

**Econometric Modelling**  
**Prof. Rudra P. Pradhan**  
**Department of Management**  
**Indian Institute of Technology, Kharagpur**

**Lecture No. # 19**  
**Matrix Approach to Econometric Modelling (Contd.)**

Good evening, this is doctor Pradhan here. So, welcome to NPTEL project on econometric modelling. So, today we will discuss the same matrix approach to econometric modelling. So, in the last class, we have given the basic framework of matrix approach to econometric modelling that means, how you have to have use matrix to solve the econometric modelling that to get the estimated model.

So, basically we use matrix for a multivariate problem, but that does not mean that matrix application is always meant for multivariate problem. It can be also use for bivariate also. So, you **you** can use matrix in the bivariate, you can use trivariate or you can use multivariate but the use of matrix multivariate is the more interesting, more meaningful or it is more utility, but it is very difficult to start with your multivariate problem and that to apply of matrix approach here it is very difficult.

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MATHEM

$$Y = [1, 4, 3, 8, 9]$$
$$X = [0, 1, 2, 3, 4]$$
$$Y = X\beta + U \quad \cdot \quad Y = [Y_1, Y_2, Y_3, Y_4, Y_5]$$
$$X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & \beta_{11} \\ 1 & \beta_{21} \\ 1 & \beta_{31} \\ 1 & \beta_{41} \\ 1 & \beta_{51} \end{bmatrix}$$
$$Y = \begin{bmatrix} 1 \\ 4 \\ 3 \\ 8 \\ 9 \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

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So, what we have to do here? We have to take a very simple problem and we will highlight how matrix can be used to get the estimated model. So, that is **that is** our today's observations only. So, I will take a simple problem. So, the problem consist of like this Y is 1 4 3 8 9. So, then corresponding to Y we have X concerned to 0 1 2 3 4.

So that means, we are taking a bivariate systems here. So, there is one dependent variable Y and there is another independent variable X. So, we can **we can** regress or we can estimate by simple application of various techniques, which we have discussed long back. But, my point is here, how matrix can be applied to solve this particular problem, so that we can get the estimated equations.

So, now in this particular structure there are two variables, one dependent and one independent and sample size is equal to 5. So, artificially we are keeping the system very simple so, that we can **we can** discuss very quickly otherwise, you will **you will** start with very complex problem it is very difficult to finish within this particular class timing. So, that is why we have taken a very simple problem that too very small sample size. So, that our aim is here how matrix can be applied to get the estimated econometric model.

So, as usual for this particular problem so, in the mean times it is not required to know what is exactly Y? What is exactly X? And what is theory behind Y and X? Whether Y and X are related to each other? So, in the mean time just we are just testing the pure mathematics. So that means, with having Y and X how we will get the estimated model that to  $\beta_0$  hat and  $\beta_1$  hat and then we have to test the significance of this parameters and the overall fitness of the model. That is the basic agenda.

So of course, we need theory, we need literature so, that we can justify properly or we can properly interpret the estimated model. But in the mean times it is not required here. It is required when we will go for something else. So, in the meantime we just apply the matrix and have the estimated model that is our main agenda. So, let us assume that our simple model is  $Y = X\beta + U$ . So, here Y consists of  $Y_1 Y_2 Y_3 Y_4$  and  $Y_5$  where these are the concept and this particular structure can be represented like this.

So, X represents here, 1 1 1 there are 5 observation 4 5. So, then 0 1 then 2 3 4 this is X; that means, what is exactly the structure this is like this. So, what we are doing 1 1 1 1 1 this is  $\beta_0$  and for  $\beta_1$  the samples are  $X_1 X_2 X_3 X_4 X_5$ . So, this is X and similarly, Y can be represented like this 1 4 3 8 9 so, this is Y.

So, then we like to know what is beta? So, beta is simply beta 0 and beta 1. So now, this is our model and this is called as a matrix approach to bivariate econometric models. So, the Y equal to X beta plus U it is very interesting. So, if the system is 2 then we can write Y equal to X beta plus U, if it is 3 we can also write Y equal to X beta plus U, if the system consist of 4 variable we can also write Y equal to X beta plus U. Even the system is 5 variables you can write Y equal to X beta plus U.

That means, whatever step you have the problem setup is all about we will always represent in unique format that is the specialty of matrix approach. So that means, Y is simply equal to X beta plus U. However, when we interpret or when we will justifying the variable Y, X beta and U that time it will highlight the details. For instance, if it is bivariate then there are only 2 columns here. So, if it is trivariate then there are 3 columns here, if it is 4 variables then another column you have to add.

So, this is how you have to proceed further. So, this is the basic setup here. So, this is for bivariate so, once trivariate then another column then if it is 4 variable system then there is another row like this it will continue. So similarly, it will add one after another beta 0 beta 1 beta 2 beta 4. So, by the way it will also increase continuously. So, this is how the entire structure so that means, Y is like this, X is like this and beta is like this. So now, we like to know the structure. How will we get the estimated model?

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Handwritten mathematical derivation on a blue background showing the calculation of the OLS estimator  $\hat{\beta} = (X'X)^{-1} X'y$ . The derivation includes the following steps:

- Model:  $Y^1 = X\beta^1$
- Estimator:  $\hat{\beta} = \frac{(X'X)^{-1} X'y}{X'X}$
- Matrix  $X$ :  $X = \begin{bmatrix} 1 & 0 \\ \vdots & 1 \\ \vdots & 2 \\ \vdots & 3 \\ \vdots & 4 \end{bmatrix}$
- Matrix  $X'$ :  $X' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$
- Matrix  $X'X$ :  $X'X = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 1 \end{bmatrix}$
- Calculation of  $X'X$  elements:
  - Top-left element:  $1*1 + 1*1 + 1*1 + 1*1 + 1*1 = 5$
  - Top-right element:  $1*0 + 1*1 + 1*2 + 1*3 + 1*4 = 10$
  - Bottom-left element:  $0*1 + 1*1 + 2*1 + 3*1 + 4*1 = 10$
  - Bottom-right element:  $0*0 + 1*1 + 2*2 + 3*3 + 4*4 = 30$
- Final result:  $X'X = \begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix}$

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So, what is  $\hat{Y}$ ?  $\hat{Y}$  is equal to  $X\hat{\beta}$ , where  $X\hat{\beta}$  equal to  $X(X^T X)^{-1} X^T Y$ . So, this is our estimated structures. So, forget about the variance of  $\hat{\beta}$  and other things because that will come later. So, once we get the estimated model then we will look into the significance of the parameter and overall fitness of the model. So, our **our** basic objective is here to get this  $\hat{\beta}$ . So, how do we get this  $\hat{\beta}$  through this matrix application only that is what?

So,  $X$  equal to simply  $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$ ; so five ones, then  $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$  this is  $X$ . So, then  $X^T$  equal to  $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$  then  $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$  this is how the  $X^T X$  all about. So that means, we like to know what is  $X^T X$  first we calculate  $X^T X$ . So,  $X^T X$  means. So, this is  $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$  multiplied by  $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$  this is then  $\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 5 & 4 & 3 & 2 \\ 3 & 4 & 5 & 4 & 3 \\ 2 & 3 & 4 & 5 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$  this is how a complete structure.

So, now we have to multiply. So, this is nothing, but, 1 into 1 plus this way it will connect like this **this** will connect like this **this** will connect like this and this will connect like this. So that means, 1 1 1 1 this and this **this** and this **this** and this **this** and this and finally, this and this.

So, this is how the entire structure; so, 1 into 1 plus 1 into 1 plus 1 into 1 plus 1 into 1 high multiplication 1 2 3 4 then, plus 1 into 1. So, because it is 2 into 2, this is 2 into 4, this is 4 into 2. So, this is 2 into 5 and this is 5 into 2. So, we will get simply 2 into 2. So, this is 1. So obviously, there will be another one so that means, this particular 5 has to be multiplied by this one.

So that means, 1 into 0 plus 1 into 1 plus 1 into 2 plus 1 into 3 plus 1 into 4. So similarly, this is how we have received. So, this will be multiplied by this and this will be multiplied by this. So, that means, 0 into 1 plus 1 into 1 plus 2 into 1 plus 3 into 1 plus 4 into 1. So, this is one structures then another is 0 into 0 plus 1 into 1 plus 2 into 2 plus 3 into 3 plus 4 into 4 this is how the entire structure result. So, now this is if we will do simplify this will simply 5 and if we will simplify this one this will be simply 10 and if we will simplify this one again this will be 10 and if we will simplify this one this will be remains 30. So now, we will transpose so that means,  $X^T X^{-1} X$  equal to simply 5 10 then 10 30. So, this is how the  $X X^T$  transpose. So, we have received  $X^T X$  is this much.

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Handwritten mathematical derivation on a blue background:

$$X'X = \begin{pmatrix} 5 & 10 \\ 10 & 30 \end{pmatrix} \quad \beta^1 = (X'X)^{-1} X'y$$

$$(X'X)^{-1} = \frac{A^{-1}}{|A|}$$

$$= \frac{\begin{bmatrix} 30 & -10 \\ -10 & 5 \end{bmatrix}}{\begin{matrix} 30 \times 5 - 10 \times 10 \\ 150 - 100 \end{matrix}} = \frac{1}{50} \begin{bmatrix} 30 & -10 \\ -10 & 5 \end{bmatrix}$$

$$(X'X)^{-1} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.1 \end{bmatrix} \quad X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$X'y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 8 \\ 9 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 4 + 1 \times 3 + 1 \times 8 + 1 \times 9 + 1 \times 7 \\ 0 \times 1 + 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 7 \end{bmatrix} = \begin{bmatrix} 25 \\ 70 \end{bmatrix}$$

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So similarly, we like to know what is X transpose Y so that means, X transpose X equal to 5 10 then 10 30 at a time we can finish these (( )) so that means, we need beta hat, beta hat equal to X transpose X inverse into X transpose y. So, we have now X transpose x. So, we have not received X transpose X inverse. Let us first do the X transpose X inverse. So, which is nothing but adjoint of matrix A divided by mode A, this is A matrix. So, we like to find out the adjoint of that particular matrix and divide by value of that matrix that is what the simple formula of inverse matrix.

So, for that here we have to know the cofactor and (( )) etcetera. So, I am not writing in detail. So, I am just calculating the value only. So now, if we will simplify this one then it becomes 30. So, then this is minus 10, this is minus 10 and this become 30. So, divide by value of the matrix this is 30 into 5 minus 10 into 10. So, this is nothing but 100 and this is 150 that means, it is equal to 1 by 50 by 30 minus 10 then minus 10 5 this is the inverse matrix.

So, inverse matrix of X transpose X inverse; so X transpose X inverse equal to 1 by 50 30, minus 10, minus 10, 5. So, that is the inverse matrix. So, now we like to know if we will simplify this one then X transpose X inverse is equal to 30 by 50 so that means, 3 by 5 this is 0.6 and this will be minus 0.2 then this is minus 0.2 then, this is simply 0.1. So, this is X transpose X inverse.

So similarly, we like to know  $X^T Y$ . So, we know what is  $X^T$ . So,  $X^T$  is here equal to  $1 \ 1 \ 1 \ 1 \ 1$  then  $0 \ 1 \ 2 \ 3 \ 4$  and we have to multiply by  $Y$ .  $Y$  is how much?  $Y$  is column vector here. So,  $Y$  represents  $1 \ 4 \ 3 \ 8 \ 9$ .

So that means,  $1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 2 \ 3 \ 4$  multiplied by  $Y$ .  $Y$  is how much? it is  $1 \ 4 \ 3 \ 8 \ 9$ . So, this is how the  $X^T Y$ . So, we will simplify again. So that means, this is  $2$  into  $5$  this is  $5$  into  $1$ . So, we will get  $2$  into  $1$  simply. So,  $2$  into  $1$  this multiplied by this and this multiplied by this. So that means, it is nothing but,  $1$  into  $1$  plus  $1$  into  $4$  plus  $1$  into  $3$  plus  $1$  into  $8$  plus  $1$  into  $9$ . So, this is one row, then second row is  $0$  into  $1$  plus  $1$  into  $4$  plus  $2$  into  $3$  plus  $3$  into  $3$  plus  $4$  into  $9$ . So, this is another row. So now, if you will simplify, then this will be  $25 \ 25$  and  $70 \ 25$  this is nothing but,  $25 \ 75$ ; that means, this particular sum is  $70$  and this particular sum is equal to  $25$ .

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Handwritten mathematical derivation on a blue background:

$$Y = [1, 4, 3, 8, 9]$$

$$X = [0, 1, 2, 3, 4]$$

$$X^T X = \begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix} \quad (X^T X)^{-1} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.1 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 25 \\ 70 \end{bmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 25 \\ 70 \end{bmatrix} \quad \begin{matrix} 2 \times 2 \\ 2 \times 1 \end{matrix} = 2 \times 1$$

$$= \begin{bmatrix} 0.6 \times 25 + (-0.2) \times 70 \\ (-0.2) \times 25 + (0.1) \times 70 \end{bmatrix}$$

$$= \begin{bmatrix} 15.0 - 14.0 \\ -5.0 + 7.0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

So, we will just summarize. So, we have  $Y$  equal to  $1 \ 4 \ 3 \ 8 \ 9$ , then  $X$  equal to  $0 \ 1 \ 2 \ 3 \ 4$ , then  $X^T X$  equal to  $5 \ 10 \ 10 \ 30$  this is  $X^T X$ . Then  $X^T X$  inverse is equal to  $0.60$ , then minus  $0.2$ , minus  $0.2$  then this is  $0.1$ . So, this is  $X^T X$  inverse. Then we need  $X^T Y$ .  $X^T Y$  is equal to how much?  $X^T Y$  is equal to  $25 \ 70$ . This is the summary of this particular system.

So, now you need to know, what is  $\hat{\beta}$ ? So,  $\hat{\beta}$  equal to  $X^T X$  inverse into  $X^T Y$ . So now,  $X^T X$  inverse is here then  $X^T Y$  is here. So

that means, 0.6 minus 0.2 minus 0.2 0.1. So, this is 1 matrix another matrix is 25 70. So, this is order of 2 into 1 and this is order of 2 into 2.

So obviously, this will be crossed out. So, ultimately the result of matrix should be 2 into 1 because beta hat means which is nothing but beta 0 hat and beta 1 hat. So, this is how the entire structures. So, we have to multiply again. So, this multiplied by this and this multiplied by this. So obviously, it is 0.6 into 25 plus minus 0.2 into 70. So, minus 0.70, then another column will be minus 0.2 into 25 plus 0.1 into 70. So, this is another **another** row.

So, this particular structure will be equal to how much? If we will simplify then this is 0.6 into 25. So, 15 then minus 14.0 then, minus 50 then this is plus 70.0.70 0.50. So that means, it is equal to 1 and 2. This is how it is coming. So that means, beta 0 hat equal to 1 and beta 1 hat equal to 2. Again we will **we will** summarize this entire result.

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Handwritten mathematical derivation on a blue background:

$$X'X = \begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix} \quad X'Y = \begin{bmatrix} 25 \\ 70 \end{bmatrix}$$

$$(X'X)^{-1} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.1 \end{bmatrix} \quad (X'X)^{-1}X'Y = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\beta^0 \hat{=} (X'X)^{-1}X'Y \Rightarrow \begin{bmatrix} \beta^0 \\ \beta^1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y^1 = 1 + 2x$$

$$= \beta^0 + \beta^1 x$$

$$(\text{Var}(\beta^0)) = \sigma_u^2 (X'X)^{-1}$$

where  $\sigma_u^2 = \frac{\sum e^2}{(n-k)}$

total Sample obs'n

Total no. of variables

2

So that means, X transpose X what we have obtained X transpose X is equal to 5 10 10 30, then X transpose Y equal to 25 70, then X transpose X inverse is equal to 0.6 minus 0.2 then minus 0.2 0.1. So, this is X transpose X inverse then we have X transpose X inverse into X transpose Y which is coming 1 and 2. So, this is how the system.

So now, what is our beta hat? So, beta hat equal to X transpose X inverse into X transpose Y; so which is nothing but beta 0 hat and beta 1 hat; so which is equal to 1 and

2. So that means, the estimated equation will be  $\hat{Y}$  equal to  $1 + 2X$ . So that means, this is **this is** nothing but, how we will write here; so that means, it is  $\hat{\beta}_0$  plus  $\hat{\beta}_1 X$ . So,  $\hat{\beta}_0$  is 1 and  $\hat{\beta}_1$  equal to 2.

So now, once you get the estimated model, then you have to go by as usual procedures. So, to test the significance of the parameter that  $\hat{\beta}_0$  and  $\hat{\beta}_1$  and again overall fitness of the model that is  $R^2$  and followed by F statistic. So, again go by  $\hat{\beta}_0$  separately and  $\hat{\beta}_1$  separately whether you will get  $\hat{\beta}$  direct value then, automatically it will indicate the significance of the parameters.

So let us see here. So now, to know the significance of the parameters then once this is **this is**  $\hat{\beta}$ . So, we like to know variance of  $\hat{\beta}$ . So, variance of  $\hat{\beta}$  equal to that is the formula is  $\sigma^2 U X^T X^{-1}$ . So, this is the formula which we calculate for you can say variance of  $\hat{\beta}$ . So, now once you have variance of  $\hat{\beta}$ . So, then accordingly by the procedure you have to calculate the sigma squares, then once you have the sigma square, then you have to multiply with  $X^T X^{-1}$ . Then you can get to know the variance of  $\hat{\beta}$ .

So, accordingly we have to make a square root you will get the standard error of  $\hat{\beta}$ . So, then you have to connect with null hypothesis ultimately you will get the t statistics. So, let see here how is the structure. So, before **before** going to sigma square summation  $X^T X^{-1}$  its better, you have to first calculate this particular structure. So, this is sigma square e square. So, sigma square u is equal to summation e square by  $n - k$ ,  $n$  represents total number of observations, total sample observations **sample observations** and this  $k$  represents total number of variables. Here  $k$  represents 2, because two variables are there only  $Y$  and  $X$  so,  $n - k$ . So now, we like to know what summation e square is.



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$$\sigma_u^2 = \frac{\epsilon^2}{(n-k)}$$
$$\sigma_e^2 = \sum y^2 - \sum \hat{y}^2$$
$$y'y = \sum y^2 = y'y - n\bar{y}^2$$
$$y'X'y = \sum \hat{y}^2 = \beta'X'y - n\bar{y}^2$$
$$e'e = \sum e^2 = y'y - \beta'X'y$$

So, summation e square is here. So that means, sigma square U equal to summation e square by n minus k. So, here summation e square equal to summation Y square minus summation Y hat square. So, what is summation Y hat square? So, summation Y square **summation Y square** is equal to Y transpose Y minus n Y bar square. So, this is how the structure about n minus n Y bar square and summation Y hat square equal to, but this is ESS **this is ESS** that will beta hat transpose X transpose Y minus n bar Y square.

Then summation e square equal to summation Y square minus n square. So, this is summation Y square equal to explained sum square that is residual sum square that will be Y transpose Y minus beta hat transpose X transpose Y. This is summation e square this is summation Y square, and then this is summation Y hat square.

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	Y	X	YY	XX	YX
1	1	0	1	0	0
2	4	1	16	1	4
3	3	2	9	4	6
4	8	3	64	9	24
5	9	4	81	16	36

So, then next step is you have to calculate summation Y square, summation Y hat square and summation Y square. You see, since we have only simply Y and X. So, you can just put in the excel sheet and you can calculate immediately by transferring **transferring** into deviation format this is all about say original format. We can transfer into deviation format and you can get the answer.

But here, our aim is not to get the result immediately. Here our aim is how matrix can help you to get this result done. So, that is about the structure. So that means, once you have this particular estimated beta. So, you have to find out the significance of the beta; then obviously, we have to find out the variance of beta hat and for that we need to know sigma square U and for that you need to know summation e square, summation Y hat square, summation Y square. So, these are the formulas which help you to calculate all these statistics.

So, that means what we have to do now. So, we know what is Y and we know what Y hat is? So, accordingly we have to calculate. So, summation Y square is nothing but, Y transpose Y means we have to multiply just like we have got here e transpose e. So, similarly, Y transpose e will get it here. So similarly, summation Y hat square equal to Y hat transpose you can say Y. So, this is how the entire structure can be built. So, let us see how we will calculate summation Y square first. So, summation e square **summation e square** we have to calculate like this.

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$$Y = \begin{bmatrix} 1 \\ 4 \\ 3 \\ 8 \\ 9 \end{bmatrix} \quad Y' = (1 \ 4 \ 3 \ 8 \ 9)$$

$$Y'Y = (1 \ 4 \ 3 \ 8 \ 9) \begin{bmatrix} 1 \\ 4 \\ 3 \\ 8 \\ 9 \end{bmatrix} =$$

$$= [1 \times 1 + 4 \times 4 + 3 \times 3 + 8 \times 8 + 9 \times 9]_{1 \times 1}$$

$$= 171$$

$$Y' = X' \beta = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

So, what we have to do. So, let say Y is there Y is how much? So, Y is 1 4 3 8 9. So, this is this is Y **Y** structures and we like to know Y bar Y transpose equal to 1 4 3 8 9. So, this is in square matrix. This is square matrix, this is column matrix. So, now we like to know what is Y transpose Y? So, Y transpose Y is nothing but, 1 4 3 8 9 then multiplied by column vector 1 4 3 8 9. So, this is how will give you summation Y square.

So, now this will be equal to 1 into 1 plus 4 into 4 plus 3 into 3 plus 8 into 8 plus 9 into 9. So, this is 1 into 5 and this is 5 into 1. So, the matrix orders we can receive 1 cross 1 only. So that means, summation Y square equal to 171. So, similarly, we like to know Y hat square. So, for that we need to know Y hat. So, hat equal to actually X beta hat. So, Y hat equal to X beta hat, but we **we** have beta hat and we have also X.

So, we can again multiply. So, what is X here? So, Y hat equal to X beta hat. So, X beta hat means what is X here? So, X is here is 1 1 1 1 1 five numbers. So, then 0 1 2 3 4 then multiplied by beta hat, **beta hat** equal to simply 1 and 2. So, this is 5 into 2 and this is 2 into 1. So, we will have the order 5 into 1.

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The image shows a whiteboard with handwritten mathematical work. At the top right, there is a small logo that says "© CET I.I.T. KGP". The main work is as follows:

$$Y^T = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \times 1 + 0 \times 2 \\ 1 \times 1 + 1 \times 2 \\ 1 \times 1 + 2 \times 2 \\ 1 \times 1 + 3 \times 2 \\ 1 \times 1 + 4 \times 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{pmatrix}$$
$$Y^T = (1 \ 3 \ 5 \ 7 \ 9)$$

At the bottom left of the whiteboard, there is a circular logo with a star-like pattern and the text "NPTEL" below it.

So, how will you do that? So, we will find out here. So, we like to know  $Y^T$ . So,  $Y^T$  equal to 1 1 1 1 1 then this is 0 1 2 3 4 then multiplied by 1 and 2. So that means, 1 into 1 then, 1 into 1 plus 0 into 2. So, this is first element then 1 into 1 plus 1 into 2 then 1 into 1 plus 2 into 2 then, 1 into 1 plus 3 into 2 then 1 into 1 plus 4 into 2.

So, this is the  $Y^T$ . So, if you will simplify then you will get this is nothing but, 1 this is nothing but, 3 this is nothing but, 5 and this is nothing but, 7 this is nothing but, 9. So, this is  $Y^T$ , but we need  $Y^T$  transpose  $Y$ . So,  $Y^T$  transpose means. So, this will be like this 1 3 5 7 9.

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$$Y^T Y = (1 \ 3 \ 5 \ 7 \ 9) \begin{bmatrix} 1 \\ 4 \\ 3 \\ 8 \\ 9 \end{bmatrix}$$

$$= \left( \frac{1 \times 1}{1} + \frac{3 \times 4}{12} + \frac{5 \times 3}{15} + \frac{7 \times 8}{56} + \frac{9 \times 9}{81} \right)$$

$$= [165]_{1 \times 1}$$

$$e = Y - \hat{Y}$$

$$= \begin{bmatrix} 1 \\ 4 \\ 3 \\ 8 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

So, we like to calculate  $Y^T Y$ . So, now,  $Y^T Y$  is nothing but, 1 3 5 7 9. So, this is 1 into 5 order then multiplied by  $Y$  what is  $Y$  here. So,  $Y$  here is 1 4 3 8 9 with the order of 5 into 1. So, 5 5 cancels. So, ultimately this has to be considered; that means we will get 1 into 1 matrix. So, for that we have to 1 into 1 plus 3 into 4 plus 5 into 3 plus 7 into 8 plus 9 into 9 9 into 9 so that means, this is completed.

(Refer Slide Time: 27:26)

So, this **this** is 1, this is 12, this is 15, this is 56, this is 81. So, if we will make a sum then it will be coming 165. So, this is 165 1 upon 1 this is  $Y^T Y$ . Similarly, summation  $Y^T Y$  we have just calculated summation  $Y^T Y$  here. This is summation  $Y^T Y$  **this is summation  $Y^T Y$** . So, this is summation  $Y^T Y$ . So, we **we** like to have again summation  $e^T e$  **summation  $e^T e$** .

So, what is  $e$ ? So, we like to know the  $e$  first. So,  $e$  equal to  $Y$  minus  $\hat{Y}$ . So, what is  $\hat{Y}$  here. So,  $\hat{Y}$  is 1 3 5 7 9. So, we will formulate like this. So,  $Y$  equal to what is  $Y$  here? So,  $Y$  is 1 4 3 8 9 so, then  $\hat{Y}$  minus  **$\hat{Y}$**  what is  $\hat{Y}$ ?  $\hat{Y}$  equal to 1 3 5 7 9 which just receives  $\hat{Y}$  equal to  $X \hat{\beta}$ . So, which means we have to multiply  $X$  vector with  $\hat{\beta}$  then you will get the  $\hat{Y}$ .

So, just like this. **This** is  $Y^T Y$  so obviously, 1 3 5 7 9 will be in a column shape. So, now this is 5 into 5 plus 1 this is 5 plus 1. So obviously, subtraction can be

possible. So that means, we will get e is nothing but, error is nothing but, 0 1 minus 2 1 0 because 1 minus 1 0, 4 minus 3 1, then 3 minus 5 minus 2, 8 minus 7 1, then 9 minus 9 0. So, this is e component but, our requirement is e transpose e.

(Refer Slide Time: 34:55)

$$e = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \quad e' = [0 \ 1 \ -2 \ 1 \ 0]$$

$$e'e = [0 \ 1 \ -2 \ 1 \ 0] \begin{matrix} 1 \times 5 \\ \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \\ 5 \times 1 \end{matrix}$$

$$= [0 \times 0 + 1 \times 1 + (-2) \times (-2) + 1 \times 1 + 0 \times 0]$$

$$= 6$$

So, now we have e equal to 0 1 minus 2 1 0 this is e component. So, we like to calculate e transpose. So, e transpose equal to 0 1 minus 2 1 0. So, this is e transpose. So, we like to e transpose e. So, e transpose e means 0 1 minus 2 1 0 multiplied by 0 1 minus 2 1 0. So, this is another structure. So now, this is 1 into 5 and this is 5 into 1. So, obviously we have system 1 cross 1. So that means, it is equal to 0 into 0 plus 1 into 1 plus minus 2 into minus 2 plus 1 into 1 plus 0 into 0. So, this is of 1 into 2 order this is 0, this is 1, this is 4, this is 1, this is 0. So, that will be coming actually 6 that will be coming actually 6.

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RSS —  $e'e = 6$ .  
ESS —  $y'ny = 165$ .  
TSS —  $y'y = 171$ .  
TSS = ESS + RSS  
 $171 = 165 + 6$   
 $= 171$   
 $\sigma^2 = \frac{SSE}{(n-k)}$   
 $= \frac{6}{(5-2)} = 2$ .  
 $\text{Var}(\hat{\beta}) = \sigma_u^2 (X'X)^{-1}$   
 $= 2 \begin{pmatrix} 0.6 & -0.2 \\ -0.2 & 0.1 \end{pmatrix}$   
 $= ?$

So, we will just simplify all these structures. So that means, our  $e$  transpose  $e$  equal to 6 then  $Y$  hat transpose  $Y$  is equal to 165 then  $Y$  transpose  $Y$  is equal to 171. Let's check it here. So, because this is nothing but, TSS this is nothing but, ESS and this is nothing but RSS. But, by default we have TSS equal to ESS plus RSS. So that means, total sum square equal to explained sum square plus residual sum squares. So that means, if you just connect here. So, this is 171; so that means, 165 plus 6 this is equal to 171. So that means, this and this are equal. So that means, we are in the right track.

So, this is RSS, this is ESS, this is TSS. But, we need ESS only because sigma square equal to summation  $e$  square by  $n$  minus 2. So that means, this is 6  $n$  minus 2 because  $k$  equal to 2 here. So obviously, 6 by  $n$  is 5 here. So, 5 minus 2 this is 3. So that means, sigma square equal to 2.

(Refer Slide Time: 20:14)

So, the moment you will get sigma square 2. Then obviously, let us see here this is 6. So, R square **R square** will be coming. So that means, sigma square equal to 2. So now, variance of beta hat equal to sigma square  $U$   $X$  transpose  $X$  inverse. What is  $X$  transpose  $X$  inverse let us see here. So, this is  $X$  transpose  $X$  inverse. So, now, we have to write here. So that means, sigma square  $U$  equal to 2 here then,  $X$  transpose  $X$  inverse means, it is multiplied by 0.6 minus 0.2 minus 0.2 into 0.1 This is how the entire structure all about.

So, this is variance of sigma square  $\sigma^2$  and this is  $X^T X^{-1}$ . So, now, 2 have to be multiplied with this one. So, we will get the variance of beta hat  $\text{variance of beta hat}$ . So, this in fact, is transpose? So, there is no point to write it here again transpose  $X^T X^{-1}$  into  $X^T Y$ . Variance of beta hat equal to sigma square  $X^T X^{-1}$ .

So now, we have to calculate this value. So, you have the variance of beta hat that has to be calculated. So, this is one part of the story so, that means what we have received here. So, our model will be like this.

(Refer Slide Time: 40:05)

$$\hat{Y} = 1 + 2X$$

$$H_0: \hat{\beta}_0 \neq 0 \quad \hat{\beta}_0 = 1 \quad \hat{\beta}_1 = 2$$

$$H_A: \hat{\beta}_0 = 0$$

$$t_{\hat{\beta}_0} = \frac{\hat{\beta}_0}{\text{SE}(\hat{\beta}_0)} \quad t_{\hat{\beta}_1} = \frac{\hat{\beta}_1}{\text{SE}(\hat{\beta}_1)}$$

So,  $\hat{Y}$  equal to  $1 + 2X$  so that means,  $\hat{\beta}_0$  equal to 1 and  $\hat{\beta}_1$  equal to 2. So, now we need to know whether  $\hat{\beta}_0$  is significant or  $\hat{\beta}_1$  is significant. So that means, we like to calculate  $t_{\hat{\beta}_0}$  accordingly, we have to set the null hypothesis. Null hypothesis are such that  $\hat{\beta}_0$  not equal to 0 ok it will be equal to 0 and against the alternative hypothesis  $\hat{\beta}_0$  not equal to 0 this is equal to 0; so that means,  $t_{\hat{\beta}_0}$  divide by standard error of  $\hat{\beta}_0$ . So similarly, we have to calculate variance of beta hat that will be single figure the moment will get this multiply. Then obviously, you will get this value. So, similarly,  $t_{\hat{\beta}_1}$  is equal to  $\hat{\beta}_1$  hat by standard error of  $\hat{\beta}_1$  hat. This is  $\hat{\beta}_0$  hat with standard error of  $\hat{\beta}_1$  hat.



So, this is calculated statistics and you have to compare with the tabulated statistics with respect to this degrees of freedom  $n$  minus 2 then, you can get to know whether it is statistical significant or not statistical significant.

So, this is first part of the story that is with respect to significance of the parameters. So, the moment you know the significance of the parameter; obviously, there must be definite conclusion whether beta 0 is significant or beta 1 is significant or you can say both are non significant or vice versa, one is significant, another is not significant.

So, means by the way in that particular process you get to know whether this particular item is statistically significant or not. So now, what you have to do? So, you like to know **you like to know** the other part of the story that is overall fitness of the model.

(Refer Slide Time: 42:27)

$$\hat{Y} = 1 + 2X$$

$$= \hat{\beta}_0 + \hat{\beta}_1 X$$

$$\hat{\beta}_0 = 1 \quad \hat{\beta}_1 = 2$$

$$R^2 = 0.965$$

$$\bar{R}^2 = 0.953$$

$$R^2 = \frac{ESS}{TSS} = \frac{1.65}{1.71} = 0.965$$

$$R^2 = 0.965 \text{ (i.e. } 97.1\%)$$

$$\bar{R}^2 = 1 - \frac{(1 - R^2)(n-1)}{n-k}$$

$$= 1 - \frac{(0.035)4}{3} = 0.953$$

0.965
0.035
1.00

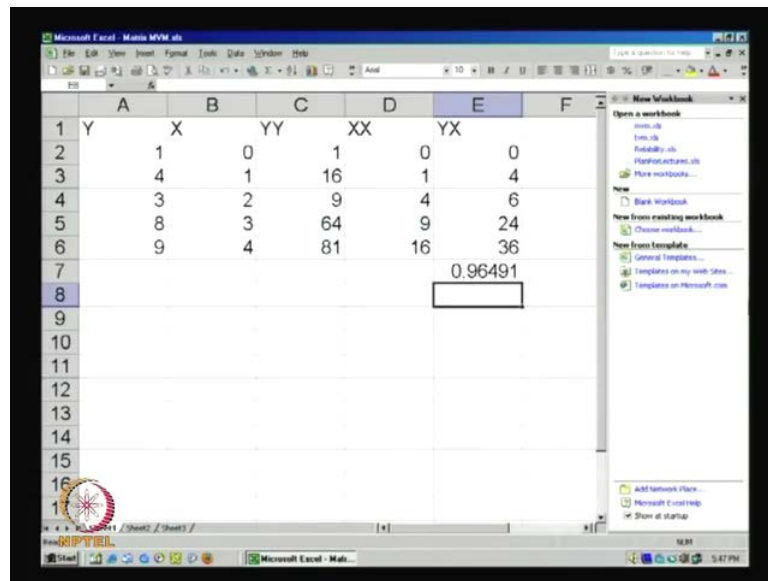
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So, overall fitness of the models; so that means, once you have estimated models  $\hat{Y}$  equal to 1 plus 2  $X$ . So that, this  $\hat{Y}$  equal to 1 plus 2  $X$  which we have just now received. So that means, this is nothing but, beta 0 hat plus beta 1 hat  $X$ . So that means, beta 0 hat is equal to 1 and beta 1 hat equal to 2. So, you like to know  $t$  of beta 0 hat and  $t$  of beta 1 hat. So, this is how you have to obtain. So, then you check it whether statistical significant or not. So that means, this is first part of the model that is significance of the parameter.

(Refer Slide Time: 20:14)

So, the second part of the model is the R square that is overall fitness of the model. So, what is R square? **R square** here is ESS the ratio between ESS by TSS. What is ESS here? So, ESS is 165 then, this TSS is equal to 171. So, this is R square value so that means, we will put it here. So, the calculation will be like this.

(Refer Slide Time: 44:14)



	A	B	C	D	E	F
1	Y	X	YY	XX	YX	
2		1	0	1	0	0
3		4	1	16	1	4
4		3	2	9	4	6
5		8	3	64	9	24
6		9	4	81	16	36
7					0.96491	
8						
9						
10						
11						
12						
13						
14						
15						
16						

So, just one minute here 1 equal to 165 divided by 171. So, this will be coming 0.965. So, means R square value R square value is 0.965 or otherwise or it is nothing but, 97 percent. If 97 percent means the reliability of the model is very high that means, the overall fitness model is very high. Since **since** it is very close to 1. So that means, this model can be use for forecasting or policy use, but in the same time we have to check whether these parameters are statistically significant or not.

So, the moment both parameters are statistically significant, then obviously, this R square can be valid. Still it is doubtful because in a bivariate system if you are getting high and high R squares then obviously, it may be criticize because mostly the multivariate system is there and R square value is high; then it can be justified.

But, here in this bivariate system even if parameters are statistical significant still it is doubtful case until and unless you clarify that no other variables are significantly influence in the system. Till if you are getting like this systems then, you have to find out

whether the other variables which can influence this model then you enter and you check it whether the model parameters or model **model** reliability will be getting affected. If it is not, then you can say that this model is absolutely with you. This is R square similarly, we will get adjusted R square **adjusted R square** equal to  $1 - \frac{1 - R^2}{n - k}$ . So, this is this is  $n - k$ .

(Refer Slide Time: 47:01)

	A	B	C	D	E	F
1	Y	X	YY	XX	YX	
2		1	0	1	0	0
3		4	1	16	4	4
4		3	2	9	4	6
5		8	3	64	9	24
6		9	4	81	16	36
7					0.96491	
8					0.03509	
9					0.04678	
10					0.95322	
11						
12						
13						
14						
15						
16						

So, now  $1 - \frac{1 - R^2}{n - k}$  means 0.965. So, this will be coming 0.035. So, 0.035 into  $n - 1$ ,  $n$  is 5 here. So, this is  $4$  divide by  $n - k$   $5 - 2$  this will be 3. So that means, 0.035. So, equal to  $1 - \frac{1 - 0.9673}{5 - 2}$  which is equal to 0.035 this is equal to  $e^{-8}$  multiplied by 4 by 3, then it is coming 0.46. So, 0.046 point 8.

So, it is equal to  $1 -$  this much. So, this is coming 0.95. So, this is coming 0.963. So that means,  $R^2$  equal to 0.965 and adjusted  $R^2$  equal to 0.95. So, there is difference and the difference mostly because of **because of** the adjustment of variables and the number of sample size that means, mostly it is with respect to degrees of freedom only.

So obviously, adjusted  $R^2$  is more reliable component which can test the reliability or best fitness of the model. But, still we are not confident whether this particular model is absolutely significant because we have received just  $R^2$  and that has to be again statistically significance. So, for that we need to have F statistics. So, we **we** need to know, what is F statistic here?

(Refer Slide Time: 48:36)

The image shows a handwritten derivation of the F-statistic formula on a blue background. The first part shows the calculation of the F-statistic as the ratio of ESS to RSS, resulting in 165/6 = 27.5. The second part shows the derivation of the F-statistic formula in terms of R-squared, TSS, and n-k.

$$F = \frac{ESS}{RSS} = \frac{165}{6} = 27.5$$
$$F = \frac{ESS}{RSS} = \frac{ESS/TSS / (k-1)}{RSS/TSS / (n-k)} = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)}$$

So, F statistic is the ratio between ESS by RSS, but ESS we have already calculated that is 165 divide by RSS. RSS is just coming 6 **6** so that means, 165 by 6.

(Refer Slide Time: 48:56)

The screenshot shows a Microsoft Excel spreadsheet with the following data:

	A	B	C	D	E	F
1	Y	X	YY	XX	YX	
2		1	0	1	0	0
3		4	1	16	1	4
4		3	2	9	4	6
5		8	3	64	9	24
6		9	4	81	16	36
7						0.96491
8						0.03509
9						0.04678
10						0.95322
11						27.5
12						
13						
14						
15						
16						

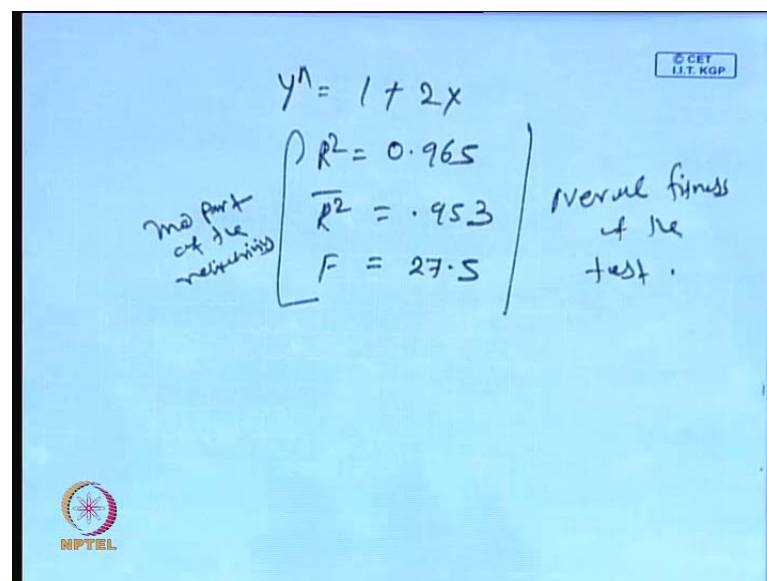
So, this is equal to 165 divide by 6 is 27.5. So, it will be coming 27.5. So, R square is 27.5. So, there is another chance that this particular structure is highly statistically significant. There is no doubt about it if you are getting high and high R square. Then obviously, F statistic will be very high because F is derived from the R square only. Since F is equal to ESS by RSS you just divide TSS in both the sides. So that means,

ESS by TSS divide by RSS by TSS. So, it will be R square followed by k minus 1 and this is followed by n minus k.

So, this means R square by k minus 1 divide by 1 minus R square because RSS is TSS minus ESS. So, TSS by TSS equal to 1 then ESS by TSS again equal to R square. So, 1 minus R square by n minus k. So that means, there is another possibility that, it is justified that if R square is substantially high. Then obviously, F will be substantially high, if R square is substantially low then F will be substantially low.

So that means, there is huge connection between R square and F. The significance of the F depends upon the value of R square, if R square is high then there is another chance that F will be statistically significant. If R square is very low then there is another possibility that the F will be not significant. So, means it is very closely connected to each other.

(Refer Slide Time: 50:55)



The slide shows handwritten mathematical results on a light blue background. At the top, the regression equation is  $\hat{y} = 1 + 2x$ . Below it, a large bracket groups three statistics:  $R^2 = 0.965$ ,  $\bar{R}^2 = 0.953$ , and  $F = 27.5$ . To the left of the bracket, the text "2nd part of the reliability" is written. To the right, "Overall fitness of the test." is written. In the top right corner, there is a small logo for "CET I.I.T. KGP". In the bottom left corner, there is the NPTEL logo.

So, now if we will summarize all these details then finally, the estimated model will be  $\hat{y}$  equal to 1 plus 2 X and R square equal to 0.965 then, adjusted R bar square equal to 0.953 and F statistic is equal to F statistic equal to simply 27.5. So, this is the second part of the reliability.

So that means, this is overall fitness of the test **overall fitness of the test**. So that means, we are now come to conclusion that matrix **matrix** can be applied to estimate the

bivariate model, it can also be helpful for estimating the trivariate model and it can also be helpful for estimating the multivariate model.

So that means, it is just like the system of old wine in a new bottle, nothing new. So, there **there there** are several ways to get the estimated model and followed by the checking of significance of the parameter and overall fitness of the model. But, of course, sometimes you must have some knowledge means various techniques, various methods, various tools through which the estimation can be possible. It is not suggestive that you have to always know one technique or one tool or one path through which you will get the job done.

You should know various techniques, various tools and various methods through which how you have to receive the estimated models. Till now we are discussing about the application of various techniques only. Even this matrix approach also we are transferring the system into matrix ultimately we are differentiating with respect to corresponding parameters, but that is nothing, but, the application of **(( ))** again. Methodology wise you can apply various other techniques like GLS technique, WLS technique, maximum likelihood estimate technique. So, many other methods are there by which we can get the estimated model.

But we will discuss later how GLS and WLS or maximum likelihood estimator can be applied to get the estimated model. Till now we are in a position that there are various ways to get the estimated model. So, now one part of the econometric modelling is over. So, in the next class onwards we assume that there is estimated model. So, once you have the estimated model; what are the things we have to look? and what are the things we have to do further to get the model best fitted and we have to look the model should be very best for forecasting use and for policy use.

So, for that the job is not so simple, the complexity will be more interesting here, because we have to look various aspects of estimated models before you use for forecasting and policy use. So, when we look for best models with respect to forecasting and policy use. So, we have to compromise lots of things, we have to go through various aspects. So, sometimes one will be very, very much helpful for you and on the same time other will not be helpful for you.

So, with this type of circumstance that means, sometimes you are in the gain position you are in the lost position. So, we have to find out a balance approach through which the model can be judge as a best model and can be used for forecasting and policy use. So, we will discuss in details in the next class. So, till now we have to (( )). Thank you very much, have a nice day.