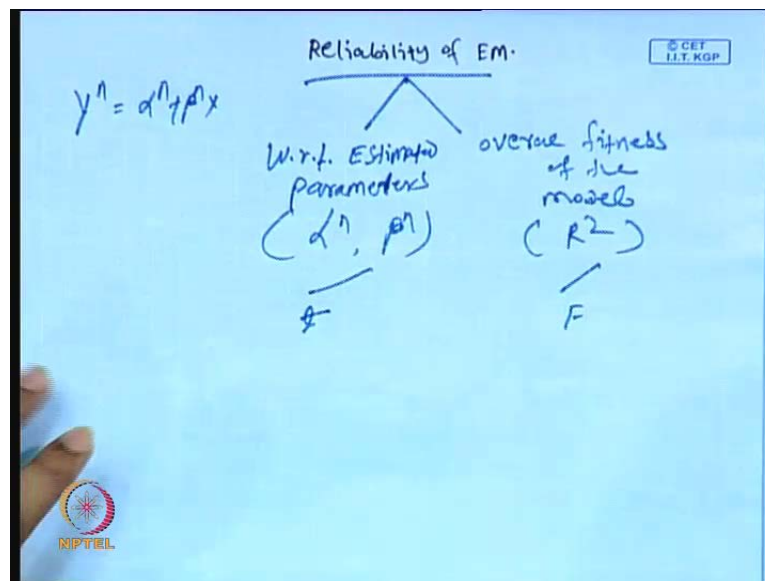


**Econometric Modeling**  
**Prof. Rudra P. Pradhan**  
**Department of Management**  
**Indian Institute of Technology, Kharagpur**

**Module No. # 01**  
**Lecture No. # 11**  
**Reliability BEM (Contd.)**

Good morning; this is Doctor Pradhan here. Welcome to NPTEL project on econometric modeling. Today, we will continue on reliability of bivariate econometric modeling. So, today's - our main agenda - is to take a problem and highlight the details about its reliability part. So, that means, we have to see how this you know, econometric modeling bivariate, econometric modeling can be fitted and how you have to go for this reliability check. So, before I, you know, start with this particular solutions, let me briefly highlight once again this reliability part of this bivariate econometric modeling.

(Refer Slide Time: 01:17)



Now, for bivariate econometric modeling, we know the system must be with respect to 2 variables; say Y and X and for this we have to, we have fitted a regression equations  $\hat{Y}$  hat equal to alpha hat plus beta hat X. So, this is our you know, estimated models which we have received through the **you know** through the help of a certain information. So, we must have information on Y and X and by the process or some applications. So, we have received this  $\hat{Y}$  hat equal to alpha hat plus beta hat X.

Now, our agenda is here; the moment you get the estimated model it should be perfectly... so; that means, otherwise it is called as a best fitted model. Otherwise, it is called as a best fitted model. **Now, this** to get these best fitted models. So, to know whether this model is a best fitted one, we have go for or we have to go for this reliability of you know, estimated model. So, reliability of you know, econometric model or estimated model has two parts; that is, with respect to estimated parameters and basically here is alpha hat and beta hat and second part is the overall fitness of the models overall fitness of the overall fitness of the models and that is nothing but, r square.

So, this is this is to be investigated by t statistic and this is investigated by f statistics. This is how we have already discussed; let me highlight how the structure of this first part that is you know, the reliability of parameters. So, for this we have to prepare a table; here it is.

(Refer Slide Time: 02:59)

1. Reliability Est. Parameters.

EST. param.	EST. Value.	Var. EP	SE EP	t EP	Level of S.
$\alpha^1$	$\bar{y} - \beta^1 \bar{x}$	$\frac{\sigma_{e^2}}{n \sum x^2}$	$\frac{\sigma_{e^2}}{\sqrt{\sum x^2}}$	$\frac{\alpha^1}{SE(\alpha^1)}$	—
$\beta^1$	$\frac{\sum xy}{\sum x^2}$	$\frac{\sigma_{e^2}}{\sum x^2}$	$\frac{\sigma_{e^2}}{\sum x^2}$	$\frac{\beta^1}{SE(\beta^1)}$	—

2. ANOVA

Sources of Variation	Sum of Sq.	mean	Df	F	P
ESS	$\sum y^2$	$\frac{\sum y^2}{(k+1)}$	k-1	$\frac{ESS}{ESS}$	—
RSS	$\sum e^2$	$\frac{\sum e^2}{(n-k)}$	n-k	$\frac{RSS}{RSS}$	—
TSS	$\sum y^2$	$\frac{\sum y^2}{(n-1)}$	n-1		

Reliability of estimated parameters estimated parameters. So, the problem is like this. You have estimated parameters; then estimated value, then variance - variance of estimated parameters, then standard error of estimated parameters, then t of estimated parameters, then level of significance. This is how we have to you can say **sorry** estimated value variance of X standard error t and like this.

In fact, this is estimated parameters value we will put it like this; estimated parameters here then, this is value of estimated parameters, then this is variance of estimated parameters. So, now, for this you have alpha hats and you have beta hats **alright**. So, you have alpha parameters and beta parameters. So, the estimated value equal to  $\bar{Y}$  minus  $\hat{\beta}_0$  and this is  $\sum X Y$  by  $\sum x^2$ . So, this is variance means it is  $\sigma^2 u \sum X^2$  by  $n \sum x^2$  and this is  $\sigma^2 u$  by  $\sum x^2$ .

Now, this is variance of alpha hat and this is variance of beta hat to the power 1 by 2 and this is you know, alpha hat by alpha hat by standard error of alpha hat and this is beta hat by standard error of beta hat this is how we have receives. So, the probability level of significance we have to just accordingly. So, now, the second part of the model is the a ANOVA that is you know analysis of variance ANOVA this is first part and this is second part. So, this second part is the there are certain structure here there are sources of variations sources of variations. So, the sources of variations are you know explained sum square then and then residual sum square and then total sum square this is total sum square.

So, now, sum of square here  $\sum \hat{y}^2$ ; this is a  $\sum e^2$  and this is  $\sum y^2$ . So, mean sum square; mean sum square is nothing but,  $\sum \hat{y}^2$  by  $k - 1$ . Then, this is  $\sum e^2$  by  $n - k$  and this is  $\sum y^2$  by  $n - 1$  and next is degrees of freedom degrees of freedom then f statistics and probability level of significance degree of freedom. Here,  $k - 1$  this is  $n - k$  this is  $n - 1$ . So, f is nothing but,  $ESS$  by  $RSS$ . So, that is nothing but,  $\sum \hat{y}^2$  by  $\sum e^2$ . So, p will be defined accordingly.

(Refer Slide Time: 06:36)

$$R^2 = \frac{ESS}{TSS} = \frac{\sum \hat{y}^2}{\sum y^2} = \frac{(\sum xy)^2}{\sum x^2 \cdot \sum y^2}$$

$$\sum y^2 = \sum \hat{y}^2 + \sum e^2 \quad \sum \hat{y} = \sum \hat{\beta} x$$

$\swarrow$                        $\swarrow$                        $\swarrow$   
 TSS                      ESS                      RSS

$$\sigma^2_u = \frac{\sum e^2}{(n-2)}$$

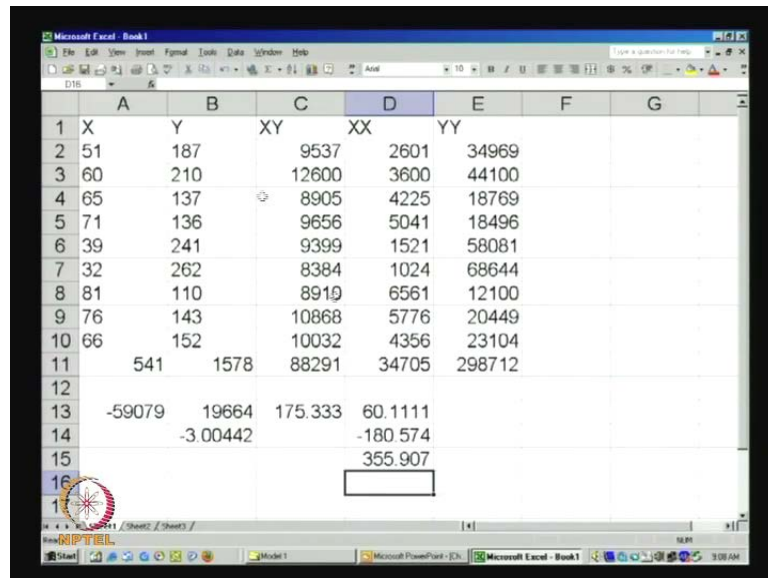
$$\bar{R}^2: \text{Adjusted } R^2$$

$$R^2 = 1 - \frac{(1 - R^2)(n-1)}{(n-k)}$$

Now, you see here is the fundamental principal is here is. So, we like to know we like to know what is R square, which is nothing but ESS by TSS. So, in other words, it is nothing but, summation y hat square by summation y square. It is nothing but, summation xy whole square by summation x square into summation y square; because, y here equal to y here equal to beta **sorry** summation y hat square equal to beta hat summation beta hat and x; so, summation beta hat x. This is how we have received these particular items. Now, here summation y square equal to summation y hat square plus summation e square.

So, this is TSS this is ESS this is RSS total sum square explained sum square residual sum square. Now, sigma square equal to summation e square by n minus 2. So, now, there is corresponding to r we have another reliability factor called as a adjusted R square this is what we call it a adjusted R square. So, adjusted R square is nothing but, 1 minus 1 minus R square into n minus 1 divided by n minus k n minus 1 into n minus k. So, this particular is we really **want to** when you will go for multivariate models; but for the time being, still you know it is very important because it adjusted the degrees of freedom.

(Refer Slide Time: 08:26)



The screenshot shows a Microsoft Excel spreadsheet with the following data:

	A	B	C	D	E	F	G
1	X	Y	XY	XX	YY		
2	51	187	9537	2601	34969		
3	60	210	12600	3600	44100		
4	65	137	8905	4225	18769		
5	71	136	9656	5041	18496		
6	39	241	9399	1521	58081		
7	32	262	8384	1024	68644		
8	81	110	8919	6561	12100		
9	76	143	10868	5776	20449		
10	66	152	10032	4356	23104		
11		541	1578	88291	34705	298712	
12							
13	-59079	19664	175.333	60.1111			
14		-3.00442		-180.574			
15				355.907			
16							

So, now you know for particular discussion, what you have to do? We have to solve a particular problem with regards this, you know particular structures. So, let me highlight here **is. So**, we will put it here is some items let you know I will call it here X and this sides I will call it Y. So, let it be first we enter this data. So, this is 51 then **60** then 65 and this is 71, then this is 39 then 32 then 81 then 76 then 66 and 51, 60 this is 60 then 65 then 71 then 39 and 39 then you have to cut it here paste. So, this is X series and corresponding X series Y here 187 then 210 then 137 then 136 then 241 then 2 **60** 2 then 110 then 153 then 152.

So, there are we have taken altogether 9 observations view let me I will write it little bit here. So, view zoom get it 200. So, now,. So, this is how we have received the entire structure here. So, the a series is XY you know and this is what **alright**. So, now, this is X series and this is Y series there are 9 observations. So, how do we proceed for this particular structure?

(Refer Slide Time: 10:54)

The whiteboard contains the following handwritten text:

$Y = \alpha + \beta X + u \rightarrow Y^{\wedge} = \alpha^{\wedge} + \beta^{\wedge} X$   
 $\alpha^{\wedge} = \bar{Y} - \beta^{\wedge} \bar{X}$   
 $\beta^{\wedge} = \frac{\sum XY}{\sum X^2}$

Step 1: Better Descriptive Stat.

$\bar{X} : ?$        $G_{11}$   
 $\bar{Y} : ?$        $G_{22}$

Max: 81 (X)      Min: 32 (X)      med.  
         262 (Y)           110 (Y)

$\sum X = 541$        $\sum Y = 1578$

$n = 9$

A hand is visible at the bottom right, pointing with a white marker towards the variance-covariance matrix  $\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$ .

So, now, the model is 0 th with respect to Y and X. We assume that the regression model equal to Y equal to alpha plus beta X plus u and through which by the process of wireless applications we have received Y hat equal to alpha hat plus beta hat X and where alpha hat equal to Y bar minus beta hat X bar and beta hat equal to summation X Y by summation X square **alright**.

Now, after getting this much information we have to go for reliability test; that means, we have to judge whether this particular model is the best fitted one. So, now, for this we have you know stepwise procedures. So, we have to **we have to** proceed step by step. So, this is step 1. So, how do we go for this data? Means, here our agenda is to check whether this particular estimator is the best fitted or not. So, for that we have to proceed in a stepwise process. Now, first step is to know the descriptive statistic descriptive; first you have to know the descriptive statistic descriptive statistic.

The descriptive statistic basically deals with X bar and Y bar. So, X bar, Y bar, sigma X, sigma Y; then, sigma this is you can say variance covariance matrix. Then, we have to know variance covariance matrix sigma 1 1; then sigma 1 2, sigma 2 1, sigma 2 2. So, then of course, we like to know maximum of the series. We like to know the minimum of the series. Then, we like to know the median of the series. So, many things we like to know. So, now, you check it here. These particular items it will take. We have to fill up this particular gap here is. So, this is equal to...in fact, here there is a lot of application.

We like to know lot sum and this is this is sum. So, first is the summation X summation X equal to 541. So, then 1578. So, summation Y equal to 1578. So, to know maximum and minimum, you can put it here in a ascending order or descending order. So, you can get to know very easily. So, you put it here ascending to descending order. So, you get to know so; that means, for X for X the maximum is 81 this is for X and the minimum is 32. So, this is 32 for y. So, similarly this series it can also arrange in ascending order.

This way, this is how you will get it a 262 is the 262 is you know, Y series this is Y series. So, this is X and minimum is here is 110 110 this is for Y. So, this is required means, this much of information is required because we like to know, what is the variation inside the particular game boundary? So, if you know in detail then; obviously, the model can be much more you can say, feasibility or you can say reliability. So, anyway you have to now you have to go back to the original position where we have started. So, this is are the observations. So, how do you proceeds. So, this you know step 1 step 1 information is just to know the descriptive nature of the statistics.

So, because I have not highlighted details here,  $\bar{X}$  and  $\bar{Y}$ . So, that means, here n equal to 9; divided by n. So, you will get  $\bar{X}$  and you divided by again 9. Here, you will get  $\bar{Y}$ . Similarly, you will get say  $\sigma_X$  and  $\sigma_Y$ . So, that is  $\sigma_X^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$ . So, we have to create a series  $X_i - \bar{X}$  and we have to create again a series  $Y_i - \bar{Y}$ . So, then you create a series summation  $\sum (X_i - \bar{X})^2$  into summation  $\sum (Y_i - \bar{Y})^2$ . Accordingly, you will get you know several items. So, that it can be explained the variance covariance matrix because, here our main agenda is not to describe this entire situation; because, it is you know it is a relevant. But, in the mean times we are skipping all these details because our main agenda is to check the you know, reliability of the model.

(Refer Slide Time: 15:42)

Step:  $\hat{\alpha} = ?$        $\hat{\beta} = ?$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$        $\hat{\beta} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

$\sum x : 541$      $\sum xy = 88291$      $\sum y^2 =$   
 $\sum y : 1578$      $\sum x^2 = 34705$      $n = 9$

$\sum y^2 = 298712$      $\sum x^2 = 34705$

$\hat{\beta} = \frac{9 \times 88291 - 541 \times 1578}{9 \times 34705 - (541)^2} = \frac{-59079}{19664}$

$\hat{\beta} = -3.004$

So, what we have to do in the step 2? You have to get the first alpha hat component and we have to get the beta hat component. So, alpha hat equal to Y bar minus beta hat X bar and beta hat equal to n summation X Y minus summation X into summation Y then n summation X square minus sum X whole square. Now, what is our requirement? What we quickly calculate the... We need to calculate summation X equal to summation, how much summation Y and summation X y?

Then we have to know summation X square and we like to know again summation Y square. Then, you like to know in the mean time this is summation X square and we like to know summation Y square value and this is n value n equal to 9 here. So, summation X equal to here 541, then summation Y is here 1578 then summation X y. So, what we have to do we have calculate a series here is. So, which is a X Y X y. So, now, this is nothing but, equal and this a multiplied by this. So, now we will get it this 9537. So, you just call it. So, you will fill this particular column. So, this enters...

Now, we like to know what is the sum of this particular series. So, this is interest; the sum is X 88291 88291 and the summation this is 88291. Similarly, we like to know summation X square; what you have to do is X x and another column, we will calculate Y y. So, now, this is equal to 51 again multiplied by the same items. Then, you scroll this series particular series then you will get it here is the enter items. So, now, again this



should be copy this should be copy that enters **alright** . So, now, here Y y we have to fill the series Y multiplied again y. So, we need to know the summation Y square.

So, again we have to scroll this items. So, this item . So, now, what we have receives. So, now, this is summation Y square; that means, summation Y square is equal to 298712 then summation X square is equal to 34705. So, n is already here is now you come to the pictures. So, what you have to do we start with beta hat first then we will get it alpha hat. So, beta hat equal to n summation X Y so; that means, n equal to 9 here. So, 9 into summation X Y is a 88291 minus summation X into summation y. So, this is 50, 541 multiplied by 178. So, divided by n summation X square. So, this is 9 into summation X square equal to 34705 minus sum X whole square sum X equal to 541 . So, this is whole square

So, we like to know what is this exactly value. So, so what you have to do here. So, this is nothing but, we like to know 9 this is equal to 9 you will put it like this. So, 9 into 9 into 88291 88291 . So, this is minus then 541 541 what you have to do 541 and 541 multiplied by 1578 1578 . So, this is how you will receive it. So, so denominator is like this. So, 9 into once again we will check it. So, 9 into 8 8 88291 then minus 541 into 1578 . So, this is coming minus 59079. So, this is equal to minus 59079.

So, divided by divided by a another item. So, we will create. So, which is equal to minus **sorry** 9 into 9 into 34705 minus 541 into 541 this is our k now 34705. So, this is ( ). So, this is how 1 9 6 4. So, now, what you have to do? We have to just, we have to find out to equal to this divided by these particular items. So, now, this is, **if will** we solve this one then it is coming minus 3.004. So, this is how beta hat; that means, beta hat component is a minus 3.004. So, beta hat is in this is negatively slogged so; that means, this relationship is a a inverse related to others. So, Y and X are inversely related to each other.

(Refer Slide Time: 21:46)

The image shows a handwritten derivation on a light blue background. At the top right, there is a small logo for '© CET I.I.T. KGP'. The derivation starts with  $\beta^{\wedge} = -3.004$ . Below that,  $\alpha^{\wedge} = \bar{Y} - \beta^{\wedge} \bar{X}$  is written. This is followed by a calculation:  $= \frac{1578}{9} - (-3.004) \times \frac{541}{9}$ . The next line shows  $= 175.33 - (-160.574)$ , and the final result is  $= 335.907$ . Below this, there is a correction:  ~~$\beta^{\wedge} = -3.004 + 35$~~  and a boxed final equation:  $\hat{Y} = 335.91 - 3.004 X$ . In the bottom left corner, there is a logo for 'NPTEL'.

Corresponding to beta hats **corresponding to beta hats**, we like to know what is alpha hat here. So, beta hat I am repeating here once again, beta hat is coming minus 3.004 and we like to know what is alpha hat alpha hat equal to Y bar minus beta hat X bar. So, that means, what is I bar? Here, it is now 1 1578 divided by 9 minus beta hat equal to minus 3.004 multiplied by X bar X bar is a 541 divided by 9; which is equal to... So, what we are able to do here is, this is equal to 1578 divided by 9. So, we will get 175.3.

So, then wherein we like to know 541 by 9. So, this is 541 divided by 9. So, you will get **60** so; that means, and this is this is 175.33 minus what you will do this is minus 60. So, this is minus and this is minus this is minus and minus plus. So, what you have to do. So, this is this is minus equal to minus 3.004 multiplied by multiplied by this particular item. So, this is coming 1 8 y. So, now, 170 this is equal to 170 175 minus this particular item. So, it is 355 a .907. So, it is coming. So, minus into minus 1 1 8 Y 8 Y point 5 7 4; this will be coming 355.907. This is how Y bar means alpha hat equal to Y bar minus beta hat X bar and. So, y Y bar equal to Y bar equal to 175.33 and X bar equal to X bar equal to 18.574. Obviously, if you simplify then you will get it a you will get it, the term 355907. So, the estimated equation equal to minus 3.004 plus 35 **sorry**, this is not correct. So, Y hat equal to 355.91 minus 3.004 x. So, this is what the estimated lines. So, this is how the estimated line.

(Refer Slide Time: 24:47)

	B	C	D	E	F	G	H
1	Y	Y (H)	e	Y-Y			
2	187	202.706	-15.706	11.66	135.956	27.3698	749.105
3	210	175.67	34.33	34.66	1201.32	175.67	30859.9
4	137	160.65	-23.65	-38.34	1469.96	160.65	25808.4
5	136	142.626	-6.626	-39.34	1547.64	142.626	20342.2
6	241	238.754	2.246	65.66	4311.24	238.754	57003.5
7	262	259.782	2.218	86.66	7509.96	259.782	67486.7
8	110	112.586	-2.586	-65.34	4269.32	112.586	12675.6
9	143	127.606	15.394	-32.34	1045.88	127.606	16283.3
10	152	157.646	-5.646	-23.34	544.756	157.646	24852.3
11	175.333	175.336	-0.026	-0.06	22036	1402.69	256061
12					2448.44		28451.2
13							26002.8
14							
15							
16							

So, what you have to do? We will we go through this particular copy, these items then, you go to next sheet and again you paste it here. So, now, go to view zoom second 2 hundred **alright**. So, we have X series we have Y series. So, we have to create here Y hat. So, Y a Y hat; we will create Y hat what is Y hat here. So, Y hat equal to 355.91 minus 3.00 x.

This is equal to this is equal to 355 a point 9 1 minus 3.00 3.004 multiplied by multiplied by a Y into Y hat equal to into x. So, this is what we have received. So, now, you just scroll it. So, this is what the Y hat series this is what the Y hat series. So, corresponding Y hat we have to get e hat error components. So, e equal to Y minus Y hat. So, this is equal to Y minus Y hat this is what we have this here. So, now, just you scroll it. So, this is how you just check it what is the error sum. So, this error sum is should be coming around close to 0 this is.

(Refer Slide Time: 26:24)

$$\hat{Y} = \hat{\alpha} + \hat{\beta}X$$

$$\hat{Y} = 355.91 - 3.004X$$

$$\text{var } \hat{\alpha} = \frac{\sigma^2 \sum X^2}{n \sum X^2}$$

$$\sigma^2 = \frac{\sum e^2}{n-2}$$

$$\sum e^2 = \sum Y^2 - \sum \hat{Y}^2$$

$$\sum Y^2 = \sum (4-9)^2 = 22036/9 = 2448.4$$

$$\sum \hat{Y}^2 = \sum (4-\hat{Y})^2 = 28451.2$$

$$\sum e^2 = 2448.4 - 28451.2 = -26027.8$$

$$\sigma^2 = \frac{26027.8}{7} = 3718.3$$

So, now, now what we have received. So, in this shall we started with the in this shall we started with the you can say Y information X information in the mean times we had received Y hat and we had received the error component we have received the error component. So, now, So, what you have to do means what how you have to proceed further. So, now, we have receives we have receives this Y hat component. So, this Y hat component. So, this is Y hat equal to 355.91 minus 3.004 x. So, now, we have this is how we Y hat we have received. So, now, Y hat is this much.

So, what you have to do? So, this is otherwise known as  $R^2$  in other words, this is Y hat equal to alpha hat plus beta hat X so; that means, alpha hat is this much and beta hat is this much. So, now, we like to know the reliability part of the model. So, that means, we like to know whether this alpha hat is a statistical significant or beta hat is the statistical significant. So, now, for this we have to proceed further to calculate the variance of alpha hat and variance of beta hats. Let me highlight here is, you know, variance of alpha hat variance of alpha hat equal to sigma square summation X square by a summation X square **alright**.

Now, m summation X square this is n summation X square and summation X square . So, now, what we have to do here is you first calculate the sigma square sigma square is the summation e square by n minus 2. So, what is summation e square? Summation e square equal to summation Y square minus summation Y hat square summation

summation  $\hat{Y}^2$ . So, now, basically here is summation  $Y^2$  means summation  $(Y - \bar{Y})^2$ . So, similarly summation  $\hat{Y}^2$  is nothing but, summation  $(\hat{Y} - \bar{Y})^2$  so; obviously, summation  $e^2$  square we will get it accordingly.

Now, we have to calculate summation  $Y^2$  first. So, how do you go for that. So, we have to create here is we first we first like to know what is the mean of this particular series. So, mean of this particular series you can get it here is. So, this is what they mean. So, 60.11; is it 60.11 mean of the series 60.11? So, this is how we have to received 75.33 **alright**. This is mean of X this is mean of Y this is mean of Y here. So, we like to know summation  $Y^2$  summation  $(Y - \bar{Y})^2$ . So,  $Y - \bar{Y}$ ; this is how we have to make it square; so, **alright**.

So, this is equal to  $\sum (y - \bar{y})^2$ . So, this minus 1 minus 1 point, so, 175.34. So, 175.34. So, **alright** 175.34. This is what we have received  $Y - \hat{Y}$ . So, this is copy now it is **alright**. So, this is we have to make it square again. So, so this is what we have to do multiplied by this 1's. So, this is 1356. So, we can. **So, alright**.... So, now, this is  $\sum \hat{Y}^2$ . So, this particular this is  $\sum Y^2$  no this is  $\sum Y^2$ .

So, summation  $Y^2$  equal to. So, 22036; 22036 summation  $(Y - \hat{Y})^2$  equal to 22036 and obviously, it is divided by  $n$   $n$  is  $n$  is you can say by this is divided by divided by 9. So, 2448 this is 2236 divided by 9. So, we will get, because this is summation  $Y^2$  means square means. So, it is to be divided by  $n$ . So, it would been coming 2448.4 **alright**. So, similarly summation  $\hat{Y}^2$ . So, what we have to do instead of writing  $\hat{Y}$  here. Similarly, we have to create another series here is  $\hat{Y}^2$  square.

What will we do, summation? Similarly, we have to find out here is  $\hat{Y}^2$ . So, this is a this minus this minus seventy 5.36. So, I need to look on like this. So, scroll it **alright**. So, now, you just move it through **alright**. So, this multiplied by this particular item again. So, this is coming like this. So, now, this summation  $\hat{Y}^2$ . So, is equal to 28451.2. So, this is 2444 2448.4 so; obviously, summation  $\hat{Y}^2$  equal to summation  $Y^2$ . So, 2 4 4 8 point 4 minus 28 28451.2

So, this is this is somewhat summation  $Y^2$  summation  $Y^2$  divided by  $n$  2236. So, this is  $\hat{Y}^2$  2 448 2 2 0 1 9 this is 9. So, this is how you have to receive the

summation Y square. So, 2448; this is coming equal to this minus this. So, this is 26002.8, **alright**. Now, this is how you have to fill this area here is. So, variance of alpha hat accordingly you have to calculate.

(Refer Slide Time: 33:16)

The image shows handwritten mathematical derivations on a blue background. At the top right, there is a small box containing the text 'ECEET D.T. RGP'. The main derivations are as follows:

$$\text{var}(\hat{\alpha}) = \sigma^2 \cdot \frac{\sum x^2}{n \sum x}$$

$$= 3715.7 \cdot \frac{54705}{9 \times 347}$$

$$\sum x^2 = \sum (x - \bar{x})^2$$

$$\sum y^2 = \sum (x^2 + \bar{x}^2 - 2x\bar{x})$$

$$\sum xy = \sum x^2 + n\bar{x}^2 - 2\bar{x}\sum x$$

$$\sum x^2 + n\bar{x}^2 - \bar{x}^2 \cdot \frac{\sum x}{n} \cdot n$$

$$\sum x^2 + n\bar{x}^2 - 2n\bar{x}^2$$

$$\sum x^2 = \sum x^2 - n\bar{x}^2$$

At the bottom left, there is a small circular logo with the text 'NPTEL' below it.

Variance of alpha hat equal to alpha hat equal to sigma square; so, sigma square into summation X square by n summation X square. So, now, here, summation e square by n minus 2. So, which means summation sigma square equal to summation e square 9 2 means 2 6 00 2 divided by n minus 2 this is 7.

So, this is equal to divide... no sorry, this equal to 3715.7. So, this is sigma square value. So, now, variance of alpha hat equal to sigma square u. So, 3715.7 into summation X square. So, summation X square we have received already here; summation X square equal to this is 347. So, this is into 347 347 0 5 divided by n summation X square into into a 9. So, what we have to do actually? We like to calculate **first** summation X square.

So, we like to calculate summation Y square then, summation X y. So, there is a standard formula before means unless we are wasting time. So, what we have to do here. So, this is nothing but, summation X minus X bar whole square. So, this if will we simplify furthers this particular extension. So, this is nothing but, summation X square plus X bar square minus 2 2 into X into X bars. So, if we if we will calculate further then this is summation X square plus n X bar square minus X bar into 2 summation x. Now, 2

summation x. So, this is summation X square plus n X bar square minus X bar into 2 into summation X divided by n into n. So, this is nothing but, X bar.

So, summation X square plus n X bar square minus X bar this is X bar means 2 n X bar square. So, this is n X bar square this n ys. So, this is summation X bar square minus m X bar square. So, this is what these summation X square value.

(Refer Slide Time: 36:06)

The image shows handwritten mathematical derivations on a light blue background. At the top right, there is a small logo for 'CET LIT. KGP'. The derivations are as follows:

$$\begin{aligned} \sum x^2 &= & \sum x^2 &= \sum x^2 - n\bar{x}^2 \\ \sum y^2 &= & \sum y^2 &= \sum y^2 - n\bar{y}^2 \\ \sum xy &= & \sum xy &= \sum xy - n\bar{x}\bar{y} \end{aligned}$$
  

$$\begin{aligned} \sum x^2 &= 34705 - 9 * (60.11)^2 \\ &= 2186.1 \\ \sum y^2 &= 296712 - 9 * (175.33)^2 \\ &= 22046. \end{aligned}$$
  

$$\begin{aligned} \sum y^2 &= \sum (r^2 x^2) = r^2 \sum x^2 = \\ r^2 &= \frac{(\sum xy)^2}{(\sum x^2)(\sum y^2)} \end{aligned}$$

Accordingly, we have to transport all these things; that means, we have information summation Y square, summation Y square summation X Y. Then, we need to calculate summation X square; we need to calculate summation Y square. We need to calculate summation X Y; these are **very** you know essential to you can say to consume less times.

This is summation; X square is in capital format and these are all small format. Summation X square is something; summation X square minus n X bar square. Then, similarly, this is summation Y square minus n Y bar square. So, this is how summation X Y minus n into n into X bar and Y bars summation this sorry summation X Y summation X Y minus n X bar into Y bar . Now, to get summation X square to get summation X square, we have summation X square; so, we have summation X square; that is you know 34705 minus n X bar square. So, minus n is 9 into what is X bar X bar equal to simply what is X bar X bar.

(Refer Slide Time: 37:15)

	A	B	C	D	E	F	G
2	51	187	9537	2601	34969		
3	60	210	12600	3600	44100		
4	65	137	8905	4225	18769		
5	71	136	9656	5041	18496		
6	39	241	9399	1521	58081		
7	32	262	8384	1024	68644		
8	81	110	8910	6561	12100		
9	76	143	10868	5776	20449		
10	66	152	10032	4356	23104		
11	60.1111	175.333	88291	34705	298712		
12							
13	-59079	19664	175.333	60.1111		2186.091	
14		-3.00442		-180.574		22046.52	
15				355.907		-6560.78	
16						43043791	
17						19689.76	

This particular, I thought instead of putting sum we will take average directly. So, now, minus  $\bar{X}$  square. So, this is nothing but,  $\sum (X - \bar{X})^2$  alright. So, if we simplify you will get this summation  $\sum X^2$  value. Similarly, summation  $\sum Y^2$  equal to summation  $\sum Y^2$  equal to summation  $\sum \bar{Y}^2$ . So, this is this is a summation  $\sum \bar{Y}^2$  this is  $298712$  minus  $9$  into this is  $\bar{Y}$  square; so,  $175.33$  whole square. So, this is our summation  $\sum Y^2$  will be get it. So, summation  $\sum X^2$  summation  $\sum Y^2$  similarly with the help of you know summation  $\sum X^2$  and summation  $\sum Y^2$  we can able to get means we can able to calculate the summation  $\hat{Y}^2$  also.

So, these the detailed structure is like this and this is equal to  $34705$  minus minus  $9$  into  $9$  into  $60.11$  into  $60.11$   $2186$ . So, this will be coming  $2186.86$  .1  $28186.1$ . So, this is what we have received the summation  $284186$   $2186$   $2186.1$  this is summation  $\sum Y^2$  square then similarly  $\hat{Y}^2$   $\bar{Y}^2$  equal to this is nothing but,  $298712$  minus  $9$  into  $175.33$  multiplied by  $175.33$  . So, we will get  $220220$   $22046$   $22046$ .

So, this is how we have to received the... you can say summation  $\sum Y^2$  summation  $\sum Y^2$  square. So, this is summation  $\sum X^2$  this is summation  $\sum Y^2$  then with the help of this we can also get get to know  $\hat{Y}^2$  summation  $\hat{Y}^2$  is nothing but, summation  $\hat{\beta}^2$   $\hat{X}^2$  whole square so; that means, its  $\hat{\beta}^2$  square summation  $\sum X^2$ . So, what is  $\hat{\beta}^2$  square. So,  $\hat{\beta}^2$  square is nothing but,



summation  $\sum XY$  whole square divided by summation  $\sum X^2$  into square summation  $\sum X^2$  into  $\sum X^2$ .

Altogether, this is summation  $\hat{Y}$  square beta hat square, summation  $\sum X^2$ . So, summation into summation  $\sum X^2$ . So, this will be coming summation  $\sum XY$  by summation  $\sum X^2$  simply this is summation  $\sum XY$  whole square by summation  $\sum X^2$  square. So, this is what the beta beta you know beta hat square or summation  $\hat{Y}$  square. We like to know what is summation  $\sum XY$  and we like to know summation  $\sum X^2$  because, summation  $\sum X^2$  is already calculated here. So, we like to know, what is summation  $\sum xy$ ?

(Refer Slide Time: 41:21)

$$\begin{aligned} \sum xy &= \sum XY - N\bar{X}\bar{Y} \\ &= 88291 - 9 \times 60.11 \times 175.33 \\ &= -6560.8 \end{aligned}$$

$$\sum y^2 = \frac{(\sum xy)^2}{\sum x^2}$$

$$= \frac{42043791}{2106}$$

$$= 19609.76$$

$\sum x^2 = +$   
 $\sum y^2 = +$   
 $\sum xy = +/-$

Summation  $\sum xy$  summation  $\sum xy$  is nothing but, summation  $\sum xy$  minus  $N \bar{X} \bar{Y}$ . So, what is summation  $\sum xy$ ? Here, summation  $\sum xy$  is, here is, 88291 minus **n into** 9 into  $\bar{X}$  bar is 60.11 multiplied by and then 175.33. So, which is a. So, this is equal to this is equal to 88291; 88291 minus. So, minus 9 into 9, into 6.11 multiplied by 175.33; this is coming negative; this is coming negative and this is coming negative. So, minus 65 minus 6560.8 before, one thing you must be very careful. Actually, we need to know this summation; small  $\sum x^2$  we need to know; summations small  $\sum y^2$  we need to know, summations small  $\sum x$  into summation small  $\sum y$ .

So, you must be very careful that this must be always positive and this must be always positive. This can be positive, this can be negative because, this particular item is nothing

but, the covariance item and this is variance item this is variance item. So, variance cannot be negative; variance means variance of X cannot be negative; variance of Y cannot be negative. So, whatever formula you have to use, it should be always positive. So, that is why that is our summation small x square is always positive. **Small** summation small y there is also always positive. But, summation xy can be positive; can be negative. If they are negative, related to each other that means, if x and y are negative related to each other then; obviously, summations small xy will be negative; if they are positive related then summation xy equal to positive. So, that will decide whether this coefficient will be negative or positive because, it will be, it will indicate the, you know, negative association and positive. What is positive association between these two variables? So, now, the moment you get summation x square summation y square and summation xy then with the help of with the help of all these three items we can able to get the summation Y hat square also.

So, summation y hat square is equal to summation xy whole square divided by summation x square. So, this is 6560. So, what you have to do. So, you make it again square multiplied by this particular item. So, this is how it is coming like this. So, now, divided by summation x square what is summation x square. So, now, So, this is this is 43043791 divided by summation x square summation x square is nothing but, 2186 2186. So, 2186 means a we need to divide this divided by 2186.1.1. So, this will be coming summation y hat square is coming 19689.76; **19689.76**; this is y hat square.

(Refer Slide Time: 44:58)

**Analysis of ANOVA**

With Estimated parameters      With overall statistics of the model

Source of Variation	Sum of Squares	Mean Sum Sq	df	F	P
ESS	$\sum y_i^2$	$\sum y_i^2 / (n-1)$	$n-1$		
RSS	$\sum e_i^2$	$\sum e_i^2 / (n-4)$	$n-4$		
TSS	$\sum y_i^2$	$\sum y_i^2 / (n-1)$	$n-1$		

$R^2 = \frac{ESS}{TSS} = \frac{\sum y_i^2}{\sum y_i^2 + \sum e_i^2}$   
 $R^2 = \frac{ESS}{TSS} = \frac{\sum y_i^2}{\sum y_i^2 + \sum e_i^2}$   
 $R^2 = 1 - \frac{(1-R^2)(n-1)}{(n-4)}$   
 $F = \frac{ESS}{TSS} \frac{(n-1)}{(n-4)}$   
 $F = \frac{RSS}{TSS} \frac{(n-1)}{(n-4)}$   
 $F = \frac{R^2 - (R^2)^2}{(1-R^2)^2} \frac{(n-1)}{(n-4)}$

Now, our standard structure is almost all. So, what you have to do? You go to this particular table here. So, now, you see we get to know this  $y$   $\hat{Y}$  we get to know this  $\beta$   $\hat{\alpha}$ . Then, we are in the process of calculating variance of  $\alpha$   $\hat{\alpha}$  and we are in the process of calculating the  $\sigma^2$   $e$   $y$   $\sum x^2$ .

(Refer Slide Time: 45:21)

Handwritten mathematical derivations on a whiteboard:

$$\sum x^2 \quad \sum y^2 \quad \sum xy$$

$$\sum y \hat{y} \quad \underline{\underline{\sum e^2}} = \sum y^2 - \sum y \hat{y} \quad \frac{\sum e^2}{n-2} \quad \sigma_u^2$$

$$\text{Var}(\hat{\alpha}) = \sigma_u^2 \cdot \frac{\sum x^2}{n \sum x^2} \quad \text{Var}(\hat{\alpha}) = \sigma_u^2 / \sum x^2$$

$$\text{SE}(\hat{\alpha}) = \sqrt{\text{Var}(\hat{\alpha})} \quad \text{SE}(\hat{\alpha}) = \sqrt{\frac{\sigma_u^2}{\sum x^2}}$$

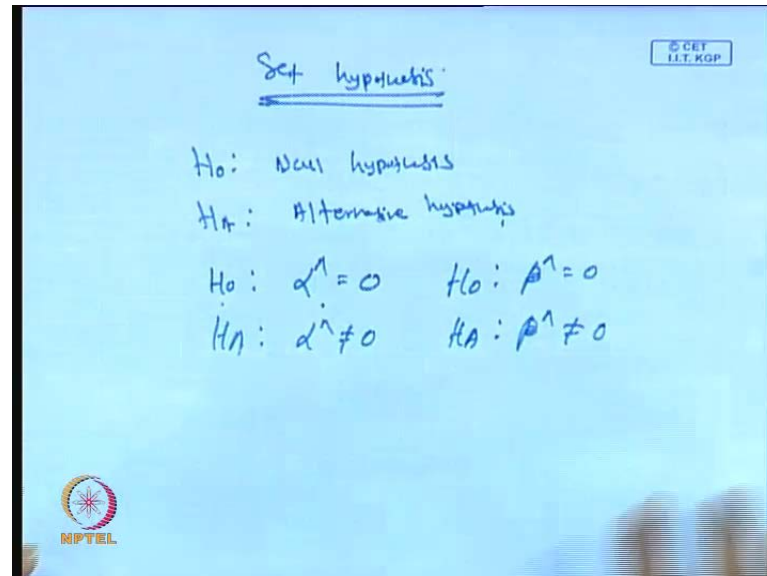
Once you get **this** all these details, what you have to do? So, we have summation  $x^2$ ; we have summation  $y^2$ ; we have summation  $xy$ ; we have summation  $y \hat{y}$  square. So, now we **calculate** we need to calculate summation  $e^2$  summation  $e^2$  equal to summation  $y^2$  minus summation  $y \hat{y}$  square.

So, the moment you will get all these things then you have to go for **variance** you know variance of  $\alpha$   $\hat{\alpha}$ . So, variance of  $\alpha$   $\hat{\alpha}$  variance of  $\alpha$   $\hat{\alpha}$ . In fact, once you will get summation  $e^2$  square. So, you can also in the same times you can calculate summation  $e^2$  square by  $n - 2$  which is nothing but,  $\sigma^2$   $u$ . So, now, variance of  $\alpha$   $\hat{\alpha}$  equal to  $\sigma^2$   $u$  then this is summation  $x^2$  square by  $n$  summation  $x^2$  square. So,  $\sigma^2$   $u$  you have to put it here is. So, then you have to calculate the variance of  $\alpha$   $\hat{\alpha}$  then standard error of standard error of  $\alpha$   $\hat{\alpha}$  is equal to simply square root of variance of  $\alpha$   $\hat{\alpha}$  variance of  $\alpha$   $\hat{\alpha}$ .

Similarly, we have to calculate the variance of  $\beta$   $\hat{\beta}$ . So, variance of  $\beta$   $\hat{\beta}$  here, variance of  $\beta$   $\hat{\beta}$  equal to  $\sigma^2$   $u$   $\sigma^2$   $u$  divided by summation  $X^2$  square. So, now, standard error of standard error of  $\beta$   $\hat{\beta}$  equal to  $\sigma$  variance of

variance of beta hat variance of beta hat. So, now, once you get standard error of alpha hat and standard error of beta hat then next procedure is to get the t t statistics.

(Refer Slide Time: 47:01)



Before you move to t statistic there is a need to set the hypothesis; there is a need to set the hypothesis **there is a need to set the hypothesis.**

What is this hypothesis? There are two hypotheses actually. One is called as a null hypothesis and second one is called as a alternative hypothesis. Alternative hypothesis, first of all, what is hypothesis? Hypothesis means it is a statement which is not verified; we like to verify it; that means, here our observation is that alpha, alpha hat and beta hat has to be significant and we have to verify that whether this alpha hat is significant or you know not significant. Similarly, beta hat is significant or not significant. So, we have to set the hypothesis there means, how it is. How you can justify that whether it is significant or not significant? So, that means, alpha hat must have some value and beta hat must have some value.

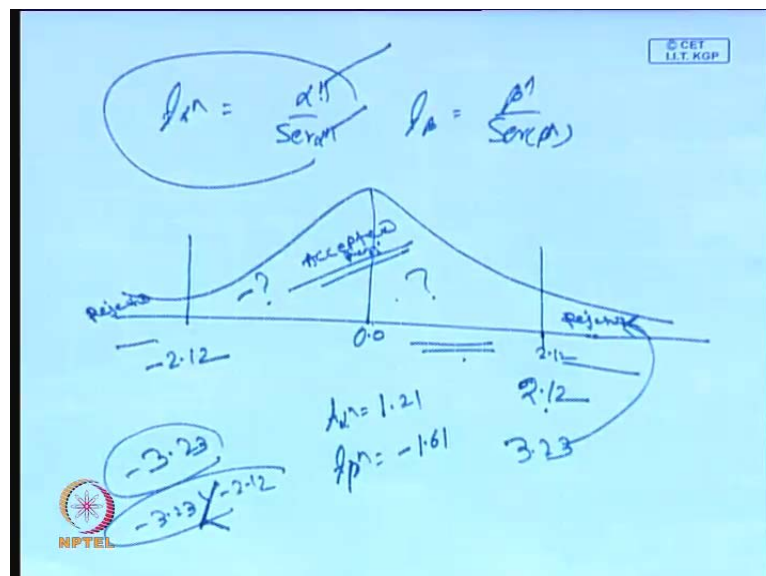
So, the moment you have a value then accordingly you can you can go for the statistical test; that means, in the standard rules which we have discussed in the last class that we need to calculate the t statistic and we have to compare with a tabulator statistics. That means, if you have some value then you can calculate the statistic and you can compare with the tabulated. If there is a no value, it is very difficult to calculate. So, what you have to do? So, in this particular contest, null hypothesis means a hypothesis which you

have to set to verify **your to** you know, to verify a particular objective which in then generally we set null hypothesis negatively. That means, for this h you know this particular problem. So, we said that alpha for alpha hat alpha hat equal to 0.

We start with you know, negative assumption so that, we have to... our objective is you **do reject that** assumption. So,  $H_0$  equal to **alpha**  $H_0$  such that alpha hat equal to 0 against  $H_A$  **k**  $H_A$ ; such that, alpha hat not equal to 0. Similarly, for beta  $H_0$  such that, beta hat equal to 0 and against alternative hypothesis beta hat not equal to 0. So, that means, we start with the standard assumption that alpha hat equal to 0. So, that means, there are no **derivatives**. So, we have to justify that alpha had some value. So, the moment you have to get alpha had some value then automatically the null hypothesis will be rejected.

So, our **every time** objective is to reject the null hypothesis. Null hypothesis means the statement we start with a negative direction. So, then we have we have to reject that negative direction then come to the perfect you know, means perfect accuracy that is the significance level of the tests. So, what is all about this particular assumption? So, the standard rule is that, we have to calculate the test statistics.

(Refer Slide Time: 49:56)



Now, t alpha hat equal to you know alpha hat by standard error of alpha hat and t beta hat is equal to beta hat by standard error of beta hat. So, now, this is calculated statistic. So, we have to derive then we have to compare with the tabulated statistics.

So, what is the structure? Actually, it is structured by this I mean, it is clearly followed by the normal distributions. So, now, the hypothesis statistics hypothetical means testing of hypothesis based on these particular structures. So, this is how we will classify the entire diagram into three parts. This is one part; this is another part and this is another. This is the mean of this particular series. So, now, what you have to do? This particular region is called as an accepted region. So, this particular region is called as an accepted region. So, this particular idea is called as a rejected region. So, this is also rejected region.

Now, let us start with this some value; let us say there are  $t$  edges, certain value with respect to different sample size and degrees of freedom. So, let us assume that you know when you will go for hypothesis testing. There is two different setup; it is called as one tailed test and two tailed test. So, one tailed test means this is remaining close and this will be expand. So, this means, this particular structure is called as a right tailed test and if you close this one and you will expand this one it is called as a left tailed test. So, now, what you have to do? It will be divided equally; then there is called as a 2 tailed tests.

So, that means, 50 percent this side and 50 percent this side. **So, now,** let us assume that there is a certain value set to at this side everything will be positive. So, the  $t$  value is 1.2; so, this side minus 2.12. So, now, with the basis on some you know, value  $\alpha$  hat you know how what is the  $\alpha$  hat and we have also calculated standard  $\alpha$  hat. So, now, the ratio between  $\alpha$  hat and standard error of  $\alpha$  hat will give you the  $t$  of  $\alpha$  hat. So, now, we have to position the  $t$  of  $\alpha$  hat that is the calculator this is what the tabulated statistic all is all about a certain degrees of freedom or you can say sample size.

Now, let us assume that **at a** for the particular problem the  $t$  value tabulated  $t$  value is a 2.12 for right tail and 2 minus 2.12 for the left tail. So, now, we have to see what is the  $t$   $\alpha$  hat; that is calculated statistic? So, now, the moment you will get  $t$   $\alpha$  hat the calculated statistics then you have to represent... we have to just check it whether it is negative or positive. If it is negative then this structure will coming this side if it is positive then this structure will be coming this side. So, now, let us assume that this particular item say 3 minus 3.23; so, that means, this particular item is coming this side so; that means, it is nowhere in this particular region.

So, now if this is the structure then the structure is minus 3.23 is greater than to minus 2.12 means it is not a greater than; obviously, it is a less than. So, that means, it is coming this side. So, this side **coming** means we are rejecting the null hypothesis the moment you are rejecting the null hypothesis; that means, alpha hat is the statistically significant. Similarly, suppose we will get the component called as a 3.23 positive. So, 3.23 means this particular value will be coming this side so; that means, it is greater than to this item. So, that means, it is in the rejected region.

The moment it will be in the rejected region; that means, we are rejecting the null hypothesis the moment you are rejecting the null hypothesis. So, the observation is that or we have to represent that this particular statistically particular item is statistically significant. So, now, you take in otherwise. So, this is 2.18. So, I will be take I I will get the t is t statistic theta alpha hat is equal to simply 1.21. **So, now, 1.21.** So, this is 0 point 0 origin. So, 1.21 means this will coming in this particular range this is 2.12. So, this will coming this range. Obviously, this particular means this particular will replace in this region this; is accepted region.

That means, we are accepting the null hypothesis. The moment you are accepting the null hypothesis; that means, by default we will call it as a... this will this particular parameter is not statistically significant. Similarly, let us assume that we are getting t alpha hat t alpha hat or t beta hat equal to **minus** 1 point you can say 61 s. So, minus 1.61 means it will come left side and here the boundary is minus 2.12. That means, it will not cross these ones. So, minus 1.61 means it will place somewhere here is all somewhere here only so; that means, it is still in accepted regions.

So, the moment it will in accepted region, we are rejecting the means, sorry, we are accepting the null hypothesis. The moment you are accepting the null hypothesis, that means, our observation is that we are rejecting the means, we are in the position to say that this particular statistic is a not significant. That means, what is this? we have two steps here; first step is you get the calculated statistics for alpha hat and beta hat and again you have to know the alpha hat. The tabulated statistic and beta hat tabulated statistics then you piece the **(( ))**. Then, you just check whether the calculated statistic means, what is the position of calculated statistic if it is coming this side; negative and this side after the **(( ))** line then we have to reject the null hypothesis and statement put the statement that it is significant.

Similarly, this side if it is positive and it will go beyond that one then, we are rejecting null hypothesis and putting this statement. That it is the statistically significant; otherwise, the conclusion is that the parameter is not statistically significant. So, this is how we have to justify the hypothesis testing. Of course, we have not considered here the level of significance. This is based on two things; first is with respect to number of sample size, that too degrees of freedom and second thing is the probability level of **significance** - probability level of significance. It may be 1 percent, it may be 5 percent, it may be 10 percent.

The values are coming different with respect to 1 percent, 5 percent and 10 percent corresponding to different levels of probability and the sample size. The items will be different. So, you have to compare with a calculated statistic with different levels of you can say probability level of significance accordingly make a judgment. So, whether it is statistical significant at 1 percent or statistical significant at 55 percent or statistical significant at 10 percent. If not then, you will simply... that the statistic is or the parameter is not statistical significant.

With this, we will conclude this session. In the next class **will we** go in details about this ANOVA part? How you have to proceed with ANOVA and you have to justify the significance of the particularly outer fitness of the model; that too r scale. Thank you very much, have a nice day.