Advanced Financial Instruments for Sustainable Business and Decentralized Markets

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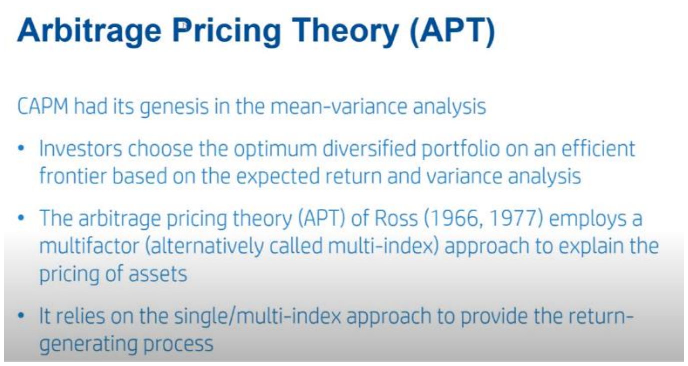
Week 4

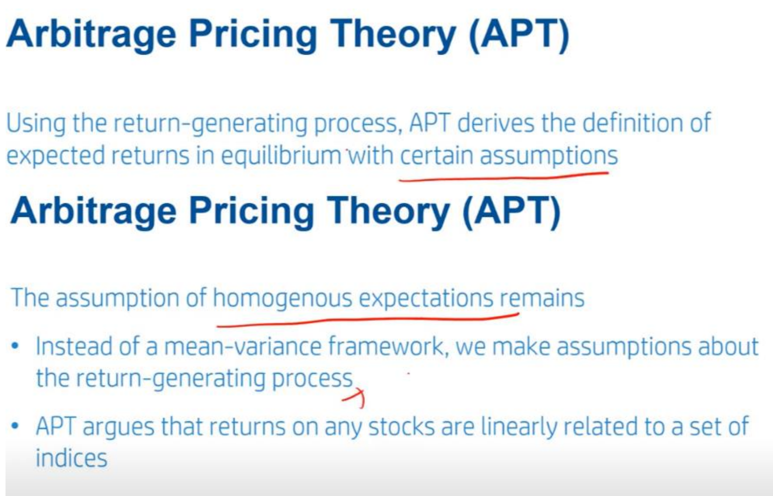
Lesson 13

In this lesson, we will introduce arbitrage pricing theory as application of index models in portfolio management. We will discuss a simple graphical proof of APT. Next, we will discuss various approaches to test APT. We will also try to understand whether CAPM becomes inconsistent in the presence of APT. Lastly, we will discuss the application of these asset pricing models such as APT in active and passive fund management. We will go through a few interesting examples to demonstrate the application of these models in fund management industry.



Arbitrage pricing theory APT. In this video, we will introduce arbitrage pricing theory of ROS. Before we start with APT, please remember that CAPM had its genesis in the mean variance analysis. That is, investors choose the optimum diversified portfolio on an efficient frontier based on the expected return and variance analysis.

In contrast, the arbitrage pricing theory of ROS which is APT employs a multi-factor approach also called multi-index model approach to explain the pricing of assets. It relies on the single and multi-index approach to provide the return generating process. Using this return generating process, APT derives the definition of expected returns in equilibrium with certain assumptions. The definition is as good as these assumptions are held. At the heart of this approach is the arbitrage argument and therefore the name APT or arbitrage pricing theory.



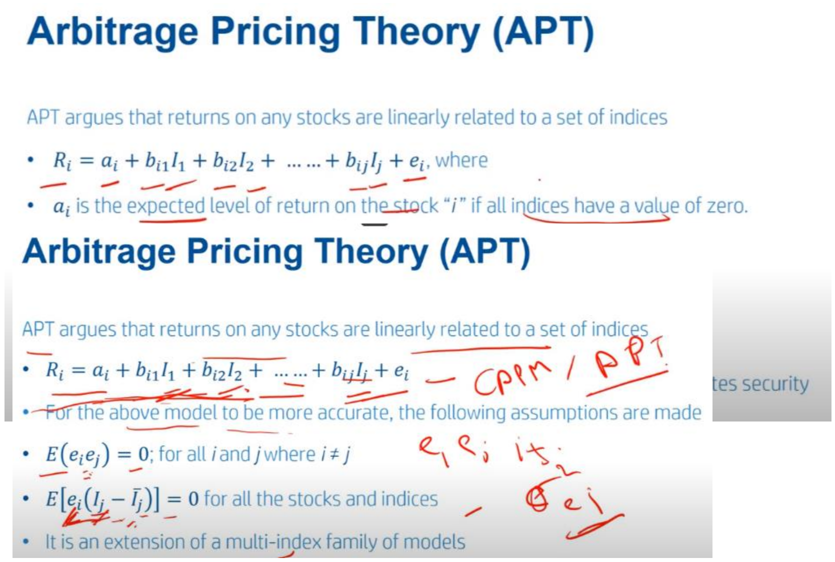
This APT is based on the law of one price. That is two items with the same risk profile and cash flows cannot sell at different prices. This is at the heart of APT. APT is more general in the sense as compared to CAPM. It is more general that unlike CAPM, it does not restrict the forces that affect pricing to expected mean and variance.

So, it is more generic in that sense. Nonetheless, the assumption of homogeneous expectations is also there, which is similar to CAPM. That assumption remains. The assumption of the mean variance framework is replaced by the assumption about the process generating stock returns and that process comes from the single and in multi-index models that we have already seen. For example, APT argues that returns on a stock are linearly related to set of indices like I1, I2 and so on Ij.

Here these I1s can be indices like oil and gas sector index or banking sector index or broad macroeconomic influences like market influences. So, Ri equal to alpha i plus bi1 into I1 plus bi2 into I2 and so on up till bij into Ij plus ti. This equation is precisely the return generating process that we are talking about. Alpha i here is the expected level of return on stock i if all the indices have zero value of the sulphide which is also stock specific term. Ij are the values of jth index for example I1, I2 and so on that affects the return on stock i.

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots b_{ij}I_j + e_j$$

Now these are the broad influences that affect a large number of stocks at the same time



in a similar manner. Bij is the sensitivity of stock i to the jth index. It is something similar to beta that we have studied in the context of CAPM. So, for example, Bij here is the sensitivity of stock i to influence ij. Again, Ei is the random error term with a mean zero and variance equal to sigma Ei square.

This Ei or the residual or error term is what reflects the idiosyncratic or stock specific risk which is captured by this variance sigma Ei square. So, essentially this above mentioned equation that we can see here this equation describes the process that generates security returns. So, as we have said already this APT theory argues that returns on any stocks are linearly related to a set of indices like I1, I2 and so on and this relationship is reflected in this process which is shown here by this equation Ri equal to alpha i plus Bi1 into I1 and so on as return generating process. So, for the above model to be accurate description of reality there are certain assumptions and the assumptions are not alien to us. For example, expected value of Ei into Ej equal to zero which means Ei and Ej are correlated.

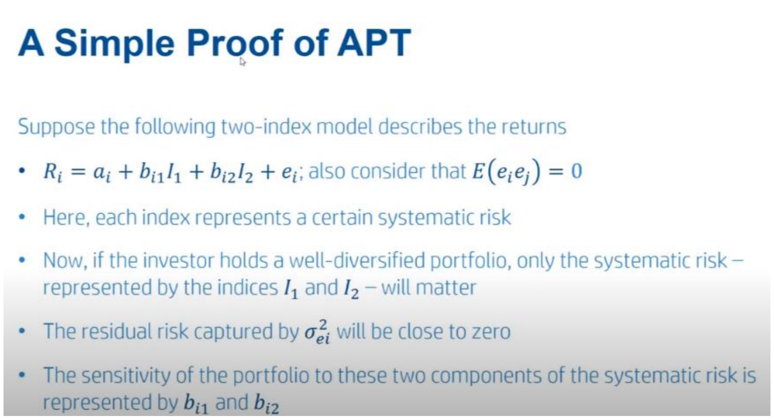
$E(e_i e_j) = 0$

$E[e_j (I_j - \bar{I}_j)] = 0$

Here i is assumed to be not equal to j that means these Ei and Ej are stock specific components they are not supposed to be correlated. Any correlation between Ei and Ej would be sort of violation of these index models. The idea is that any variation in stock which is common across different stock is captured by these broad market-bound basis. So, there should not be any correlation left between Ei and Ej and should be captured by this model itself. And therefore, there should not be any correlation explicitly between these residuals stock specific component of variance.

$E[e_j (I_j - \bar{I}_j)] = 0$

If you remember we call the variance of these Ei s as sigma square Ei which is the stock specific component of variance or variance which is specific to stock. We also assume that expected value of Ei into Ij minus Ij bar is zero that means the correlation between residual and indices is also zero. So, this Ei is purely stock specific and should not be having any relationship with these stock indices. This is Ei stock specific and these Ij s are indices that are broad market-bound indices affecting a large number of securities. So, the stock specific component which is this Ei is supposed to be uncorrelated with the indices Ij s.



$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots b_{ij}I_j + e_j$

This is to a large extent ensured by regression model. So, these models are estimated with the help of some kind of regression. It is assumed and by the design of regression model to a large extent this property is held. And we would have noticed by now that this is a familiar expression this equation which is a part of multi-index family models. So, to summarize we have understood that this model is basically the definition of multi-index family of models.

Essentially APT is the description of expected returns when the process that generate these returns can be defined by single or multi-index model as seen here. The precise contribution of APT is in reaching the equilibrium expected returns from a given single or multi-index model family return generating process. Lastly, in this video we also discussed or compared made a comparison between CAPM and APT. A simple proof of APT part 1. In this video, we will discuss a simple visual proof of arbitrage pricing theory APT.

Let us consider a simple two-index model shown here that is Ri equal to alpha i plus Bi1 into i1 plus Bi2 into i2 plus Ei. In this model, we also know based on the assumptions of Ei Ej that expectations of Ei Ej is 0 that means Ei and Ej are not correlated. Here each index i1 and i2 represents a certain systematic risk like market risk or industry sector specific risk. Now if an investor holds a well diversified portfolio, then only systematic risk like i1 i2 should matter and idiosyncratic stock specific risk which is driven by this Ei residual is 0 that means the variance of this Ei is sigma Ei square. The risk component is supposed to be idiosyncratic and stock specific so it should be close to 0.

And therefore the sensitivity of the portfolio towards these two components i1 and i2 that represents systematic risk which is also called Bi1 Bi2. This is something similar to beta in the context of capital. Here each index i1 i2 represented a certain systematic risk. It may be related to for example oil and gas firm or a firm in energy or the risk of recession or risk of FMCG firm. And if the investor held a well diversified portfolio only the systematic risk represented by i1 i2 will matter.

The residual risk as we discussed sigma Ei square will be close to 0. Now let us take a simple example here. Consider three well diversified portfolios as shown here A portfolio B and C their expected returns are given to us and their sensitivities towards these indices i1 and i2 are also provided to us. For example expected return of security C is 10% its sensitivity towards index 1 is 0.3 which is the beta or Bi1 and its beta or Bi2 with respect to security index i2 index i2 is 0.

Please note that these returns that we are discussing are returns at equilibrium that means there is no arbitrage. Also please remember our discussions over CAPM where the sensitivity towards a single index which was market portfolio in the context of that single index in CAPM that single index was market portfolio. Although securities that were in equilibrium as per CAPM they were defined by a line called SML security market line which had two axes R for expected return and B1 which was the sensitivity with respect to market or you can call it beta. Now in this case since there are two indices and we have two sensitivities B1 and B2 therefore we can safely assume that these portfolios will lie on a plane like this where one axis is expected return axis on security i one is Bi B1 and one is B2 or you can call them Bi1 or Bi2 as well. If we believe in that plane the generic equation of any plane which passes through three axis can be those axis are Bi1 Bi2 and expected return Ri bar that plane can be easily described by this equation.

A Simple Proof of APT

Consider the three well-diversified portfolios shown below

Portfolio	Expected Return (%)	b_{i1}	b_{i2}
A	15	1.0	0.6
B	14	0.5	1.0
C	10	0.3	0.2

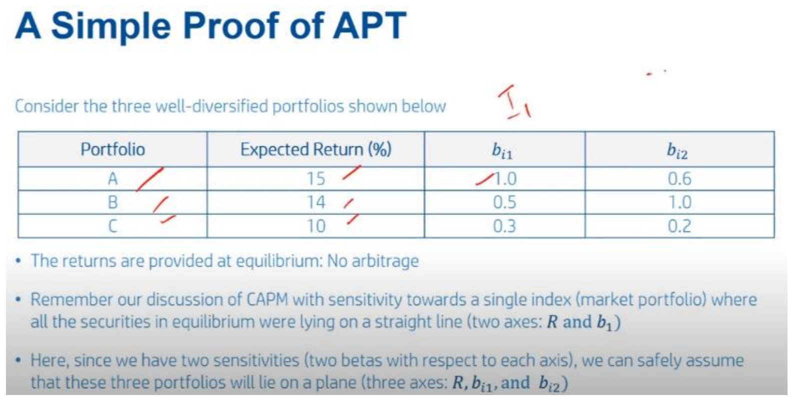
- The generic equation for a plane is as follows: $\bar{R}_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$
- Can we solve for the values of λ_0, λ_1, and λ_2 using the values provided in the table?

The mathematical proof of this equation is not discussed here Ri bar equal to lambda naught plus lambda 1 times Bi1 plus lambda 2 times Bi2. Here lambda naught lambda 1 and lambda 2 are unknowns we have a equation three unknowns and three observations we have A B and C. So therefore we can find a deterministic solution we can find a solution for this equation using the information from these points and we can solve for the values of lambda naught lambda 1 and lambda 2. Let us do that. Once you solve the equation of plane using these three information point you get this equation Ri bar equal to 7.

$$\underline{Ri} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i12}$$

unknowns lambda 1 lambda 2 which are obtained here. Now consider a third portfolio E with expected return of 15 percent Bi1 of 0.

6 and Bi2 of 0.6. Let us compare this portfolio E with another portfolio D that places one

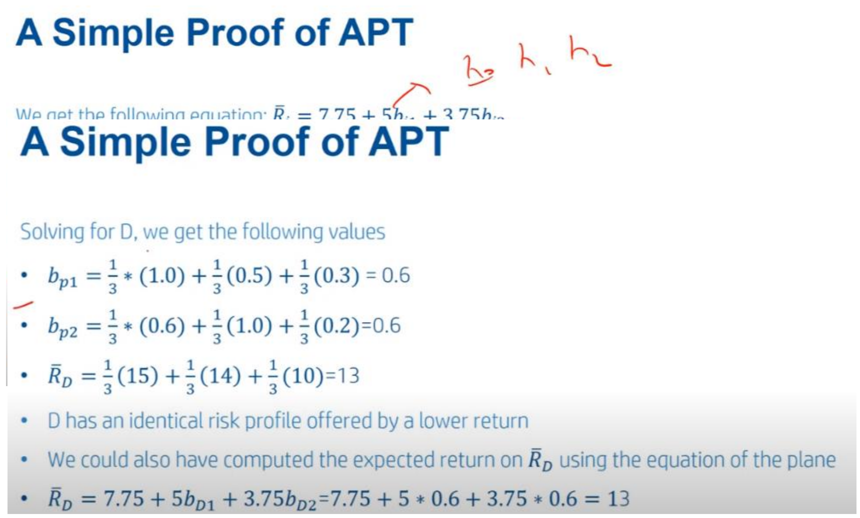


third of the amount in portfolio A one third in portfolio B and one third in portfolio C. How do I compute my expected return on portfolio D and its Bi's noting that it invest one third in all the securities A B and C it is quite easy knowing that this is a one third portfolio of ABCD we can provide sorry ABC we can provide one third of the expected returns which is 13 percent and we can similarly we can compute the averages of Bi1 and Bi2 which is shown here for example for this portfolio we can call it BP1 since the portfolio BP1 is 0.6 which is nothing but a simple average of these three values and similarly for Bi BP2 we can compute the simple average of these three values to get this value 0.6 although we could have already computed this with the help of this equation of plane by substituting values we could have computed the expected return easily but a simple average would also do so the expected return is 13 percent and sensitivities are 0.

6 and 0.6 with respect to index I1 and I2. Let us compare this portfolio D with portfolio E notice that portfolio E has same Bi's that means Bi1 and Bi2 are same as 0.6 as portfolio D but the expected return it offers is higher than portfolio D which was 13 percent and therefore it appears that portfolio E is undervalued because it offers a higher return as compared to portfolio D despite the fact that the risk profile is same since these are diversified portfolios only the systematic risk will matter which are represented by Bi1 and Bi2 which are the sensitivities of portfolios I and D with respect to the only two possible indices as assumed I1 and I2. Since E offers a higher expected return by the arbitrage argument or law of one price these two portfolios cannot sell for a different price for a very long time or have expected returns that are different from each other which in case of portfolio E is 15 percent and in case of portfolio D it is 13 percent that means portfolio E is undervalued and offers a higher expected return. So the arbitrageurs or in

general you can say investors as soon as they observe this kind of mispricing they would buy E they will go long in E buy it and sell D short which guarantees a riskless profit of 2 percent where riskless because the risk profile of portfolio E and D as represented by Bi1 and Bi2 is identical so when they go long in E and short in D they essentially eliminate this risk that is coming from Bi1 and Bi2 and therefore as more and more people will buy E, E will start to fall its expected return will fall its price will rise and relative the price or expected return related to portfolio D will change for example relative to D its expected returns will fall and relative to D its price will rise till the time that both of them will reach the equilibrium plane.

For example if E is away from plane and D lies on the plane and E is much further away then E will be driven towards this plane and will meet point D within a very short while. It will continue to fall until it reaches this plane which is defined by points A, B and C. We have already identified a point which is point D corresponding to point D and E will fall on this point. This plane that we are discussing it is made on three axis. Axis 1 is



expected return Ri bar, 1 is Bi1 which is the risk or sensitivity with respect to index i1 and Bi2 which is the risk or sensitivity with respect to index i2.

$$b_{p1} = \frac{1}{3} * (1.0) + \frac{1}{3}(0.5) + \frac{1}{3}(0.3) = 0.6$$

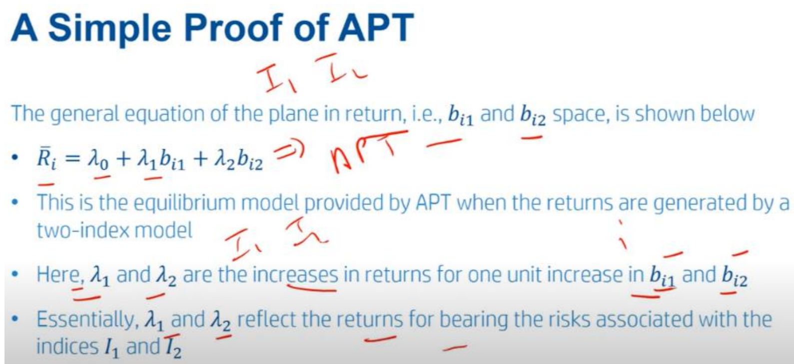
$$b_{p2} = \frac{1}{3} * (0.6) + \frac{1}{3}(1.0) + \frac{1}{3}(0.2) = 0.6$$

$$\underline{R_D} = \frac{1}{3} * (15) + \frac{1}{3}(14) + \frac{1}{3}(10) = 13$$

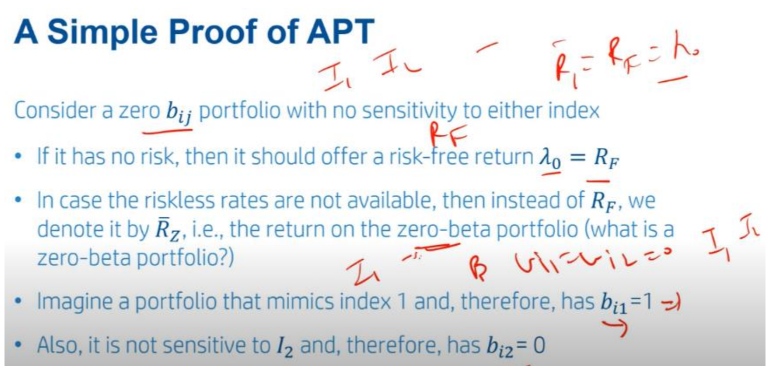
So any securities whether it was undervalued or overvalued at equilibrium it will be driven towards this equilibrium plane if it is overvalued then it will be below and if it is undervalued it will be above. It will be driven towards this plane by arbitrage argument because if it is not on the plane this will lead to arbitrage opportunity and such securities that are undervalued or overvalued they will converge to this plane. To summarize in this video we discussed the simple visual proof of arbitrage pricing theory. We saw that arbitrage pricing theory is derived by law of one price or arbitrage argument that is if a security is underpriced it will fall above that equilibrium plane. If it is overpriced it will fall below that equilibrium plane.

This equilibrium plane is defined by three axis. What are these axis? These axis are return expected return axis Ri bar, Bi1 sensitivity towards index i1 and Bi2 sensitivity of the security towards index i2. By arbitrage argument or law of one price if a security is mispriced and not on this plane whether up or down it will be driven by arbitrage trading activity it will be driven towards this plane let us say if it is underpriced and it falls above its expected returns are higher then because of arbitrage argument it will be driven towards this plane. If it is overpriced it will be below this plane and driven upwards towards this plane because of arbitrage opportunity and ultimately within a short frame it will lie on this arbitrage equilibrium plane.

A simple proof of APT part 2. In this video we will conclude our discussion on the proof of APT with our broad understanding of various properties related to PPT and the equilibrium model definition. In the previous video we discussed that the general equation of plane includes three axis which are return Bi1 Bi2 space which is shown here Ri bar equal to lambda 0 plus lambda 1 into Bi1 plus lambda 2 into Bi2 which is precisely the equation of this plane equilibrium plane. This definition is what is the contribution of APT to the asset pricing model as defined by multi index and single index model. So, this equilibrium model is precisely provided by APT where the returns are generated by two indices I1 and I2 and lambda 1 and lambda 2 here are the increase in returns or you can call them risk premia for one unit increase in or one unit change in Bi1 and Bi2. So, these



are the risk premia associated with indices I1 and I2 where sensitivities with respect to these indices for a given security I is Bi1 and Bi2.



So, to summarize lambda 1 and lambda 2 essentially reflects the returns the extra returns for bearing the risk that are associated with indices I1 and I2. Let us consider a 0 Bij portfolio that means a portfolio which has no sensitivity to either of these two indices I1 and I2 that means it does not have any risk. Since it does not have any risk therefore it should only offer a risk free rate which is RF. Now, in this previous expression we have already seen that this Bi1 is 0 Bi2 is 0 then in that case Ri bar equal to lambda 0 and we are saying that lambda 0 is equal to RF. So, Ri bar is equal to RF which is same as lambda 0.

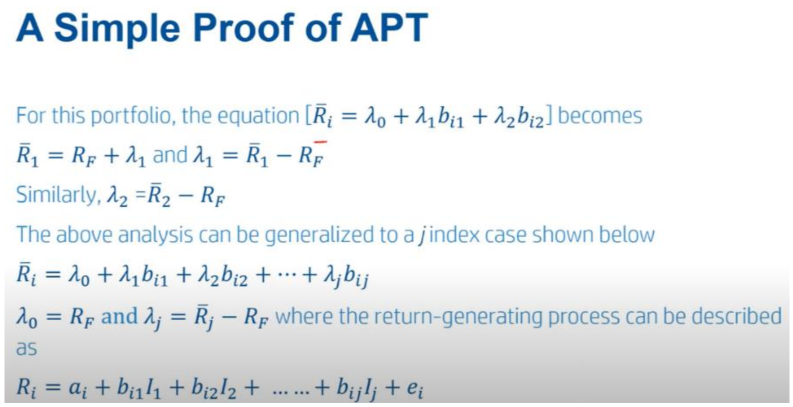
$$\underline{Ri} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i12}$$

$$\underline{R_1} = R_F + \lambda_1$$

$$\lambda_1 = \underline{R_1} - R_F$$

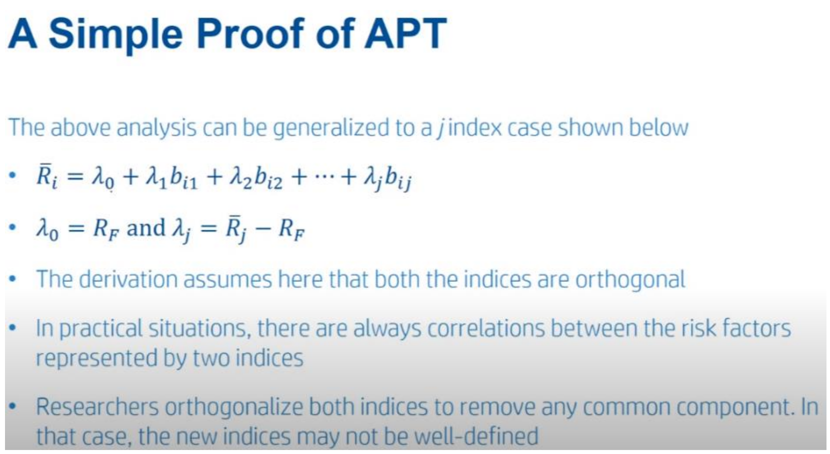
$$\lambda_2 = \underline{R_2} - R_F$$

$$\underline{Ri} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i12} + \cdots + \lambda_j b_{ij}$$



So, we have the value of lambda 0 one parameter. However, many times risk less rates are not available. So, you tend to use 0 beta portfolios portfolios which have no sensitivity towards any indices. So, that is why we are calling them 0 beta. The 0 beta portfolio will have a beta of 0 that means Bi1 of 0 Bi2 of 0 with respect to indices I1 and I2.

So, these will be proxied if risk less rates are not available. Now, let us imagine a portfolio that mimics index I1 or index 1 that means it has only one sensitivity which is with respect to index 1 and that is Bi1. So, Bi1 equal to 1. It has no sensitivity with respect to the other index which is I1 and I2 and therefore Bi2 equal to 0.



$$\underline{Ri} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i12} + \cdots + \lambda_j b_{ij}$$

Let us discuss the properties of this portfolio. In the equation that we saw earlier for two index equilibrium returns Ri bar equal to lambda 0 plus lambda 1 Bi1 plus lambda 2 Bi2. In this particular case where Bi2 equal to 0 and Bi1 equal to 1 this portfolio becomes Ri bar equal to RF plus lambda 1. Remember lambda 0 equal to RF. So, we substitute that value and therefore here we get lambda 1 equal to R1 bar minus RF. Similarly, we can also obtain lambda 2 equal to R2 bar minus RF by simply suggesting that there exists a portfolio with only sensitivity towards this index I2, Bi2 equal to 1 which is the sensitivity with respect to index I2 and all the other Bij as 0.

For example, if there are only two indices as we have assumed then Bi1 equal to 0. So, using that property we can find lambda 2 equal to R2 bar minus RF. This analysis can be generalized to a J index case also. For example, the equilibrium model for a J index case can be written as this Ri bar equal to lambda 0 plus lambda 1 Bi1 so on up to lambda j Bij. Here, if we keep assuming that lambda 0 equal to RF and lambda j equal to Rj bar minus RF where the return generating process is given by this equation, we can define APT very simply by this equation.

This is our APT equation where lambda 0 equal to RF lambda j, generic term for lambda j is Rj bar minus RF and this return this equilibrium model, this APT equilibrium model is driven by this multi index return generating process. So, this is our multi index return generating process which leads to this equilibrium model as defined by APT. So, with this simple derivation, we find this equilibrium model and these values for lambdas that are risk premia for different thesis. This derivation assumes that all these indices in this case if there are two indices, then both the indices are orthogonal, if there are multiple indices or these indices are orthogonal. When we say orthogonal that means all these indices I1, I2 and so on are not correlated with each other.

However, in practical situations, there are always correlations between these indices or risk factors. For example, if one of the risk factor or index I2 is oil and gas index, chances are it will be heavily correlated with one of the other index let us say I1 is market index. So, chances are these two will be definitely correlated. Researchers try to orthogonalize these indices mathematically by removing any common component and create new indices I1 dash and I2 dash. However, these I1 dash and I2 dash may not have the exact concrete meaning that was earlier.

For example, if I1 was earlier market index, I1 dash may not necessarily be proxy of market or I2 dash, for example, I2 was earlier oil and gas industry index, then I2 dash may be slightly different in its behavior as compared to oil and gas industry, maybe something different. So, these indices, these new indices that are orthogonalized not correlated. So, mathematically they have a good property, but they are not well defined. To summarize the APT model, we started with the simple CAPM. In a straight line CAPM, there was only two axis return and beta axis.

We translated this and found two coordinates corresponding to each point that denoted a portfolio. Now, we translated this to a two index APT model where two indices were I1 and I2 and there was one return index Ri bar and two beta indices, beta axis corresponding to each of the I1, I2 indices, we had two beta axis and therefore we had to use three coordinates one corresponding to the return and two corresponding to Bi1 and Bi2, betas. This led to the definition of plane and that plane also defines our efficient frontier. So, in the context of CAPM, we had the efficient frontier defined by only two coordinates. Here we have three coordinates, one corresponding to return and one, two corresponding to two beta axis, which define the efficient frontier as well.

If any portfolio which is represented by a point is above or below this plane, that means that security or portfolio is under or overpriced. And therefore, this violates the law of one price. That means at equilibrium, all the securities that have similar risks should fall on the same point. In this case, that point becomes a plane so they should fall on that equilibrium equity plane. If this law of one price is violated, then arbitragers and other market participants may conduct a riskless arbitrage, they will buy the undervalued security and sell the overvalued security.

And as more and more participants keep on doing that, these securities will drive towards that equilibrium plane and become fairly priced. So, if they are over or above or below this plane, they will be driven towards this equilibrium plane and become efficiently priced because of this arbitrage activity. The implication of this riskless arbitrage is that in

equilibrium, all the security should lie on this plane. And this plane is precisely defined by three coordinates expected return, BI1 and BI2. This is in the case when the process that is generating return is assumed to be driven by two indices I1, I2.

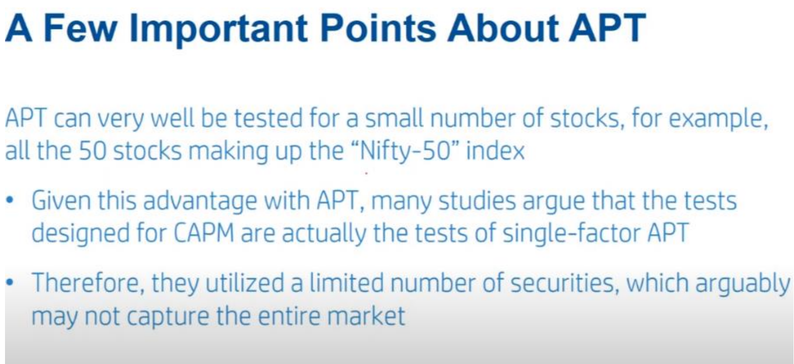
If the process is a return index model I1, I2, Ij and so on, then the return generating process will lead to an APT model which will also reflect the sensitivities towards all these j indices BI1, BI2 and so on up till Bij. A few important points about APT. In this video, we will talk about some interesting and contrasting aspects about APT that differentiated from other asset pricing models such as CAPM. In the context of CAPM, we need to identify the market portfolio and identification of market portfolio requires identification of all the risky assets that comprise market portfolio. While testing CAPM, one can always come

back and question whether all those securities are truly captured in this risky asset.

That means have we identified all the risky assets? This is a critical question and difficult one at that. So, have we truly achieved and constructed market portfolio? This question needs to be answered while identifying and working with CAPM. However, in the context of APT, arbitrage conditions can be applied to any set of securities or portfolios. And therefore, when it comes to APT, it is not necessary to identify all the securities in the market and therefore it is not necessary to identify the market portfolio.

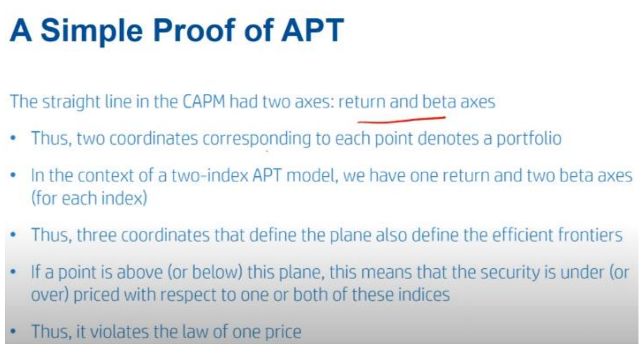
That makes APT a very generic one. So, for example, APT can be tested for a very small number of stocks. For example, a simple index like Nifty 50 which comprises 50 stocks can be utilized to test APT. So, only 50 stocks making up Nifty 50 can be used and tested for APT. Given this advantage, many studies and research papers argue that us while tests of CAPM are simply nothing but test of single factor APT where that single factor is market, market factor. Because one can always question the validity of market portfolio which is a set of all the risky asset in that given set of securities.

So, one can say that if we do not have all the securities whatever form or shape we tested that was single factor APT where we had the market factor as part of that single index return generating process that we have discussed. Because these studies utilize a limited number of securities which arguably may not capture the entire market. One caution needs to be highlighted here. The systematic influences that are used as a part of return generating process that we saw which is Ri equal to alpha i plus bi1 i1 plus bi2 i2 and so on plus error

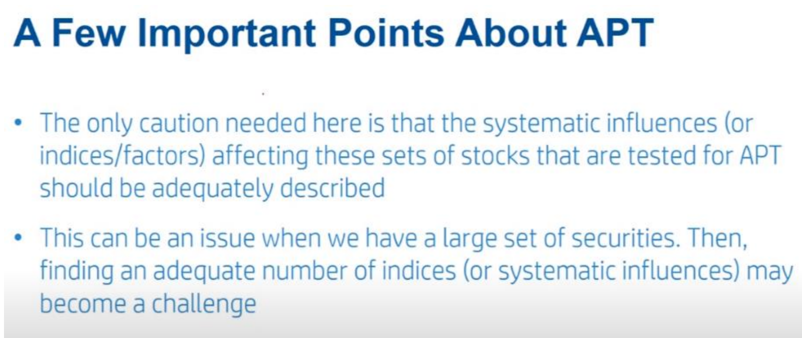


term. All these indices or systematic influences that one has used should be adequately described.

That means there can be an issue when you have large number of securities. Possibility is that there are large number of indices or systematic influences that may affect the set of securities and therefore identifying all the relevant set of indices or systematic influences or factors may be an issue while testing APT. To summarize, in this video, we discussed that APT is extremely generic in nature. APT allows us to describe equilibrium with the



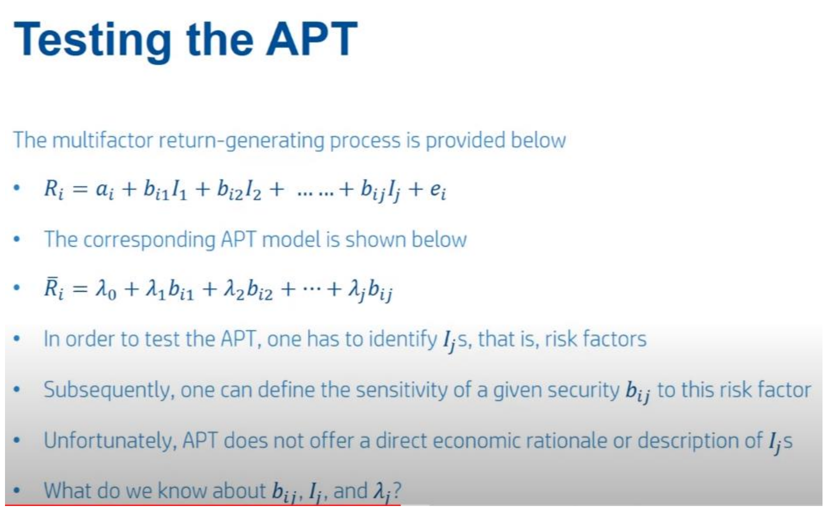
help of single and multi index models as return generating process. However, APT does not tell us what are these relevant influences or indices that should be used to model a set of securities. We do not know the lambda, the risk premia and the influences i1, i2 and so on.



Generally, researchers identify these influences through data generated models like factor analysis. For example, these risk factors like inflation risk, market risk and all. Once the mathematical model has provided with set of indices, these are tested for their correlations with real macroeconomic factors such as market risk, inflation risk and so on. However, the model itself does not provide these definitions. One does not have the direct specific economic rationale for a given factor that is thrown to us by mathematical models and that requires a lot of efforts in identifying the relevant influences.

In this video, we will examine how to test the APT. As per the APT, the multi factor return generating process is provided here. In this model, the corresponding APT model is provided here. This model Ri bar equal to lambda 0 plus lambda 1 Bi1 plus lambda 2 Bi2 and so on up to lambda j Bij is the equilibrium return as per the APT. In order to test this APT model, one needs to identify these indices ij s.

These are risk factors or broad market wide influences. Once these ij s are identified either through some mathematical procedure or maybe through a priori theoretical underpinning. Once these are identified, one can obtain sensitivities that is Bij. Bij here is the sensitivity of a security i to a risk factor ij. One identifies these Bij s which is the sensitivity of a given security to the risk factor.



$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \cdots . + b_{ij}I_j + e_i$$

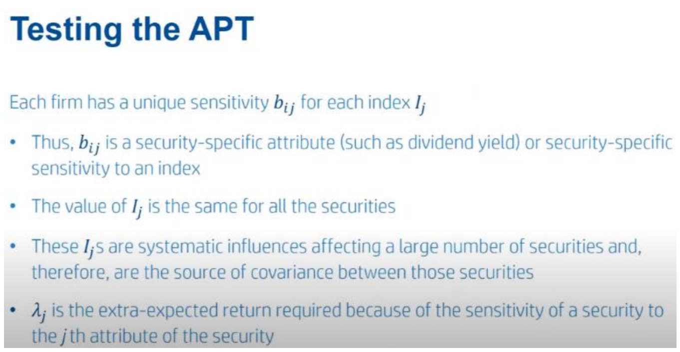
$$\underline{R_i} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots . + \lambda_j b_{ij}$$

However, APT does not offer any economic rationale for these ij s. And therefore, in order to define the complete model, we need to understand what is Bij here, what is ij, what is lambda j and what are their roles in this return generating process and APT model. Each form or a security has a unique sensitivity Bij. Each form or security i has a unique sensitivity Bij for a given index ij. Thus, this Bij is a security specific attribute.

For example, beta of a firm or dividend yield. This is specific to security and act as a sensitivity towards an index ij. The value of ij which is the broad market wide influence does not change with security. It is same for all the securities. For example, let s say ij is the bank index.

This bank index will remain same for each of the security. These ij s or risk factors are supposed to be systematic influences that affect a large number of securities at the same time in a similar manner and therefore these influences act as a source of covariance between these securities. Lambda j here is the extra risk done, extra expected return required because of the sensitivity of security to the jth attribute. So, this is the extra return or return premia that is offered because a security is sensitive to a given index. For example, if one looks at CAPM, in CAPM, this Bij would be something similar to the beta which is the sensitivity to market.

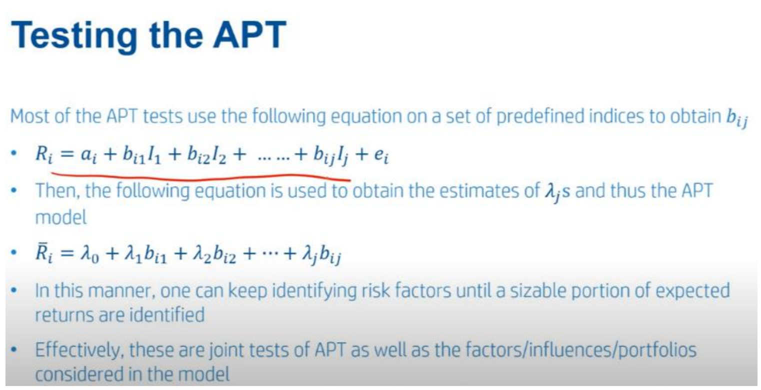
ij is the market index here. For CAPM, ij would be something similar to market index and Lambda j which is the risk premia that is additional return is Rm minus Rf which is because the extra return that is offered on account of this sensitivity beta towards market index. These are well defined for CAPM. However, for APT, these are not well defined in the model and one has to test the model below to establish APT. However, in order to test this model, you need these ij's and Bij's so that you can test this model.



Once you have ij's and Bij's, then only you can obtain Lambda j's. So, this testing, testing of this APT model requires estimates of Bij and Lambda j which essentially also requires inputs of ij's. As per the modern market theory, most of the tests of APT use this kind of return generating process, this model. They use a predefined set of indices like i1, i2 and so on ij's and obtain sensitivities Bij by estimating this model where i1, i2 and ij's and so on are predefined. Once Bij's are identified, then the following equation is used to obtain the estimates of Lambda j and thus completely identify the APT model. So, first we start with ij's indices, then we obtain sensitivities Bij's through some kind of regression estimation and once Bij's are estimated, then putting these values of Bij, one estimates the return generating process and therefore the Lambda's.

In this model, you can keep on adding more and more risk factors that is ij's and keep on increasing the explanatory power of the model. However, after a certain time, the incremental or marginal contribution of explanatory power by adding more factors is very less. So, it is a trade-off between how much explanatory power you want and how many factors you want to introduce. Introduction of large number of factors can make model less parsimonious and also introduce noise in the model. Moreover, effectively these tests are not only tests of APT, but they are essentially joint test of APT as well as the factors or influences or portfolios that are ij's that are involved in the model.

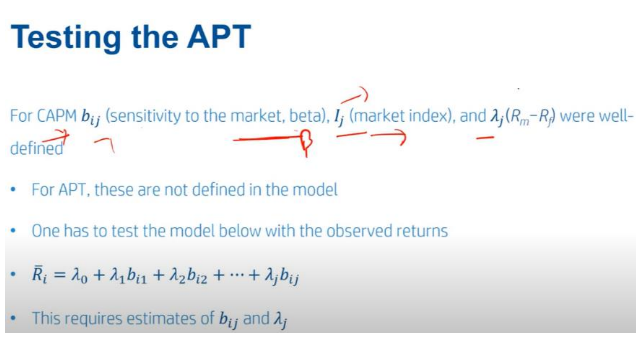
So, it is a sort of joint test between APT as well as the ij's or the factors. Because there is no generalizable theory that explains all the factors, a number of mathematical approaches are available. For example, factor analysis approach or specifying the attributes of security like dividend deal, beta and so on. Specifying the influences like some industries in the sector specific influences that affect the return generating process that is single or multi-index models or specifying a set of portfolios that capture the return generating process.



$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \cdots . + b_{ij}I_j + e_i$$

$$\underline{R_i} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots . + \lambda_j b_{ij}$$

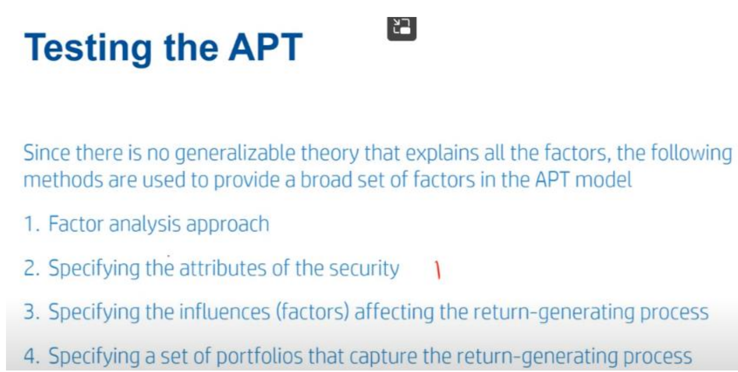
on which leads to identification of complete APT model. There are certain approaches like factor analysis approach specifying the attributes of security, specifying the influences or specifying a set of portfolios that are often employed to test the APT.



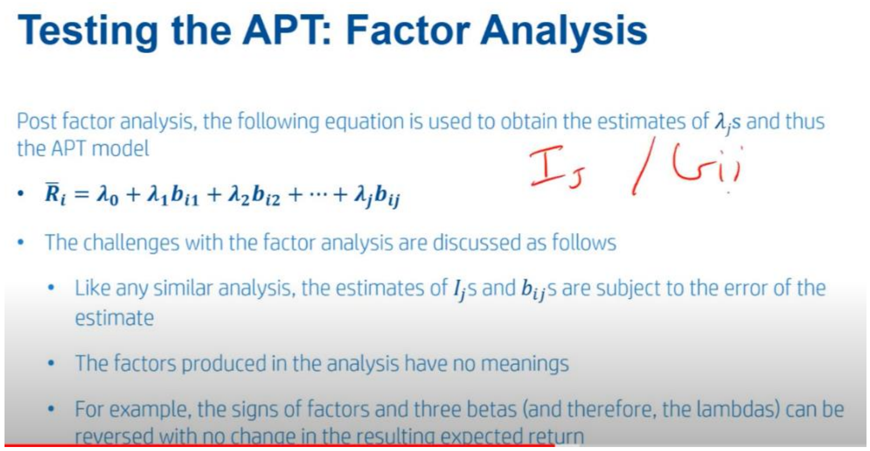
$$\underline{R_i} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots . + \lambda_j b_{ij}$$

These tests are essentially not only test of APT but sort of joint test of APT along with the factors that are employed in the model or return generating process. Generating APT Factor Analysis In this video, we will discuss a very important method to generate APT factors with the help of factor analysis. A slightly purer and advanced method to generate factors and factor sensitivities is conducted through factor analysis. As a part of factor analysis, one obtains mathematically determined ij's or factors and bij's or sensitivities in a manner that reduces the covariance of residuals. Remember, these single or multi-index return generating process have residuals that are stock specific components and it was assumed that components residuals ei ej should have minimum or almost zero correlation or covariance with each other.

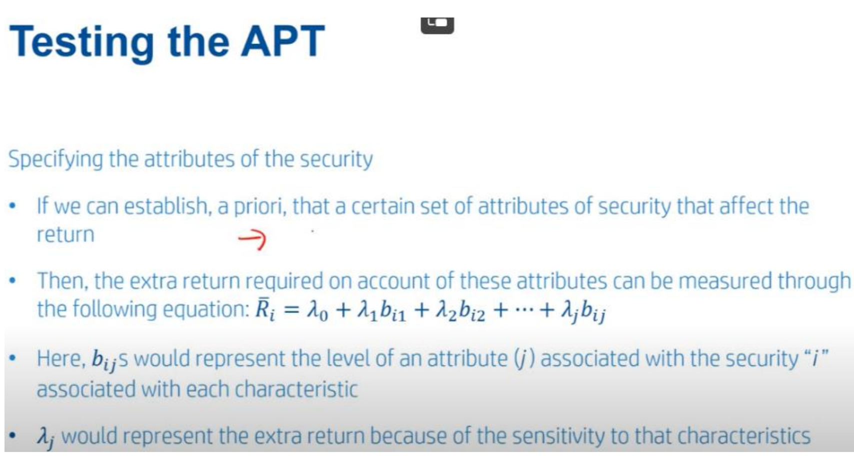
The way factor analysis works here, it tries to minimize this correlation and generate a set of factors ij's and their factor loadings which are also referred to as sensitivities bij. So, these factor loadings are referred to as factor sensitivities. Now, one can keep on adding factors here or these ij's in the model in order to increase the explanatory power of the model so that these factors can explain the covariance matrix or in simple terms the explanatory power of the model and decrease the residuals. However, this presents a trade-



off in a sense that more and more factors will have lower or lower marginal contribution to the explanatory power, but at the same time may increase or involve certain noise, introduce certain noise to the model. Once we are done with the factor analysis, we obtain ij's, factors and bij's factor loadings in the parlance of factor analysis or sensitivities of the security.

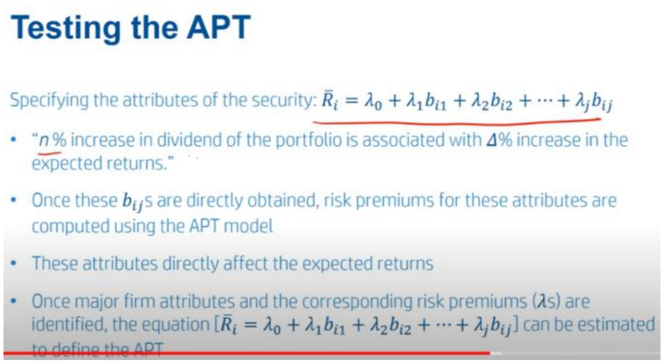


Now, once you have the bij's using the APT model that we discussed earlier, we can obtain lambda j's or return premia in the following form. Through this model, we estimate lambdas once we have bij's or factor sensitivities. However, this analysis also involves certain challenges as we can see here. First and foremost, the estimates of ij's and bij's that are provided by factor analysis, they are subject to the error of estimate, which is a part of any statistical analysis of this kind. Moreover, APT does not provide any universal theory of these factors and therefore the factors produced here, these ij's do not have any special macroeconomic meaning.



For example, once you have ij's and bij's from factor analysis, you can very well change their sign or multiply and discount their magnitude. For example, ij's can be doubled and bij's can be halved without creating any effect on the mathematical analysis or ij's can be put negative and bij can be put negative, signs can be changed without affecting the model or analysis. So, in a sense, these ij's and bij's do not have any economic interpretation that is driven by factor analysis. To summarize, in this video, we discussed how we can obtain ij's and bij's as a part of factor analysis. However, we also noted that these estimates are subject to error of estimation and second, these estimates do not have any special economic meaning as provided by factor analysis that economic intuition or interpretation has to come from theory or outside factor analysis.

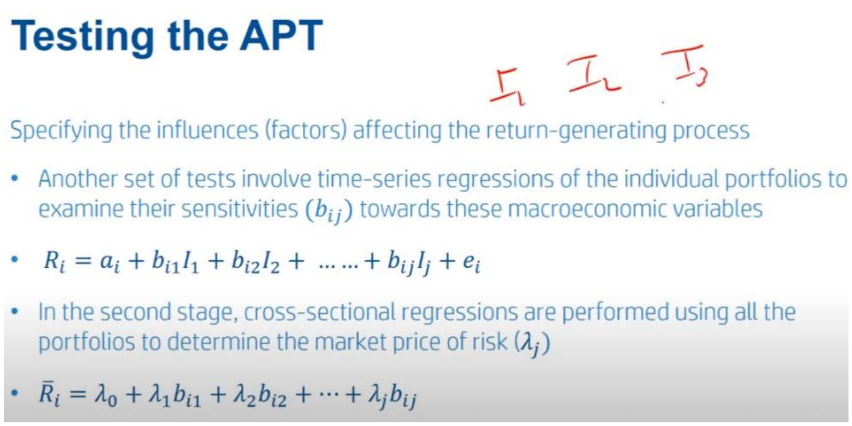
Testing the APT, Specifying the attributes of the security. In this video, we will discuss one more method of testing the APT which is through specifying the attributes of the security and thereby defining the APT model. To begin with, if we can establish that there are certain attributes of a security that can affect the returns expected from that security, then one can specify and define APT model more easily. These attributes, for example, beta of a security or expected dividends from a security, such attributes do affect expected returns and once we specify these attributes, for example, we can specify the expected



level of dividends from security as bij. Once you specify the level of attributes, here bij's are not necessarily the sensitivities but certain specific attributes of a security and then extra return required on account of these attributes can be measured or estimated with the help of this APT equation.

Again, remember bij here is a level of attribute associated with security for that characteristic. For example, a security may have 5% of dividend level or a security may have certain level of liquidity and the return associated expected return associated with that level of liquidity will become the lambda corresponding to that attribute and therefore lambda j would represent certain extra return because of the sensitivity of that security towards that jth attribute. More precisely, while specifying the attributes of the security and estimating the lambdas and bijs through this APT equilibrium model, let's look at this statement. N% increase in the dividend of the portfolio is associated with delta percent increase in the expected returns. Here, dividend is that security specific characteristic which also leads or determines the expected return from the security. So, we can quantify a certain level of dividend or the change in level of dividend with a certain change in expected returns.

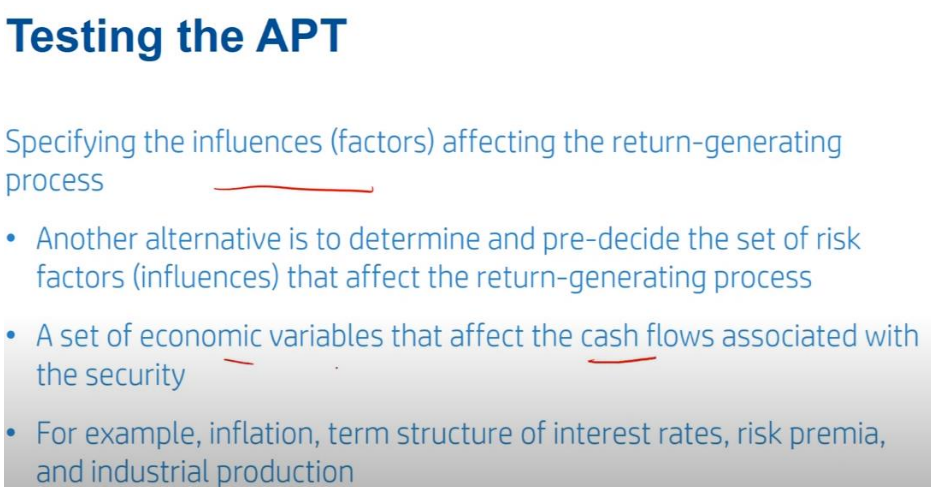
Once these bijs are directly obtained in the form of security specific characteristic, then lambdas or risk premiums of these security specific characteristics or attributes can be estimated very easily from the APT model through a methodology such as regression. We must remember that these attributes are important because they directly affect the expected returns. Once some of the major attributes of the securities and corresponding risk premiums that is lambdas are identified, we can easily establish or define this APT return equilibrium equation which defines the expected return from the security as per the APT model. To summarize, in this video, we discussed that there are certain major attributes of



a security that determine expected returns of that security.

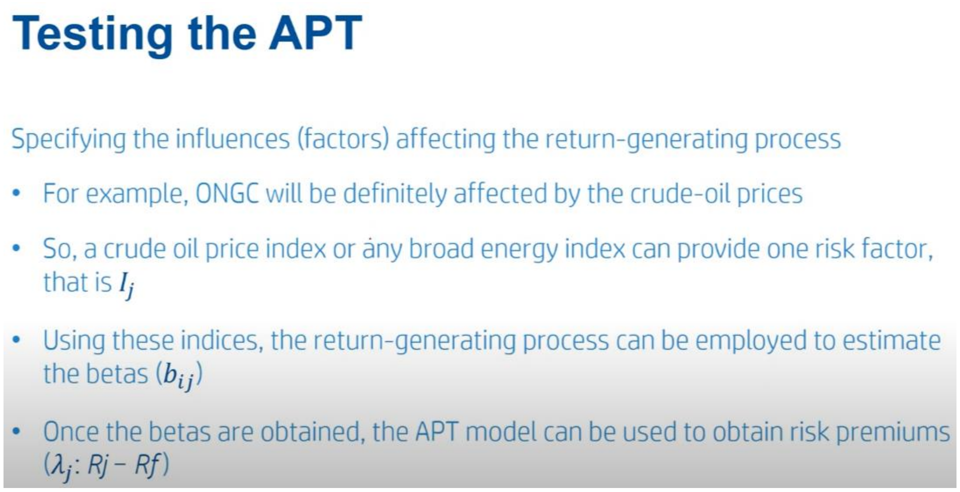
These attributes can be utilized as bijs in the APT model. Once these bijs are obtained, we can obtain the estimate of lambdas or return premiums for that security as defined in the APT model and thereby, state establishing and defining the complete APT model. Testing the APT by specifying a set of systematic influences or portfolios. In this video, we will discuss an interesting method of testing and specifying APT with the help of a set of portfolios or a priori systematic influences. In this method, we try to specify a set of influences or factors that affect the return generating process.

These influences are determined and pre-decided which affect the return generating process. These are set of economic variables that can affect the cash flows associated with the security. Some of the most prominent influences or factors include inflation, term structure of interest rates, risk premia and industrial production. As a part of this method,



first, we a priori we specify these influences which are I1, I2 as we discussed these can be inflation, inflation, industrial production and so on. Once these influences or factors are decided a priori, pre-decided, then using this return generating process, this model Ri equal to alpha i plus bi1 into I1 and so on. Using this model in a cross sectional regression that means for each security i a cross sectional regression over time is run equal to 1, t equal to 2 and so on.

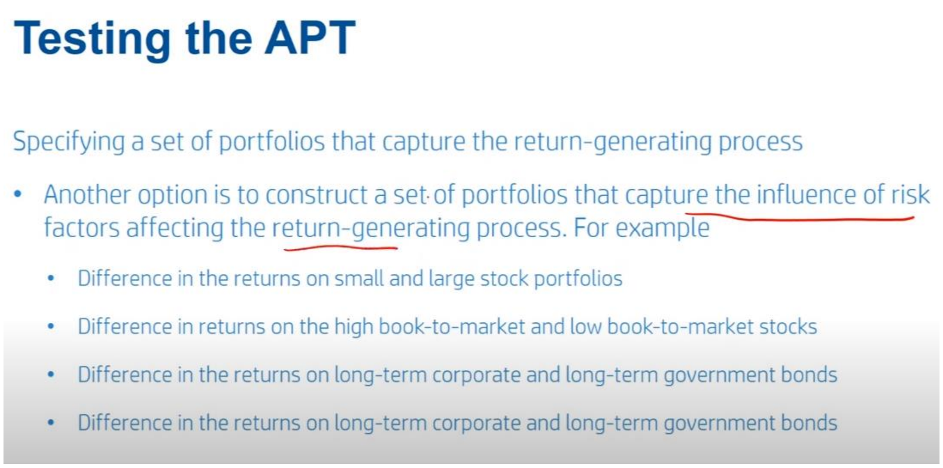
With security specific time series regressions, one obtains the sensitivities of the security with respect to these indices bi1, bi2 and so on. Once we have these sensitivities of security with respect to these indices through these time series regressions for each cross section equal to 1, t equal to 2 for each cross section for a number of securities, we obtain the estimates of lambda. So, while obtaining the estimates of lambda, we conduct cross sectional regressions, where in each cross section a set of securities are employed, their sensitivities bi's are known, they are obtained from time series regressions as we discussed earlier in a time series. Now, through cross sectional regressions, we estimate these lambdas from the given or identified bi's.



Thus, once we have the estimates of lambdas and bi's, we have completely specified the APT model. For example, let us take case of ONGC. Now, crude oil prices affect ONGC and a number of similar securities that are part of oil and gas industry. And therefore, any broad energy index that captures the movement in crude oil prices can provide one such risk factor ij related to oil and gas industry. Similarly, various such indices such as inflation, interest rates and so on can be decided and in the return generating process, these ij's can be specified to obtain estimates of sensitivities of the security towards these factors ij's. So, these bij's are the sensitivities of the security towards these pre decided or specified factors such as oil and gas index, inflation and so on.

These are time series regressions of individual securities over time, as we discussed equal to 1, equal to 2 and so on for each of the security. Once we have estimates of these bij's, we can rely upon the APT model through cross sectional regressions. In cross sectional regressions at each cross section equal to 1, let us say or equal to 2, in each cross section, multiple securities with their bij's, we have estimates of bij's for all the securities, we estimate lambdas from the model APT model, which is lambda naught plus lambda 1 v1 plus lambda 2 v2 and so on up to lambda j bj, we specify and obtain the estimates of lambda. This lambda is the risk and once we have the estimates of lambda and sensitivity, we have the APT model completely specified. Another very similar method is to employ a set of portfolios that capture these influences, risk influences or factors as we discussed earlier, these risk influences that affect the return generating process, not directly but through certain portfolios, we try to capture these risk influences.

For example, difference in returns between small and large stocks can be one such factor which captures the influence of size factor, this is often referred to as size factor, which is the difference in large and small stock returns or difference in high book to market to low book to market stocks. This is often referred to as value versus growth factor. So, you have difference in long term corporate long term government bond, which is the greatest factor. So, similarly, you can have portfolios that capture the risk of these influences or factors that can be employed instead of predefining or determining a factor itself.



So, this is another method where you can specify portfolios and use their returns as ij's, i1, i2 and so on. Once you have these i1, i2, you can estimate the sensitivities that are bij's through cross time series regressions. And once you have the estimates of bij's in cross sectional regressions, you can estimate lambdas or the risk premium. To summarize, in this video, we discussed how to estimate APT model by either specifying the set of influences or risk factors or specifying the portfolios that capture these influences or risk factors. Once we have identified these portfolios or influences, we have estimates of i1's, ij's. Using these ij's in time series regression, we obtain the sensitivities of securities towards these portfolios or risk influences.

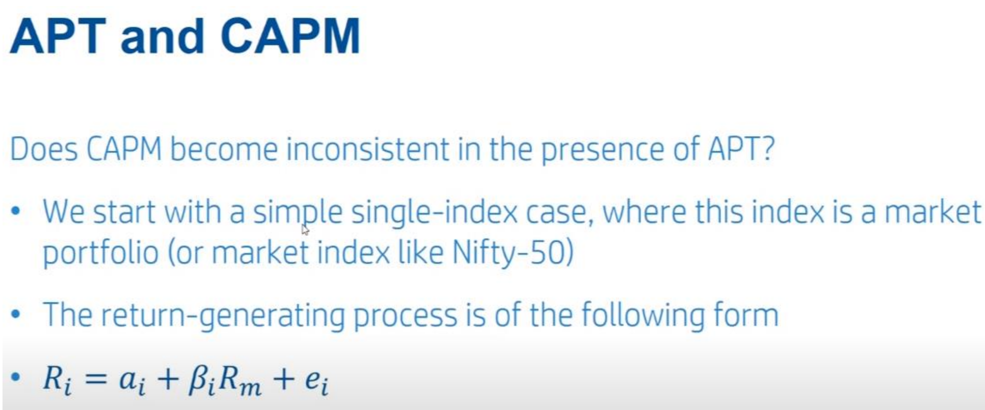
These are bij's. Once we have the estimate of these bij's, we can estimate lambdas in the APT model thus completely specifying the APT model. APT and Kappan single market index. In this video, we will try to answer the question whether Kappan becomes inconsistent in the presence of APT where single market index is available. To answer this question whether Kappan becomes inconsistent in the presence of APT, we start with a simple single index case.

Here, this single index is market portfolio like NIFTY50 or S&P500. Here, the return generating process that we have discussed for a single index case takes the following form. Ri which is the return on security i, alpha i plus beta i times Rm plus ei. ei here is the residual or error term. Now, if we go back to our earlier discussions about APT, we said that this return generating process can be taken to the equilibrium return form by APT when it is written like this. Ri bar which is the expected return equal to lambda 0 plus lambda 1 into bi1 plus lambda 2 into bi2 and so on up till lambda j into bij.

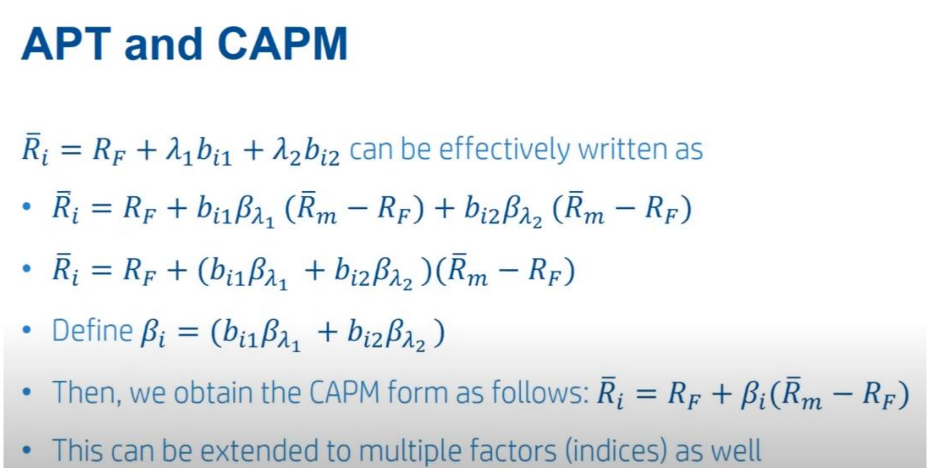
Notice there is no residual term because this is equilibrium equation of return as defined by APT. Here, lambda 0 if Rf risk free rate is available then lambda 0 equal to Rf, lambda j equal to Rj bar minus Rf. Now, lambda j here is the risk premium with respect to index j. If there is only one single index available and risk free rate Rf is also available therefore, this single index where the single index is market index and in the presence of risk free rate, we can derive the APT expression as follows. Here, Ri bar equal to Rf plus beta times Rm bar minus Rf where Rj bar becomes Rm bar where Rm bar is the return on market.

This is the equation for expected return as per APT model which is also similar to that provided by CAPM. This equation, this derived APT equation suggests that when single index return generating process is the true depiction of APT, CAPM is indeed consistent with APT. But what about multi index forms? So, in the next video, we will talk about multi index case whether APT is consistent with CAPM or not. To summarize, in this video, we discussed that if single index model which is market model is there, CAPM and APT are not necessarily inconsistent. APT and CAPM multi index case.

In this video, we will examine whether APT and CAPM become inconsistent in the presence of multi index return generating process. Let us start with a simple return



generating process with two indices. The process looks like this. Ri equal to alpha plus Bi1 into I1 plus Bi2 into I2 plus Ai. Ai here is the residual. The corresponding APT equilibrium model for this return generating process is Ri bar equal to RF plus lambda1 into Bi1 plus lambda2 into Bi2 where assumption is that risk free rate RF is available.



$$\underline{R_i} = R_F + \lambda_1 b_{i1} + \lambda_2 b_{i2}$$

$$\underline{R_i} = R_F + b_{i1}\beta_{\lambda 1}(\underline{R_m} - R_F) + b_{i2}\beta_{\lambda 2}(\underline{R_m} - R_F)$$

$$\underline{R_i} = R_F + (b_{i1}\beta_{\lambda 1} + b_{i2}\beta_{\lambda 2})(\underline{R_m} - R_F)$$

$$\beta_i = (b_{i1}\beta_{\lambda 1} + b_{i2}\beta_{\lambda 2})$$

Please remember here, lambda j is the price of risk for a portfolio where Bij equal to 1. That means the portfolio has a sensitivity of 1 for one index and 0 for all the other indices. And for that index j, lambda j equal to Rj bar minus RF, that price of risk. If you assume that CAPM holds, then it also holds for all the securities as well as portfolios like I1, I2 that is broad market wide portfolio such as I1, I2 which are the risk factors in the model as well. And since we assume that this CAPM holds for portfolios I1, I2 as well, industry portfolios like I1, I2 may have some sensitivity to the market portfolio.

Let's call this sensitivity beta lambda j. If you remember, the risk premium in simple CAPM was beta times Rm bar minus Rm when the sensitivity to the market was beta i. So in this case, where sensitivity is beta times beta lambda j, the risk premium lambda j can be written as beta times lambda j into Rm bar minus RF. Assumption here is that the industry or in I1 portfolios like I1, I2, they are also sensitive to market. And therefore, if the return generating process for two indices is this one, Ri equal to alpha i plus Bi1 into I1 and so on, the corresponding equilibrium model for this return generating process can be written as Ri bar equal to RF plus lambda 1 into Bi1 plus lambda 2 into Bi2, where lambda 1 equal to R1 bar minus RF. But we said that we also believe in CAPM and therefore R1 bar minus RF can be written as beta times Rm bar minus RF, beta lambda 1 times Rm bar minus RF in fact.

Similarly, this is for index I1 and therefore similarly for index I2, we can write the value of lambda 2 as R2 bar minus RF, which is equal to beta lambda 2 into Rm bar minus RF. This is for index 2. With these expressions, we can simplify our APT model.

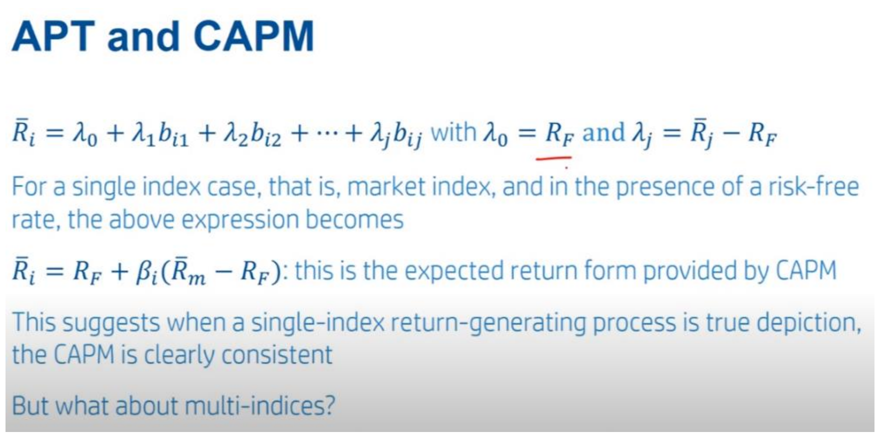
APT and CAPM

Now, refer to our earlier discussions on APT, where we said that the above return-generating process could be written in terms of sensitivities of the securities to index and the price of risk in the following form

- $\bar{R}_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots + \lambda_j b_{ij}$ with $\lambda_0 = R_F$ and $\lambda_j = \bar{R}_j - R_F$

$$\underline{R_i} = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots . + \lambda_j b_{ij}$$

Our original APT model is Ri bar equal to RF plus lambda 1 Bi1 plus lambda 2 Bi2. We can simplify by substituting the values of lambda 1. Lambda 1 was beta lambda 1 into Rm bar minus RF and lambda 2 was beta lambda 2 times Rm bar minus RF. Simplifying this expression, we can take out Rm bar minus RF, which is common to both these terms and therefore the resulting expression is Ri bar equal to RF plus this combined term into Rm bar minus RF. Let's simplify this and call it beta i. And this results in an expression which is very similar to CAPM, which is Ri bar equal to RF plus beta i, where beta is this times Rm bar minus RF.

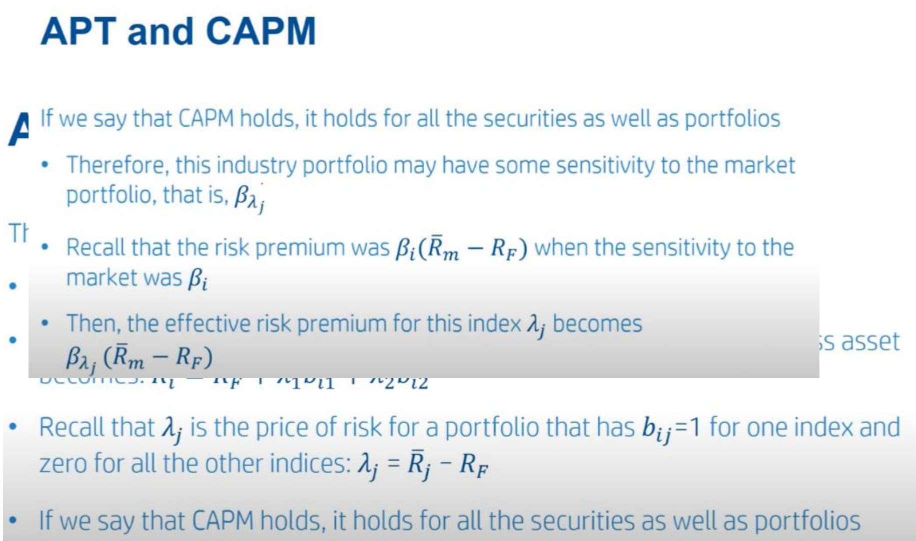


And this analysis can be extended to multiple factors or indices as well. Based on this, we can say that the APT solution, the APT solution to the return genetic process, even with multiple factors is consistent with CAPM. So even if we assume that CAPM holds, it is not inconsistent with multiple indices as well. So with multi-index return generating process, the APT model that is derived with multiple indices that explain the covariances or com movements between the stock returns, even there, if we assume CAPM holds, there is nothing that violates that assumption. That means CAPM is consistent with APT even when the return generating process is one with multi indices.

To summarize, in this video, we examined whether CAPM and APT are consistent in the presence of multi-index generating process. We found that CAPM and APT can remain consistent with each other even in the presence of multi-index generating process. Application of asset pricing models in passive management. In this video, we'll discuss the applications of asset pricing model in the context of passive management. A very simple and intuitive application of APT models is to construct a portfolio of stocks that closely tracks a certain index. This index can be a market index like Nifty Fifty or a certain industry sector specific index like Bank Nifty, which represents the risk of banking stocks.

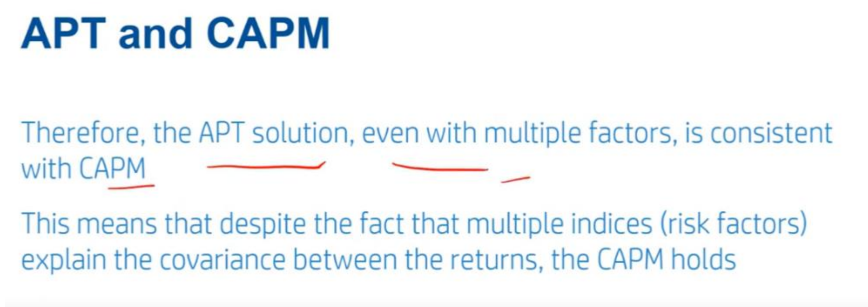
$$R_i = a_i + I_1 b_{i1} + I_2 b_{i2} + \cdots . + e_i$$

$$\underline{R_i} = R_F + \lambda_1 b_{i1} + \lambda_2 b_{i2}$$

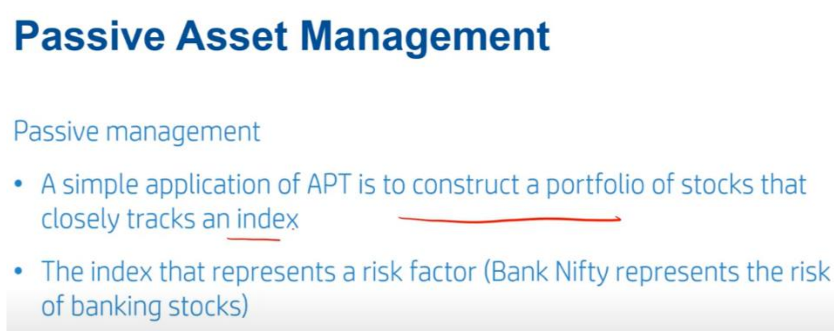


The core idea behind this kind of passive management is to use a rather small number of stocks to construct a diversified portfolio. If you have large number of stocks that will involve a lot of transaction costs, so you tend to use small number of stocks and you try to track a certain index like market index. However, please remember it is difficult to track an index which has large number of stocks. One cannot track all the stocks in a certain index or market. So the strategy here is to hold only to that extent the number of stocks that is sufficient to diversify the portfolio.

However, there's one challenge with this kind of strategy. If there are multiple indices that affect your portfolio and your model is not well matched, for example, if you have a certain portfolio where a number of indices are influencing that portfolio and you miss one of the important indices, then the shocks to those indices may appear in your portfolio or in your model as residuals EI. That is residuals EI. Now if your portfolio is not explicitly matched to some of these indices, then let's say there is a shock to one of these indices which you have not matched explicitly to your portfolio and the moment in your portfolio, the price



moment of your portfolio may appear as over or under performance depending upon the nature of shock in terms of performance evaluation.



That is your portfolio performance is exposed to these changes. Let us discuss this aspect in more detail. The benefit of using multi indices instead of single indices. For example, if your portfolio has five indices, energy, banking, inflation, cyclical stocks and government bond portfolio. Compare this kind of portfolio matching strategy. Instead of five indices, you can also select only one index which is market portfolio.

Both of these portfolios will capture the sensitivity to market risk. Since many of these indices that we discussed like energy, banking, inflation, they tend to capture the impact of market movement as well and therefore affect the risk of market. So both of these portfolios, the multi index and single index, will capture the sensitivity to market risk. However, if there is an oil price shock or some unexpected changes in inflation, the market portfolio with its sensitivity matched to Nifty Fifty or some kind of broad market index may not be very efficient in tracking that index. This is so because while constructing the market portfolio, you are indifferent whether the stocks in your portfolio are sensitive to inflation or unexpected changes in oil price movements. And therefore, if there are any shocks, if let's say the stocks in your portfolio are indeed sensitive to oil price index or inflation index, your portfolio which is explicitly designed and match its sensitivity with

market portfolio may not be very efficient in tracking those shocks and their impact on your portfolio.

Because for the simple reason that objective of your portfolio construction was just to construct a portfolio with its sensitivity matched to some kind of market index like Nifty Fifty and no diversifiable risk. And therefore, while analyzing the performance of this market index matched portfolio to another portfolio which is matched with multi index APT kind of model that is explicitly matched to the sensitivity of oil price index or let's say inflation index, the evaluation of performance can be very different. So that portfolio which is only matched to market index can incorrectly or inefficiently give you a sense of over or under performance which actually is not the case. That is that additional over or under performance is driven by one of the indices that is not matched properly or its sensitivities are not properly matched. If you would have matched the sensitivity of that portfolio with that additional oil and gas or inflation index, then probably the performance would have appeared appropriate with the risk of portfolio and not as a over or under performance.

To summarize, in this video we discussed how multi index and single index models can be applied in the context of passive portfolio management. In this video, we'll discuss the application of asset pricing models in the context of active portfolio management. As a part of active portfolio management, one would like to hold a particular portfolio which is well diversified but at the same time one would also like to take bet on certain risk factors. For example, let's say you hold a market portfolio or a portfolio that mimics market indices like Nifty Fifty or S&P 500. Still, you would like to take some active bets on sectors.

For example, let's say you believe that currently the oil and gas sector is undervalued and the prices may go up in future. For example, there is some kind of regulation which is going to benefit which is coming and going to benefit this particular sector. And therefore, in order to benefit from this kind of bet, what would you like to do is you would like to add more securities or more stocks that are sensitive to oil and gas index and the prices of these securities are likely to go up. However, at the same time you'd also not like to lose your beta or change your beta of your portfolio vis-a-vis market index that means market beta.

So, you will add oil and gas sector sensitive stocks in a manner that your market beta remains indifferent. For example, if it was earlier 0.5, you'd like it to remain intact while adding certain oil and gas stocks and removing certain other stocks so that net market beta remains constant. That is one. Second, as the gains are materialized, for example, as the market has risen, oil and particularly oil and gas sector has risen and the prices of these securities have risen, you have benefited, gains are realized and now you can liquidate those holdings. And now you can liquidate those these additional securities from oil and gas sector and go back to the original position while your beta still remains, market beta still remains intact. In this fashion, you can apply these multi index or single index model to take active bets in the market while at the same time benefiting from them and maintaining your portfolio diversified.

In summary, in this video, we discussed the application of multi index and single index model in the context of active portfolio management. While CAPM has its genesis in the mean variance framework, APT relies on the arbitrage argument. APT utilizes the return generating process provided by the single and multi index models to generate the equilibrium asset pricing model. Under the APT, riskless arbitrage drives prices towards the equilibrium plane. The equation of this plane is determined by the systematic risk influences affecting the set of securities under consideration.

APT can be tested with the help of a factor analysis, specifying the attributes, specifying a set of systematic influences and specifying a set of portfolios. In the presence of APT, CAPM does not necessarily becomes invalid as long as the APT factors are influenced by the market factor and have a well specified beta with respect to the market factor. Some of the most widely employed applications of asset pricing models include active and passive management and factor investing. Thank you.