

Advanced Algorithmic Trading and Portfolio Management

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Lecture-13

Week 4

In this lesson, we will discuss portfolio performance evaluation with one parameter measures. Some of the important measures we will discuss include Sharpe ratio, Treynor's measure, Jensen's measure and Information ratio measure. In addition, we will also discuss performance measurement with downside risk using Sortino's ratio measure. We will conclude the discussion by comparing portfolio performances using multiple measures with applications in fund management industry. Portfolio Performance Evaluation In this video, we will discuss the basics of portfolio performance. While evaluating the performance of a portfolio, there are certain questions that are very important and should be asked.

Portfolio Performance Evaluation

While evaluating the performance of a portfolio, the following questions are asked

- What are the policies that the fund has pronounced for itself, and how well those policies are followed?
- How diversified is the fund?
- What is the asset allocation?
- The portfolios being evaluated must be comparable
- For example, if a fund has restricted that its managers should invest only in AA-rated instruments or better should not compare with those funds that invest in funds that have no such restrictions

For example, what are the policies that the fund has pronounced for itself and how well these policies are followed? For example, whether it is value versus growth style? Or how well diversified is the fund? What is the quantum of idiosyncratic risk? And in the past, how well the fund has managed this risk? What is the nature of us at a location? Is it more towards equity or more towards debt? Therefore, the portfolios that are being evaluated

must be comparable with each other in terms of their risk profile. For example, if a fund has restricted its managers that they should invest only in double A minus or double A and above kind of category rated instruments, then they should not be comparing the returns from these assets with those funds that can invest in lower rated instruments such as double B or single B. To put it more precisely, the returns earned by a security or a fund is directly linked to the amount borne by it, the security of the fund. And therefore, the problems arise when funds that are being compared have different risk profiles.

Portfolio Performance Evaluation

- Therefore, the return earned is directly linked to the amount of risk borne by the fund
- But problems arise where the funds that are compared have different risk levels
- In the ensuing discussions, we will focus on one-parameter measures that are most commonly employed in the literature

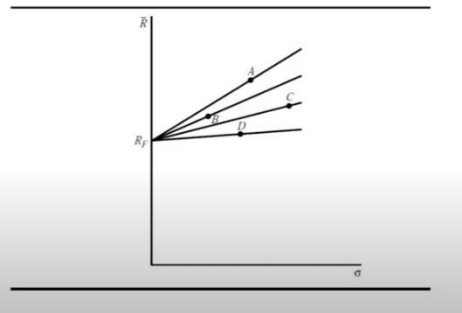
In this lesson, therefore, we will try to discuss some of the measures that are used or employed to compare the performance of funds. And these are often referred to as one parameter measures, which account for the risk profile of the fund or security while comparing their returns. To summarize, in this video, we discussed that performance of a fund or any security for that matter should be only compared when their risk profiles are comparable. One parameter measures Sharpe-Erinci. In this video, we will discuss a very important one parameter measure of portfolio performance, which is Sharpe ratio.

Sharpe ratio is very easily computed with the help of this formula, which is $\frac{R_a - R_f}{\sigma_A}$, which is the expected return from the portfolio or security, R_f which is risk free rate and σ_A which is the standard deviation representing total risk of the asset or security. Essentially, this ratio Sharpe ratio measures excess return, excess over risk free rate against the risk or standard deviation borne by the security or fund. This can be easily understood through the following diagram here. Notice that on y axis we have excess returns, on x axis we have risk in the form of standard deviation σ . In that case, Sharpe ratio is nothing but the line joining risk free asset and the asset itself.

Sharpe Ratio

Sharpe ratio of point A = $\frac{\bar{R}_A - R_f}{\sigma_A}$

- The ratio measures excess return over risk-free rate against the risk borne by the fund
- The portfolios on line joining the investment A and R_f offer the highest slope and, therefore, the best Sharpe ratio measure of performance



$$A = \frac{\bar{R}_A - R_f}{\sigma_A}$$

For any asset, this Sharpe ratio will represent the slope of this line, the line that is joining R_f and the asset on expected return in sigma space. As you can see here, for security A, this slope is highest and as compared to other securities like C, B, D and therefore, this security A here offers the highest Sharpe ratio or the highest performance if the Sharpe ratio is the appropriate measure. Let us look at this simple example where three portfolios, DEF along with market and risk free rate of interest are provided to us. In addition, we also have the standard deviation or total risk of these portfolios, DEF and market. We can use securities or portfolios interchangeably.

Sharpe ratio for these measures are computed here. If you look at the values of the Sharpe

Sharpe Ratio

Sharpe ratio of point A = $\frac{\bar{R}_A - R_f}{\sigma_A}$

- Compare the examples of three portfolios that follow the Sharpe measure

Portfolio	Average Annual Rate of Return	SD	Sharpe Measure
D	13%	0.18	$(0.13 - 0.08) / 0.18 = 0.278$
E	17%	0.22	$(0.17 - 0.08) / 0.22 = 0.409$
F	16%	0.23	$(0.16 - 0.08) / 0.23 = 0.348$
Market	14%	0.20	$(0.14 - 0.08) / 0.20 = 0.300$
Risk-free	8%		

- Here, portfolio D performs the worst, even as compared to the market portfolio
- Portfolio E performs best

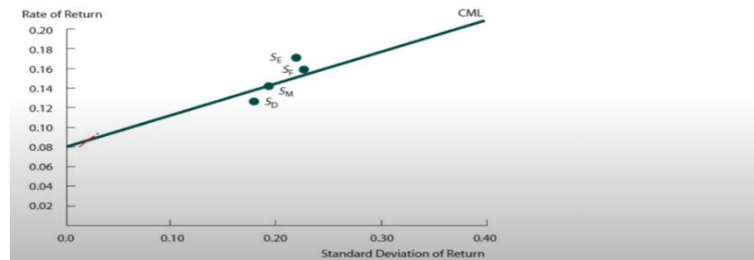
ratio, we find that security or portfolio D performs worst as compared to not only E and F but also market. While portfolios E and F performance is better than market and performance of security E is the best. Let us plot them on expected return and standard deviation space. On this plot, their performance can be easily judged without even the numbers.

We can see that security E and F, their Sharpe ratios are above the capital market line CML. While portfolio lies on the CML while portfolio or security D lies below CML, it seems that security E and F are undervalued and therefore offers higher expected return and perform better in terms of Sharpe ratio while portfolio D is overvalued and performing badly if the Sharpe ratio is appropriate measure. These performances are obviously measured on risk adjusted basis. We are using Sharpe ratio as the appropriate measure. There are some key points to note here.

First, when we are talking about Sharpe ratio, the appropriate measure of risk that is considered here is sigma standard deviation which represents the total risk of the fund. The total risk of the fund or security includes market risk which is systematic risk and stock specific risk which is idiosyncratic or diversifiable risk. If the fund is well diversified, then in that case, most of the funds risk will be coming from the systematic part of risk. In most of these situations, there are small investors who are invested in these mutual funds. These are small retail investors who invest a sizable portion of their wealth in such funds individually.

Sharpe Ratio

The performance of these portfolios can be plotted on the capital market line (CML)
Portfolios E and F are above the CML line, indicating better risk-adjusted performance



Sharpe Ratio

- The measure of risk considered here is the standard deviation, i.e., total risk of the fund
- This includes the market risk (systematic risk) and stock-specific risk
- Please note that if the fund is well diversified, then most of the fund's risk will be systematic risk
- In most of the situations, the investors invested in the fund are small retail investors, who invest a sizable portion of their risk in the fund

Sharpe Ratio

- For the investor, the entire risk of the fund is important, not only the market risk part of it
- Since these investors rely precisely on the ability of the fund to diversify on behalf of them
- The Sharpe measure looks at the decision from the point of view of an investor choosing a mutual fund to represent the majority of his investment

And therefore, for this kind of a small retail investor, the entire risk of the fund is important, not only the market part of it because we are saying that his most of the or large proportion of his wealth is invested in this fund or portfolio. And therefore, essentially they are relying on the ability of fund or that portfolio to diversify on behalf of them. Therefore, we can say that Sharpe ratio measure or criteria looks at the investment decision from the point of view of an investor who has chosen this fund to represent majority of investment. That means most of his or her wealth is invested in this fund or portfolio and therefore depending upon the level of diversification, whether it is diversified or not so well diversified, the measure of this that matters to this investor is the total risk of that particular portfolio where she has invested most of her wealth. To summarize, in this video, we discuss Sharpe ratio.

Sharpe Ratio

- An investor choosing a mutual fund to represent a large part of her wealth would likely be concerned with the full risk of the fund, and the standard deviation is a measure of that risk
- The measure computes risk-premium earned per unit of total risk
- This measure uses CML to compare portfolios

We discussed that Sharpe ratio measures the risk adjusted performance by adjusting for

risk free rate against total risk which is standard deviation, which includes market risk as well as diversifiable or idiosyncratic risk. This measure is appropriate for a small retail investor who has invested majority of her wealth in one particular fund or mutual fund or portfolio and therefore the entire risk of that fund or portfolio is what is important for her and therefore it is the standard deviation or total risk of the portfolio that is the appropriate risk measure. To put it more precisely, an investor choosing mutual fund to represent a large part of her portfolio or wealth would likely to be concerned with the full risk of the fund and standard deviation therefore becomes the appropriate measure of risk. This measure computes the risk premium earned per unit of total risk which is market risks or systematic risk plus diversifiable risk or idiosyncratic risk. Also this measure uses Capital Market Line, CML to compare portfolios as we saw in this video earlier.

Treynor's Measure

$$\text{Treynor's measure} = \frac{\overline{R_p} - R_f}{\beta}$$

- Treynor's measure examines excess return with risk measure being beta
- For the diversified investors who only consider the systematic risk for performance evaluation, Treynor's measure is the appropriate measure
- Treynor's measure is applicable to majority of the investors irrespective of their risk preferences
- Treynor argues that rational, risk-averse investors would always prefer the portfolios on security market line

$$\text{Treynor's measure} = \frac{\overline{R_p} - R_f}{\beta}$$

In this video, we will discuss Treynor's measure which is a very important one parameter measure of portfolio performance. The formula for Treynor's measure is $\overline{R_p} - R_f$ upon beta. It is very similar in construction to Sharpe measure. The only difference here is that we examine the excess returns that is $\overline{R_p} - R_f$ against the risk measure which is beta. Beta remains here which is the sensitivity of any security towards market as the appropriate risk measure.

For any diversified investor who considers systematic risk or market risk to be the appropriate risk for performance evaluation, Treynor's measure is the appropriate measure.

It is applicable to majority of investors irrespective of their risk preferences as we will see shortly. Treynor argues that for risk averse investors, they would always prefer their portfolios on Security Market Line. Here, if I plot all the securities that are in equilibrium on two axis, one is beta and one is expected on RI bar, Security Market Line would look something like this. Here this is the RF and there is one point which is RM.

Treynor's Measure

$$\text{Treynor's measure} = \frac{\bar{R}_p - R_f}{\beta}$$

- The equation of SML is shown here: $\bar{R}_p = R_f + \beta_p * (\bar{R}_M - R_f)$
- Combined with Rf, this portfolio will generate the SML
- Any higher value would indicate that the portfolio offers excess risk-adjusted returns and plots above SML



$$\text{Treynor's measure} = \frac{\bar{R}_p - R_f}{\beta}$$

$$\bar{R}_p = R_f + \beta_p * (\bar{R}_M - R_f)$$

All the other securities at equilibrium will fall on this line and therefore, investor would not like to be either below, he would like to be on this line. Because at equilibrium all the securities are on this line, there is no security which is expected to be above or below in equilibrium. In short term due to friction some security may be above or below but ultimately they will be driven towards this line because of arbitrage argument. The slope of this curve, the slope of this curve is the Treynor's measure. The slope is this form coming from this formula.

Now in short term there may be securities that are above as well as below. For any investor, any security that charts the highest slope is always preferred. Any security that offers the highest slope or this Treynor's measure is always preferred. Here the appropriate risk measure as we discussed is beta which measures the systematic risk component of any security. Now this measure assumes that investors hold diversified portfolio and all the investors are risk averse and would like to maximize this Treynor's measure.

Treynor's Measure

Consider the information about three investment managers below (w, x, and y). In addition, we are given the market rate of return and the risk-free rate

Investment Manager	Average Annual Rate of Return	Beta	Treynor's Measure
w	12%	0.90	$(0.12 - 0.08)/0.90 = 0.044$
x	16%	1.05	$(0.16 - 0.08)/1.05 = 0.076$
y	18%	1.20	$(0.18 - 0.08)/1.20 = 0.083$
Market	14%	1.00	$(0.14 - 0.08)/1.00 = 0.060$
Risk-free	8%	0.00	

- These results indicate that Manager w not only performed worst among the three managers but performed worse than the market as well, on a risk-adjusted basis

Let us examine this Treynor's measure for market portfolio. For expected return \bar{R}_M , risk free rate R_F and beta M where beta M is 1 because its sensitivity of market with itself and therefore, the slope or Treynor's measure becomes \bar{R}_M minus R_F itself. If we believe that in equilibrium all the securities lie on this security market line that is R_F connecting R_F and \bar{R}_M , all the securities lie on this line. So the slope of this line remains same which is \bar{R}_M minus R_F which will be equal to \bar{R}_P minus R_F upon beta P for any security that lies on this SML. And therefore, this formula of SML becomes \bar{R}_P equal to R_F plus beta P times \bar{R}_M minus R_F which is very similar to what we saw about CAPM, capital asset pricing model which is nothing but this equation itself.

Treynor's Measure

These results indicate that Manager w not only performed worst across the three managers, but performed worse than the market as well, on risk-adjusted basis

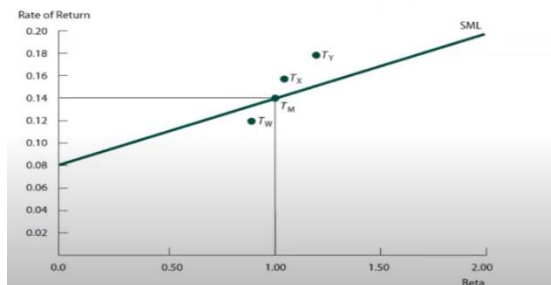
- While x and y performed better than the market, y performed best

Investment Manager	Average Annual Rate of Return	Beta	Treynor's Measure
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y	18%	1.20	$(0.18 - 0.08)/1.20 = 0.083$
Market	14%	1.00	$(0.14 - 0.08)/1.00 = 0.060$
Risk-free	8%	0.00	

Please note, if we examine this SML, any portfolio that lies above this SML offers excess research adjusted return and therefore, a large number of investors would like to hold this portfolio. This portfolio is obviously undervalued as per SML. If any portfolio is below this line, that means it is overvalued and lot of investors would like to either liquidate or short this portfolio. Let us understand this through a simple example. Consider three managers W, XY and their performances shown here in the form of average annual returns and the beta of their portfolios W, XY and market.

Treynor's Measure

Their performance on SML can be plotted as follows



We can compute Treynor's measure for them. Here these Treynor's measure for W, XY and market are computed. We can clearly see that manager W performed worst while X and Y performed better as compared to the market. So, when we are comparing it in market, that means X and Y are above SML line, W is below SML line and Y has performed in fact best. These results clearly show that manager W not only performed worst across all the three managers but also performed poor than market and therefore, he will lie below SML while managers X and Y, they perform better, Y performed best and both of these X and Y will be above the SML.

As we can see here on this chart, W here is below market portfolio while X and Y are above market which is SML line. So, X and Y are above SML line, W is below SML line, their performances are easily plotted and that can be understood. However, there is a very fundamental problem with this measure which is as follows. Consider two portfolios. First portfolio which offers a very poor performance and its R_p bar is even less than R_f and therefore, if it has positive beta assuming that it has positive beta, this measure would be negative less than 0.

Treynor's Measure

What is the challenge with this measure?

- Consider two portfolios: one, which offers a return that is below risk-free (although with the positive beta) \rightarrow
- The negative measure would indicate poor performance
- Even when plotted on SML, this point would indicate a very poor performance
- Second, consider a security with a negative beta that offers a very high return above the risk-free rate
- This would also offer a negative measure despite good performance

At the same time, this measure while this measure indicates a very poor performance at the same time consider another security with a negative beta while it offers a very high R_p bar, high R_p or portfolio returns. Given that it has a negative beta, the Treynor's measure would indicate a poor performance despite this portfolio offering a very high R_p and therefore, a very good performance. Let us understand this with a simple numerical example. For example, consider a portfolio of gold mining stocks with a beta of minus 2, it performed very well and offered 10% return. Therefore, the performance measure is 0.

1 minus 0.08 divided by minus 0.2 which is equal to minus 0.10 which is negative. So, it

suggests, the measure suggests a very poor performance, but if we plot it on SML, notice the expected returns from this portfolio R_f plus beta gold into R_M minus R_f which is 0.

Treynor's Measure

What is the challenge with this measure?


- For example, a portfolio of gold mining stocks with a beta of -2 performs well and offers a 10% return
- Then the measure would be $(0.10 - 0.08)/(-0.2) = -0.10$
- However, if plotted on SML, this will be above SML and indicate exceptional returns
- See for example. $E(R_{gold}) = R_f + \beta_{gold}(R_M - R_f) = 0.08 + (-0.2)(0.14 - 0.08) = 6.8\%$ expected returns, which is lower than the actual return of 10%. Thus, the point will be above SML

$$E(R_{gold}) = R_f + \beta_{gold}(R_M - R_f) = 0.08 + (-0.2) * (0.14 - 0.08) = 6.8\%$$

08 plus minus 0.2 into 0.14 minus 0.08 which is 6.8% which is much lower than the actual return of 10% which suggests this point is way above on the SML line it is much much above. The expected performance is 6.8% while it is offering 10% and therefore, it has performed much better and Treynor's measure incorrectly suggests that this is a poor performing portfolio. To summarize in this video, we discussed that Treynor's measure is appropriate measure for those investors who hold diversified portfolios and the appropriate risk measure for these investors is beta.

Treynor's Measure

What is the challenge with this measure?

- Consider two portfolios: one, which offers a return that is below risk-free (although with the positive beta) 
- The negative measure would indicate poor performance
- Even when plotted on SML, this point would indicate a very poor performance
- Second, consider a security with a negative beta that offers a very high return above the risk-free rate
- This would also offer a negative measure despite good performance

However, Treynor's measure also has certain problems as we saw with portfolios and securities having negative beta this measure incorrectly shows poor performance. In this video, we will discuss a very important one parameter measure of portfolio performance which is Jensen's alpha. Jensen's alpha essentially is the differential in the return as predicted by CAPM or security market line. If you remember our security market line is represented as \bar{R}_p expected return minus R_f equal to beta times \bar{R}_M minus R_f . \bar{R}_p is the expected return on security or portfolio while \bar{R}_M is the expected return on market, beta is the sensitivity to what is market and R_f is the risk period.

If we run this kind of regression model, we are expecting to obtain SML, this SML line. However, if a security is above or below which indicates over or under performance as per the Jensen's measure, how do we measure it? We run this kind of regression model where \bar{R}_p minus R_f is our dependent variable by \bar{R}_M minus R_f is our independent variable. If we run this kind of regression and we find a slope coefficient, this slope coefficient to be significantly positive, theoretically we are expecting a zero intercept that is we are expecting this kind of model to have a zero intercept. However, if we find a significant value here, let us call it alpha p, if this alpha p is significant and positive that indicates that differential indicates measure of performance, positive performance by that fund manager. One here is that CAPM is the appropriate model which is represented by the security market line as shown here.

Jensen's Measure (α)

- Jensen's measure is the differential in the return as predicted by the CAPM model
- $R_p = \alpha_p + \bar{R}_p = \alpha_p + R_f + (\bar{R}_M - R_f)\beta_p$
- \bar{R}_p is the expected return. Then $R_p - \bar{R}_p$, this differential return is called the Jensen's measure of performance
- The key assumption here is that CAPM is the guiding model

$$R_p = \alpha_p + \bar{R}_p = \alpha_p + R_f + \beta_p * (\bar{R}_M - R_f)$$

If we find this alpha p to be significantly positive that indicates a higher performance as per Jensen's alpha measure. Theoretically, in this kind of model, we are expecting alpha p to be zero which suggests that CAPM is appropriately followed or SML line, all the securities are on SML line, that SML line. If security is above this SML line that is it is undervalued, then in that case for that kind of security we are expecting to find the positive and significant alpha. The presence of this positive intercept term, a constant alpha j or alpha p would suggest the ability of security selection or higher security selection ability or predicting a better performance ability to generate higher performance by a portfolio manager. A negative alpha would similarly indicate a poorer performance especially if it is significant as well.

Jensen's Measure (α)

- $R_{pt} - R_f = \alpha_p + \beta_j [R_{mt} - R_f] + e_{jt}$
- In this model, we expected $\alpha_p = 0$
- Presence of positive intercept (constant term) α_j would indicate the ability of security selection or predicting the market performance by a portfolio manager
- A negative Alpha would indicate poor performance

$$R_{pt} - R_f = \alpha_p + \beta_p * (R_{mt} - R_f) + e_{jt}$$

To summarize, in this video, we discussed Jensen's alpha measure which compares the performance of a security or portfolio along the SML or CAPM, capital asset pricing model or security market line. If a security is above this security market line, then in that case, there is a significant additional risk adjusted performance which is measured by this alpha p measure. If it is positive and significant, then definitely that manager has overperformed. If it is negative and significant, then that manager has underperformed. In this video, we will discuss a very important one parameter measure of portfolio performance that is information ratio measure.

Information ratio measure is computed with this formula $\overline{R_p} - \overline{R_b}$ upon σ_{ER} . Here, $\overline{R_p}$ is the return on a portfolio that is under evaluation. $\overline{R_b}$ is the return on a benchmark portfolio against which this $\overline{R_p}$ is compared. And therefore, $\overline{ER_B}$ is basically expected excess return on portfolio as compared to the benchmark. σ_{ER} is the standard deviation of excess returns.

Information Ratio (IR) Measure

Information ratio (IR) measure: $\frac{\overline{R_p} - \overline{R_b}}{\sigma_{ER}} = \frac{\overline{ER_B}}{\sigma_{ER}}$

- Here, $\overline{R_p}$ is the return on a portfolio, $\overline{R_b}$ is the return on the benchmark portfolio
- $\overline{ER_B}$ is the excess return. σ_{ER} is the standard deviation of excess returns
- The numerator here measures the ability of the portfolio manager to perform better than a given benchmark (e.g., Nifty)

$$\text{Information ratio measure: } \frac{\overline{R_p} - \overline{R_b}}{\sigma_{ER}} = \frac{\overline{ER_B}}{\sigma_{ER}}$$

The numerator here, this $\overline{R_p} - \overline{R_b}$ measures the ability of this portfolio manager to perform better than a given benchmark. Benchmark portfolio can be a market portfolio like Nifty Fifty or any other such portfolio of interest which is taken as benchmark. The denominator part σ_{ER} which measures a sort of residual or incremental risk because it is the difference between, it is the standard deviation of returns or difference between the returns in portfolio versus benchmark return which is $\overline{R_p} - \overline{R_b}$ standard deviation of that those excess returns. So it represents the excess risk that manager took to obtain these excess returns. And therefore, in a way this IR can be interpreted as benefit to cost ratio.

It sort of evaluates the quality of information with the manager or stock selection ability

which is adjusted by the non-systematic risk measured by the standard deviation of these excess returns. Let us examine this with the help of a simple example. Quarterly returns for a particular portfolio are provided to us and we are supposed to calculate the information ratio for this portfolio. So these are the quarterly returns for the portfolio and the quarterly returns for benchmark portfolio. In order to compute IR ratio, we will compute first the differences between portfolio and benchmark returns.

Information Ratio (IR) Measure

$$\text{Information ratio (IR) measure: } \frac{\bar{R}_P - \bar{R}_B}{\sigma_{ER}} = \frac{\overline{ER}_B}{\sigma_{ER}}$$

- The denominator measures the residual (or incremental) risk that the manager took to obtain these excess returns
- Thus, IR can be interpreted as a benefit to cost ratio
- It evaluates the quality of information with the manager (or stock selection ability) adjusted by the non-systematic taken by the investor

$$\text{Information ratio measure: } \frac{\bar{R}_p - \bar{R}_b}{\sigma_{ER}} = \frac{\overline{ER}_B}{\sigma_{ER}}$$

These are the differences. Then subsequently once we have these differences which is \bar{R}_P bar minus \bar{R}_B bar, we will compute the average of these differences that is \bar{R}_P bar minus \bar{R}_B bar. We have the average \bar{R}_P bar here and \bar{R}_B bar here, average of returns. And then we will compute the standard deviation of these differences. The standard deviation works out to 1%.

Information Ratio (IR) Measure

- IR = 0.2%/1%=0.20; this represents the manager's incremental performance (Alpha, relative to the index) per unit of risk incurred in the pursuit of those active returns
- IR will be only positive when the manager outperforms his benchmark

Quarter	Portfolio Returns	Benchmark Returns	Difference
1	2.30%	2.70%	-0.40%
2	-3.60%	-4.60%	1.00%
3	11.20%	10.10%	1.10%
4	1.20%	2.20%	-1.00%
5	1.50%	0.80%	1.10%
6	3.20%	2.80%	0.40%
7	8.90%	8.10%	0.80%
8	-0.80%	0.60%	-1.40%
Average	2.99%	2.79%	$\bar{R}_P - \bar{R}_B = 0.20\%$
SD			$\sigma_{ER} = 1.00\%$

So the information ratio here is 0.2% upon 1% which is 0.20. This information ratio here represents manager's incremental performance sort of alpha relative to a given index, the benchmark returns per unit of the risk or sigma ER incurred in pursuit of these returns. Now this IR will be positive when the manager outperforms the benchmark. This RP bar minus RB bar, so when he outperforms the benchmark then this IR will be positive and it is adjusted by those sigma ER, the non-systematic or additional risk which is the standard deviation of excess returns here.

To summarize, in this video we discussed information ratio measure. The measure compares a portfolio with a benchmark index or portfolio. In the numerator we have excess returns as compared to the benchmark divided by additional risk which is the standard deviation of these excess returns in the pursuit of generating or obtaining these excess return that standard deviation or total risk. Performance measurement with downside risk, sortiness ratio. In this video we will discuss a very important measure of portfolio performance which is sortiness ratio.

Sortino's Ratio

Sortino's ratio: $\frac{\bar{R}_p - T}{D_R}$

- Here, D_R is the downside risk. Total risk, i.e., SD, includes upside and downside both risks
- T is the target rate of return
- In most of the computations, $T = R_f$ (risk-free rate) or some target return as set by fund management

• $D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$

$$\text{Sortino's ratio: } \frac{\bar{R}_p - T}{D_R}$$

$$D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$$

The measure captures the downside risk of the portfolio. To begin with, sortiness ratio formula shown here $\frac{\overline{R_p} - T}{D_R}$, D_R here is very important term. Unlike other performance measure that we have discussed till now, D_R only captures the downside risk. The idea here is that often there are instances where return on a portfolio is higher and sometimes lower as well. However, investors may be particularly sensitive and a lot of behavioral finance literature suggest they are indeed sensitive and aberrant to these lower performances and therefore for them it is this lower performances or negative performances in stock return that are of more importance.

Its computation we will shortly discuss. The standard deviation which is total risk includes both upside and downsides but this D_R is more focused on the downside. T here is some kind of target return. It can be a benchmark like market index or some sort of benchmark where the performance of $\overline{R_p}$ is compared. In many computations T is also considered as risk free rate as may be said by the some kind of target return as said by the management.

Sortino's Ratio

Sortino's ratio: $\frac{\overline{R_p} - T}{D_R}$; $D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$

- MAR is the minimum acceptable rate of return, often considered as the target return
- Also, in most of the computations, $\text{MAR} = \text{Average returns } (\overline{R_p})$
- Risk-free returns (R_f), also in some cases, MAR can be = 0
- Sortino's measure measures returns in excess of a pre-defined target rate

$$D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$$

DR formula is interesting. It is the standard deviation but here if the return is lower than some kind of minimum acceptable rate MAR is here is the minimum acceptable rate. If return is lower than that only then it is considered. So only those return instances where returns are lower than MAR are considered for computation of this standard deviation while there RI is greater than MAR then the observations are taken as 0. So while computing this standard deviation or measure of risk DR we take all those observation where RI is greater than some kind of acceptable rate then we take those observations as 0 and only those instances where it is less than MAR then those instances are only considered. In the formula of Sautino's ratio this DR here is the standard deviation but it captures only the downside risk.

Sortino's Ratio

$$\text{Sortino's ratio: } \frac{\bar{R}_p - T}{D_R}; D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$$

- This excess return is not adjusted by the total risk (SD) but only by the downside risk
- The downside risk is computed against some minimum acceptable returns
- This kind of downside risk is often considered more appropriate because the downside volatility is often associated with a shortfall
- Thus, this downside risk can be considered to capture the fear of investors more efficiently

$$\text{Sortino's ratio: } \frac{\bar{R}_p - T}{D_R}$$

$$D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$$

For example, MAR employed in this is some kind of minimum acceptable rate of return which is often considered some kind of target return where we are comparing our returns with this particular return. It may be a particular average of the security return historical average of the portfolio or may be risk free rate or can be 0 as well. The idea here is that numerator measures excess return against a predefined target this target can be again some kind of benchmark. So the numerator is the excess return over a benchmark while the denominator the risk part only measures those instances where return is lower than some kind of minimum acceptable return.

So only those downside instances are considered in this formula. The idea here is that this excess return which is $\bar{R}_P - T$ is not adjusted by the total risk or standard deviation but only the downside risk which is those instances where the return achieved is lower than some kind of minimum acceptable rate. This downside risk is computed against some kind of minimum acceptable rate MAR and it is considered more appropriate because often for investors the downside volatility is more important because it is associated with their drawdown levels or shortfall in their portfolio portfolios. Often this downside risk captures the fear of investors and it is more efficient in capturing the fear of investors. To summarize in this video we discussed Sortino's ratio. Sortino's ratio is a measure of performance where only downside risk of the portfolio is considered.

Sortino's Ratio

Consider the example below, where we compare the Sharpe and Sortino measures with a risk-free rate of 2%

Year ($R_f = 2\%$)	Portfolio A Return (%)	Portfolio B Return (%)
1	-5	-1
2	-3	-1
3	-2	-1
4	3	-1
5	3	0
6	6	4
7	7	4
8	8	7
9	10	13
10	13	16
Average	4	4

Then downside risk measures are more appropriate in capturing investor fears more efficiently. In this video we will understand Sortino's ratio with a simple example where we will compare the performance of two portfolios based on Sharpe ratio and Sortino's ratio. Consider the following example here where we have annual returns for a given portfolio A and portfolio B are provided along with the risk-free rate of 2%. We have computed the average values of returns for portfolio A and portfolio B and their standard deviations. Since we have the average returns we have R_f , we have the average returns \bar{R}_P and their standard deviations we can easily compute their Sharpe ratio as 0.

Sortino's Ratio

The Sharpe ratios of portfolios A and B are computed as follows

- $S_A = \frac{4-2}{5.60} = 0.357$ and $S_B = \frac{4-2}{5.92} = 0.338$
- Based on these numbers, it appears that portfolio A outperformed portfolio B
- Let us see what happens when we only consider the downside risk
- Let us use the average return of 4% as MAR to compute the downside return, and target return T as a risk-free return

0.357 for portfolio A and 0.338 for portfolio B. Based on these figures it appears that portfolio A outperforms portfolio B. Let us see what happens when we only consider downside risk. Let us use a figure of 4% as minimum acceptable rate of return to compute the downside risk and target return T as risk-free return. Also in this example we will consider all those instances where return on portfolio is greater than 0 as 0. In order to compute the Sortino's ratio the following formulas for portfolio A and B will be employed are $\bar{R} - T$, T has the target return as we said we will use the target return as risk-free return 2% and DR will be computed against MAR minimum acceptable return of 4% while keeping all those instances where RI or portfolio returns R_P for portfolio A and B are greater than 0 we will consider those situations as 0.

So here MAR is 2%, target rate is 4% and wherever the portfolio returns are positive we are considering those instances as 0. So our downside measures of risk can be easily computed with this formula. Notice that we are only considering positive returns here for computation of downside risk. We are considering all positive returns as 0, those instances as 0s only negative returns are considered and based on that the computation becomes simple. For example here you have minus 5, so minus 5 minus 4 which is MAR raised to the power 2 similarly you have minus 3 so minus 3 minus 4 raised to the power 2 this is for portfolio A.

Similarly for portfolio B also we can compute that downside risk. Now it appears that downside risk measure for portfolio A is 4.1 and for portfolio B is 3.41. So the Sortenos ratio becomes, the Sortenos ratio here is 4 minus 2 which is RP minus T upon 4, T is RFI which is 2% and downside risk measures we have computed for these 4.

10 and 3.41. So the figure for Sortenos ratio 0.488 for portfolio A and 0.587 for portfolio B. So based on these results it appears that as per Sortenos ratio portfolio B appears to have performed better. This may be the case because portfolio A appears to have a lot of negative returns. So lot of large negative returns is related with portfolio A and lot of risk averse investors would probably be uncomfortable with this kind of aspect of portfolio A. To summarize in this video we discussed Sortenos ratio measure we understood with an example we compared the performance as per Sharpe ratio for two portfolios A and B and well as Sortenos ratio.

Sortino's Ratio

The Sortino's ratios of portfolios A and B are computed as follows

- All the positive returns are considered zero

- Sortino's ratio: $\frac{\bar{R}_p - T}{D_R}$

- $D_R = \sqrt{\frac{1}{N} \sum_{i=1}^N (\text{Min}(0, R_i - \text{MAR}))^2}$

Sortino's Ratio

Let us see what happens when we only consider the downside risk

- Let us use the average return of 4% as MAR to compute the downside return
- All the positive returns are considered zero

$$DR_A = \sqrt{\frac{[(-5-4)^2 + (-3-4)^2 + (-2-4)^2 + (3-4)^2 + (-3-4)^2]}{10}} = 4.10$$

$$DR_B = \sqrt{\frac{[(-1-4)^2 + (-1-4)^2 + (-1-4)^2 + (-1-4)^2 + (0-4)^2]}{10}} = 3.41$$

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We found that while on the overall risk measure sigma as per the Sharpe ratio portfolio A performed better but when we only considered the downside risk measure of Sortenos ratio portfolio A performed poorly as compared to portfolio B. This happened because in a number of instances portfolio A offered large negative returns which may be not very suitable or some of the risk averse investors may be uncomfortable with this kind of property. To summarize one parameter measures are extremely simple and intuitive measures of portfolio performance. These measures evaluate the performance after adjusting for the risk of the portfolio.

However different measures account for the risk in a different manner. Therefore their utility to investors depends upon the investment profile. For example if portfolios are well diversified then Sortenos measure and Sharpe give the same results. However for poorly diversified portfolio one can get a high rank on the Sortenos measure as it ignores the systematic risk despite performing poorly on Sharpe measure. Also to be noted that these measures provide comparisons and produce relative rankings not absolute performance rankings. In this regard the advantage of Jensen s alpha is that it produces an absolute measure.

For example an alpha value of 2% would indicate that manager generated an excess return of 2% per period more than the expected returns. Also the result from Jensen's alpha has certain statistical significance. Moreover Jensen's alpha has the flexibility to compute the alpha with respect to any given model. Another class of measures capture the downside risk dimension only. For example Sortino's ratio. Thank you.

Sortino's Ratio

With Sortino's ratio, portfolio B appears to perform better

- This happens because portfolio A appears to have more extreme negative returns
- Various risk-averse investors would be uncomfortable with this aspect of portfolio A

Year ($R_f = 2\%$)	Portfolio A Return (%)	Portfolio B Return (%)
1	-5	-1
2	-3	-1
3	-2	-1
4	3	-1
5	3	0
6	6	4
7	7	4
8	8	7
9	10	13
10	13	16
Average	4	4
SD	5.60	5.92