

Security Analysis and Portfolio Management

Prof. J. Mahakud

Department of Humanities and Social Sciences

Indian Institute of Technology, Kharagpur

Module No. # 01

Lecture No. # 25

Arbitrage Pricing Theory

In the previous class, we discussed about the capital asset pricing model or the first step of the market capital theory. And what are those things we found there or if you talk in a holistic **view from** point of view, then what we can say that beta is the sole factor, which affect the expected return of a particular stock.

Or whenever we deal with in a particular market and basically, we are always anxious to find out an optimum portfolio or basically in general, we can call an efficient portfolio. Here always we decide the expected return of that particular portfolio or the expected return of that particular stock on the basis of the beta. And already you know that the beta is nothing but it is the market risk or what kind of risk we face in the market with respect to any kind of market portfolio.

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$$E(R) = R_f + \beta (R_m - R_f)$$

$\beta \Rightarrow$ Sole factor, which determines the expected return of the stock

$\beta =$ Regression between the individual stock and Market Portfolio

$E(R_i) = f(R_m)$ Reg. coefficient $\rightarrow \beta$



For example, whenever in the previous class, if you go back, then what we have seen that, basically your expected return of a stock is equal to the R_f plus beta into R_m minus R_f . So, here the whole theory was referring beta is the sole factor, beta is the sole factor which determines the expected return of the stock. which determines the expected return of the stock. But gradually what we can see that or when we can say how this beta is calculated, the easiest way or the popular way of calculation of the beta is, whenever we regress between the individual stock individual stock and market portfolio.

So, what basically it means? It basically means that the expected return of an individual stock is a function of the market return. So, if you regress between these two, whatever coefficient the regression coefficient that basically is defined as beta and this beta is that significant variable which always determines the expected return.

In this context, what here we are trying to show or what we are trying to determine? the capital market theory The basic sole or basic theme of the capital market theory is to find out those factor or to find out those factors which can affect the expected return of a particular stock.

So, therefore, in the previous class what we have concluded? The limitations of capital asset pricing model is that to find a particular market portfolio, which is a better proxy or which portfolio basically we can define as the best portfolio or the optimal portfolio, where we can maximize our return with a given amount of risk.

Other limitations whatever we have, some people have argued that beta may not be the sole or unique factor which can affect the stock return in a particular time; maybe there are some other factors, we should take into account to examine or to determine the expected return of the particular stock.

So, to argue these things and after arguing and after exploring certain possibilities, the other people have tried to find out the factors which can affect the stock return at a particular time. In this context, the theory called the arbitrage pricing theory has been developed.

So, here before going to discuss about what arbitrage pricing theory talks about the expected return of the stock, let me discuss what you mean by the arbitrage. If you ask

what is the meaning of arbitrage in a financial market or more specifically the stock market. In general, if you see that if somebody is getting some return without any risk, then we can say there is some arbitrage opportunity is prevailing in the market in that particular time.

How it is possible? It is possible whenever the price of a particular asset is not uniform across the various markets available in that particular period. You take the example, if you have taken the position both in Bombay stock exchange and as well as the **Nastak** stock exchange and that particular stock is traded there as well as in **Nastak**. So, let there is a different price differential between these two exchanges about this particular stock, let the price of the stock in Bombay is fifty rupees and the price of the stock in **Nastak** is seventy rupees.

So, in that time, what the investor can do? Immediately they can buy the stock in Bombay stock exchange and they will sell it in **Nastak** stock exchange. So, at a particular time, they will pay fifty rupees and they will get seventy rupees. So, in this context what you can see?

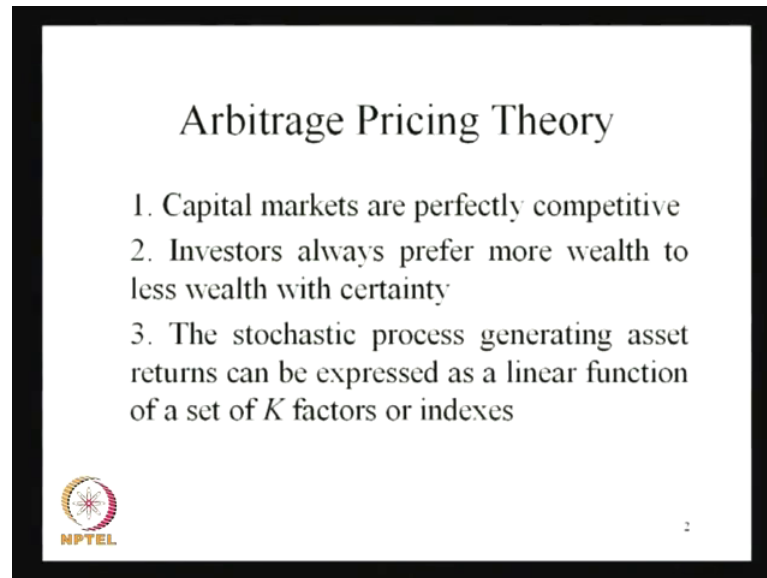
That if there is no uniform price without any kind of extra risk what the investor is going to take, he will get some extra return from the investment. If that kind of situation prevailed in the market, then **what we can say** in conclusion what we can say? There is some arbitrage opportunity is prevailing in that particular period in this particular market.

So, here **what** in this context we can say or in generally in totality what we can say, that basically if there is no law of one price, if the law of one price does not hold good, then we can say that there is a chance of there is a probability of arbitrage opportunity in the market and what do you mean by the law of one price? Already I told you that the law of one price means the price of a particular asset in a particular time in various markets should be same at this investment process.

If this is not happening, then there is a big chance of arbitrage opportunity in the market and **how** the investors can earn some extra amount of the return without taking any kind of extra risk. So, that is what **should** arbitrage is talking about.

And taking into account this particular concept into the mind, the researchers have tried to explain some certain theories or some certain factors which can also affect the stock return or the expected stock return in a particular time period.

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So, therefore, the concept of arbitrage pricing theory has been developed. So, if you go through this arbitrage pricing theory, there are certain assumptions against this arbitrage pricing theory. The first assumption is Capital markets are perfectly competitive; Investors always prefer more wealth to less wealth with certainty; the stochastic process generating asset returns can be expressed as a linear function of a set of K factors or the indexes. You just minutely observe those assumptions what this arbitrage pricing theory is taking.

You go by this first assumption what it talks about? It talks about the capital markets are perfectly competitive. What do mean by this perfectly competitive? We already know that **perfect** perfectly competitive means there are enormous amount of buyers or enormous amount of the sellers will be available.

If there is a stiff competition between the different buyers and sellers, then what will happen, that there is a perfect price discovery can prevail in the market and always there is a market clearing mechanism will work there.

That means the demand will be always equal to supply in that particular time, because equilibrium means always we should feel that the aggregate demand in that market should be equal to this aggregate supply.

But here the question arises, whenever we talk about this competitiveness in this particular market and here we always refer to number of buyers and sellers and as well as also we have seen that, we should also take into account some sort of transaction cost what we bare in this particular market to go for buying and selling in a particular time.

So, therefore, what we can conclude, that in a capital market or in a any of the markets whenever we talk, if there is a general equilibrium or whenever there is equilibrium position is prevailing in this particular market, so here in-doubted or undoubtedly we can say that there is no discrepancy between the demand and the supply situation and there is a perfect price discovery which is prevailing in this particular situation.

Number two is general assumption what everybody takes into account, whenever we derive any kind of theory. The second assumption talks about that, investors always prefer more wealth to the less, that means, all the investors are the rational investors and they all want to maximize their return and they always prefer more to less, which is the general sense of the common man and as well as the common investor; they always want to maximize their return that is always there in the back up of their mind that, if I am getting something more in the market with some level of the risk and I should switch over my investment to that particular asset instead of investing the asset where I am investing now.

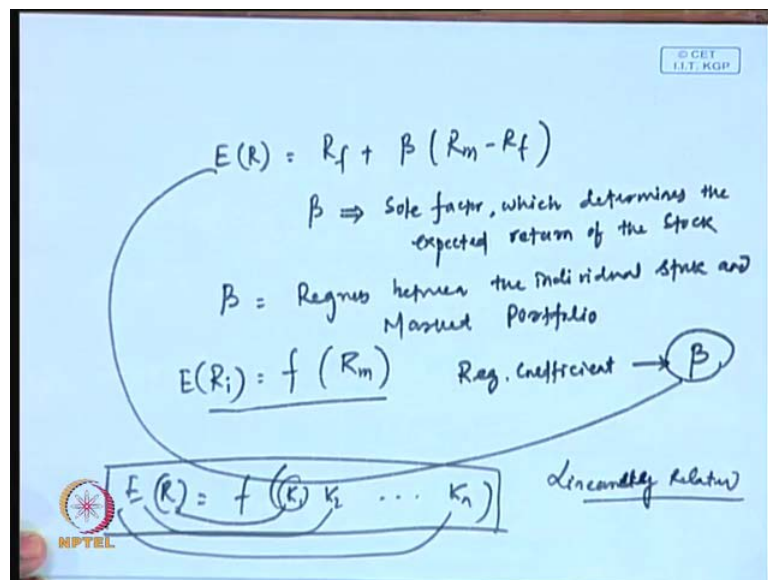
So, therefore, this is the basic assumption always we should take, that the all the investors or rational investors therefore, they always prefer more to less. Then the third point **is** talks about the stochastic process generating asset returns can be expressed as a linear function of set of k factors, what does it mean?

Already in the introduction what I told that, some of the researchers have argued that it is not the beta which affects the stock return only or beta is not the sole for unique factor which can determine the expected return from a stock, it is basically some other factors which also plays a significant role.

So, these other factors basically in the assumption we always refer it to the K factor. So, there are **K amount of the factors K factor** K number of factors which basically talks about or which basically shows the particular factors which affect the stock returns.

And another basic assumption, if you remember, it is a linear function; that means, the particular variables which are affecting the stock return and the particular process through which this function or this particular variables is going to affect the stock return, these are linear in nature; that means, there is no such kind of non-linear relationship should be existing between these two.

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That means what you can say if you take this expected return of the stock, it is basically is a function of let K_1, K_2 like that K_n and whole function is basically a linear function. The relationship between K_1 to $E R$, K_2 to the $E R$ and K_n to the $E R$ all are linearly related. So, there is no such kind of non-linear dynamics or non-linear process will be existing between the different assets **with the expected** or different factors with the expected return of the stock or any other asset.


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Arbitrage Pricing Theory (APT)

$$R_i = E_i + b_{i1}\delta_1 + b_{i2}\delta_2 + \dots + b_{ik}\delta_k + \varepsilon_i$$

For $i = 1$ to N where:

R_i = return on asset i during a specified time period
 E_i = expected return for asset i
 b_{ik} = reaction in asset i 's returns to movements in a common factor
 δ_k = a common factor with a zero mean that influences the returns on all assets
 ε_i = a unique effect on asset i 's return that, by assumption, is completely diversifiable in large portfolios and has a mean of zero
 N = number of assets



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So, these are the three major assumptions what this arbitrage pricing theory takes. So, now, already what I have derived here or what I have shown to you here in this case, if you see already it is known to you that return from a stock is basically **your** during a specified time, (Refer Slide Time: 14:10) **it is your** it is your **mention** E_i plus b_{i1} this δ_1 , then b_{i2} δ_2 like **like** that plus b_{ik} into δ_k .

This is b_{i1} , b_{i2} like kind of thing and finally, one error term should be included here. So, your R_i represents $i = 1$ to n , and R_i is equal to return on asset i during a specified time period, then the expected return E_i represents your expected return for asset i , then b_{ik} is basically the reaction in asset i 's return to movements in a common factor.

δ_k is basically a common factor with a zero mean that influences the return on an asset, then your ε_i term it is basically that unique effect on assets i 's by assumption is completely diversifiable in a large portfolio and has a mean of zero and n is equal to your number of assets what we are going to incorporate into your model.

So, therefore, this δ_1 to δ_k , these are the different factors which are going to affect your return of the **return of the** stock or return of **the** any other asset in a particular time period.

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$R_t = E_t + b_{i,1}d_c + b_{i,2}d_i + \dots + b_{i,k}d_k$
 $E(\epsilon_i) = 0$
OLS
 Ordinary Least Square Regression
 We can't incorporate
 Statistical function (Stochastic functions)
 $Y = \beta_0 + \beta_1 X_1 + u$
 $u \rightarrow$ Error term
 Impact of other variables which can affect Y.
 Mathematical function
 $Y = \beta_0 + \beta_1 X_1$ (Deterministic function)
 $E(u) = 0$
 $Var(u) = \text{Constant}$

Some of the things if you observe here, **this is your** you always refer that the expected epsilon i is equal to 0 and also we assume, that means, what you can say the expected mean of this particular term will be 0, then as well as also we assume that whenever we refer to any kind of estimation process, for example, if you are running this OLS or we can say this ordinary least square regression and here whenever we take all those assumption, in this assumption first of all we assume that the expected mean of the error term basically will be 0; that means, this part basically talks about the particular factors what we are not incorporating **we are not incorporating in your** in our model.

If we **will** incorporate all the variables, may be there are thousand variables which are going to affect your stock return, but in practical sense it is not possible. If you recall, I already told you these things in the some of the previous sessions that, always whenever we use any kind of model in econometric or statistical way in to the finance literature, always we use this statistical function and this statistical function basically is different from the mathematical function.

If you say that **how it is difference** how the difference can be distinguished between these two or what is the difference, then **your** whenever you talk about mathematical functions, we can write that Y is equal to let **beta 1 plus beta 2** beta 0 plus beta 1 X 1. Now, this is your mathematical function and if you **draw** this line, this is basically **this is basically**

your beta 0 and this part is the slope, which basically your beta 1, and X_1 is equal to the variables which is affecting your Y.

But whenever we... That is why this function is basically a deterministic function **this function is a deterministic function**, but whenever we go to the statistical function, statistical functions are basically the stochastic functions. **statistical functions are the stochastic functions**. And what do you mean by the stochastic functions? So, here why we call this as deterministic function, because X is the only variable, which is affecting Y, which is possible in terms of some other aspects, but it is not possible for say particularly finding out the behavioral signs objects we deal with or the particular model which incorporates the behavioral part.

So, this is not possible in this case, but this is possible whenever we talk about any other thing. So, what do you mean by stochastic functions? In the stochastic functions, it is not possible to identify all the deterministic variable which could have the impact on the dependent variable.

So, here, how **this function** if the same function if you take, then here your stochastic function will be $\beta_0 + \beta_1 X_1 + u$ and that u basically is your error term and this term generally reflects the impact of impact of other variables impact of other variables which can affect Y, but it is not possible to incorporate into the model.


Therefore to overcome this problem, we have included one error term in that and the error term will have certain properties and the mean value of error or expected u should be equal to 0 and the variance of u should be constant. So, these are the two major assumptions always we take whenever we deal with this kind of analysis.

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Arbitrage Pricing Theory (APT)
Multiple factors expected to have an impact on all assets:

- Inflation
- Growth in GNP
- Major political upheavals
- Changes in interest rates
- And many more....

Contrast with CAPM insistence that only beta is relevant

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Then go to this second step, which are those variables or the K factors what we are going to incorporate in our model? Here if you observe these factors, in general have been identified as inflation, growth in GNP or the GDP, major political upheavals, change in interest rates, inflation and many more.

Here, if you see, that the various factors which could have the impact on expected stock return directly or indirectly, so those things will be taken into account. But one basic observation if you find in arbitrage pricing theory, that arbitrage pricing theory never talks about the deterministic number of factors which are going to affect the expected return of this particular stock. Either there are ten factors or twenty factors or fifteen factors, those things has not being identified **those things has not been identified** by the arbitrage pricing theorist people, that exactly how many variables which can have the impact.

In this context, if you remember or if you analyze your concepts, then what you can find that any of the variables can affect the stock return; it can be company specific variable, it can be a industry specific variable, it can also a macro economic variables.

But most of the literatures were not talking about this, but here, if you observe, that the variables what we have identified either it is inflation or it is growth in GNP or the GPD

or it is major political upheavals or change in interest rates, these are more or less variables which are related to the whole macro economic variables.

That means all the companies are compulsorily are going to be affected by this, may be the degree of impact will be different from company to company or from industry to industry, but those factors are the responsible factors which determines the expected return of a stock in a particular time. Therefore, what we can say here, that we have to know how those variables could have the impact on the stock return.

You can start with the concept of late inflation; how inflation can affect the expected return of a stock. You see **whenever** the inflation could have the impact on the stock return in various ways, But if you observe in the real market or practical market, what you can find? **inflation will always** If inflation rate is quite high, then you will find the expected return of a stock also should be quite high.

Why it should be quite high? Because the investor who wanted to invest in the market in that particular time, they definitely will have the expectations to get more premium, because already the inflation rate in the market in that particular time is quite high that is why the real cost what they are going to face that is quite high.

So, therefore, unless they do not get the high return from the market in that particular time, it is very difficult for them to invest in the market. Therefore, **their expectation will be the inflation rate would be** if inflation rate will be quite high, then the expected return of the stock also should be quite high.

And another logic is, why this inflation rate will be higher? The inflation rate will be higher, whenever the money supply will be higher; and if the money supply will be higher, then already we know when the money supply will be higher, it is basically whenever the interest rate would be lower.

If the interest rate would be lower, the money supply will be higher, and then already we know that if interest rate will be lower, then it is good for the investor to invest in the market. So, the demand for investment in the market will be higher, if the interest rate will be lower. So, in that context also we can find if the demand will be more in that

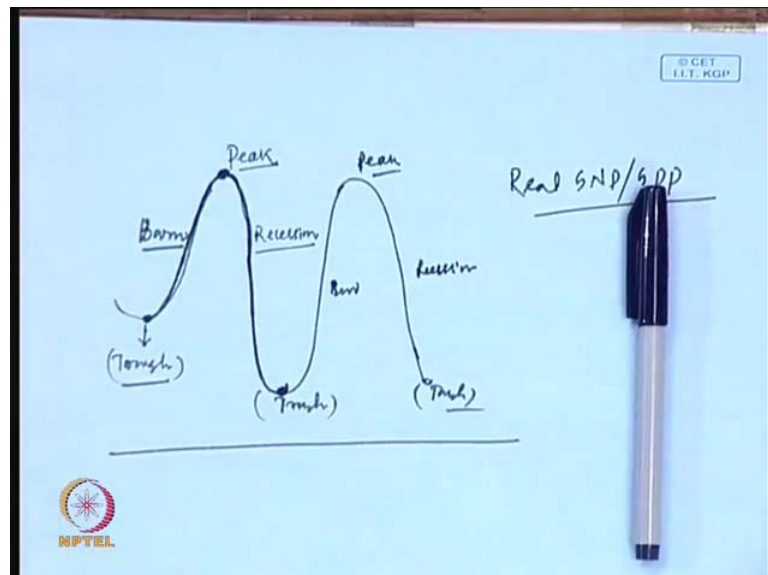
particular market in that particular period, then the expected return from the stock also will be higher.

Therefore, what we can say? Always we should assume that inflation will have very unique impact on or always there would be certain impact on the stock return in a real market situation and this the linkage through which the inflation can have the impact on the stock returns.

Then another factor is basically the growth rate of the GNP or GPD. Already you know that it is the growth rate GNP means gross national product or it is GDP means gross domestic product, whatever it may be, these are the different proxies for the strength of the economy or **the** we can say the **growth of this** growth variable, which talks about the condition of the economy or generally always we use this variable also **the** one of the indicator's of a cycle or the business cycle.

I hope you must have the ideas, which are the different phases we go through in a particular cycle. If **you if** you see that whenever we talk about the different phases, the phases are like this.

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So, where it starts, for example, **this is your** this is one phase; this starts with a inclining train and it will reach here, then it starts declining, then finally, again it will reach here, then again it start inclining. So, here this particular point we call it the turf, then this

particular period, whenever it start inclining, we call it the boom. This is the boom or we can say there is economic boom, this is very household word in everybody's mind **that how this when you can call it this is the boom**; then finally, it will reach the peak. So, this is your peak, then once it will reach the peak, it will start declining.

So, this phase, we call it recession, then once it recession, it goes down and again it will reach in the turf, then again it will **be** boom, then finally, it will reach in the peak, then again it will go down to there is a recessionary phase, then finally, it will reach the turf.

So, these are the different aspects, these are the different cycle and how those cycles have been identified? There are various parameters or there are various indicators through which we can identify the different periods and one of the indicators is the real GNP or the GDP.

So, here what we can see that, obviously, always we should expect the return in the boom or we can say in the boom period in a business cyclic process, the return from the stock or return from the asset should be higher and in the recessionary trend, always we can expect that the return will be lower.

But if you see one particular thing, that whenever we talk about the different types of the return or different types of the impact of the business cycle on the return, we can see that where it will be boom, whenever the growth rate of the GDP will be higher and when will be recession, the growth rate of the GDP will be very lower or in the recessionary the biggest definition or **the** we can say that continuously in the two quarters or three quarters the growth rate should be negative.

Here what generally we can say that. If you identify the different business cycle through any of the indicators either it is growth rate of the GDP or it is **(())** spread or it is default spread, these are the different indicators what we used to measure the business cycle.

What do mean by **(())** spread? **(())** spread is basically the difference between the long **long** term bond to short bond; the return from the long term bond to return from short bond and default spread is basically difference between the return between the high rated bond and low rated bond.

So, in this context what you can see, this GPD is a popular variable which is a better reflected, which basically better reflects this impact or the business cyclic process. So, once we can identify this business cycle or the particular phase of the business cycle, then it will be easy for the investor to decide whether the expected return should be higher or the expected return should be lower.

Then another factor is basically the major political upheavals. If there is no political stability, then the country risk also will be higher; if **if** you find the trend, there is a political instability, then what will happen, that **it** this particular market will not attract the foreign investors.

If it will not attract the foreign investors and gradually it will also lose the confidence of the domestic investor; if both the confidence of the foreign investors and the domestic investors will not be there in the market, then obviously, what will happen, the expected return will go down, because there will be no demand in the market for that particular assets.

So, to overcome that problem always we should ensure that there is a political stability is prevailing in the market in a particular time; if there is instability in the market, then what will happen, that it will have the adverse impact both in the short run and the long run on the different pricing of the different assets in the financial market.


So, obviously, the stock market will be highly affected by that, because the stock investment or the aggregate demand in stock market also will go down because of the lack of confidence or the lack of opportunities in this particular scenario.

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Arbitrage Pricing Theory (APT)
Multiple factors expected to have an impact on all assets:

- Inflation
- Growth in GNP
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- And many more....

Contrast with CAPM insistence that only beta is relevant

 4

Then another variable basically what we have that is your change in interest rates. So, if you observe that the change in interest rates already in the literature what we can say? In the literature what we always see that the change in interest rate is the most important variable for the investment process.

That means, what here we trying to say that if there is a change in interest, then automatically it will affect the whole investment process; if the investment process will be affected or the total amount of the investment will be affected, then what will happen? It will affect adversely the growth rate and as well as the expected return of the stock.

Because once the interest rate will be more, already what I told you it will affect adversely the investment and if the investment will be affected by that, then what will happen that ultimately it will affect the aggregate demand of the economy and once **once** the aggregate demand of the economy will be affected, then the investment will be affected, then finally it will affect the growth rate.

And if it will affect the growth rate, already we know definitely all the market should be affected by that. And already in the previous sessions also to some extent, whenever the economic analysis we discussed we also discussed about little bit the different factors which can have the contributing factor for deciding where to invest where not to invest.

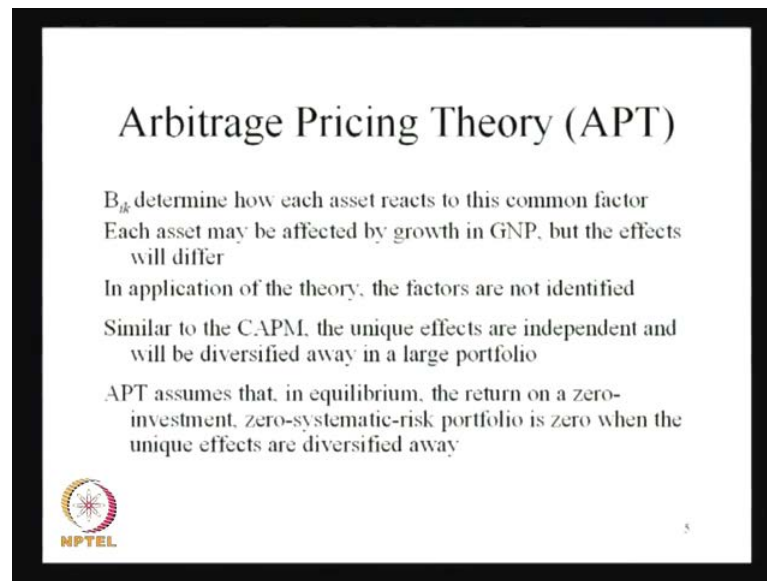
But here if you see that investment **is basically the key role** has the key role for the development of the economy and as well as the investment is totally driven by the interest rate changes. So, if the interest rate will have the adverse impact or the high interest rate will have the adverse impact on the investment, then the whole investment process and as well as the whole growth rate of the economy also will be affected, then the return also will be affected.

Then many more which is the problem with arbitrage pricing theory it is because it does not talk about all the other variables which could have the impact on stock return, it totally neglected the impact of the industry specific variable; it does not talk about the some other variables which could have like some **(())** specific variable or the compress specific variable, but they could have the impact in the further session in the multi factor pricing model; we will be discussing extensively about this.

So, this is one of the limitations what the arbitrage pricing theory has. Arbitrage pricing theory says that only there are k factors, but which are those k factors is not identified by them.


But still it is an advanced theory than the capital asset pricing model, because capital asset pricing model was talking about only the **beta is the sole factor** beta is the only factor which can affect the stock return. But which is not the true phenomena **which is not the actual phenomena** always whenever we talk about in market situation to decide the expected return of a particular stock.

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Arbitrage Pricing Theory (APT)

- B_{jk} determine how each asset reacts to this common factor
- Each asset may be affected by growth in GNP, but the effects will differ
- In application of the theory, the factors are not identified
- Similar to the CAPM, the unique effects are independent and will be diversified away in a large portfolio
- APT assumes that, in equilibrium, the return on a zero-investment, zero-systematic-risk portfolio is zero when the unique effects are diversified away

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So, already we discussed about these things, but that is why what we can say this is how each assets react to the common factor, which is represented by the coefficients of the different factors **whenever we (()) the regression the expected return of the stock.**

Each asset may be affected by growth in GNP, but the effect will differ already I told you, because the impact of the business cycle or impact of the growth rate of the economy on all the assets are not same, but in application of the theory, the factors are not identified properly.

Then similar to the capital asset pricing model, the unique effects are independent and will be diversified away in a large portfolio which is sometimes we should also use in the arbitrage pricing theory case.

Arbitrage pricing theory assumes that in equilibrium the return on zero investment or zero systematic risk portfolios is zero when the unique effects are diversified away, because there is no risk, there would be no return. So, it is not possible to diversify the systematic risk. So, if you assume that there is no systematic risk in a particular portfolio or a particular asset, then definitely we can assume that there is one zero return in this particular portfolio.

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Arbitrage Pricing Theory (APT)

The expected return on any asset i (E_i) can be expressed as:


$$E_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \dots + \lambda_k b_{ik}$$

where:

λ_0 = the expected return on an asset with zero systematic risk where $\lambda_0 = E_0$

λ_1 = the risk premium related to each of the common factors - for example the risk premium related to interest rate risk $\lambda_1 = E_i - E_0$

b_i = the pricing relationship between the risk premium and asset i - that is how responsive asset i is to this common factor K

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So, that is why the expected return of a stock that is basically of an asset basically expressed as this, then this lambda 0 is basically the expected return of an asset with 0 systematic risk, where already we know that, it may be zero, then the lambda 1 is equal to the risk premium related to the each of the common factors.

For example, the risk premium related to interest rate risk or the risk premium related to the inflation risk or the risk premium related to the growth rate, then the b_i represents the pricing relationship between the risk premium and the asset and that how responsive asset i to the common factor K .

So, this is basically the model what we formulate to establish the arbitrage pricing theory in a regular phenomena or what we can say that, we should take care of this arbitrage pricing theory case or very minutely we should identify the actual variables which could have the impact on the expected stock return.


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Example of Two Stocks and a Two-Factor Model

λ_1 = changes in the rate of inflation. The risk premium related to this factor is 1 percent for every 1 percent change in the rate ($\lambda_1 = .01$)

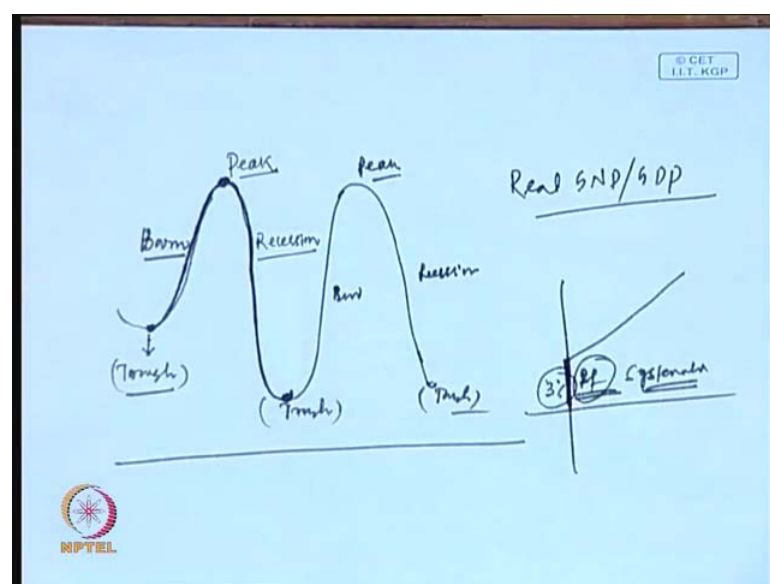
λ_2 = percent growth in real GNP. The average risk premium related to this factor is 2 percent for every 1 percent change in the rate ($\lambda_2 = .02$)

= the rate of return on a zero-systematic-risk asset (zero beta: $b_{0j} = 0$) is 3 percent



You take the example of a true factor model, let the lambda 1 is change in rate of inflation. The risk premium related to this factor is 1 percent for every 1 percent change in the rate that means lambda 1 is equal to point .01. If a percent is growth in real GNP, let lambda 2, the average risk premium related to this factor is 2 percent that means, for every 1 percent change in the rate, there is 2 percent increase in risk premium that is why lambda 2 is equal to 0.02.

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The rate of return on zero systematic risk asset, that means, 0 beta is 3 percent, that means, that is **your** without any kind of factor if the return is changing, that means, **that is your**, this part **this part** which is basically your R f or systematic, there is no risk involved in this.

So, that is why we call it, let this is 3 percent. So, then what you can find that, another asset is (Refer Slide Time: 38:07) let two stocks in a two factor, let the response of asset X to change in the rate of inflation is .5; the response of asset Y to changes in rate of inflation is 2; the changes in growth rate is 1.5; the response of asset y to the changes in the growth rate of the GNP is 1.75, then it is 1.75. It is for one stock, it is for second stock; for stock one, it is two stocks and two factor models. Therefore the beta x 1 is .5, beta y 1 is equal to .5; beta x 2 is equal to 1.5, but the y 2 is equal 1.77, then how this model will look like?


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Example of Two Stocks
and a Two-Factor Model

$$E_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$$

$$= .03 + (.01)b_{i1} + (.02)b_{i2}$$

$$E_x = .03 + (.01)(0.50) + (.02)(1.50)$$

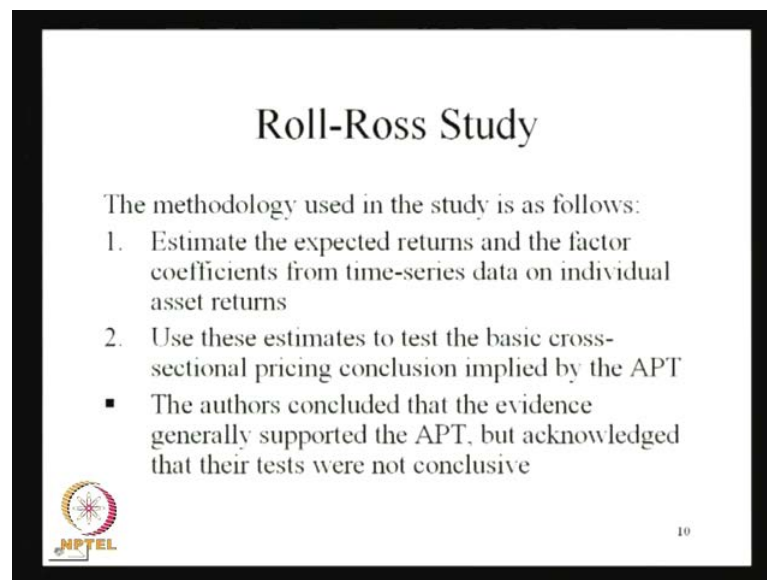
$$= .065 = 6.5\%$$


The expected return already we know that the E i is equal to lambda 0 plus lambda 1 b i 1 plus lambda 2 b i 2, that means, it will be 0.03; 0.03 is this one which is 3 percent, which **is** basically talks about the zero risk level or we can say without any kind of factor there is a change in returns that is why we call it risk rate of return, then we have 0.01 b i 1 plus 0.02 into b i 2, because this is the responsiveness **to** with respect to lambda 1 and lambda 2. Then this will be your E x is equal to b i 1 is equal to **0.05** 0.5, then b i 2 is equal to 1.5, then expected return from the stock X is equal to 0.065 for the 0.5 percent.

So, like that if you talk about $E y$, the expected return from the stock y that will be again 0.03 which is same for both the assets or both the stocks and 0.01 which is your λ and b_{i1} is equal to 2, and b_{i2} is equal to here also 0.75, then the expected return from the stock is 8.5 percent.

So, in this way, basically the responsiveness of change in particular stocks due to the change in this b_{i1} and b_{i2} , these are basically the factors which basically affect the expected return of the stock and the coefficients, which basically regression coefficient, which indicates the responsiveness of this particular factors towards the expected return of the particular stock.


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Roll-Ross Study

The methodology used in the study is as follows:

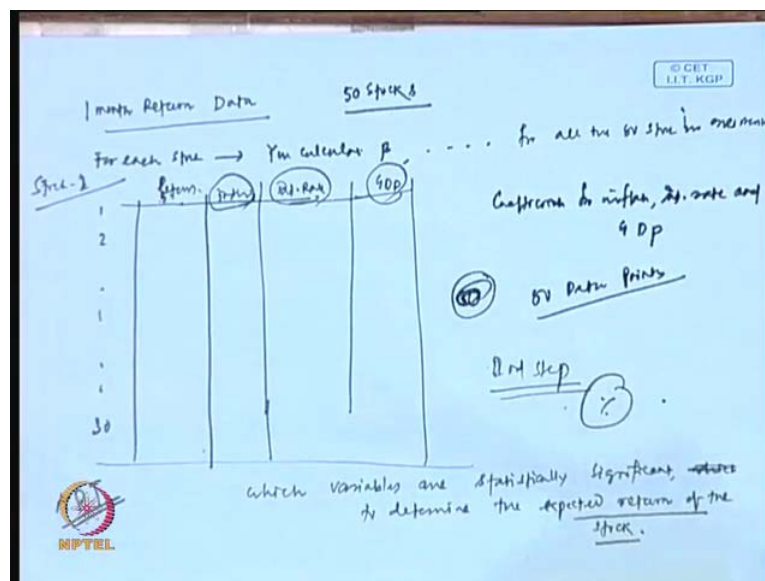
1. Estimate the expected returns and the factor coefficients from time-series data on individual asset returns
2. Use these estimates to test the basic cross-sectional pricing conclusion implied by the APT
 - The authors concluded that the evidence generally supported the APT, but acknowledged that their tests were not conclusive

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There are various studies which have used this arbitrage pricing theory and what is the result they found? First study is very famous study given by Roll and Rose, what is the methodology and what is the results they found that, first they said that they estimated the expected returns and the factor coefficients from the time series data on individual asset returns and use this estimates to test the basic cross sectional pricing conclusion implied by the APT or arbitrage pricing theory.

And the authors concluded that the evidence generally supported the arbitrage pricing theory, but acknowledge that the test were not conclusive. What basically here the Roll and Rose is trying to say?

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You see that we have certain, let we have one month return data (()) **One month return data (())** and we have let **five stocks** 50 stocks. So, what basically the Roll and Rose were trying to say? They said that for each stock, **for each stock** you calculate beta and other variables for all the 50 stocks in one month.

What does it mean? You take 1 2, let you have 30 returns we have, then we regress it **with a...** there are different variables; we have inflation; we have this return inflation interest rate, then we have GDP, etcetera.

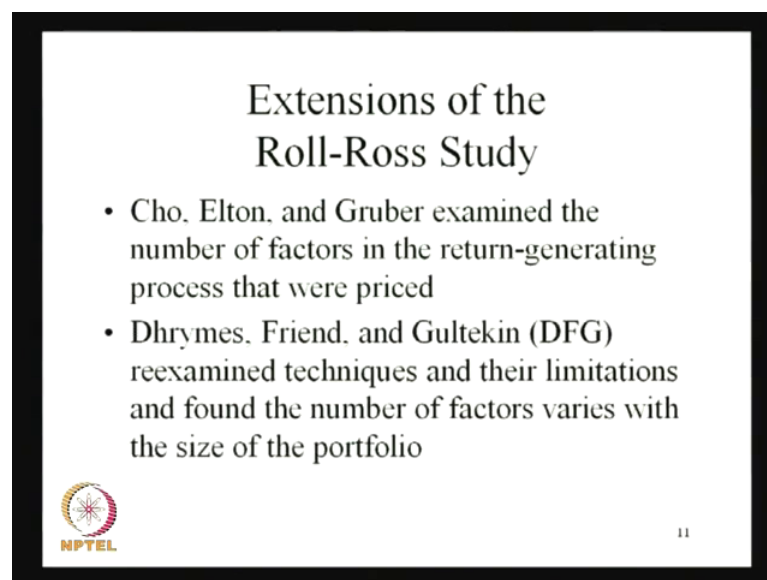
Then **if** for each stock, for a stock 1, you take all those thirty days data and regress it and after doing the regression, you will find the coefficients **coefficients** for inflation, interest rate and GDP and after that what you can do, by that you will have the **thirty** fifty **fifty** data points.

Fifty data points for each stock and for each coefficient and whatever way we have done the calculation previously, then this fifty data point for each stock what we have calculated about the inflation, about the interest rate, about the GDP, then use it in the second step and see how the expected returns are calculated.

And what is the conclusion they found? They found that the expected return of the calculated and as well as out of them, they can say which variable which variables are statistically significant **which statistically significant** to determine the expected return of the stock **statistically significant to determine the expected return of the stock** that we can get it from here.


But Roll and Rose have concluded that to some extent, they found the support for the APT or the arbitrage pricing theory, but they said that their test were inconclusive or we can say it is not conclusive; it is because **that** may be some sort of other mechanism has been left out from there and in the further studies, those answers will be given by the multi factor pricing model.

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**Extensions of the
Roll-Ross Study**

- Cho, Elton, and Gruber examined the number of factors in the return-generating process that were priced
- Dhrymes, Friend, and Gultekin (DFG) reexamined techniques and their limitations and found the number of factors varies with the size of the portfolio

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Then to do this some other people have extended this analysis, out of them the famous analysis has been made by Cho, Elton and Gruber who examined the number of factors in the return-generating process that were priced. And another who is Dhrymes, friend and Gultekin reexamined the techniques and their limitations and found the number of factors varies with the size of the portfolio.

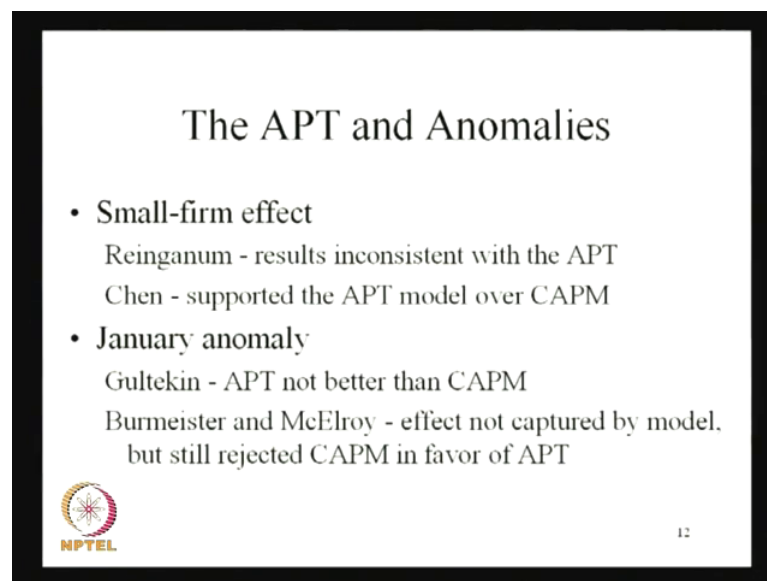
So, basically this DFG have reexamined the different techniques which are used to test this arbitrage pricing theory and they said that, if you increase the number of stocks in your analysis, if you increase the number of observations in your analysis, then the

results will be different and we will find that varieties of results on the basis of the data what we are going to use.

So, in this context what we can say? It is not only the financial variables or financial factors **which may** which may have the significant role, sometimes whenever we make this studies practically or academically, we can find that sometimes the result varies depending upon the data set, depending upon the number of years what we have taken into account and as well as we have also the number of data points.

That means how many assets have been included in our analysis and how many stocks we have taken into account, these have also the played the significant role to decide where to get this particular significant factor, which could have the important role for determining the stock return. That is why those parts also should be taken into account whenever we go for calculating or testing this arbitrage pricing theory.

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The slide is titled "The APT and Anomalies" and contains the following content:

- **Small-firm effect**
 - Reinganum - results inconsistent with the APT
 - Chen - supported the APT model over CAPM
- **January anomaly**
 - Gultekin - APT not better than CAPM
 - Burmeister and McElroy - effect not captured by model, but still rejected CAPM in favor of APT

The slide also features the NPTEL logo in the bottom left corner and the number 12 in the bottom right corner.

Then which are the problems here, some people said that, already I told you there are some factors which has not been taken into account in the arbitrage pricing theory; out of them is one specific factor, somebody said that the size of the company basically will have the significant role for determination of the expected return of a stock.

So, whenever they have incorporated the size or we can say this Reinganum has made this study, that is, he said that the small size affects the results inconsistent with the

arbitrage pricing theory. That means whatever assumptions and whatever implications the arbitrage pricing theory was trying to explain, these are not consistent with the small firms whenever we take into account.

Chen also supported the APT model over CAPM, because it is a better model; it is a more comprehensive model and what CAPM model was trying to answer? The APT model is overcoming that thing and more factors have been incorporated and more analysis have been made and that is why, **what** we can say **if** it is a better model according to chin.

And in USA, particularly this part is possible in the January anomaly, because you will find lot of discrepancy between the January and the December, because there financial year starts with January and end with December and to get this (()) tax revert most of the investors basically do not keep the stocks in December in that the price goes down and January again whenever they buy this stock in a particular heavy amount the prices goes up.

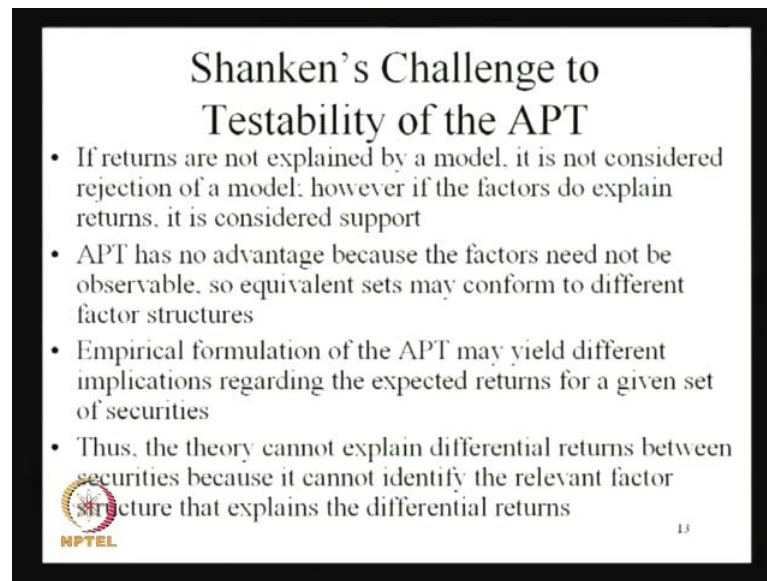
So, that is why there is unusual behavior; we always see or some kind of unusual price pattern always we observe in these two periods. In Indian context, you may say it is April anomaly, because we have the different anomaly, but it is not that much significant consistent in the Indian context to whatever way it is applicable in the US context.

When popular in the finance literature, we have referred it the January anomaly. So, whenever this January anomaly has been taken into account, they found that this Gultekin has found that APT is not better than CAPM.

They said that, if we consider the January anomaly into the consideration, may be beta is the sole factor which affects your stock return; it is not the other factor that is why they said that CAPM is a better model than the arbitrage pricing model.


Then the Burmeister and McElroy, they found that effect not captured by the model, but still rejected CAPM in favor of APT; that means, they were favoring APT over this CAPM model and they said that, it is not the anomaly which is reflecting that particular impact on this model.

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**Shanken's Challenge to
Testability of the APT**

- If returns are not explained by a model, it is not considered rejection of a model; however if the factors do explain returns, it is considered support
- APT has no advantage because the factors need not be observable, so equivalent sets may conform to different factor structures
- Empirical formulation of the APT may yield different implications regarding the expected returns for a given set of securities
- Thus, the theory cannot explain differential returns between securities because it cannot identify the relevant factor structure that explains the differential returns

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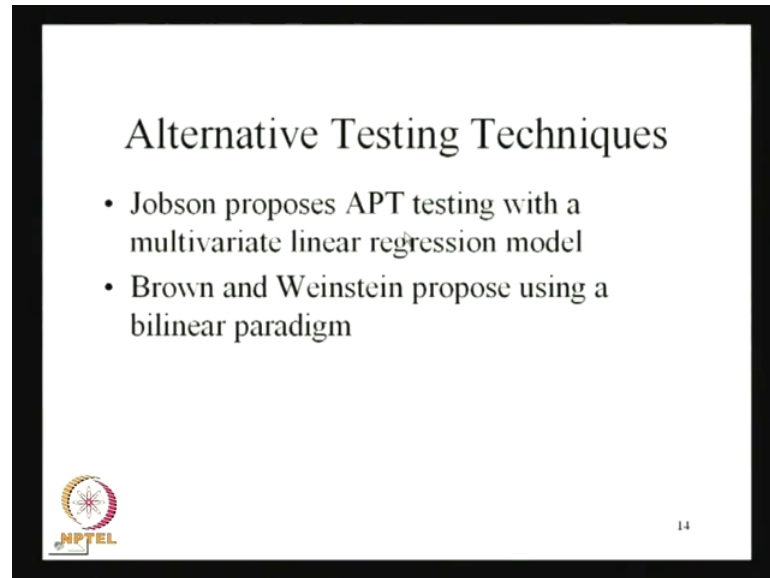
Then there are other people who have challenging this test ability of this APT theory. The Shanken's challenge to test ability of the APT if you refer, **what they say** what is said that, if returns are not explained by a model, it is not considered the rejection of a model; however, if the factors to explain returns it is considered the support.

Then APT has no advantage because the factors need not be observable. So, equivalent sets may confirm to different factor structures. Empirical formulation of the APT may yield different implications regarding the expected returns for a given set of securities.

That means empirical formulation modeling basically also is important consideration we should take into account whenever we are using any kind of theoretical models in the finance literature.

So, thus the theory cannot explain the differential returns between the securities, because it cannot identify the relevant factor structure that explains the differential returns. Why this differential returns will be there and why this factor gives eight point return and another factor gives ten point returns that identification is quite important, whenever we are using this kind of model. So, identify the factors which affect the expected return of a particular asset or particular stock. So, it is the more tasks or it is more logical to identify those things as compared to the other variables.

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So, that is why these other things we should consider whenever we talk about the alternative testing techniques whatever we have. We have the Jobson proposes another technique, which is called APT testing with a multivariate linear regression model.

Brown and Weinstein proposes using a bilinear paradigm, but more or less if you conclude or if you want to say something about the advancement of APT, this is basically the multifactor pricing model and the multifactor pricing model basically talks about the different factors which affect the key role, which affect the expected return of a stock in a particular framework.

So, if you consider those factors, if you take into account those factors and you can make a **comprehensive view**, comprehensive study to formulate that, taking into account both the firm **specific industry** specific factors as well as the macro economic factors and make a linear formulation or the non-linear formulation, may be it will have better reflection; it will have the better prediction of expected return than the normal APM or the CAPM mode.

So, we will be discussing this multifactor pricing model in the next session.

Thank you.