

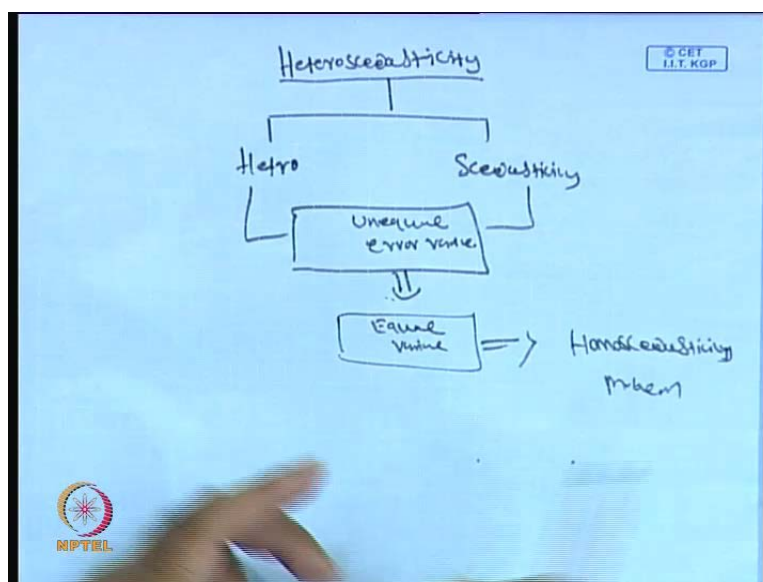
**Econometric Modeling**  
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**Module No. # 01**

**Lecture No. # 25**

**Heteroscedasticity Problem (Contd.)**

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Good evening! This is Dr. Pradhan here. Welcome to NPTEL project on econometric modeling. So, today we will continue the heteroscedasticity problem. In the last lectures we have discussed we have started little bit about the heteroscedasticity issue. So, how it is coming to that in the econometric model and how it is very useful for discussing that particular issue. So, we highlighted that heteroscedasticity has two specific aspects. So, this time I am bringing in little bit more here is. So, this is hetro then this is scedasticity.

So that means, briefly it is the equation of unequal error variance. So, if it is come to equal variance then that will lead to homoscedasticity issue homoscedasticity problem. So that means, we start with a fit a model first. So, the moment you have estimated

model then like you know. So, many other econometric problem like multicollinearity autocorrelation etc. So, we have to all such agree heteroscedasticity problem.

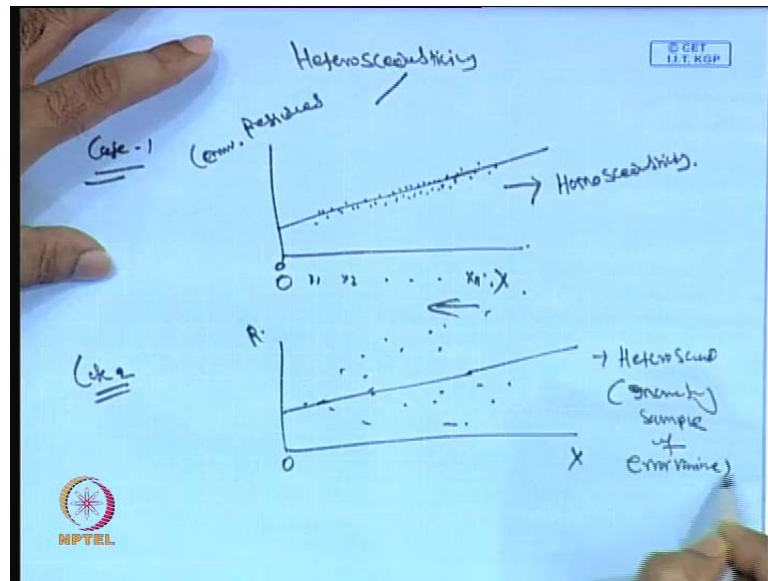
So, heteroscedasticity basic objective is to check the whether the error variance are equal or unequal. If it is equal then it is a good sign then you have to go ahead with the econometric problems. You can use that model or you can say that this model is best fitted and can be used for forecasting or policy use. But, if the error variance are not equal then you have to redesign reformulate or you can say restructure till you get the estimated model which is the purely homoscedasticity in nature. That means, there should not be any heteroscedasticity. So, the what is; that means, there should not be any unequal error variance.

So, we start we means the starting point is you we must have some estimated model. Then in the mean time you must have find out the error component. Then find out error variance for each and every sample point for  $U_1 U_2 U_3$  like this. Then if it is equal over the different time periods or different samples then obviously, if it is equal then it is homoscedasticity if not equal then it is a heteroscedasticity. Then you have to you know recycle it till you get the homoscedasticity. So, that is how the problem is all about. So, that is how I have started.

So, let us assume that the problem has a heteroscedasticity. So, then your objective is to make the unequal variance to equal variance and that will lead to homoscedasticity problem. So, this is the basic structure. So, now, that means, theoretical means say by statistics heteroscedasticity means the presence of unequal variance in the econometric models.

So, unequal variance means it is related to error terms unequal error variance in the particular econometric models. So, now, if it is unequal error variance then that is one of the negative point for econometric modeling that will give you the red signal. So, you turn in to green signal that is the homoscedasticity problem then, you have to go ahead with the forecasting issue.

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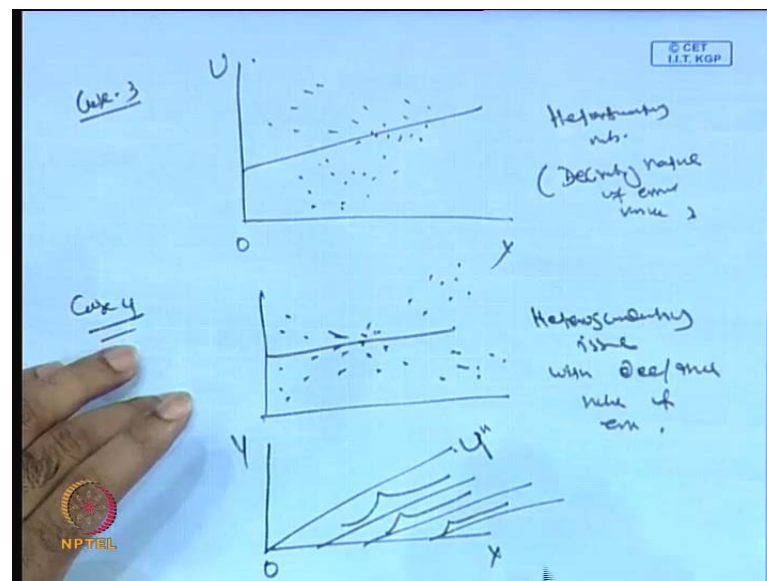
So, now, the moment you have a heteroscedasticity, so, now, like you know degree of autocorrelation degree of multicollinearity. So, similarly, you can have a framework or you know structure nature of this heteroscedasticity. It is too much problematic or very small problem all together. So, then you have to see how is that actually safe. So, there are various ways we can represent the heteroscedasticity problem. So, let us say case 1 here is. So, what is the issue of heteroscedasticity? Heteroscedasticity means I will take  $X$  observation here. That means,  $X_1 X_2$  up to you say  $X_n$  these are the  $X$  observations all right.

So, now this side I will put residuals. So, that is means error terms error variance. So, now, this will I will plot like this. These are all error you know error items. So, now, this error items are in a you know it is more or less equal spread very close to each other. You see the gap is very close to each other. So, that means, if this is the case then it we can call it as a not heteroscedasticity problem whether it is a homoscedasticity problem.

So, this is the variance issue only. So, if the variance is large then it will affect the models if the variance is low it will be less effect only model. So, we try to build a structure where there is a less error variance. So, that is how that is the objective of this particular process. I will take a case two another case. So, like this in the same structures I will put it here  $X$  and this side is residuals error terms.

Then, obviously, I will take like this. The plot will be like this. So, this is means it is the if error variance will be errors are coming like this way then there is a heteroscedasticity problem. This is heteroscedasticity problem and that that too a it is the increase in sample of error variance. This is the increase in sample of error variance increase sample of error variance.

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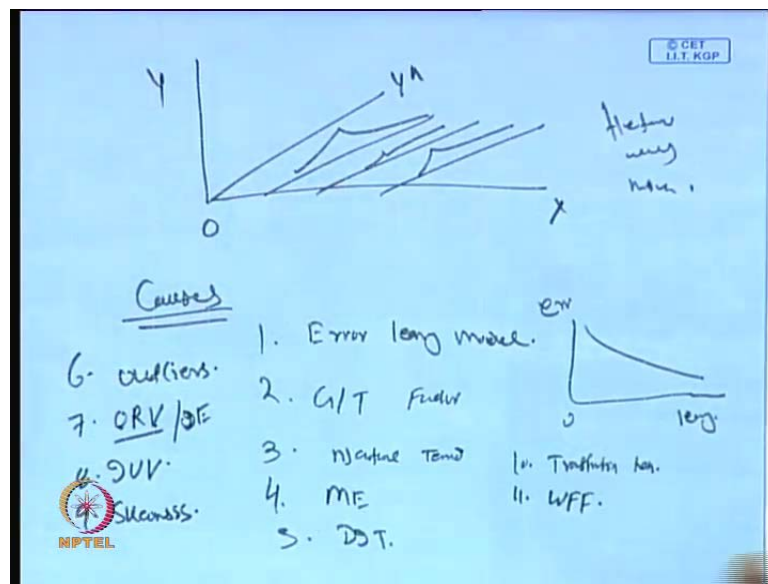


I will take here case 3, I will take similar like this way and this side X, this side residuals U. Then the problem will be like this. So, this is also heteroscedasticity problem the nature is a heteroscedasticity problem and here it is a decreasing issue decreasing nature of error variance. So, now, I will take case four another case. So, there may be like this, it may be like this, it may be like, this then it will be like this, then it will be like this. So, this is also heteroscedasticity issue with both decreasing and increasing nature of error variance.

So, there are all together there are four different steps of heteroscedasticity problems. Case 1 where there is a no such heteroscedasticity say in minor labels. So, it is not a problem for econometric modeling that model can be considered as the best models and can be used for forecasting or policy use. But, in the second third fourth cases sometimes the error variance is increasing and sometimes both can be go simultaneously. In that case it is problem for econometric you need to remove it.

So, exactly how is the step if will take another type of structures. You see here is this side I will take you know 3 dimensional picture. If I will plot then, obviously, the picture will be coming like this 0 X this side Y and this side U or we can say Y head alright. So, like this you know you plot all these things like this way. So, this type of sequence is called as a it is looking like homoscedasticity. So, this type of structure is called as a homoscedasticity issue.

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But, you see I will take here is another 3 dimensional picture. So, this is a X this is a 0 Y this is Y hat. So, then I will draw like this. This is purely heteroscedasticity problem.

So, now we come to know that the basic definition of heteroscedasticity is that unequal error variance are in a increasing trend, sometimes error variance are in a decreasing trend. Sometime error variance are very constant with respect to whatever step of the sample and sometimes there may be increasing decreasing and constant it can go together. So, that depends upon what is your sample observations. If the sample observation is exclusively very high then you will find there is a everything is there increasing, decreasing and constant.

So, anyway if that is the case, we have to being the sequence or structures where there is a error variance are homogeneous in nature. So, that is that should be our aim and that should be our requirement for econometric modeling. So, now, once we know the nature of the heteroscedasticity, next obvious question is what are the causes behind this

heteroscedasticity? Why heteroscedasticity? You know seems to be there in the econometric model there are various reasons for that, causes behind this heteroscedasticity problem.

First is there is a concept called as a error learning models. You know I have mentioned in the last class the quotation like success is the pillar of failure is the pillar of success. So, that means, if you fail then, obviously, it will give you learning lessons and that lessons are learning will be very useful for future. So, error learning model also behaves like that way. So, means it will give you a equation like this way. This is you know error and this is how it is called as a learning ability or you can say experience or you can say committing mistakes with respect to sample observation different time priors.

So, over the time priors when will we do things after different time interval then, obviously, you will find your error variance will be start decreasing. So, it cannot be uniform or it cannot be constant. So, today you have no experience that is why you are doing and you are committing lots of mistakes. So, tomorrow you have little bit experience then, obviously, you do commit errors, but, the amount of error will be less than to first case.

Again next stage you have experience you have you gain lessons or gain message knowledge etc and obviously, error will be committing less and less. So, that is how error cannot be constant over and over time. So, it will be in a declining trend. As a result it will lead to heteroscedasticity issue. So, this is first causes behind this (( )) and this is very interesting very important 1 then growth and trend factors.

Ah you see, if we you know in the econometric model itself is going to discuss the interdependence between a dependent variable and independent variables. So, when will we approach a particular problem you know societal problems then we are tackling various variables to a particular issue. So, variables behaviors are much you know not constant it will be behave with respect to different condition different situation different places different time framework. So, that is why you know over the times the variables behavior is somewhat increasing decreasing increasing decreasing etc.

So, this is how it cannot be constant. As a result all the variables are behaving in a different way. So, it will give you signal that it cannot that means, it will give you indication that error variance cannot be constant, it will be unequal buy default. So, then

third point is that this is called as a growth and trend factors then natural tendency. It is a natural tendency of the variables for instance some of the cases variable itself by nature it will be in a increasing or decreasing trend. For a case of you know earnings is a variables.

So, I will take few different industry the software industry then corporate financial world then I will take some you know agricultural sector then I will just compare the earnings of agricultural peoples and earnings of industrial people and earnings of you know say finance people say banking industry. Then, if will you compare then, obviously, by default the earnings of 3 such industry are completely different. If you while club all these variables together then I will ask you what is the impact of earnings on say savings then, obviously, you will find there is a heterogeneous problem means heteroscedasticity problem.

Because, the sample itself give you the a problem setup. It will give you indication that there is a you can say a heteroscedasticity problem. Because the variable itself behaving as a current way. In fact, there is a also measurement error so like you know measurement error means a sometimes you are asking some question to respondents. So, some respondents are behaving very accurately and they are very unbiased. But, some of the respondents are behaving some other way round it is a bias. So, if will we club these 2 respondents then, obviously, it will lead to heteroscedasticity problem.

So, you know by default, by techniques you will find some heteroscedasticity. It is not artificial instrument. It is by this principal or structure it will come automatically. So, this is how the you know the measurement error also have a problem of a heteroscedasticity issue. So, this is you know measurement errors then fifth data improvement techniques. So, this is another important variable. So, generally over and over and above if will you apply you know different techniques with respect to a particular problem then you know by technique itself it will be minimize the error.

For instance, if will we apply o l s g l s w l s and maximum likelihood estimator of a particular problem. Then, obviously, we will find the error cannot be constant in by each and every technique. So, by technique itself it will generate lots of you can say errors or you can say it will give you heteroscedasticity issue.

So, obviously, even if the technique will be also minimized the heteroscedasticity issue. for instance, if you have a problem and if you are applying o l s then there may be heteroscedasticity problem. But, if you in the same problem if you will apply w l s or g l s then heteroscedasticity problem can be solved. So, that is how technique itself can create heteroscedasticity problem. It can solve the heteroscedasticity problem. So, different techniques also have you know cause for you know heteroscedasticity problem.

Then sixth a outliers is a problem of heteroscedasticity. You see outliers means it is a data point which is highly distance from outdoor data points. That means, it is going against the homogeneous clusters. So, suppose there are ten you know ten items here is and 2 items in the few other places. It is very highly distance from other data points. So, then in that case it will lead heteroscedasticity problem. So, you have to be very careful about how you have to remove this.

In fact, in that case you can either you transfer this data or you can say go for this structural way testing etc to normalize this situation. Otherwise you if will you go by simple modeling by integrating all these samples at a time then, obviously, you can say a heteroscedasticity will be coming in to the picture. Then, you need to have a solution for that. So, this is outliers problem.

Then similarly, a omission of relevant variables also another items which can solve the means which can add the heteroscedasticity problems. For instance, last class I have discuss the issue like you know stock price with industrial productions. Then money supply then wholesale price index exchange rate etcetera.

So, in that case I have mentioned very clearly if will you drop a particular variable important variables say money supply or you can say wholesale price index then, obviously, it will give you message that either it give autocorrelation problem or it may be a heteroscedasticity problem or it may be a both also. So, that is why inclusion of your relevant variables means exclusion of a relevant variable also very problem.

Similarly, inclusion of unnecessary variable also problem for this heteroscedasticity and autocorrelations. So, you must be very careful what are the relevant variables you are putting in the system and what are the unnecessary variables you are putting in the systems. So, necessary variables should be included and unnecessary variables should be excluded.



If it is other way around then, obviously, it will be problem for heteroscedasticity. So, this is how omission of relevant variable or in inclusion you know exclusion of this is omission of relevant variables or inclusion of a unnecessary variables. So, that is how it is called as a inclusion of unnecessary variables this is another point you can consider here is skewness of the distributions.

For instance it will take a very group of peoples in a particular setup. Let us take a trace of income. So, there is a income distribution, income itself is a inequality in natures. So, as a result it will affect the heteroscedasticity problem. Some of the variable by default by nature they are you know unequal variance. So, it cannot be have some of the variables are very equal in nature. For instance take a case of you know very well deployed financial market or capital market and you observe this stock price or you can say observe the exchange rate etc then you will find it is a very less volatility.

So, then if the market is very troubles then you will find a error variance will be very less. If the market is in struggle then you will find huge you know error variance. In fact, if is a very volatility that is called as a volatility modeling. If volatility is very high then, obviously, the  $(( ))$  will be coming in to the picture and. In fact, it is a very interesting for different players, but, it may be effecting the economy systems. So, it cannot be equal variance should not be exactly equal for each or for different problem setup of different condition for financial market or you can say capital market case.

So, error variance should not be equal perfectly. But, we our objective should make struggle market, but, if there is struggle market then, obviously, people cannot have the reach or people cannot go for investment etc. So, there need of something, but, it should not be extremely high it should be very optimum one. So, this is how the structure of you know skewness of the distribution. So, that means, it is a skewness issue.

So, similarly, there is another concept called as a data transformations. In fact, heterogeneous when there is a heterogeneity then, obviously, or heteroscedasticity then, obviously, one of the standard trick is to go for transformation. If will you go for transformation then, obviously, it may turn to homoscedasticity problem. But, the thing is the transformation should be very accurate, very perfect, very feasible one. If the transformation rule is or the transformation technique is a not unbiased, if it is totally

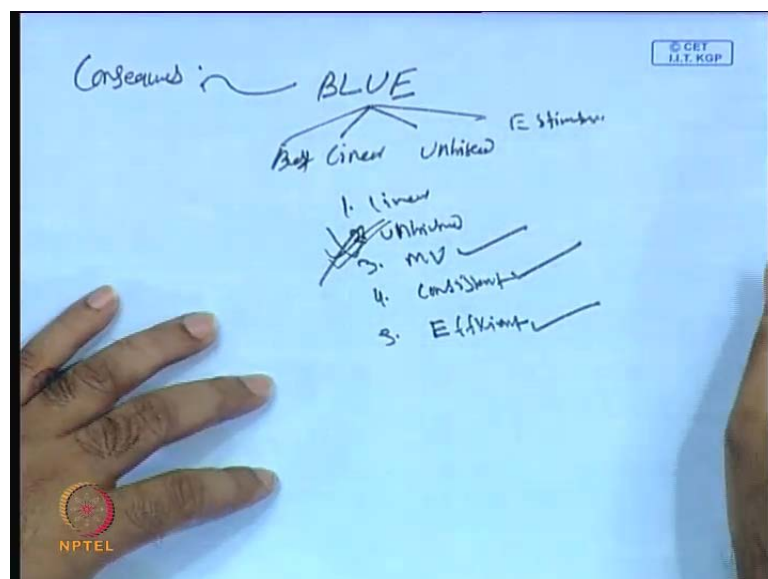
biased then, obviously, it will add heteroscedasticity problem later then it will reduce the heteroscedasticity problem.

So, the transformation technique should be perfectly for this particular problems. So, that means, in tenth there is a transformation technique. Similarly, another point is where very beginning itself wrong functional form. So, that is also another factors which are the autocorrelation problem also, but, you know transformation means if you go for wrong functional form say linear model to non-linear or non-linear model to model and if you go for solving the particulars thing then, obviously, it will add a heteroscedasticity problem. So, you must be very careful about that one. So, that means, this is called as a wrong functional form of the model.

So, like this there are several you can say several ways you have to find out heteroscedasticity can be coming to the picture. So, the course is not single ones. So, there are multiple causes and these are the means. Whatever factors we have discussed these are the relevant pair causes through which heteroscedasticity is always be there in the econometric problem. Always means in most of the cases. You know every case it cannot be, but, most of the cases you will find heteroscedasticity is a issue.

. So, now, you get to know what is what is heteroscedasticity problem? what are its nature and what are causes? Then, finally, you we like to know what is the consequences?

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So, we like to know consequences. Obviously, so, far as the consequences is concerned we like to highlight you know the blue property. So, far as consequence is a effect of then we have to see the blue factors best linear unbiased estimators. So, that means, it should be linear, it should be unbiased minimum variance then consistent and finally, not least is a efficient very parameter should be very efficient.

So, now, so, far as heteroscedasticity is there, it does not affect the unbiased, unbiasedness property it will go with a unbiased property. But, it is not efficient. So, it will effect a it is not effect this one's. So, it is effect this one, this will effect this one this will effect this one. So, that means, when heteroscedasticity is a present in the model then you know estimated parameters will not follow the minimum variance will not follow consistent property. It will not fall the follow the efficient property. So, that means, because the variance is a very high and that is totally unequal.

So, if the variance is low and you know equal for every sample points or for different sample points then, obviously, So, this will lead to you can say blue property. So, our aim is to transfer this particular inefficient or you can say inconsistent or larger variance to minimum variance or consistent or you can say efficient. That is the consequence part of the model. So, if it is not going to maintain the minimum of variance property consistent property and efficient property.

So, as a result if this model cannot be considered as the best and it cannot be used or it should not be used for you know for forecasting and policy use. You need to have a solution for that. That means, again you have to read it then formulate the structure till you get the model where all the estimated parameters having minimum variance unbiased consistent and efficient. This is how the consequence part of this heteroscedasticity problem.

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Detection Criteria

1. Spearman Rank Correlation method

Y	X	$\hat{Y}$	e	
$y_1$	$x_1$	$\hat{y}_1$	$e_1$	$D_1^2$
$y_2$	$x_2$	$\hat{y}_2$	$e_2$	$D_2^2$
$y_3$	$x_3$	$\hat{y}_3$	$e_3$	$D_3^2$
$\Sigma D^2$				

$$r_{yx} = 1 - \frac{6 \Sigma D^2}{n(n^2-1)}$$

$D = e \cdot x$   
 $n \rightarrow \text{total no of obs}$

$$r = \frac{\sigma^2}{\sigma^2 + \frac{1}{n}(\sigma_1^2 - \sigma_1) + \frac{1}{n^2}(\sigma_2^2 - \sigma_2) + \dots}$$

Next is the detection criteria. So, like you know multicollinearity and autocorrelation heteroscedasticity or can be detected by various you know methods. So, there are large number of methods available to detect the heteroscedasticity problem. Accordingly you have to find out it is also, solution for that. So, what is that particular solutions you see here is means forget about first solution. Lets first we detect then we will go for a solution. So, detection criteria there are several methods.

I will just highlight you few methods here because of time constraint and a mostly these 2 two 3 techniques are usually applied to detect the you can say heteroscedasticity problem. It is very difficult to go the way we have discussed till now, but, we will we find out some simple way how to detect the heteroscedasticity problem. So, that simple way we have to structures here is.

So, one method is called as this spearman rank correlation method. So, that formula is  $r_{yx}$  upon X equal to 1 minus six summation D square divide by n into n square minus one. So, what is e and what is r e x? That means, e is another 1 variable X is another variable you know we start with a Y X then Y hat and e, but, in this spearman rank correlation to detect the heteroscedasticity we like to integrate X and y.

So, we like to know what is the correlation associated with between X and e if there is a such correlations. In fact, it looks like you know heteroscedasticity multicollinearity type

situation. So, if it is a if the particular correlation is a statistical significant then it will be have the you know heteroscedasticity issue. So, that has to be removed.

So,  $1 - 6 \sum D^2$  into divide by  $n$  into  $n^2 - 1$ . So,  $D$  is the difference between  $e$  and  $x$ . So, we have series of points. So, like  $e_1 e_2 e_3$  like this then  $X_1 X_2 X_3$  like this. So, we have to find out the difference is called as a  $D_1 D_2 D_3$  like this. So, this is how you have to make summation find out again  $D_1^2 D_2^2 D_3^2$  like this. So, this is how you have to make summation find out again  $D_1^2 D_2^2 D_3^2$  like this. So, the moment you will get summation  $D^2$  then you put it here  $n$  equal to total number of observations and this  $r_{eX}$  represents correlation coefficient between  $e$  and  $x$ .

So, now we will get you know  $r$  value. So, now, you have to apply  $t$  statistics. So,  $t$  is nothing, but,  $r^2$  by  $1 - r^2$  followed by  $n - 2$ . So, now, if it is statistically significant then you know there is a presence of heteroscedasticity that is to be solved, but, you know then correlation has a it is a very beautiful technique. But, it is lots of you know different structures and different step.

So, we have to very careful about that one of the difference ones. One of the interesting structure is that having a same range in various cases if there is same range then this particular formula cannot be used. So, that is then if the some cases the ranks are not ranks are same for different observations. Then, obviously, we use this formula  $1 - 6 \sum D^2$  then  $1 + 12$  into  $m - 1$  to the power  $3 - m$  that means,  $m - 1$  into  $m - 1$  square minus  $1$  then plus  $1$  by  $2$ ,  $1$  by  $12$  then  $m - 2$  square  $m - 2$   $3 - m$  two.

So, it will continue like number of equal ranks. So, divide by  $n$  into  $n^2 - 1$ . So,  $n$  into  $n^2 - 1$ . So, this should be the criteria when equal ranks are there. For instance, let us say  $D_1 D_2$  are equal. So, that means,  $2^2$  is there is. So, it is in  $1$  case  $1$  by  $12$ . So, it is  $2$  to the power  $3 - 2$  like this you have to calculate the rank correlation coefficient. Again once you have a rank correlation coefficient then you have to again test statistic if it is significant then, obviously, there is heteroscedasticity problem.

If not then it is a homoscedasticity problems. So, it means the moment  $t$  is significant then, obviously, you have to redesign the structure and reformulate it and till you get the a homoscedasticity transformation or homoscedasticity problem. So, if it is not

significant then by default the presence of heteroscedasticity is not a serious problem for the forecasting or policy use. So, you can go ahead with that particular heteroscedasticity.

Of course, heteroscedasticity cannot be totally 0. Sometimes it may be available at a minor level, but, that is not a problem. But, if it is at a major level then you have to remove for that you must have some solution for that otherwise it cannot be used for forecasting and policy use all right.

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Handwritten notes on a blue background:

- Top right: © CET I.I.T. KGP
- Equation:  $r =$
- Equation:  $-1 \leq r \leq 1$
- Text: 2nd Goldfeld-Quandt Test
- Equation:  $F = \frac{ESS_2 / \left(\frac{n-k}{2}\right)}{ESS_1 / \left(\frac{n-k}{2}\right)}$
- Diagram: A vertical line labeled 'X' on the left. To its right, error terms  $e_1$  and  $e_2$  are grouped under  $ESS_1$ . Below them,  $e_3$  and  $e_n$  are grouped under  $ESS_2$ . A circle with a 'G' is drawn around  $e_3$ .
- Bottom left: NPTEL logo

The thing is when we will calculate r here is r you calculate then you know this is correlation coefficient, r is lies between minus 1 plus one. So, it is you know it is the r square technique so, obviously, you know that is how you need t can give you the data signal significance of the correlation coefficient. You only calculate getting the r is not a sufficient because it will give you the degree of association means obviously, like you know multicollinearity it will give you the linear association between these 2 variables X and u. So, now, if the degree is high then there is a presence of heteroscedasticity if the degree is low it is a presence of low heteroscedasticity.

So, accordingly you have to restructure and redesign till you get the situation where it is. That means, the situation where the degree of association is at the 0 level or it is close to 0 if it is at the higher level close to 1 then it is a serious problem. So, you have to restructure the model. So, this is 1 technique through which you have to solve the

particular problem second technique is called as a goldfeld and Quandt test  $g$   $q$  test. This is another test through which you have to detect the heteroscedasticity problem. What is this test in that case we use  $f$  statistic  $f$  statistic is explained sum square to divide by  $n - c - 2 - k - k$  divide by  $ESS_1 / (n - c - 2 - k)$ .

So, this is  $ESS_2$  and  $ESS_1$  so, that means, this is explained sum squares at the level 2 and explained sum square at level one. So, what is the level 2 level 1. So, that means, you see here. So, we have  $X$  and we have  $e$ . So,  $e_1 e_2 e_3$  and up to you can say  $e_n$  this is how the structure, but, you know when the sample size is very less. So, this particular method may not be very perfect fit for a sample small sample then you go for  $n$  correlation then you find out the solution.

You know these techniques are there are various techniques various means all techniques are not similar structure the similar shape there are different structure and different shape they are also very strong in a particular problem. This particular technique is very useful for large number of samples. So, what is that structure here. So, the moment you will get a this type of error structures let us say it is the five hundred errors. So, for 500 observation points. So, you what you have to do you first arrange in an ascending to descending orders then you remove some negative points.

So, that negative point removal is called as a  $c$  common points. So, now, say 500 you drop fifty in the middle then you it will drop 50 then it will reduce to 450 then you know first you divide the entire structure into 2 equal parts then accordingly it will go like this. So, you remove this particular point then this you will find out  $ESS_{SS_1}$  and you will find out here  $ESS_2$ .

So, this is  $c$  the unit of variables then accordingly you find out the  $f$  statistic. If the  $f$  statistic is you know statistically significant then there is a heteroscedasticity problem. So, if the  $f$  statistic is not significant then that that will be creative. You can say that will give you an indication that it is a homoscedasticity problem. So, if it is homoscedasticity then there is no issue you have to go ahead. If it is a  $f$  is found statistically significant then you have to redesign restructure till you get the homoscedasticity issue means transformation of homoscedasticity.

So, similarly, there is another test called as a White test through which you can check the detect the heteroscedasticity.

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The slide is titled "White Test" and contains the following handwritten content:

- Top left:  $\hat{Y} = X\beta$
- Top middle:  $\hat{Y} = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \gamma_3 X_3 + u$
- Top right:  $\hat{Y} = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \gamma_3 X_3$
- Below that:  $e = Y - \hat{Y} = R$
- Center:  $e = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_1^2 + \alpha_5 X_2^2 + \alpha_6 X_3^2 + \alpha_7 X_1 X_2 + \alpha_8 X_1 X_3 + \alpha_9 X_2 X_3 + u$
- Bottom center:  $R^2 \approx \frac{k-1}{n-k-1}$

In the white test so, we need to integrate with a Y hat and e. So, first you get Y hat and then you find out e; that means, followed by Y and X information. So, you process it to get Y hat. So, Y process you will get Y hat then you have to obtain e then you have to build a model here is like this let us say the sample point there are 3 variables in the system. So, you use step like this you know gamma 1 gamma 0 plus gamma 1 X 1 plus gamma 2 X 2 plus gamma 3 X 3 plus u. So, estimated model will be gamma 0 hat plus gamma 1 hat X 1 plus gamma 2 hat X 2 plus gamma 3 hat X three. So, you find out error terms. So, Y minus Y hat.

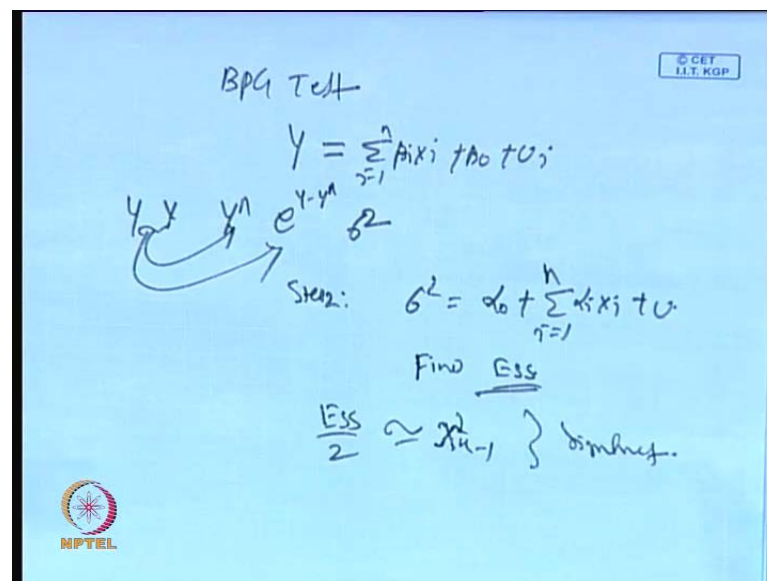
So, obviously, you here is the moment you will get e. Then, obviously, of course, the r square is highest and f is statistically significant gamma 0 is significant gamma 1 significant gamma 2 significant gamma 3 are significant. Means all these parameters are statistical significant in the higher level and r square is very high and f is also highly significant at the higher level. So, still you need to have the heteroscedasticity set.

To go for heteroscedasticity set you have to go for again to pick another model say e equal to you know alpha 0 plus alpha 1 X 1 plus alpha 2 X 2 plus alpha 3 X 3 plus alpha 4 X 1 squares then plus alpha 5 X 2 squares plus alpha 6 X 3 squares plus alpha 7 alpha 7 X 1 X 2 plus alpha 8 X 1 X 3 then plus alpha 9 X x 2 3 plus U. U is an another error term.



So, now you estimate this model and have the r square value find the r square. So, now, you multiplied n with r square then it is followed by chi square is the k minus 1 degrees of freedom. If this is statistically significant then it is the presence of heteroscedasticity if it is not significant then there is no such problem of heteroscedasticity. You can say heteroscedasticity. So, you have to proceed with the means we can use that models for policy use and you can say forecasting, but, if it is significant then again you have to redesign restructure till you get the better fitted model ok.

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Now another way you will be get the model there is a test called as the b p g test. What is the b p g test? b p g test is the you first design a model like this Y equal to summation beta I X i. i equal to 1 to n plus beta 0 plus U i. So, now, you have Y hat you have e followed by Y and X followed by e. So, Y minus Y hat. So, then once you have Y minus Y hat then, obviously, you find out error variance sigma squares. So, now, in the step 2 you try to regress sigma square equal to alpha 0 plus summation alpha I X i equal to 1 to n plus U. Then you find out then you find out explained sum squares you find out explained sum squares.

So, now explained sum square divide by 2 followed by chi square k minus 1 chi square k minus 1. If it is significant then there is a problem of heteroscedasticity if it is not significant then there is no such heteroscedasticity problem. So, that means, you can use that particular model for forecasting or policy use. So, these are the detection criteria

through which heteroscedasticity can be observed or it can give you the indication message that whether there is a presence of heteroscedasticity in the system or in that particular problem or not.

If there is such problem then, obviously, you have to find out its alternative or you can say restructure design then till you get the model best fitted which is free from heteroscedasticity. But sometimes you will face lot of difficulty. So, to solve a particular problem there is a introduction of another problem. For instance, if you solve heteroscedasticity problem which is already clear through autocorrelation problem let us start with a some estimated models which is not a correlations.

But, when you will go for a heteroscedasticity it you can find that there is a some problem of heteroscedasticity. So, you have to redesign restructure. So, in that case you will find that that redesign restructure will give you the heteroscedasticity problem means solution. So, the moment you solve that heteroscedasticity then a in that particular case you know redesign model may be you know extra problem create extra problem like it can have a again autocorrelation problem again. You have to try to solve the autocorrelation if you are success then it is very if it is not success then you have to find out some compromise rules.

So, how quick how close autocorrelation have in the model and how close heteroscedasticity have been in the model. It should be because presence of autocorrelation is not a serious issue it may be because it is allowed 1.5 to say 2.5, but, presence of heteroscedasticity should not be allowed it should be removed.

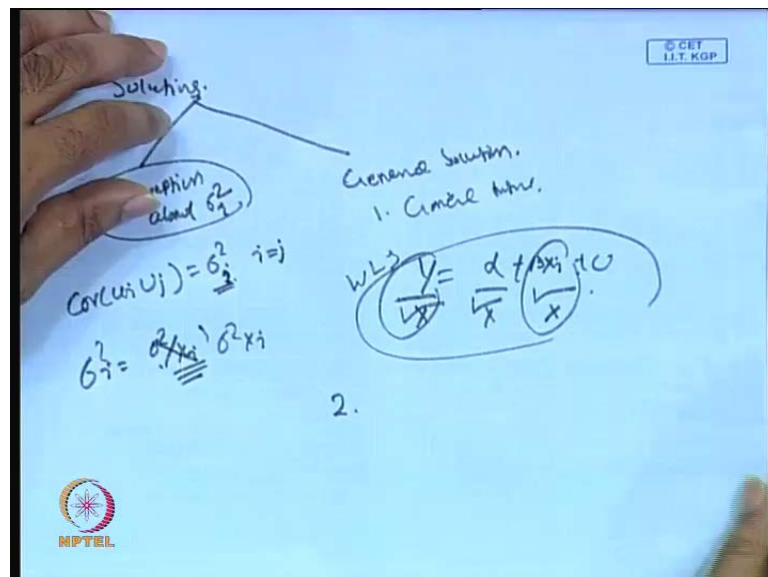
Because it is the variation of data points should be very much negative effector for the econometric modeling that in that case it cannot be considered as the best model. So, that is why you first solve the heteroscedasticity problem and you will come to other problems. So, that it can be a solved accordingly. So, this is how the structure of you know heteroscedasticity problem. So, that means, we get to know what is the various means the definition of heteroscedasticity, nature of heteroscedasticity, causes of heteroscedasticity, then its detection criteria. Now we like to know what are its solution criteria.

Obviously, before going to solution the obvious question is whether it is a problem for econometric modeling? In fact, most of the times we have already highlighted it is

obviously, it is a problem. So, problem means since it is not have it is you know having a blue concept means that it is not going in favor of blue theorem. So, that linear unbiased estimator.

So, obviously, it is by default it is a problem that model cannot be used for forecasting or policy use and we cannot say that this model is considered as the best model. So, that is why means it is a serious problem because it is not fulfilling the properties of blue theorem. So, so it needs solutions. So, you need to find out some solution for this scaring the heteroscedasticity.

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So, what are the solution you can find out; that means, solutions say the solutions are basically divided in to 2 parts. One is the assumption about sigma square I and you know general solutions why assumption about sigma square because it is the covariance of U upon I U I U j equal to sigma square I or [i] I i equal to j it I equal to j. But, it means it is not equal variance. So, sigma square I means sigma square I may be some functional form let us say it is sigma square by x. So, let us say this is in that particular format or you can say this type of case sigma square X i this is the form.

So, that means, what you have to do. So, U being yes transformation rule let us say U have a model Y equal to alpha plus beta X i plus u. So, what you have to do since it is coming like this way. So, you get us otherwise you take like this sigma square X i say that will be better indication. So, now, what you have to do? So, you divide the root X

both the sides. So, then you will get a particular solution by the rule this is one variable and this is another variable. So, now, you again go for estimations the error once you go get the estimations and if will you go by heteroscedasticity check then, obviously, the problem can be solved.

So, that means, here this particular structure is called as a otherwise you can apply this weighted least square methods. So, you apply some weight which is which can minimize this you know unequal variance. So, giving weight age to that particular variables will be you will solve the heteroscedasticity problem.

Similarly, best idea is here regarding assumption about sigma square I. Mean you see what is the form of sigma square it may be sigma square I equal to say alpha plus beta X i another format. So, you divide alpha plus beta X i you with Y and X then you again re estimate the means transfer variable. Obviously, you will we you will get a situation where there may be no may not be any heteroscedasticity problem.

So, that means, there we need a proper transformation with respect to sigma square i. So, you first know what is the sigma square I then with respect to sigma square I you will we go for this weight age factor or transformation rule. So, this one way to solve this particular heteroscedasticity problem. The second aspect of heteroscedasticity problem is means we categorical divided in to other part is called as a general solution.

So, general solutions means you can go for transformation rule. Also general transformation rule for instance you know, obviously, you can start very in a inspection method it will find there is a some kind of variability means huge variability. There may be some presence of outliers etc.

So, it is better you start with transformations. So, it will start with transformation then, Obviously, at the end you may not have a heteroscedasticity problem, but, if you by inspection you are getting you know some kind of variation and you are not transforming and going with you know original setup.

Then, obviously, you will again go back to the original position because of the estimated models you may not get the heteroscedasticity solution. So, it is better if you by inspection if will you get first before you handling this particular problem by inspection

you observe whether there is a heteroscedasticity suspect of heteroscedasticity problem will be there or not then you go for transformation.

But, you know any problem can be start with a transformation, but, you can do that, but, that is not good choice always because if will we apply the transformation rule a very beginning then you know you are going to lose the originality of that particular variable. So, that is why it is better you should not start with the transformation if it is not necessary or not required then you find if it is necessary then you have to go by transformation.

It is actually a just like a circle you have to move one after another structure till you get the best fitted model. In one step it is very difficult to find a solution which can be considered as the best for you know best fit way models which can be considered as a policy use or forecasting. So, it is very difficult to get in the first and you have to go step by step till you get the best fitted models.

This sigma how the ones another solution to the general transformation is either is called as a general transformation rule or transformation with respect to as per the structure of sigma square  $\sigma^2$ . So, because sigma square if it is not equal then it is obviously, sigma square  $\sigma^2$  having some functional form that functional form has to be observed very carefully. Then you have to apply or divide multiply or subtract divides or whatever you like it will get the a models which is free from heteroscedasticity this sigma another solution for this particular problem.

Second solution is in fact, this transformation rule there is a lots of transformation criteria. You can go for log transformation, you can go for exponential transformation, you can go for some mythological transformations, you can go for you know some different transformations. So, every transformation rule structure is completely different and like you know different techniques will give you different types of results and sometimes some technique will give very positive for your objective and some transformation will give you negative for the regular objective.

Similarly, in that case also in the transformation rule like technique transformation rule also give you some something positive for you something for negative for you. So, that is why what is my solution is that you go transferring the variable one after another structure. If in one transformation rule a you will get it then, obviously, its good for you

are lucky enough, but, if not then you have to sequentially go with different transformation till you get the best model which is free from heteroscedasticity and other problems.

So, transformation rule cannot be unique it should it is. In fact, by definition it is very multidimensional in natures. So, there are many ways you can transfer the variables in the current format different structure and that can be that is essential and that that is needed that is also required to do to get the best fitted model.

Because if you know model itself means econometric modeling is a decision making science if I will this define in improper way in a broad angle that means, there are various alternatives must be with you. So, within the various alternatives you have to find out the best possible solution which can be consider as a policy use or which can be consider for forecasting issue. So, that is how you must be very careful about the transformation rule.

So, we have a separate lectures for you know digit test during the digit test I. We will discuss detail about the transformation rule because you will be highlight all the issue details here is this is another means it is about the transformation rule. Then sometimes you know redesign the variables may be for instance you are using the term exports with the impact of exports on inflations. So, you are starting with the direct amount of exports with inflations and a by that way you will get you know heteroscedasticity problem. So, in the next step what you will do?

You transfer the export in to percentage of amount of export as a percentage of  $g D p$  means the another way to transformation rule. So, if will we instead of using direct exports if will you use export as a percentage of  $g D p$  then, obviously, that transformation may solve the heteroscedasticity problems. So, you must be very careful because the you way you will transfer this like this is required lots of theoretical knowledge export as a percentage of  $g D p$  is a meaningful interpretation. So, that is why it can be used, but, if there is no such meaningful interpretation you cannot use for you cannot transfer that variable arbitrarily. So, there should be logic behind it.

There should be proper theory behind this then you will go for that type of transformation. So, then similarly, there is another tricks called as a redesigning the mathematical form of the models. Since mathematical formulation model imperfection of

the model will give heteroscedasticity problem. So, you continuously redesign the model.

Because in the first hand, you cannot observe whether this particular functional form is having heteroscedasticity free from heteroscedasticity or not. So, ultimately once you have then again you have to redesign the mathematical formulation of the model then again you test for heteroscedasticity. It may be solution for the heteroscedasticity problem. So, this is another way to solve the heteroscedasticity problem.

Then another way you can solve the heteroscedasticity problem is improving the data collection technique. So, because I have mentioned. So, there may be a respondents some are very positive to your questionnaire some are very negative to your questionnaire. So, this side it is unbiased and this side it is biased and if will you clog together then, obviously, there will be heteroscedasticity problem.

So, you know it you should be very good respondents and that should be the respondents must be homogeneous in nature. They can understand a problem in an equal level or equal angles. Obviously, the sample points cannot be vary or cannot far from the other sample points. As a result heteroscedasticity problem can be solved. So, that means, what is that suggestion is that improving data technique will also solve the problem of heteroscedasticity. So, by like this there are several other rules also through which heteroscedasticity problem can be solved.

So, we have discuss the what is the exact problem of heteroscedasticity in nature then its consequences causes and detection criteria and the solution criteria and obviously, we have in between we have also discuss whether it is a problem of it is a problem you need to have a solutions and for that solution you must have a higher approach must be positive you must have a huge knowledge on technique wise transformation information etc. So, that it can be give you lots of positive things how to solve this heteroscedasticity problem. With this we will close this chapter. Thank you very much have a nice day.