

Security Analysis and Portfolio Management

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Module No. # 01

Lecture No. # 21

Introduction to Portfolio Management

Good morning. So, after discussing about the security analysis, or, in general, what we can talk? How we can take the decision in the financial market or from individual investor point of view? When we should buy, and when we should sell? Then, we can talk about the combination of the different assets, or whenever we take the investment position in the different market, whether we should invest in one individual asset or we should concentrate more on the combination of the different financial assets. So, that is the question always comes to our mind.


So, if we want to invest in a different combination of the different financial assets, then why we will take that position in the market, and why the combination is required, and why this concept of the combination of the different assets, in general, what we call it the portfolio of the different financial assets are required? So, that is the question always comes to the mind of the individual investor.

So, in this context, from this session onwards, we will talk about how, generally, we do the portfolio management, and what kind of different strategies are there through which the portfolio can be constructed, and as well as we can also talk about the different theories who deals with the construction of the optimal portfolio in the different market situations. So, today we will be talking about the introduction to portfolio management.

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Assumptions

- Investor wants to maximize the returns for a given level of risk.
- Portfolio includes all of your assets and liabilities
- The relationship between the returns for assets in the portfolio is important.
- A good portfolio is not simply a collection of individually good investments.



So, here, whenever we start with the introduction to portfolio management, some of the assumptions, always, we keep in the mind that whenever we deal with the market, always the individual investor wants to maximize the return for a given level of the risk. In other words, sometimes, also we can see some of the investors can take, also, unlimited amount of the risk to maximize the return, but that kind of situation is very rare.

Sometimes, also what we can see that with a given amount of the return, they want to minimize the risk or, for example, if you want the 15 percent or 20 percent return. So, to get this 20 percent return, so what is the risk? Actually, maximum risk I can take. So, that, actually, the investor, from the beginning, can also fix. So, that is why this is the basic background or basic assumption, always, we should take whenever we do the portfolio construction.

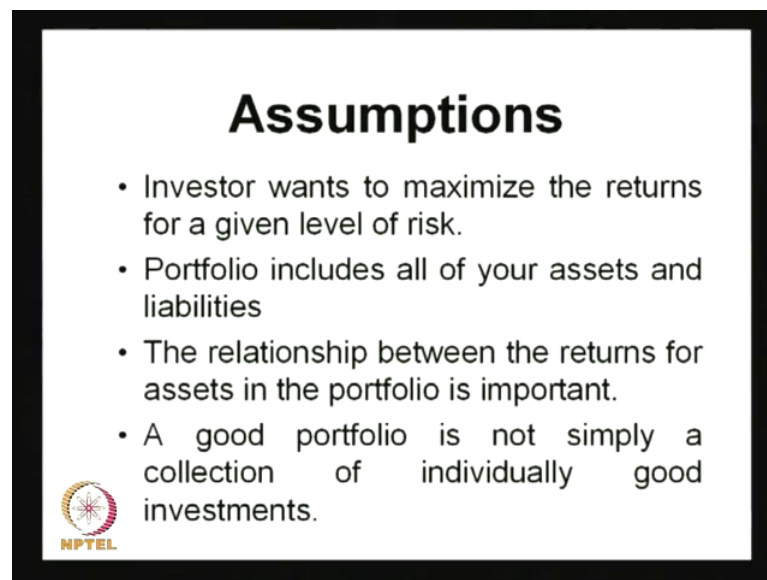
The next thing is, **always**, we already, in the beginning, what I said that the portfolio exactly includes all of your assets and the liabilities, because whenever we make the portfolio, basically, whenever we see that the investors or any of the financial institution make their portfolio, they look into the liability side and as well as asset side.

Why we look into liability side? **It is...** Basically, portfolio is nothing but a optimization concept. So, if you talk about the optimization concept, what, generally, we can see there

are certain limitations. There are certain constraints whatever we have, and always, the investor wants to minimize that constraint to maximize the return. So, the constraints can be any form– the constraint can be in terms of the risk; the constraint can be in terms of the financial position of the investor, or the constraint is also, sometimes, related to the market.


So, taking into account all the constants, always, the investor wants to maximize the return. That is why, always, we can say, the portfolio analysis or the portfolio management process will always involves with both assets and liabilities of the assets of this particular investor.

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Then, already, what we said that the relationship between the returns for assets and the portfolio in the portfolio is quite important, because already, we see that the most basic objective of the investor is to maximize the return. So, sometimes, another question arises. For example, we have different financial assets– let we have stocks, we have bonds, we have some of the fixed deposits, or we have, let we can say, any of the bank savings or we have any kind of the time deposits in the bank and as well as post office. Also, we have the mutual fund investments, and as well as also we can see there are other financial instruments which are available in the market.


So, whenever we make the portfolio, for example, we say that if we get 10 percent return from one asset and 20 percent return from another asset, and 5 percent return from another asset, then can we see that the return what we can get, or the combination what we can make from these different assets, the combination is really a only the aggregation of those assets, or is there any kind of process or is there any kind of method, through which the aggregation can takes place? The aggregation can take place only whenever we can form or we can hypothesize the different assets to a particular form.

So, but the thing is, whenever we see that different assets have different characteristics, different assets have different kind of properties. Some assets are more liquid, some assets are less liquid; some assets are easily available, some assets are not available easily. So, in this context, what we can see are some of the assets are marketable; some of the assets are non-marketable. So, looking into this perspective, it is very difficult to combine those assets to make the portfolio.

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The another objective also you can see, even if you concentrate one single asset, for example, somebody wanted to invest in only stocks. So, if there are 10000 or 20000 stocks are available in the market. So, how many stocks he should take, or why this much stocks? And second question is, whenever they take the stock of, maybe, let 20 or stock of let 30, then why these 30 stocks?


And whenever I have 100 rupees, or somebody has 100 rupees, then how that 100 rupees would be allocated between the 30 stocks? Whether this allocation will be on the basis of the risk involved in that particular asset, or it is on the basis of may be the simply weightage or the equal weightage to all the 30 assets? So, these are the different questions comes to the mind of the investor.

So, whenever we deal with all those fundamental or practical questions related to this portfolio, whenever we construct the portfolio in the market, all the relationships and all these characteristics, basically, talks about the management of the portfolio.

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So, here, if you observe this fourth point that a good portfolio is not simply a collection of individually good investments, so why it is not only the collection of individually good investments? Because just now, what we discussed about the different type of characteristics, different kind of objectives, different kind of properties of the different financial assets, then the combination– making this combination is quite difficult.

So, how to make this combination? This is basically the important step of the portfolio management. So that, actually, we will see, further, that how this combination takes place and what are those different theories, basically, talks about the different combination in the financial asset or to make the financial portfolio in the market.

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Expected Rates of Return

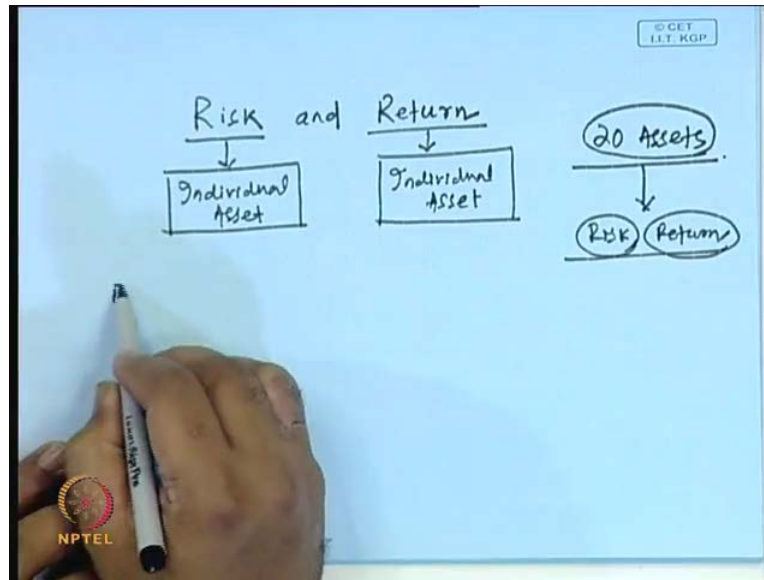
- For an individual asset - sum of the potential returns multiplied with the corresponding probability of the returns
- For a portfolio of assets - weighted average of the expected rates of return for the individual investments in the portfolio



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Then, you observed that **in the...** whenever we talk about the risk return of the different type of asset, because that, actually, we are using very frequently, what we can see that you observe that, always, whenever we make the portfolio or whenever we deal with any of the financial assets, the most important parameters, basically, what we always see, the important parameters are, basically, the risk and return.

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So, basically, this risk is involved with the individual asset; return also, always, we calculate from an individual asset. So, if you observe that the risk what you are calculating from the individual asset, and if we have 20 assets– if you have 20 assets– and we have to calculate this 20 assets risk and 20 assets return over the period of time to make our portfolio.

But here, what we have seen that how this risk is calculated, how the return is calculated for the individual asset, and how the risk and return is calculated for the portfolio, which is consisting this 20 assets. So, this is the objective from the beginning we want to see.

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Expected Rates of Return

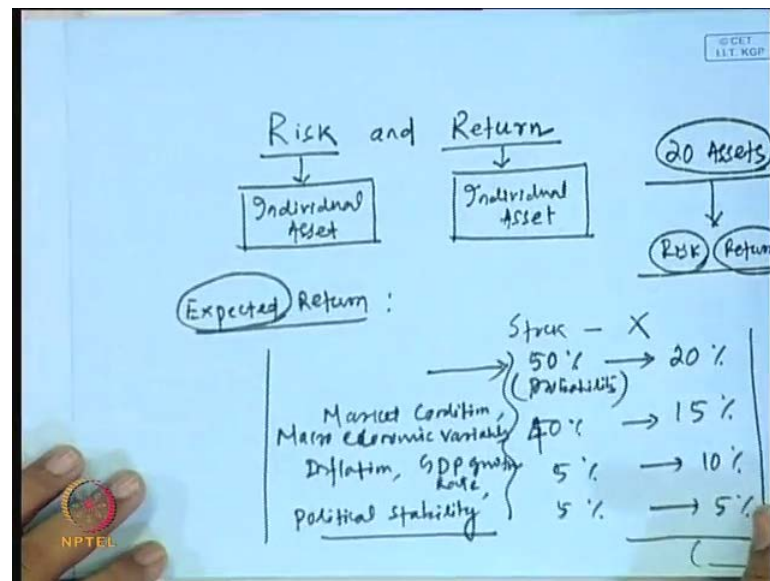
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So, that is why what we have seen that for an individual asset, sum of the potential returns multiplied with the corresponding probability of returns, that actually give you the expected return of this particular asset, but for the portfolio of the assets, it is, basically, the weighted average of the expected return of rates of return for the individual investments in the portfolio.

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What do you mean by the expected return? The question arises— what do you mean by the expected return? Why this expected return is coming into the picture? Because once we take the position in the market— whenever we take the position in the market— we do not know how much return we are going to, actually, get from that particular individual investment.

Let you start investment with a stock X. If you start the investment with the stock X, so, in this case, what we have seen there is a probability. There is, let 50 percent probability, this stock X can give you 20 percent return. There is a 50 percent probability the stock X can give you 20 percent return; there is another 40 percent probability the stock X can give you 15 percent return, or there is 5 percent probability stock X can give 10 percent return, and there is, also, another 5 percent probability stock X can give you 5 percent return.


So, this probabilistic function depends on the other external factors. The other external factors, in the sense what I refer, may be the market condition, macro economic variables. Macro economic variables, which includes your inflation, GDP growth rate, it may also include the political factors— stability, political stability, or we can say the stability of the government or the international relation, etcetera. So, these factors will determine what is the probability of getting return from the individual stocks.

So, on an average, if that particular investor wants to invest in the stock, the average return– expected return– what he is going to get from this– **this** 50 percent– 0.5 multiplied by 20 percent, then 0.4 multiplied by 15 percent, 0.5 multiplied by 10 percent, **0.5** 0.05 multiplied by 5 percent. Plus, if you add all these, then, maybe, we can say the total or the average expected return from this individual stock in this particular period will be this much.

So, this is, basically, what is the concept of the expected return, what always we going to arrive or we want to get from an individual asset, but whenever we talk about the portfolio, this scenario will be little bit different. The scenario is, for example, we have 10 assets, 20 assets. So, out of the 100 rupees, what already I told you that out of the 100 rupees or 200 rupees, that how much money should be invested in stock 1, how much money should be invested in stock 2, and how much money should be invested in stock 3. These are the different problems or different weightage, always, we want to know. So, this is, basically, if you get the weightage, and then you get the probability, then you get the return what you are expecting. From that, then will be expected return of the portfolio can be calculated.

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Computation of Expected Return for an Individual Risky Investment		
Probability	Possible Rate of Return (Percent)	Expected Return (Percent)
0.25	0.08	0.0200
0.25	0.10	0.0250
0.25	0.12	0.0300
0.25	0.14	0.0350
		<hr/> E(R) = 0.1100



You just see this— **this** is, basically, a computation of expected return for an individual risk investment because already, why it is risk investment? Because we know that any of the assets which involves both risk and return, that we can call a risky investment, because of the different situation, what we have seen. Because of the different situation, the probability- there is a 25 percent probability that the possible expected return will be 8 percent and there is a 25 percent probability the return may be 10 percent. So, there is a 25 percent probability that return may be 12 percent and there is 25 percent probability the return may be 14 percent.

So, whenever we take this possible rate of return, what we are going to expect, or what we can get it from this particular investment? So, if you get this multiplication of possible rate of return with this probability, then the expected return in each case can be calculated. Then, finally, your expected return from this particular risky investment will be 11 percent. So, this is, basically, the calculation of the individual return of one risky investment.

But again, you see, this is basically the portfolio. What is the portfolio? If you talk about this different type of portfolio, so, here, we have the portfolio of the risky assets. So, what generally we have seen, that this is the risky assets— a risky portfolio— whatever we have.


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	Weight (W_i) (Percent of Portfolio)	Expected Security Return (R_i)	Expected Portfolio Return ($W_i \times R_i$)
2	0.20	0.10	0.0200
	0.30	0.11	0.0330
	0.30	0.12	0.0360
	0.20	0.13	0.0260
			<u>0.1150</u>
			$E(R_{por i}) = 0.1150$

$$E(R_{por i}) = \sum_{i=1}^n W_i R_i$$

where :

- W_i = the percent of the portfolio in asset i
- $E(R_i)$ = the expected rate of return for asset i



Let the percentage of portfolio we have is 20 percent. Here, the expected return what we are getting 10 percent. Then, finally, the expected portfolio return will be **2 percent**.

0.2 multiplied by 0.1 that will give you the 0.02 percent, but if you see this percentage, percent of portfolio weightage will be 30 percent. Then, expected security return will be 11 percent, then the portfolio will be 3.3 percent. So, finally, what we have arrived at 11.5 percent. So, here, what we can see, that your W represents the weightage, R represents the expected return of the return for the asset 1.

So, this is the expected return from one asset; this is the expected return from another asset; this is the expected return from another asset; this is the expected return from the another asset. So, this return may be calculated from the different probabilistic function. Then, once you have this, there is a probability of 10 percent return to get from this investment or individual investment. There is 11 percent— there is expected return, maybe 11 percent— there is a probability that we can get from another investment. Then finally, if for example, this weightage should be equal to 100. So, let 20 percent we are investing in stock 1, 30 percent we are investing in stock 2. Again, this 30 percent we are investing in stock 3, and 20 percent we are investing in stock 4.

In this context, what we can see, that if you your weightage multiplied by your expected security return, that will give you the expected portfolio return from this particular

individual asset. And finally, if you take the combination of all, let **this particular security, sorry**, this particular portfolio has four assets and finally, what we have arrived that the expected return from the portfolio will be 11.5 percent.

So, that is all what, generally, these are two steps, where in the first step what we have to do– you calculate the expected return of the security using this probabilistic function. Then, once you can calculate the expected return of the security, then you assign that weightage, and in the gradual process, we will find out that how this weightage will be assigned, and how the weightage can be calculated for the individual asset in a particular portfolio.

But **once you...** if the weightage will be given to you, in this case, only for the calculation purpose, then what you can do? This weightage multiplied by the expected return– that will give you the expected portfolio return. Then finally, if you take the summation of this, then the expected return from the portfolio, basically, will be 11.5 percent.


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Variance of Returns for an Individual Investment

Variance is a measure of the variation of possible rates of return R_i , from the expected rate of return $[E(R_i)]$

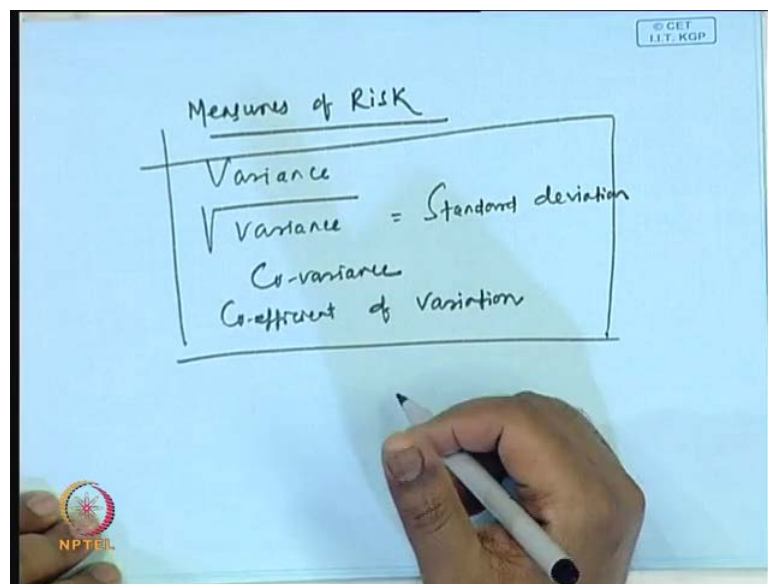
$$\text{Variance } (\sigma^2) = \sum_{i=1}^n [R_i - E(R_i)]^2 P_i$$

where P_i is the probability of the possible rate of return, R_i



Then, another thing if you observe, **this is, basically, this...** After discussing about the return of the individual's investment and as well as the return from the portfolio, then we should talk about the risk—how, generally, we can measure the risk and of this individual asset, and as well as the risk of the individual portfolio. Here, you just see how the risk can be measured. Hopefully, you must be knowing in various stages that there are various measures of the risk.

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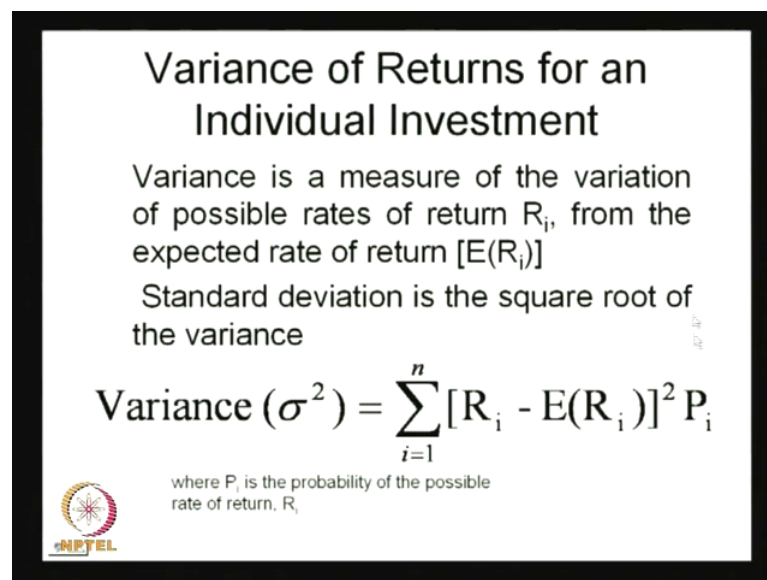
But of the easiest measure of the risk, basically, to talk about the measures of risk, the risk can be measured in various ways. What are the different type of risk, actually, we are not talking about, here. I hope, already, this has been discussed before, but the measures of risk— the popular measure is the variance, and another measure is the root of the variance— the square root of the variance, that, basically, is nothing but the standard deviation.

The standard deviation is very quite popular measure. We can also use the co-variance, or the sometimes also use the coefficient of variation. So, there are various measures which are available to measure the risk. So, how, generally, these particular measures are calculated for a particular market? So, if you can calculate this variance, co-variance, or coefficient of variation, etcetera, **etcetera**, or the standard deviation of this particular asset, and from these particular assets we can measure the variance of the return of this portfolio.

So, that is why, the first basic objective of the investor is to know that whenever he is expecting that I am going to get 10 percent return, or there is a 50 percent chance or 50 percent probability that I am going to get 10 percent return from this investment, then in this process, he should also know how much risk he is taking. That risk is nothing but, that is the variability of that particular return.

So, what is the chance, or what is the **various...** what is the probability of getting this return of 10 percent? That, basically, is also decided by the how much risk he is going to face if he wants to invest in that particular asset.

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Variance of Returns for an Individual Investment

Variance is a measure of the variation of possible rates of return R_i , from the expected rate of return $[E(R_i)]$

Standard deviation is the square root of the variance

$$\text{Variance } (\sigma^2) = \sum_{i=1}^n [R_i - E(R_i)]^2 P_i$$

where P_i is the probability of the possible rate of return, R_i



So, that is why what already I told you that variance is a measure of the variation of possible rates of return. Variance is a measure of the variation of possible rate of return, from the expected rate of return.

So, what you are expecting that this much return you are going to get, and what is the possible return or actual return we are getting. So, that variation– **variation** between these two– basically, shows you the variance of that particular stock or particular investment, whenever we take the position in the market.

So, that is why the variance is nothing but the R_i . R_i represents the possible rate of return or actual rate of return from the stock, minus how much return you are expecting. Using this probabilistic function, square of this difference into the probability, how much

probability of the variation is there. So, that basically or probability of happening this R_i or the return from this particular investment. That, basically, will give you the variance.


How generally it will be measured, that we **we** can see later, but already, I told you, the standard deviation is the square root of the variance. The standard deviation is the square root of the variance.

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Variance (Standard Deviation) of Returns for an Individual Investment

Possible Rate of Return (R_i)	Expected Return $E(R_i)$	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	P_i	$[R_i - E(R_i)]^2 P_i$
0.08	0.11	0.03	0.0009	0.25	0.000225
0.10	0.11	0.01	0.0001	0.25	0.000025
0.12	0.11	0.01	0.0001	0.25	0.000025
0.14	0.11	0.03	0.0009	0.25	0.000225
					0.000500

Variance (σ^2) = .0005
 Standard Deviation (σ) = .02236



You just see this calculation, how this variation, variance, and the standard deviation of the returns of an individual investments we can calculate. If you observe **here that...** what generally, we can see, if you observe here that this is your possible rate of return from this particular asset. This is the expected return what you are going to get using this probabilistic function. This is your difference between the actual return, or the possible return, minus expected return. This squared of this— **this** is the probability of getting this 11 percent, 11 percent, 11 percent. So, if you take this multiplication of R_i , R_i , at possible return minus expected return, square of this possible return and the expected return, then finally, you can get this, and finally, the total variance will be 0.0005.


So, the variance of this particular individual asset, which this probability of 25 percent chance of getting those returns, 25 percent chance is getting this return, 25 percent chance is getting this return, the 25 percent chance is getting this return, etcetera. Then finally, what we can see that the variance should be 0.0005. Then, obviously, this

standard deviation will be the square of this– it will be 0.2236. So, this is basically the variance or the standard deviation calculation of an individual investment.

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Covariance of Returns

- Covariance of returns
 - A measure of the degree to which two variables “move together” relative to their individual mean values over time
 - For two assets, i and j, the covariance of rates of return is defined as:
 - $Cov_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$



Then, later, when if you see this concept, already I told you that co-variance of the individual returns. So, how we can calculate the co-variance of the return, or what do we mean by the co-variance? The co-variance is, basically, nothing but it is a measure of the degree to which two variables move together, relative to their individual mean values over time.

If you minutely observe this, it is basically shows with respect to or relative to the individual mean values of that particular return of the particular stock. How these two stocks move together, or how the two assets move together? So, that, basically, is **is nothing but...**, or we can define that one is the co-variance of the returns of this particular two assets.

So, if you denote it in this way, for two assets i and j, the co-variance of rate of return is defined as the expected possible return, minus expected return, into expected return from one asset, minus expected return of one the same asset, into the expected return, into the actual return of another asset, minus the expected return of the another asset. If you take this expected function between these two, then finally, you can arrive the co-variance of the returns.

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Relation between Covariance and Correlation Coefficient


The correlation coefficient is obtained by standardizing (dividing) the covariance by the product of the individual standard deviations

$$r_{ij} = \frac{\text{Cov}_{ij}}{\sigma_i \sigma_j}$$

where

- r_{ij} = the correlation coefficient of returns
- σ_i = the standard deviation of R_i
- σ_j = the standard deviation of R_j

Correlation coefficient only in the range +1 to -1. A value of +1 would indicate perfect positive correlation. This means that returns for the two assets move together in a completely linear manner. A value of -1 would indicate perfect correlation. This means that the returns for two assets have the same percentage movement, but in opposite directions



So, in our finance literature, or particularly, the portfolio management literature, we extensively use this concept of co-variance and the correlation coefficient, which is widely used in various cases, and there is a relationship between this co-variance and correlation. What co-variance is— co-variance is how the two assets return move together with respect to their mean values, but what the correlation says? The correlation says what is the relationship between these two assets— whether the two assets are related or not; if the two assets are related, then why they related and how they are related.

How they are related— in the sense, whether they are positively related or they are negatively related. So, to know this correlation coefficient or correlation between the two assets, we have to use certain formula to find out this correlation. Hope you must have aware about these different formulas or different methods through which we can calculate the correlation coefficient. We have the Spearman correlation coefficient; we have the different type of correlation coefficient we generally calculate.

So, basically the basic objective of the financial literature is to know what is the interpretation of correlation between these two assets. If the two assets correlation is negative, what does it mean? If the correlation coefficient between the two assets is positive, then what does it mean? So, that is the basic objective of the financial analyst, to know how these two stocks or two assets move. If they move in one direction, then, definitely, we can find a positive relationship, or if that linearly related in a positive way.

That means, if we can say that the return between the two stocks or (()) return between the two assets are linearly positively correlated, then what we can say that two stocks move in the same direction.

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So, that is why there is a relationship between the co-variance and the correlation. So, what generally here, sometimes, instead of doing this, in the literature, what generally financial literature, what generally we do, we always calculate the correlation coefficient by standardizing the co-variance by the product of the individual standard deviations; that means, this is one of the ways through which the correlation coefficient can be calculated from the co-variance term.

So, what is the basic objective of doing this? The basic objective of doing this is that if the co-variance will be known to you, that means, which is basically nothing but how the very two stocks move together, and with respect to their mean values, and basically, if you know this risk involved in this particular two stocks, which is measured as standard deviation of one stock to multiplied by standard deviation of another stock, then we can

come to know how these two stocks are related; whether they are positively related or they are negatively related.

So, if you observe here, your r_{ij} is nothing but the correlation coefficient of returns, and this one is the standard deviation of one asset– standard deviation of this particular return– and this is the standard deviation of the return of another asset.

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So, if you have these two, then this has lot of implications, or lot of use in the financial terms. How these two things will be used– that we will see later, but this is the way through which the relationship between the co-variance and the correlation can be established. And what do we mean by this co-variance, and what do mean by this correlation? That, already, we have discussed. That is why, the correlation coefficient only in the range plus 1 to minus 1. Already, that we know because a value of 1 would indicate the perfect positive correlation between the two assets; that means that the returns through the two assets move together in a completely linear **matter** manner. That, already, I told you, and a value of minus one would indicate perfect correlation.

This means that these returns for the two assets have the same percentage movement, but in the opposite direction, which we desired. That means, in general, you just take the example, in this case, you want to invest in the market, and if you have started the investment in the market, that generally, what generally we do, we should take the

position in the market in such a way that if we are losing in one stock, we should gain in another stock, or if you are losing in one asset, we should gain in another asset.

So, how this particular compensatory or we can say the compensating effect will work? This will work if there is no relationship or we can say there is a perfect negative relationship between the two stocks; that means, if you because of. So, and. So, factors because of some external factors if value of the one stock or the return for the one stock is going down; that means, if even if you are losing in the market, then if there is a negatively correlated stock we have in our kitty, then what will happen? That stock will definitely perform better in that particular time.

So, if your one stock is losing, another stock is gaining. Then, automatically, the total risk or total loss what we are making in the market, that will be compensated, but if the two stocks are moving in the same direction or they are positively related, then what will happen that if we are losing in one stock, definitely, we will lose in another stock; or if you are gaining in one stock, then we are also gaining in another stock.

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Relation between Covariance and Correlation Coefficient

The correlation coefficient is obtained by standardizing (dividing) the covariance by the product of the individual standard deviations

$$r_{jk} = \frac{\text{Cov}_{jk}}{\sigma_j \sigma_k}$$

where

- r_{jk} = the correlation coefficient of returns
- σ_j = the standard deviation of R_j
- σ_k = the standard deviation of R_k

Correlation coefficient only in the range +1 to -1. A value of +1 would indicate perfect positive correlation. This means that returns for the two assets move together in a completely linear manner. A value of -1 would indicate perfect correlation. This means that the returns for two assets have the same percentage movement, but in opposite directions.

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So, to get this precautionary position, or to get a... to take a precaution in the market, what generally the investors always do– the investor always trying to find out those stocks which are basically move in the opposite direction.

If the one stock **value will incline**, the one stock value will increase, then, we should expect that the value of another stock should decline. So, there is a inclining trend of one stock and there is a declining trend in another stock. So, that basically we will work as a compensatory effect in the market.

Although it is in practice, it is difficult to find out the stocks, or find out the different type of assets, which are perfectly negatively correlated, but still as far as possible. The investors always try to find out those stocks, which are negatively correlated, or at least, the correlation between those two stocks is very less. So, in this context, what we can do, the return can be neutralized, or at least, with a given amount of the risk, we can maximize our return with other macroeconomic conditions.

(Refer Slide Time: 34:06)

Portfolio Risk

$$\sigma_{\text{port}} = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}_{ij}}$$

where :


σ_{port} = the standard deviation of the portfolio

W_i = the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio

σ_i^2 = the variance of rates of return for asset i

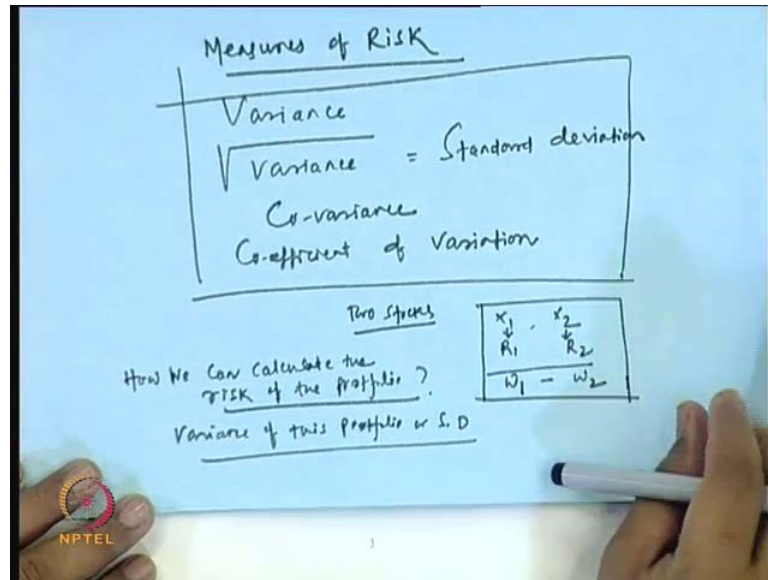
Cov_{ij} = the covariance between the rates of return for assets i and j.

where $\text{Cov}_{ij} = r_{ij} \sigma_i \sigma_j$



So, then if you observe that coming back to this is, basically, what we talked about the individual assets or individual stocks, or whatever individual financial assets which are available in the market. So, then coming back to your portfolio risk– what do you mean by this portfolio risk?

(Refer Slide Time: 34:34)



It is basically what, generally, will happen that, for example, what we have seen that you have two stocks. You have two stocks, let X_1 and X_2 , and your return you are getting from X_1 is R_1 , and return you are getting from X_2 is R_2 . And the basically, the variance, and this is the weightage you are given in X_1 ; this is the weightage you are given in X_2 . So, what generally, how, if this data will be given to you, then how we can calculate? How we can calculate the risk of the portfolio of these two assets? Or basically, how generally we can calculate the variance of this portfolio, or the standard deviation of the portfolio?

(Refer Slide Time: 35:49)

The image shows a handwritten formula on a blue background. The formula is:
$$\text{Var } P_p = w_1^2 \text{Var } R_1 + w_2^2 \text{Var } R_2 + 2 \text{Cov. } R_1 R_2 w_1 w_2$$
 Below the formula, it is written: (Variance of Two Assets Portfolio). In the top right corner, there is a small box with the text: © CET I.I.T. KGP. In the bottom left corner, there is a hand with a ring and the text: NPTEL.

So, in this case, what generally we do? So, how, generally, we measure this? The measurement of this risk or this portfolio is calculated as, or the risk of this particular portfolio is calculated as– the variance of the portfolio is nothing but it is the W 1– weightage– what we are given for the first assets; W 1 squared, with the square of this, and the variance of R 1, which is the return what we are getting from stock 1 or the variability of return what we are getting, plus W 2 square and the variance of R 2, then plus 2 co-variance your R 1 and R 2, and into W 1 and W 2. So, this is basically the **variance of, variance of two assets portfolio, variance of two assets portfolio.**

So, here, generally, what we do? We calculate the variance of R 1; we assign the weightage; we have whatever we have for the stock one or the investment one we calculate the variance of R 2, then also we decide what is the weightage we can give to the stock 2, then we have to calculate the co-variance between R 1 and R 2, then finally, we have to calculate the variance of the portfolio.

So, that is why what the here I am trying to say– the variance of the individual stock or individual asset calculation is totally different from the variance calculation of the portfolio.


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Portfolio Risk

$$\sigma_{\text{port}} = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}_{ij}}$$

where :

- σ_{port} = the standard deviation of the portfolio
- W_i = the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio
- σ_i^2 = the variance of rates of return for asset i
- Cov_{ij} = the covariance between the rates of return for assets i and j,
where $\text{Cov}_{ij} = r_{ij} \sigma_i \sigma_j$




So, here if you observe what we have taken, so this is the is why the standard deviation of this portfolio. W_i generally represents the weights of the individual assets in the portfolio, where weights are determined by the proportional value in the portfolio. Maybe there are other categories, but here, if we can take also in this way, then this is the variance rates of return for asset; i– it is the generalized term, whatever we have used. What if you have three assets, four assets, five assets? Does not matter. Automatically, the different terms will go on increasing in your equation, then the co-variance between i and j within the two investments– it is the co-variance between the rates of return for assets i and j. And here, already, we know that the co-variance is nothing but it is your correlation coefficient between the i and j– i is one asset; j is another asset. And this is the standard deviation of i; this is the standard deviation of the j. So, this is the way through which the portfolio risk is calculated in the market.

(Refer Slide Time: 38:37)

Portfolio Risk Cont...

- Any asset of a portfolio may be described by two characteristics:
 - The expected rate of return
 - The expected standard deviations of returns
- The correlation, measured by covariance, affects the portfolio standard deviation
- Low correlation reduces portfolio risk while not affecting the expected return



So, what we need whenever we calculate the portfolio risk, any asset of a portfolio may be described by the two characteristics: one is your expected return, and the expected standard deviation of the returns. Already, what we have seen in the previous calculation, and the correlation measured by the co-variance affects the portfolio standard deviation. That, already, we have seen from the previous slide that how the co-variance and the correlation are related.

Then also, we have seen that the low correlation reduces portfolio risk, while not affecting the expected return. If you observe this third point, low correlation reduces portfolio risk, while not affecting the expected return, because the correlation, **correlation** is not related to expected return from individual stock correlation is not related to the expected return from the individual stock.

(Refer Slide Time: 39:24)

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$$\text{Var } P = w_1^2 \text{Var } R_1 + w_2^2 \text{Var } R_2 + 2 \cdot \text{Cov. } (R_1, R_2) \cdot w_1 \cdot w_2$$

(Variance of Two Assets Portfolio)

Correlation is not added to expected returns from individual stocks

$$\text{Var } P = w_1^2 \text{Var } R_1 + w_2^2 \text{Var } R_2 + 2 \cdot \rho_{1,2} \cdot \sigma_1 \cdot \sigma_2 \cdot w_1 \cdot w_2$$

$$= w_1^2 \text{Var } R_1 + w_2^2 \text{Var } R_2 - 2 \cdot \sigma_{1,2} \cdot \sigma_1 \cdot \sigma_2 \cdot w_1 \cdot w_2$$

That Variance of the Portfolio (Risk) ↓

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So, therefore, what happens that even if because once the correlation will be negative between the two assets? So, here if you just expand this equation, the variance of P is your W_1 squared, the variance of R_1 plus W_2 squared the variance of R_2 , and you expand this co-variance $(())$, that is the 2, **your correlation between 1**, between 1 and 2, and standard deviation of 1 into standard deviation of the 2, and W_1 into W_2 .

So, here, what will happen if the $r_{1,2}$ will be negative? If $r_{1,2}$ will be negative then what will happen? The term will be W_1 squared variance R_1 , plus W_2 squared the variance R_2 , minus 2, minus 2. So, here, this term will be minus then $r_{1,2}$, the standard deviation 1 and standard deviation 2, and the W_1 and W_2 . So, here, in this case, once this term will be minus then what will happen there automatically the total variance—total variance of the portfolio— will go down, and why it will be down because from the total this term we have to deduct the this term.

So, once we deduction will be there, we are subtracting this variance W_1 squared variance R_1 plus W_2 squared variance R_2 . Then automatically, the total variance, or total risk of— this is the risk of the portfolio— will go down if the correlation between these two stocks are negative. That is why what we can see that the low correlation reduces the portfolio risk, while not affecting the expected return. Anyway, it is not affecting anything to return part. It is only in this variance term. Whenever it is deducting

or it is subtracting this term, then, automatically, the total portfolio variance will be reduced.

(Refer Slide Time: 42:08)

Combining Stocks with Different Returns and Risk				
Asset	$E(R_i)$	W_i	σ_i^2	σ_i
1	.10	.50	.0049	.07
2	.20	.50	.0100	.10
Case	Correlation Coefficient		Covariance	
a	+1.00		.0070	
b	+0.50		.0035	
c	0.00		.0000	
d	-0.50		-.0035	
e	-1.00		-.0070	

So, if you observe this example, so what we can see, we are combining different stocks with different returns and risk to calculate your portfolio risk. So, what generally we have seen here, if you see your is your assets, and basically, this is the expected return—this is the expected return; this is the weightage. **Sorry, there is some misalignment.**

So, here, if this is your expected return; this is your expected return; this is your weightage; this is your variance; and this is basically the standard deviation. This is basically the 0.07 is the standard deviation, and for 2— asset 2— this is your expected return; this is your weightage; this is your variance; and this is your standard deviation. The 0.1 is the standard deviation.

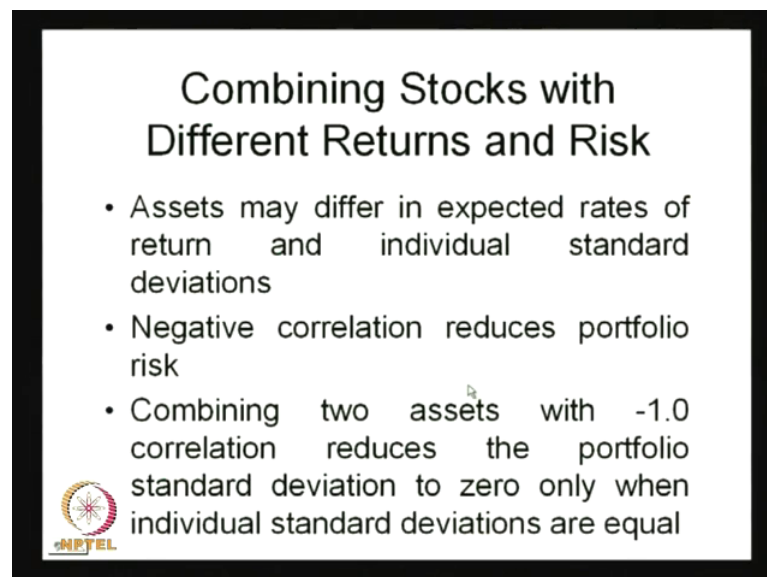
So, here, what generally we can see, that if we take this case, that whenever for the stock 1 we are getting expected return of 10 percent, putting the weightage of 50, **50** percent, variance is 0.0049 and the standard deviation is 0.07. The stock 2, the expected return is 20 percent. W_i — that means, the weightage will be 50 percent; the variance will be 0.01, and the standard deviation is 0.10.

So, there are various cases what we have taken that that case there are let we have taken five cases, and the correlation coefficient between the two assets have been given, that

the coefficient of correlation is one between the two. That means, they are highly positively correlated and the co-variance will be 0.007.


Then the for case b, let the correlation coefficient between 1 and 2– **between 1 and 2–** will be 0.5, then the co-variance will be 0.0035; for c, the correlation coefficient is 0; then automatically, the co-variance is 0. These are different hypothetical situation. What we have taken, then, for d, let the correlation coefficient is minus 0.5. Then, the co-variance will be minus 0.0035, and for case e, the correlation coefficient is minus 1. The co-variance will be minus 0.007.

(Refer Slide Time: 44:54)



Combining Stocks with Different Returns and Risk

- Assets may differ in expected rates of return and individual standard deviations
- Negative correlation reduces portfolio risk
- Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal

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So, then, if you add this, already, we have said this thing, or we have discussed these things that assets may differ in expected returns of return in individual standard deviation, but the negative correlation reduces the portfolio risk– what already we have seen here. And combining two assets with the minus 1 correlation reduces the portfolio

standard deviation to 0, only when the individual standard deviations are equal with individual standard deviations are equal. So, that, sometimes, a very hypothetical situation, what we can say.

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**Constant Correlation
with Changing Weights**

Asset	E(R _i)			
1	.10		$r_{12} = 0.00$	
2	.20			
Case	W ₁	W ₂	E(R _i)	Risk (S.D)
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700



So, here, if you observe this thing, that there is a constant correlation between there are different situation what we are talking about here. There is another situation here. If you observe, we have taken the constant correlation between the two assets, but the weightage we are changing. In the previous case, what we did– the weightage was constant for the two assets. That means, 50 percent we are investing in stock 1, and 50 percent we are investing in stock 2

But 50 percent we are investing in stock 1– asset 1– and 50 percent in stock 2, but what we have observed here that the correlation coefficient is changing in the different situation, and once the correlation coefficient is changing, how the co-variance changing, or automatically, the risk is also will be changing.

So, that is why, what generally we can do– we have seen this in the previous case– that the weightages were same, but the correlation coefficient was changing, but here, we are talking about the constant correlation between the two assets, but on the case basis, we are sending their weights. So, if we are sending their weights, then what we have

observed here, that let their expected return from stock 1 is 10 percent; the expected return from stock 2 is 20 percent; and the correlation coefficient between them let 0.

If you talk about this, what generally we can see, that case one, then, automatically, your let your W weightage in the first case will be 0; the weightage will be second case is 100 percent; then the expected return will be 20 percent, obviously. Then the risk what we can using this formula, what you can do, if you calculate the risk, **the risk** will be 10 percent

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$$w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho_{1,2} \sigma_1 \sigma_2 w_1 w_2$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho_{1,2} \sigma_1 \sigma_2 w_1 w_2$$

$$= \sqrt{0.10}$$

Because how generally the risk is calculated already I told you that this is basically W 1 square W 1 squared means W 1 squared R 1 squared, plus W 2 squared R 2 squared, plus 2 co-variance R 1 R 2 W 1 W 2.

And this term is expanded– W 1 squared R 1 squared, plus W 2 squared R 2 squared, plus 2 this r 1, 2– the correlation coefficient between 1 and 2– standard deviation one, standard deviation two, W 1 W 2. If we put those values, there W 1 is 0, the R 1 is your variance of R 1, **this** is the variance, this is the variance, and the variance of R 1 is already given **0 point how much is the.**


So, if you finally, calculate this put this values in the previous cases, then in this way, if you get the standard deviation of this series, then you will calculate the value will be 0.10 in the first case, and gradually, whenever the weightages are changing, that means,

20 percent to asset 1, 80 percent to asset 2, we are changing the expected return from the stock will be 18 percent, and the risk you are facing is 8.12 percent.

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**Constant Correlation
with Changing Weights**

Asset	E(R _i)			
1	.10		r _{ij} = 0.00	
2	.20			
Case	W ₁	W ₂	E(R _i)	Risk (S.D)
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700



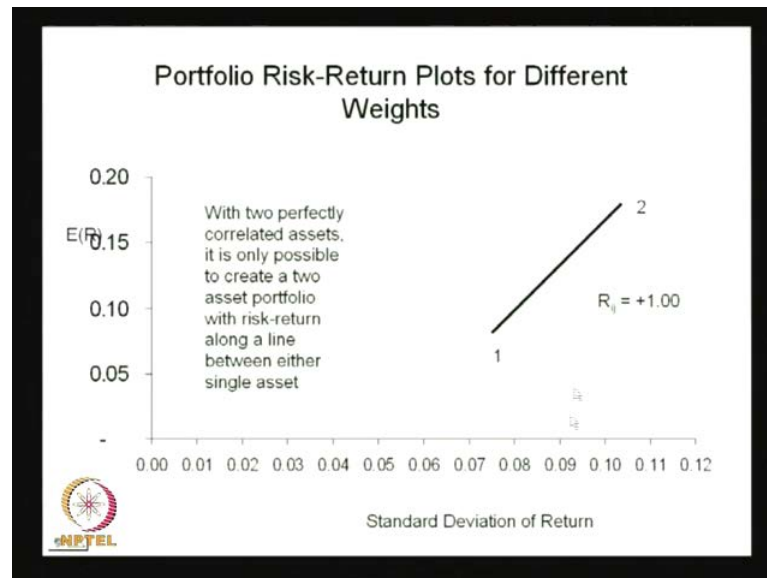
Like that if you change the 40 percent 60 percent, then your return will be expected return will be 16, and **the asset will be...** the total risk will be 6.62. So, in the like that if you taken the weightage of fifty-fifty, the return may be 6.1 percent, return will be 15, this will be this will be 6.1, and like eighty-twenty, you will find it is 12 percent expected return what we are getting, and this is 5.95, and this is if it is weightage is one-zero, then we are getting 7 percent.

So, basically what we have seen that the risk involved in this particular process, basically, depends on the different combination of the assets whatever we have. But here, in the beginning, what I said that the hypothetical weightage what we are giving to the different assets– that, basically, **talks about the...**, or that basically decided the how much return we are going to get, and how much risk we are going to get.

So, the basic step of the portfolio management, or the most important step of the portfolio management is to find out that how much weightage we should give to different assets. If you can decide, there are various theories what in the other sessions we talk we will talk about, but here, already, we have seen that this is the different step, or this is the

most important step, basically, always we should look into whenever we calculate the expected return and risk of that particular portfolio.

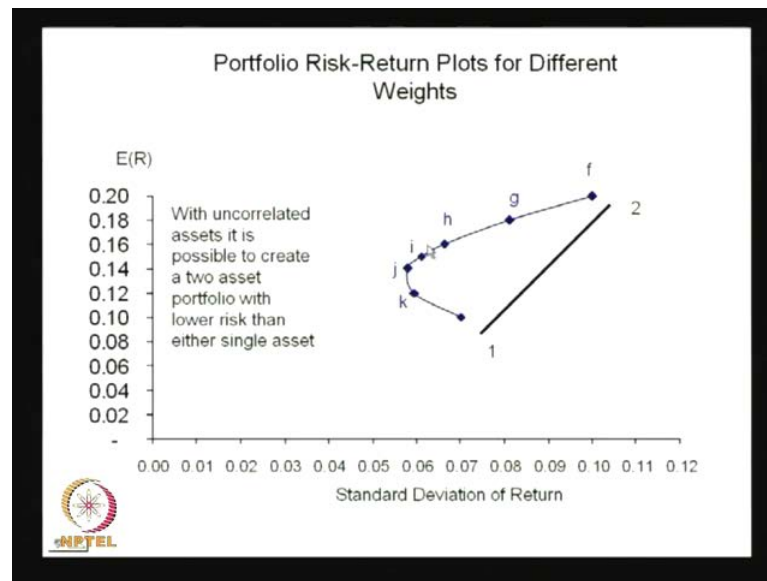
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Then if you plot this for the different weightage, then what we can see– this is your X-axis represents your standard deviation, Y-axis represents your expected return from the stock. What we have seen that if you see this, if you assume that the correlation coefficient between them it is highly positive, let plus 1, then with two perfectly correlated assets, it is only possible to create a two-asset portfolio with risk return along a line between either single asset. What does it mean? It means that if there is a perfect correlation between these two assets, then there is no point of having the two assets in your portfolio.

So, if you have only one asset in your portfolio. So, that, basically, will solve your purpose, because automatically, you are not going to minimize the risk in a market whenever you invest these two stocks in this particular portfolio. That is why if you find that the correlation coefficient between the two assets are highly positive, or we can say, it is close to 1 or it is exactly equal to 1. Then, there is no need to take into account these two assets in your portfolio. A one-asset portfolio will be enough to give you your desired return.

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Then, here the another situation. If you see, it is the with uncorrelated assets. It is possible to create a two-asset portfolio with lower risk than either a single asset, because if you go on, if you have the different assets in your portfolio, what we have seen, **we will find a...** you go on increasing the standard deviation of the return. **This is our...** let you have taken the only single asset. So, this is your risk and this is your return, but whenever you take two assets, one portfolio, let these are uncorrelated. Then, what we have seen, that you are getting more with the same amount of the risk; your return has increased.

So, that is why, what generally we have, we can say that if we have the assets which are uncorrelated, or the correlation coefficient between those assets are not that much high, then we can minimize your return by investing in those assets. So, that is the thing, generally, what we are trying to say.

So, like that, **if you...** there are different combination, also you can test with that how this changing the different assets or changing the different number of assets, and changing this different weightage to the different assets, **with varying...** with the varying this correlation coefficient will increase this value of the portfolio in the various period of time.

So, here, actually, what we can conclude that we can say that whenever we make the portfolio, it is quite important to know that how much expected return we are going to get from this portfolio, and how much assets we should have in this portfolio, and how this particular different assets will be decided, and which asset should be included, which asset should not be included, and how this different weightage should be given to different assets to make the portfolio. These are quite important aspects we should always look into whenever we decide the portfolio on our own.

So, to decide this, there are various theories, which have explain this in the different framework and different models. So, those models we will be discussing in the future sessions, and as well as we should know that how this weightage is decided, and what are those different parameters we should look into whenever we assign the weightage, and which are the different philosophies, or we can say, investment philosophies, we should follow whenever we construct the portfolio into the market.

And the basic thing is, if we talk about the return maximization for the investor, it is nothing but utility maximization. Then, how this utility of the investor is maximized with a given amount of the risk– that, actually, we should see in the next class, starting with the theory of the optimal portfolio. Thank you.