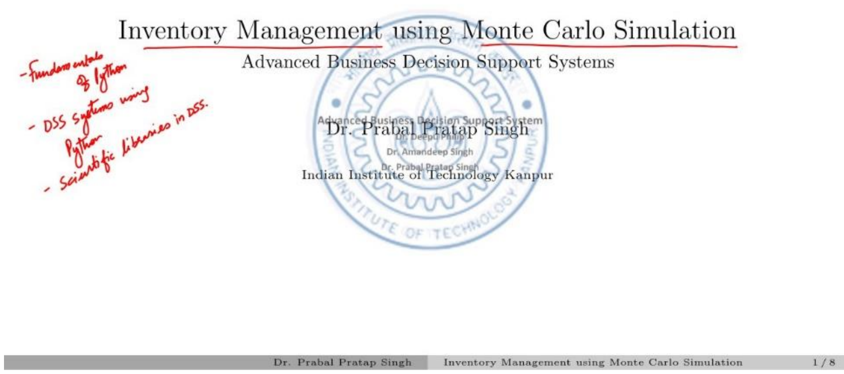
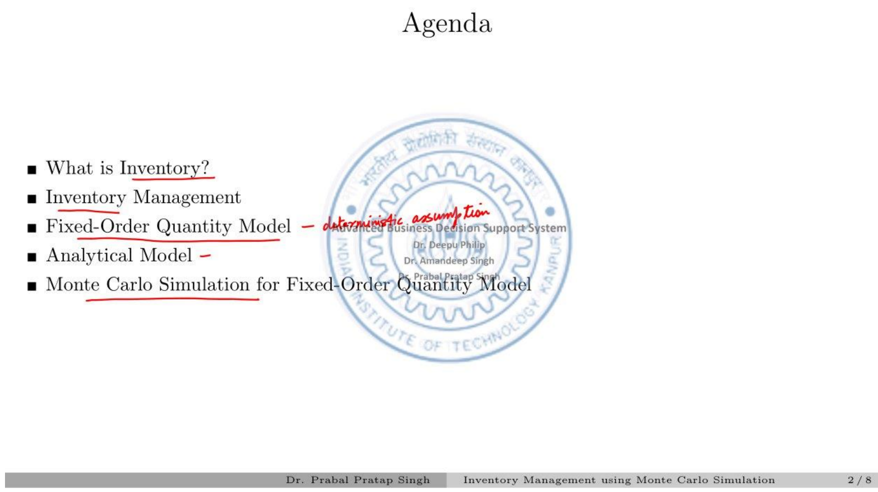
Advanced Business Decision Support Systems
Professor Deepu Philip
Department of Industrial Engineering and Management Engineering
Indian Institute of Technology, Kanpur
Professor Amandeep Singh
Imagineering Laboratory
Dr. Prabal Pratap Singh
Indian Institute of Technology, Kanpur
Lecture 35
Fixed-order Quantity Inventory Model

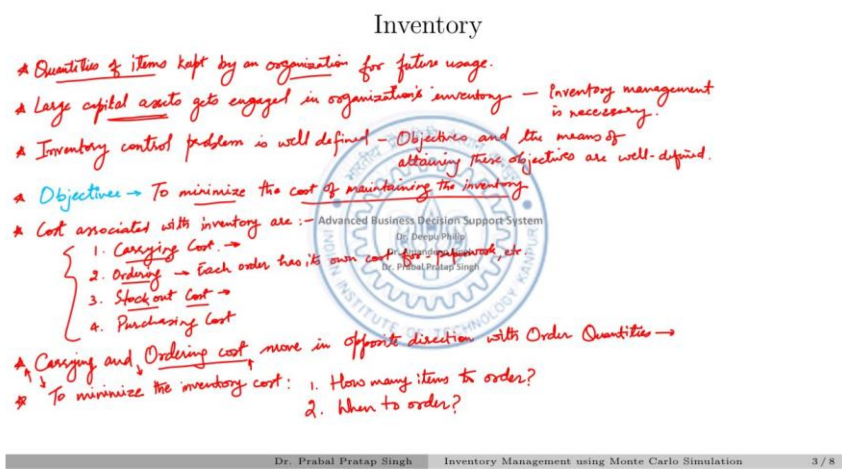
Hello everyone, I welcome you all to the lecture series on Advanced Business Decision Support Systems. I am Prabal Pratap Singh from IIT Kanpur, and we are learning about how to create various Decision Support Systems using python language.



So, till now we have covered the Fundamentals of Python. We also looked at How to Create DSS Systems Using Python, and Used Scientific Libraries for Creating Various Decision Support Systems. But now, today we will start with a different topic that is Inventory Management, and we will see what this Inventory Management Issue is, and then we will try to Develop Analytical Models for Managing the Inventory, and then we will try to see How to Manage the Inventory Problem Using this Monte Carlo Simulation. And, afterwards, we will try to Create a Decision Support System Using Python for the same thing, and we will Develop our Own Monte Carlo Simulation for this topic.



So, the Agenda for today is that we will first see What actually is Inventory, then we will look upon what are the Different Things that we Need to Manage in the Inventory, and after that we will try to learn about a very basic model which is Fixed-Order Quantity Model that has some deterministic assumptions which finally yields an unrealistic model for the real-world. And then, we will try to Develop the Analytical Model which will use various algebra, and differential calculus to calculate the most optimal quantity to keep in our inventory, and after that we will try to see How to Model the same Problem Using Monte Carlo Simulation.



So, let us start with the Inventory. So, inventory is nothing but quantities of items kept by an organization for its future usage. So, each, and every organization that develops some product or sells some products tries to keep some quantities of items in their inventory.

Now, these quantities handle large capital assets which get engaged in organizations' inventory. So, these capital assets are important for any organization, so that is why we need to manage inventory. Now, this inventory control problem is well-defined or we can say well-structured which means that the objectives, and the means of attaining these objectives are well-defined. So, we will see how to define the objective function for this inventory problem for a fixed-order quantity model. Next is what is actually the objective for here.

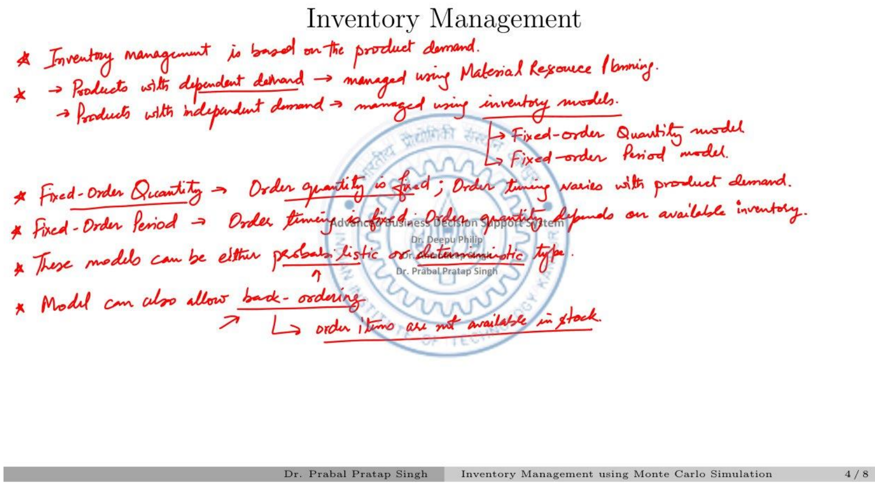
So, the objective can be to minimize the cost of maintaining the inventory. So, this is a very logical objective. Why, because an organization is holding some quantities, and they are accumulating large capital assets for these quantities, we should minimize these capital assets, so that is why the objective is this to minimize the cost of maintaining the inventory. But, to minimize any cost, we first should look at what are the different types of cost? That is included in this,while maintaining the inventory, so costs associated with inventory are, the first is the carrying cost. So, let us say if you are holding one unit of an item, and each holding of a unit of item costs, let us say, 2 rupees per unit.

This is actually the carrying cost, holding cost or carrying cost, so per unit item gets multiplied to the number of items that you are holding annually, weekly, monthly, and then we can get the aggregate carrying cost. Similarly another cost is ordering cost. So, when an organization is ordering a bulk of quantities, and then they need to first store them, and then with time they get consumed. So, every order contains its own cost, each order has its own cost for, let us say, paperwork etc.

The next is the stock out cost. Let us say some demand came for a particular item but the stock is not available. So, in that case there is some stock out cost that the company registers for each item. So, that cost is also included while maintaining the inventory. Now, the last cost could be the purchasing cost. So, this is nothing but when you are purchasing some item, what are the different kinds of cost that are available.

So, these are four major types of cost that get included while maintaining the inventory. Now, a special thing about carrying and ordering cost is, carrying and ordering cost moves in opposite direction with order quantities. So, what does this mean is, let us say, you ordered a large number of quantities in a particular order. So, your number of orders will decrease which means that your ordering cost will decrease. But, now you need to handle a large quantity of items in your inventory, so that is why the carrying cost increases but you decrease your ordering cost. Vice versa in a different scenario, if you decrease the number of quantities in a particular order, then you can decrease your carrying cost. But, your ordering cost because the number of orders will increase, so your ordering cost will increase.

So, this creates havoc while maintaining the inventory. That is why we need to look at these two costs in detail. So, to minimize the inventory cost we should ask two questions. The first is how many items to order, and the second is when to order. So, these two questions will let an organization know, how many quantities the particular order should be, and when is the optimal time to order, so that our total cost of maintaining the inventory gets minimized. So, this is the basic inventory problem.

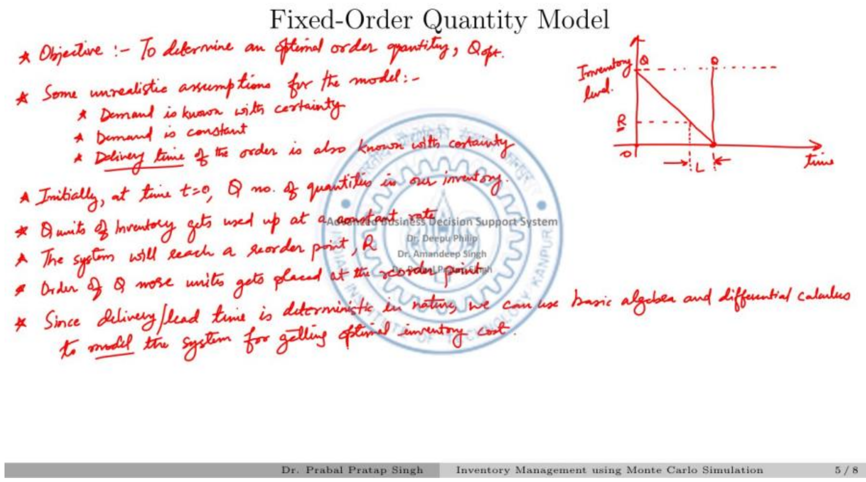


Now, how can we manage this inventory? So, Inventory Management is based on the product demand. So, if the demand is large or the demand gets lowered by the time, then the Inventory Management will be different. Now, the products with dependent demand. What I mean by dependent demand is, let us say you are developing a particular item, and the order of the item by the consumer depends on availability of other items. So, if the other items are highly available, then the need for your item gets increased. So, this is how we categorize the product into two different categories with dependent demand or independent demand.

So, those products that have dependent demand can be managed using material resource planning, and products with independent demand. These kinds of products can be managed using inventory models. Now, we will look into these inventory models. So, there are two different kinds of inventory models. The first is a fixed-order quantity model, and the second is a fixed-order period model. So, in the first one, fixed-order quantity, in these kinds of models the order quantity is fixed, and order timing, that is when to order, order timing varies with product demand.

However, in the fixed-order period model the order timing is fixed but the order quantity depends on available inventory. Now, both of these models can be either probabilistic or deterministic. So, what this means is, if a particular Inventory Management system is deterministic, then let say, for the fixed-order quantity model it says that order quantity is fixed which means that beforehand, and we know what is the particular order quantity the organization will order in its each reorder time. So, everything gets deterministic, and the order timing is also known because we know how much time the replenishment of the item will take. So, these kinds of models can be solved with an analytical approach. But, probabilistic models are those where all of your variables in a particular model are random in nature.

So, these are more closer to real-world problems, probabilistic models are more closer to real-world problems, and deterministic models are used to first define a problem, and get an optimal solution for that problem. So, these models can also allow backordering, now you may ask what is backordering? So, backordering means when we order the items when they are not in the stock. So, what it means for an organization is, let us say, a consumer asks for a particular item, and you search for that item in your inventory, and you find that the order items are not available in stock. Now, at this point of time the orders get backordered, which means that when the replenishment occurs, then the organization will provide the items, and complete your order. So, while modeling a particular Inventory Management problem, we can also include this backordering feature in them. Which will include a new cost, and it will make the model more complex.

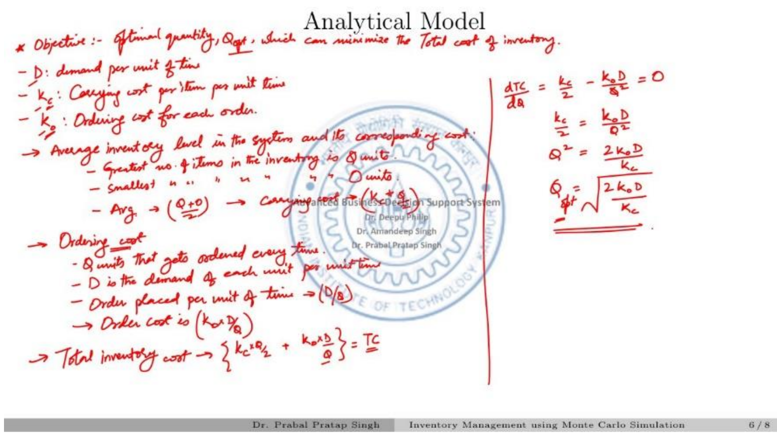


So, let us move forward, and see what is the fixed-order quantity model. So, as we have already seen that the objective here is to determine an optimal order quantity, let us denote this optimal quantity with Q optimal. Since we are talking about the deterministic approach for this model, let us do some unrealistic assumptions for the model. So, the first one is that demand is known which is rarely possible, and demand is known with certainty that means that it is not changing with time. Also, the demand is constant which means that with time, let us say, the demand is not changing in phases, and the delivery time of the order is also known with certainty.

So, this delivery time which is usually termed as lead time, is also known for this model. Now, we can model this problem by saying that initially let us say at time t = 0, we have Q number of quantities in our inventory. So, we can create a chart also for this system. Initially, this is the time, and this is the inventory level, this is the 0 point, and we have just assumed that t = 0, we have Q number of quantities in our system. Now, we have seen that demand is known, demand is constant, and delivery time of the order is also known, and we also assume in this model that the demands get consumed with time at a particular rate.

So, let us say with a particular rate demands keep on decreasing, so this will decrease in this manner, and at some point of time the number of quantities will reduce to 0. Now, before reducing to 0, we must ensure that we have already ordered our next batch of quantities, so that when we reach here we get immediately refilled with all the new orders. So, we can create a new line here, and we can say that just when we reached here, the order received, but to determine that we need to order the new batch of Q quantities somewhere between this, from here to here, we need to order between this. So, let us say, at this point we ordered, so the time taken to refill. This is the delivery time which we can denote with L, and this is again the Q number of quantities here, and this process will continue. So, let us write all these assumptions that Q units of inventory get used up at a constant rate. Let us say, the system will reach a reorder point, let us say R, and where is R in this diagram? This, when inventory level reaches at this point R, the organization will reorder.

The order of Q more units gets placed at the reorder point, and we know that since delivery time or we can say lead time is deterministic in nature. We can use basic algebra, and differential calculus to model the system, and this model is an Analytical Model, to model the system for getting optimal inventory cost.



So, now let us see how to develop this Analytical Model first. So, to develop this model we again write our objective that we need to find optimal quantity Q opt which can minimize the total cost of inventory. Now, let us define some variables like D is the demand per unit of time, let us say Kc is the carrying cost per item per unit time, K0 is the ordering cost for each order.

So, now we have our optimal variable, we have all these variables regarding our inventory model. So, let us first find what is the average inventory level in the system, and its corresponding cost. So, the greatest number of items in the inventory is Q units, and the smallest number of items in the inventory is 0 units. So, the average becomes $(\frac{Q+0}{2})$, so these are the number of quantities which are the average number of units that are present in this deterministic system. Now, we can say that carrying cost becomes, so this is the average number of quantities, and we know that for each quantity, we have Kc amount of carrying cost. So, we can write $K_c * \frac{Q}{2}$

This is very simple like there are Q units that get ordered every time the organization puts an order, and we know that D is the demand of each unit per unit time. So, the order placed per unit time is D/Q. And, now we can write the ordering cost is this K naught into D by Q. Now, we have both the ordering cost, and the carrying cost, so the total inventory cost at any particular time in this model is carrying cost which is

$(K_c * \frac{Q}{2} + K_o * \frac{D}{Q})$ we can call this as total cost. So, now we have a particular system which has this Q, which we know beforehand. And, to find the optimal value of this, we can use our differential calculus, and we can differentiate it with respect to the number of quantities, so we can write

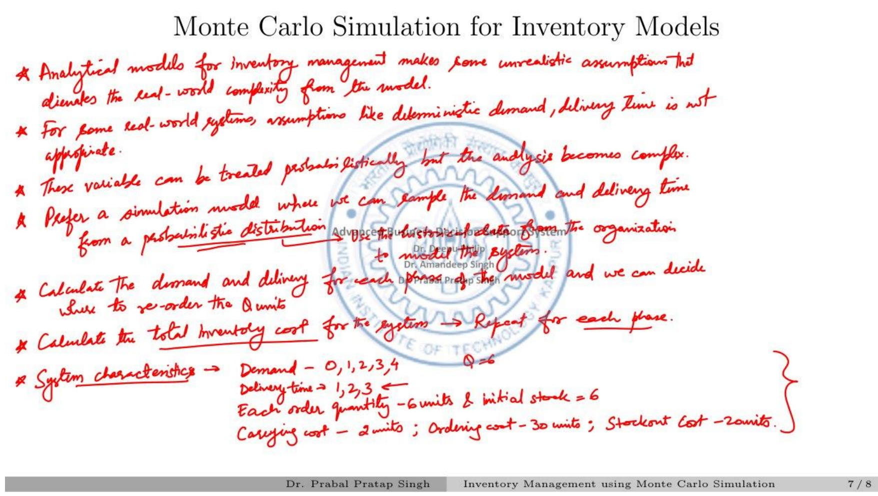
$$\frac{DTC}{DQ} = \frac{Kc}{2} - \frac{Ko}{Q^2} = 0$$

$$\frac{Kc}{2} - \frac{Ko}{Q^2}$$

$$Q^2 = \frac{2K_oD}{K_c}$$

$$Q_{opt} = \sqrt{\frac{2K_oD}{K_c}}$$

So, this is the Analytical Model, and we can easily calculate these quantities but as we have already discussed that these are dependent on some unrealistic assumptions like the demand is known beforehand, the ordering time is known beforehand, so these actually does not happen in the real-world for any organization.



So, that is why we need to use simulation or we can use other probabilistic models also but the probabilistic models becomes very complex, very easily. So, we can also run the Monte Carlo simulation for finding the optimal total cost for the complete inventory model. So, let us see, what is so Analytical Models for Inventory Management makes some unrealistic assumptions that alienates the real-world complexity from the model.

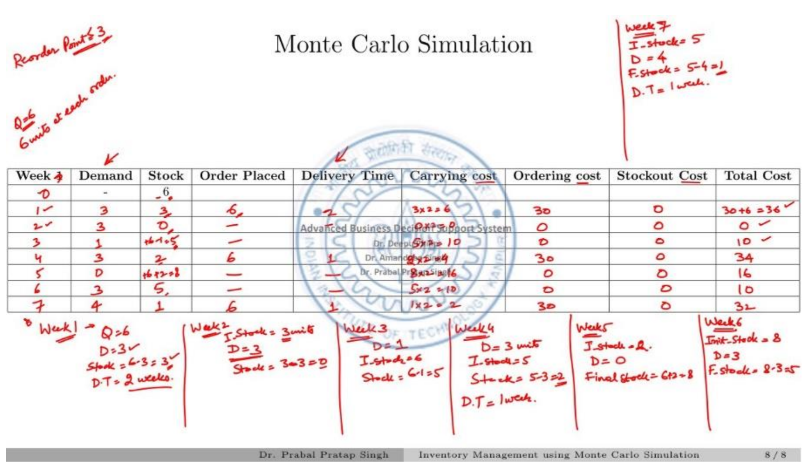
For some real-world systems assumptions like deterministic demand or delivery time is not appropriate. So, these variables can be treated probabilistically but the analysis becomes complex. So, in these situations we can prefer a simulation model where we can sample the demand, and delivery time from a probabilistic distribution, and how to determine this probabilistic distribution? We can use the historical data from the organization to model the system. Once we know that we can sample the demand, and the delivery time we need to calculate the demand, and delivery for each phase of the model.

So, with phase we mean that either if you are looking at the organization's Inventory Management in weekly time frame or in monthly time frame, then each phase is either weekly or monthly. And, then we can decide where to reorder the Q units. Once we decide where to reorder, we know the demand, and the delivery time, then we are in a position to calculate the total inventory cost for the system, and this, we will repeat for each phase. So, the whole operations, like generating a demand from the sample distribution, and the delivery time, then calculating the reorder time, and the reorder quantities, and then finally calculating this total inventory cost for the system for each phase we will get a number of inventory cost, and they will vary because we are using a probabilistic distribution.

So, based on the long-term behavior of the system, we can easily say with some confidence that this is the particular number of quantities that an organization should maintain in their inventory. So, let us now see the system characteristics. For a particular example which we are going to see, let us say the demand can vary from 0 to 4. So, the demand can be either 0 or 1 unit or 2 units, 3 or 4 units only.

So, this is the range of the demand. The next is the delivery time. So, if we are looking at the weekly time frame, the delivery time for our example model could be 1 week, 2 week or 3 weeks. The next thing is each order quantity will have 6 units. So, the Q becomes 6, and initial stock of these quantities is also 6, and let us say the organization spends on the carrying cost for each item as 2 units, and for the ordering cost as 30 units. So, these can be rupees, dollars or anything, and the stock out cost since in our example model we are not considering the back ordering cost.

So, we are only saying that if a particular item is not available when it is asked by the consumer then it will get its cost to the organization that they are not selling it. So, that stock out cost is let us say 20 units. So, we will try to simulate this system based on these system characteristics.



So, this is a sample Monte Carlo simulation that we can do by hand, but in the next lecture, we will try to see how to develop a particular python code, so that the computing machine will do all the steps for you. But before doing it with a computer, let us see how to do it by hand. So, this is week.

This is wrong, so we can be from 1, 2, 3, 4, 5, and 6, 7. So, let us see for 7 weeks, and initially, at the zeroth week, we have no demand because our system has not yet started, but initially the stock is 6 which is Q = 6, and our initial stock is also 6. So, in this order place column, we will write. Whenever, we are going to place an order, we will write here that order is placed. And, we will mention the number of quantities that we are placing. Since we are using a fixed number of quantities because it is a fixed-order quantity model, at each ordering, we will place 6 units at each order.

And, we know that in the Monte Carlo simulation, we need to generate the two things that is the demand, and delivery time, here in this model probabilistically. So, from the different number of demand units that can be possible are 0, 1, 2, 3, 4, let us say we are generating it uniformly or with some different distribution. So, we will try to generate this column demand and delivery time using some sample distributions. The next is the carrying cost.

Whatever the number of items that are remaining in the stock, we will multiply it with the amount of carrying cost for each item, and then we can get the carrying cost. Similarly, we can also mention that for each order we place. When we ask for 6 units, then we will mention our ordering cost which is a fixed-ordering cost of 30 units.

And, we have 20 units of stockout cost, so let us see how to do this for week 1. So, week 1 can be initially we have Q = 6 in the stock, this one. And, let us say the demand we generated from a random sample distribution is 3 units, so it can be either 0, 1, 2, 3 or 4. Now, we are generating this as 3, so our demand for week 1 is 3, so if the demand is 3, and 6 units are available, we can write the final stock as 6 – 3 which is 3.

Now, we also need to mention what is the reorder point for this model, so reorder point, let us say, is less than equal to 3. So, whenever your stock becomes 3, 2, 1 or 0 or anything, then you need to reorder. So, our stock here is 3 which is making our reorder point decision to true, so we need to place an order, and how much we need to place, we need to place 6 units for this order.

Now, in the first week we placed an order, when we will get this order delivered. We already discussed that the delivery time is also probabilistic, so let us say, what are the possible outcomes of delivery time is, either 1, 2 or 3 weeks. So, delivery can happen in either first week or the second week or the third week, so for this week 1, let us say, the delivery time is, let us say, 2 weeks, and we have generated a delivery time of 2 weeks, so we also assume that each order whenever placed will gets placed at the initial time of the week. So, to calculate 2 weeks, we will include this week, this first week, and the second week, so order will get placed here in the third week.

So, whenever an order will get placed, it will increase the stock to + 6 units. Now let us move forward, and see that we know the demand was 3, and our stock for the week 1 contains 3 units, so our carrying cost becomes 3 into 2 which is 6. Since we also ordered here, the ordering cost is 30 units, and there are no stock outs. That means that we have successfully delivered all the demand, so the stock out is 0. So, the total cost which is this cost, this cost, and the sum of this cost is 30 + 6 which is 36. Now, for week 2, we have in our stock as 3 units, so our initial stock says we have 3 units, and let us say that the sample distribution generated demand of 3 units again.

So, since the demand is 3, and we have initial stock of 3, so our final stock for this week is 3 – 3 which is 0 units, now you can see that the demand is 3, stock now contains 0 units, so you may think that we need to place the order again but this is also another assumption for this model is that until and unless, an active order gets replenish the stock, we will not place another order.

So, we are not going to place another order here. An active order is still not delivered because it will be delivered in this third week. So, there is nothing in our stock, so the carrying cost becomes 0, so 0 into 2 is 0, we have not ordered, and the stock out cost is also 0, so the total cost for this week is 0.

Now, let us look at week 3, and let us again generate a demand for this week, so using probabilistic distribution, let us say, the demand is 1 unit for this week, and now we know that the order placed in the first week gets delivered, so now we have 6 units in our initial stock. Initial stock has 6 units. So, the final stock will have 6 – 1 which is 5 units. so we can write here 6 – 1 = 5, and the demand generated was 1.

Now, we will again check, whether the stock reached a reorder point, but our reorder point was less than equal to 3, so we have not yet reached the reorder point, so we will not ask for another order, so there are no order placed here, no here, and the delivery time for the first week was 2 weeks, so we can write here 2.

So, the carrying cost for this week is 5 units, so 5 into 2 is 10, ordering has not done so it is 0, and we have no stock out, so it is also 0, so the total cost is 10 units for this phase. Now, again let us see for week 4, and we can generate a new demand for week 4, and let us say, it is again 3 units. Our initial stock is 5 units which is this one from the previous week, so our final stock is 5 – 3 which is 2 units, and we can see here that $2 \leq 3$, so our reorder has reached.

So, let us write our table, and this is 3, our new final stock is 2 units, and which means that we need to again place an order of 6 units here, and let us again generate the delivery time for this week which can be 1 week, so our delivery time is 1 week which means that just after this week this system will replenish the inventory with + 6 units in the next week in the week 5.

So, let us again calculate carrying cost, here we are carrying these 2 units of stock, so we will have 2 into 2 which is 4, we have placed an order so it is again 30, and the stock out cost is 0, so this becomes 34. Now, for the week 5 we can see that our initial stock is 6 units.

Let us again generate a demand for week 5, and let us say we don't need any units here in this week, so what happened is, we have increased our initial stock was 2 units, and we have replenished, so our final stock becomes 6 + 2 which is 8 units, so from the previous phase it was 2, so let us add this, and this is our 8 units, and our demand generated was 0 here, and we can see that we don't need to place any order, so we can just skip this, there is no delivery time generated here, so the carrying cost for this phase is 8 into 2, because we are carrying 8 units in our inventory that is 16, this is 0, this is again 0, and we have 16 as the final value. Now, for week 6 we can see that our initial stock is 8 units, and we can generate a demand here, let us say again, for 3 units, so demand is 3, so our final stock becomes 8 – 3 which is 5, so this becomes 5.

Now, we don't need to place an order, the delivery time will not get generated, and we can write that we are carrying 5 units in the inventory, so we can write carrying cost as 5 into 2 = 10. We have not ordered anything, stock out has not happened, it is also 0, so this is 10. Now, for the last phase we can write here as week 7.

So, our initial stock for this week is 5 units, and let us generate a demand of 4 units this time, which is the highest number of units that can be asked in a particular week, so let us write this here, and our final stock will become 5 – 4 = 1.

Now, this final stock again triggers the reorder, and we will place a reorder here, so our stock becomes only 1 unit, order will gets placed, and let us say that the delivery time generated is again 1 week, so this order will get received by the organization in the next 8th week here somewhere, so delivery time is 1, and now the carrying cost here is only 1 unit, so carrying cost becomes 2 units, and ordering cost order happened, so it is 30, there were no stock out, so it is again 0, so the total cost here is 32 units.

So, this way, we can simulate a different number of weeks for a particular model. And, once we generate a different number of weeks we can finally find the average total cost. And, by running this simulation multiple times for the longer duration, we can see that the average total cost for the system can get stable because we can see here that there are lot of variation that in the first week it was 36 units as the total cost, then it becomes 0 then 10, so all these variance will get reduced, when we simulate this model to a larger length.

So, in our next lecture, we will try to create a different Decision Support System from scratch for this model, and we will try to see how to generate different number of simulations, and after creating the complete model, we will see whether this model will reach to a stable point or not, and if it reaches to a stable point, then with confidence we can say to an organization that yes, this is the average cost which you will incur if you order this particular quantity of Q = 6 at a reorder point of 3.

Now, if you get a different reorder point, and use a different number of quantities as an initial stock, your simulation will change. So, these are the different characteristics that we need to control, and our simulation can easily give us the final average total cost that an organization can use, and can develop its own policies, so we will meet in the next lecture. Thank you.