical programming to formulate the problem statement and we try to have a optimal solution out of this. I will also put here, this also known as the Optimization models.

This is Normative. Predictive is, when you are trying to predict the future based upon the past data, majorly here forecasting comes into play. There might be regression models, there might be other models that always give you cause and effect based upon this cause. This effect would come or maybe the exponential smoothing, maybe, the moving

averages in forecasting, when you try to predict something based upon the data that you have. So, you try to pick the best decision alternative based upon the prediction.

These are Predictive models. Description is, when we are only trying to describe the data that is, you have a data, which might not be in the structured form, we have only data points, when you even calculate average out of it that is description the total average is this. You can even see the mode, you can see median, you can see standard deviation. These are all description of the data further. The data overall, what is the picture that could be plotted in a graph what does it represent? That is Descriptive models. Explanatory means, it is trying to explain something out of the data sets that you have.

That means, you are trying to further detail the data that you have. So, by connecting certain nodes of the data points, you are trying to have a representable form out of it. So, these all are kinds, which come from both the Probabilistic and Deterministic kinds. I would better draw arrows everywhere. Now, there comes the Solution procedure.

How do you try to solve the problem? The solution approach. Solution approach here could be to try to analyze the data. The big word that is being used nowadays is Big Data Analytics. So, that means, you are trying to analyze the data everything that you are trying to do, you are trying to predict, you are trying to describe, you are trying to even prescribe based upon certain analysis, that is Analytical. Analytical modeling, which comes out from these majorly Normative, Predictive, Descriptive and Explanatory.

Now, whenever you need to describe something or you need to explain something sometimes, simulation is also required. Like Monte Carlo simulation which Professor Deepu had discussed in the last lecture, comes from the data which was descriptive in nature. So, from Descriptive and from Explanatory simulation models also do come.

So, these are the decision model classification based upon structure, based upon the behavioral tricks, how to use the model, the degree of certainty of the data available, the target or aim or the process that you have, the solution approach, are you trying to use some analytics or you are trying to simulate based upon random numbers or so. There are certain symbolic models that could be maybe as I said, Verbal or Mathematical.

So, those are also kinds of different models. Also nowadays, Heuristic models are also there, other than analytical and simulation, Heuristic models also do come into play, which are applied in a consistent manner to facilitate solution of a problem. So, it is a Heuristic model, but sometimes, the heuristic background is not very clear to the user in general. For example, artificial peak holiday; for example, genetic algorithm; for example, gray ray nation analysis, these are Heuristic models. Those also do come,

which are little advanced, but we limit our talk to the Analytical and Simulation models in this course.

Now, we need to talk about the Linear Programming. Linear programming is for sure Mathematical model. As it is said it is Linear, it is Deterministic and what does it do? We will see, whether it is Descriptive or is it Analytical when we will try to solve the problem.

Linear Programming (LP), introduction

- LP is used when 'decision factors' are there in a problem
- Goal/Target function is ^(max/min) ^(max/min) based upon constraints
- 'Decision factors' could be changed
- Man, Machine, Material, Time (used by the system under study)
- Advantages
- Distribution
- Transportation
- Product Mix
- Production
- many more

Now comes, the introduction to Linear Programming. What is Linear Programming? Linear programming, is it a tool? is it a program? why do you even call it programming? Let us try to see this.

Linear programming, as the word say program is a method. It is a unique method that can be used in different kind of applications. So, it is used to take decisions. I will put Linear Programming as 'LP' is used when decision factors are there in a problem. There is a goal function and there are constraints.

Linear programming has certain decision factor that, it is said and these decision factors could be changed in the system being modeled. Because we need to have the Optimal solution, that means if it is maximizing we are trying to maximize the profit. If it is minimizing, we are trying to minimize the loss. Profit is maximized, minimizes the cost or loss. The objective function as it is called is the goal or target function here.

The example could be maybe, maximizing profits based upon man, material, any systems. I could also put time here. Any of these resources which are used by the system under study. So, it has multiple applications. These applications could come in advertising, it could be distribution, it could be transportation, it could be product mix for sales or for manufacturing, maybe in production and many more.

So, what is Linear Programming? We will be more clear by the end of this lecture, when we will try to see how is Linear Programming model developed. To understand further the Linear Programming concept and the model, let us see what structure that it has. So,

what elements does it have and how is it built? So, basically, Linear Programming has three major contributors. One is decision variables, then is objective functions, then is constraints. Decision variables are the activities or variables based upon which the decision is to be taken. Objective function is the one which is to be maximized or minimized.

Linear Programming (LP), structure of

Components of LP:

- 1) Decision variables: Action alternatives (activities)
 - Denoted by X_1, X_2, \dots, X_n
 - continuous variables
 - non-negative $X_1 \geq 0, X_2 \geq 0, \dots, X_n \geq 0$
- 2) Constraints: Limitation to the available resources
 - Linear inequalities
 - Solution of LP is possible only if it satisfies the constraints
- 3) Objective Function: $Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$
 - Maximize or Minimize it.

$C_1 = 5 \quad (2 \times 5) = 5 \text{ units}$
 $C_2 = 10 \quad (2 \times 5) = 30 \text{ units}$

So, let us try to see this structure or the components of LP. Number 1 Decision Variables- variables that is the action alternatives which are there and we need to select the best of them to arrive to the optimal values. These are action alternatives or activities based upon which the decision is to be taken.

That is why they are called as Decision Variables. These are denoted by x_1, x_2, \dots, x_n . For instance, in an advertising problem, we can see the distance at which, the advertisement is to be taken, whether it is online or offline. If it is online, we can do it from anywhere at any point globally, which can reach. If it is offline, the travel to that point and transportation cost would be there. So, that becomes the decision variable, whether that transportation cost is to be considered or not.

Number 2, the advertisement for specific product, the cost for the advertisement of one product. Number 3, the salaries for the employees. All of these could be the Decision Variables for them. This could be dependent upon the objective function, that is, we need to maximize the profit that could be the objective function.

The certain values for the Decision Variables are there. So, generally, these variables are controllable that means, these are within the control of the programmer or the person who is trying to formulate the problem. So, these are continuous variables that means, they do not take integer values, they can take values 1.1, 1.2, 1.22 and so and they are non-negative.

Non-negative means, we have $X_1 \geq 0, X_2 \geq 0$ and so on $X_n \geq 0$. If it is negative, then also we will see, how do we try to cater to that problem. Next component of Linear

Programming is the Constraints. Constraints means, there are limitations to the resources. So, this could be as I said in the previous slide, men, material, money, the number of labor hours available, the number of machines available, the amount available, the time available.

So, limitations to them are always there, which becomes our constraints. So, these are represented in Linear inequalities. When you say Linear inequalities means, it is greater than equal to or less than equal to a specific value that is set for that variable. For instance, the minimum number of labor that is to be put is 2 or we have maximum number of labor could be 10 means, 2 labor are definitely available for which the salary is fixed. But if you employ more than 2, third, fourth and so on, extra wages are to be given.

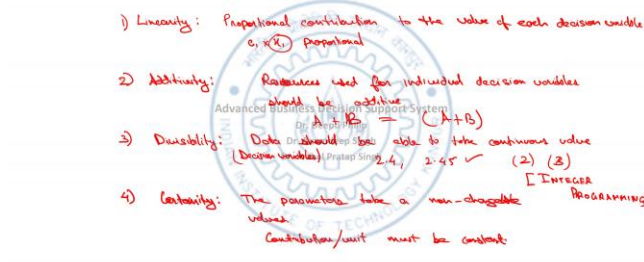
So, between 2 and 10, the decision is to be taken. Then, we can put the inequality as 2 is less than equal to X_2 is less than equal to 10. So, these Linear inequalities are put, these are known as Constraints. The solution of LP should satisfy these constraints. Third component, which is the target component is known as Objective function. When I say Objective function, it is the target function that is also carrying the variable.

It is denoted by letter 'Z', $Z = C_1x_1 + C_2x_2 + \dots + C_nx_n$. So, we need to maximize or minimize it where C_1, C_2, C_3 so on up to C_n represent the contribution of each unit x_1, x_2 so on, up to C_n respectively. That means, with one unit change in x_1, C_1 times x_1 changes. For example, if C_1 is 5 and x_1 changes from 2 to 3, 5 more units are added. If C_1 is 10, then if we change from maybe 2 to 5, that means, 30 more units are added.

In the above case, 5 more units are added that is, per unit change of the variable, this is the contribution to the respective variables. These are used to measure the performance of Z. So Graphical method or Simplex method both could be used to have the solution to the Linear Programming.

Graphical method is generally used when, we have two variables because for that the graph could be built easily and for more than two, three or more variables, Simplex method, which is a simple tabular calculation method, that is used. There are certain constraints to that as well, that we will try to see.

Linear Programming (LP), assumptions



Next comes Linear Programming. Though it seems to be very promising deterministic, there are certain assumptions. The data in realistic form is not always linear, it is not always divisible, it is not always, that the X variables could follow the non-linear value. Non-linear value means, we cannot say 2.3 machines are to be employed.

If I convert 2.3 into 2, that becomes non-optimal solution. If I convert 2.3 into 3, that also becomes non-optimal solution. For an Optimal solution, the variables could or should be able to follow the non-linear value.

This is one of the assumptions here. Let us try to see the assumptions for the linear programming. The first assumption that I could list here is, Linearity. As the name suggests of the model linear programming. It is also known as Proportionality which means, the amount of each resource used or supplied and its contribution to the profit in objective function, must be proportional to the value of each decision variable. I will put it as proportional contribution to the value of each decision variable which means, whatever resource we are using.

For example, if the production of 1 unit take 2 hours, so 2 into the number of units should be the proportional value for the number of units, which are being produced. If for 2 hours, 1 unit is produced. That is, in 10 hours, 5 units should be produced that means, these values C_1X_1 is proportional.

That is what Linearity here means. Next comes the assumption of Additivity. When I say Additivity, it means, each resource used or supplied should be additive to the other resource. For example, if we unprofit from the decision variable A and B , the amount of resource consumed for producing variable A and B , $A+B = (A+B)$. That means, source is used for individual decision variables should be additive to the other resource. Which means, resource for variable A plus resource for variable B should be equal to if I take A plus B together.

Next comes Divisibility. When I say Divisibility, as I gave you example that number of machines could not be 2.3 or 2.5 or 2.6, it should be directly divisible by a number that

is, the data should be continuous. When I say data by mean a Decision Variable, these should be able to take this value.

That means, if it is the value 2.4 or maybe 2.45, it should be acceptable 2 or 3 should not be limited. If it is 2 or 3, there is separate set of the programming, that is known as Integer Programming. Now, the solution values for decision variables are allowed to assume continuous value. For example, we need a specific amount of the product produced.

Specific amount means, 250.3 kg of product is produced. That is the Optimal solution. If I convert 250.3 to 250 or 251, that also becomes little non-optimal solution. So, divisibility is one of the assumptions.

Last assumption that I would put here is Certainty. Certainty means, the data or LP is used at the parameters, the availability of resources, the profit and loss, everything is certain and the decision variable is all known is all constant. That means, whatever we say, the prices would remain constant, the profit proportional to that provides would remain constant. That means, I would put here certainty as the parameters take a non-changeable values. That is the decision variable and consumption of resources per unit of variable must be known or constant.

That is contribution per unit of decision variable must be constant. With these assumptions, which sometimes are not 100 percent realistic, but still with Linear Programming, we can have the basic idea of what Optimal solution could we get. That is why, these models are known as Deterministic models. Let us now try to formulate a Linear Programming and try to see the formulation of a linear programming model.

Formulation of a LP Model

Objective Function: Z (min or max) = $C_1 X_1 + C_2 X_2 + \dots + C_n X_n$

subjected to the constraints: [Functionality]

$$a_{11} X_1 + a_{12} X_2 + \dots + a_{1n} X_n (\leq, =, \geq) b_1$$

$$a_{21} X_1 + a_{22} X_2 + \dots + a_{2n} X_n (\leq, =, \geq) b_2$$

$$a_{m1} X_1 + a_{m2} X_2 + \dots + a_{mn} X_n (\leq, =, \geq) b_m$$

$X_1, X_2, \dots, X_n \geq 0$

Maximize (Minimize) $Z = \sum_{j=1}^n C_j X_j$ → Contribution / Profit (Cost)

subject to:

$$\sum_{j=1}^n a_{ij} X_j (\leq, =, \geq) b_i; \quad i=1, 2, \dots, m$$

and $X_j \geq 0; \quad j=1, 2, \dots, n$

$b_i < 0$
Multiply the constraint by (-1)

b_i → Decision variable
 C_j → Decision variable
 $(a_{ij}, -b_i, 0)$ → Decision variable
 $(C_j, 0)$ → Decision variable

Since, we have discussed already, the components of Linear Programming are Decision variables, the Constraints and Objective function. And, also functionality between the constraints, whether it is less than equal to, equal to, greater than equal to, these things would be put in a formulation problem.

Now, when I say formulation of a Linear Programming problem or linear programming model, we need to jot down all the components that we have discussed. That means, there would be an Objective function which is denoted by 'Z'. This could be minimize or maximize based upon its contribution. This function is given as,

$C_1 x_1 + C_2 x_2 \dots \dots C_n x_n$ subjected to the constraints which are, $a_{11} x_1 + a_{12} x_2 \dots a_{1n} x_n$ in comparison to b_1 . Here, it could be the functionality that could be: $\leq, =, \geq$. This is known as Functionality or the Relation.

Similarly, we can have $a_{21} x_1 + a_{22} x_2 \dots a_{1n} x_n$ with the functionality related to b_2 . So, on we might have m number of constraints that is, $a_{m1} x_1 + a_{m2} x_2 \dots a_{mn} x_n$ with the same functionality and this is b_m . These are the constraints, whenever we try to formulate a problem statement, we can put the objective function in a generic form.

This is a very generic form. If I try to put it in one line, it is also possible. Here, one thing is missing that, whenever we try to solve this simple linear programming model, the variables, the decision variables assume the positive values. That means, I can write these in one line that is, maximize or minimize in the mathematical form,

$$Z = \sum_{j=1}^n C_j x_j$$

subject to the constraints which are,

$$\sum_{j=1}^n a_{ij} x_j$$

So, this is a very general formulation of a Linear Programming model in which, C_j 's, X_j , A_{ij} 's what are these? C_j are the coefficient, which represent the per unit profit or per unit cost as it is mentioned. C_j are/or is the contribution per unit profit or maybe cost.

It depends upon the problem is maximization or minimization. If it is to be maximized, the profit is to be maximized, if it is a cost or loss, it is to be minimized. Then, X_j are the Decision Variables. Now, A_{ij} , these are the technological coefficients, which are always multiplied by X_j 's. So, A_{ij} 's can assume values that could be positive, that could be negative, that could be 0, even depending upon what is the role or what is the contribution of the specific X_j to that equation.

The word contribution does not fit well here. It is only the role of the X_j 's. So, it can assume values, I would put here positive, negative or 0. Now, we are left with B_i . What is B_i ? B_i is the right hand side of the constraint.

It is assumed that, all B_i 's are greater than or equal to 0. B_i , I would say, it is positive or 0. If suppose $B_i \leq 0$, we try to multiply the whole equation by -1. Multiply the constraint by minus 1 so that, the right hand side always assume a positive value.

So, there could be certain forms of the expressions that is, less than equal to, equal to or greater than equal to that is, related to each of the constraints to the respective B_i values. So, this is very generic form. Now, if I try to formulate the Linear Programming, there are certain steps associated with it.

Formulation of a LP Model



So, let me write here steps in formulation which could be, number 1 as, identify the Decision Variables. That means, we need to identify what are the Decision Variables. So, what is the form of the expression between the decision variables, whether it is no larger than, whether it is exactly equal to, whether it is at least to that. So, these expressions for the decision variables are to be identified and these are to be put. We try to identify the problem data, which means we try to identify what are the constants, what are the parameters and then only we formulate the constraints.

Once, we have formulated the constraints, the objective function is always there and we need to formulate that. I am just quickly jotting down the steps because I will have an example in the next slide, where we will see how do we formulate. These two could be Swapped steps. Objective function calls also be jotted down first as well. There are certain examples of VM programming that we can discuss.

Formulation of a LP Model

Example

A retail store stocks two types of shirts A and B. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type A and a maximum of 300 shirts of type B.

The storage capacity, however, is limited to a maximum of 600 of both types combined.

Type A shirt fetches a profit of Rs. 2/- per unit and type B a profit of Rs. 5/- per unit.

How many of each type the store should stock per week to maximize the total profit?

Formulate a mathematical model of the problem.

$$\begin{aligned}
 & a \text{ units of Shirt A (Rs. 2)} \quad \text{and} \quad b \text{ units of Shirt B (Rs. 5)} \\
 \text{Max. } Z &= 2a + 5b \\
 \text{(Rs.)} \quad & 1a + 0b \leq 400 \\
 & 0a + 1b \leq 300 \\
 & 1a + 1b \leq 600 \\
 & \text{Both } a, \text{ and } b \geq 0
 \end{aligned}$$

So, there is one problem that I have put here, which is taken from a reference, that you can see in the reference list of this PPT. Now, a retail store stocks two types of shirts A and B. They are packed in attractive cardboard boxes. During a week, store can sell maximum of 400 shirts. You see the word maximum and try to see, what kind of expression would come here less than equal to or more than equal to or may be equal to and a maximum of 300 shirts of type B. Looks like what are the decision variables.

Then, storage capacity is limited to a maximum of 600 of both types combined. When both are combined A and B maximum 600 could be put. Type A shirt fetches profit of rupees 2. Type B shirts brings a profit of rupees 5. How many each type the store should stock per week to maximize the total profit formulate the mathematical model of the problem? Now, we can see here the word 'profit' is mentioned and also they have clearly mentioned here maximize the total profit which means, the Objective Function would be maximize.

Now, let us try to see the steps here. Identify the decision variables, identify the problem data. Decision variables are directly given here A and B which are two kinds of shirts. So, let me say let A units of shirt A and B units of shirt B are taken as the notations for the decision variables, whose contribution is rupees 2 for shirt A and this is capital B here rupees 5 for shirt B. Now, the data given here is 400 shirts of type A and 300 shirts of type B are the maximum and total storage capacity is 600 which means. Let us try to either formulate the constraints first or try to write the objective function first. Objective

function is very clear here, that is rupees 2 is the profit from shirt A and rupees 5 is profit from shirt B.

So, we can say the Objective function here is maximize $Z = 2a + 5b$, this is the maximization problem and the constraints that, we can see here directly are 400 shirts of type A is given here maximum which means, A can be maximum 400 that means, A and 400 are related to each other that is, 1A and no B, no B means 0B. Maximum could be 400 which means, the expression or the functional part here would be \leq . Similarly, maximum 300 shirts of type B which means, B that is, 1B which is added to 0A because A is not even here is less than equal to 300. Then, combine together the storage capacity, what they have is 600 that means, if we put A and B together, 1A and 1B, this is less than equal to 600 and anyway, both A and B are greater than equal to 0.

So, this is my problem formulation. I put A as units of shirt A, B as units of shirt B, contribution is given, I would say, Z is equal to this which is in rupees everything in the Objective Function is in rupees. Here, this is the capacity, which are the number of shirts into 1, so 400, 300, 600 is the problem formulation for a simple Product Mix Storage Problem or I can even call it as Product Mix Setting Problem.

X	A	D
Y	-	-

Formulation of a LP Model

Example
 A patient consults a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins, vitamin A and vitamin D.
 Doctor advises him to consume vitamin A and D regularly for a period of time.
 Doctor prescribes tonic X and tonic Y, which are having vitamin A, and D in certain proportion. Also advises the patient to consume at least 40 units of vitamin A and 50 units of vitamin D daily.
 The cost of tonics X and Y is Rs. 5 and Rs. 3 per unit respectively, and the proportion of vitamin A and D that present in X is 2 units and 3 units and in Y is 4 units and 2 units respectively.
 Formulate LPP to minimize the cost of tonics.

$\text{Min. } Z (\text{Rs.}) = 5x + 3y$
 $2x + 4y \geq 40$
 $3x + 2y \geq 50$
 Both $x, y \geq 0$ (Non-negativity constraints)

Let me try to see another problem. This is a minimization problem in which, we need to minimize the cost of the tonics. What is the problem statement here? A patient consults a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins that is, vitamin A and vitamin D, we can see two variables have come here.

Let us see whether these are the constraints or variables, doctor advises him to consume vitamin A and D regularly for a period of time, doctor prescribes tonic X and tonic Y. Let us stop here. We have A and D and X and Y tonics. Now, tonic X could have a certain value of A and certain value of vitamin D or vitamin A could be taken from tonic X of some amount and tonic Y from some amount.

So, both the ways, it looks like at this point of time, when we are stopped here, the equations could be formed. So, what are the decision variables? Vitamin A vitamin D or tonic X and tonic Y, it depends upon how easy we need to try to solve the linear program here. So, generally for the minimization problem, we tend to see that we need to have all the values, all the X size greater than equal to 0, that is an easy way. However, there is a theory in linear programming it is known as Duality.

The Duality theorem says, all the constraints could be translated into the number of variables and number of variables could be converted to the constraints. All the contributions turns to your b_i and all the b_i turns to the contributions, that also is possible in which, both the ways the problem could be stated.

Now, let us read the problem further. So, as we try to understand, how we can do it easy. Now, tonic X and tonic Y are having vitamin A and vitamin D in certain proportion that means, tonic X and tonic Y is there which is having vitamin A and D in certain proportion. Also, the doctor advises the patient to consume at least 40 units of vitamin A and 50 units of vitamin D which means, now the limits which are given here are for the vitamin A and vitamin D, these could be better to be formed as the constraints.

The cost of a tonics is rupees 5 and rupees 3 per unit respectively for X and Y. So, directly now we have X and Y as our decision variables and the proportion of vitamin A and D present in X is 2 units and 3 units whereas, in tonic Y is 4 units and 2 units. So, we need to have a minimization Linear Programming problem in which, let us see X units of tonic X and Y units of tonic Y are my decision variables because the cost is given here.

From here, I directly take these. Now, if I try to state my Objective Function, it is minimize $Z (\text{Rs}) = 5x + 3y$ taken from here. Now, the constraints are given that for vitamin A at least, 40 units are to be given which means, 40 units. At least, 40 are to be given that means, greater than equal to 40 for vitamin A. So, what vitamin A contains? 2 units of X and 4 units of Y that means, $2x + 4y$, it is taken from here, 2 units from X and 4 units of Y for vitamin A. Similarly, for vitamin D, $3x + 2y$.

This is vitamin D for which, 50 units of vitamin D are to be taken daily. At least 50 and it has to be more than or equal to 50. So, this becomes my problem formulation here and anyway non-negativity constraints that is, X, Y are both ≥ 0 which are known as Non-negativity constraints. This was simple Linear Programming formulation, we discussed about certain classification of the models to see where does the Deterministic Models fall. In Deterministic Model, we picked the Linear Programming to be discussed. We discussed about the certain characteristics of linear programming and tried to see how do you formulate the problem when the data is available in the linear form? Yes, certain assumptions are definitely given, which is not realistic all the time.

The assumptions which are that Linearity is there, Additivity should be there, Divisibility should be there and certainty of the data should be there, data should be certain, these are given. We will try to see the solution to the Linear Programming problem statements in the next lectures. We will try to see the Graphical methods and also the Excel demonstration over it and the Simplex methods and Excel demonstration on it.

Thank you. .