Security Analysis and Portfolio Management

Prof. J. Mahakud

Department of Humanities and Social Sciences

Indian Institute of Technology, Kharagpur

Lecture No. # 27

Markowitz Optimal Portfolio Selection Model

In the previous class, we discussed about the multifactor pricing models, and what was the basic objective of those models; it is to find out the expected return of the different assets, and as well as the portfolio, and the basic need for this calculation of expected return of the portfolio and the or the individual securities, or that, how to take the decision in the market for investment or in which kind of asset, we should invest, or in which kind of asset, we should not invest and as well as, how we can construct our portfolio in an effective manner.

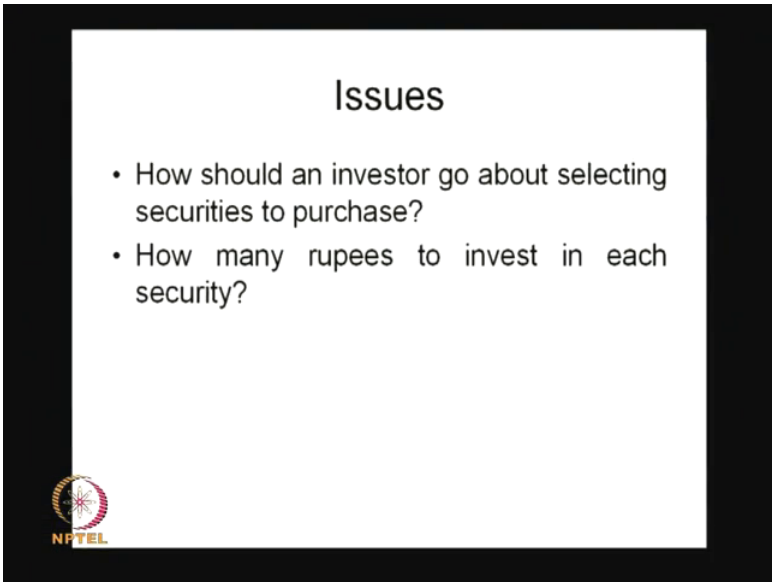
So, in some of the previous classes, we thoroughly discussed about the introduction to the portfolio. What exactly the portfolio is; how we can calculate the return of the portfolio or how we can calculate the risk of the portfolio, but after discussing about those aspects and as well as the some of the models or some of the theoretical developments related to the calculation of expected return and expected risk of the different assets or the different combinations of the assets, which generally defined as the portfolio.

Then the fundamental question always we always comes to our mind that how generally we construct the portfolios, and out of the different portfolios, portfolios, what we can construct which is the best portfolio, or in our language, what we can say, which is the optimal portfolio which can maximize the value of the firm. So, to answer this questions, there are various theories or various models have been developed, and over the two sessions, we will be discussing about those models which basically discusses about how to construct the optimal portfolio or how to allocate the funds in the different portfolios.

So, in this context, if you see that the basic theory or the foremost the funding theory basically talked about the how generally we construct the portfolio; what are those

different assets should include in the portfolio etcetera, which is given by the Markowitz, which is defined as the Markowitz optimal portfolio selection model. Then, after discussing about this Markowitz optimal portfolio selection model, then we will be discussing about the surface single index model. Then gradually, we will move forward to Lagrange multiplier theory to discuss about or to know about the portfolio allocation process.

(Refer Slide Time: 02:46)

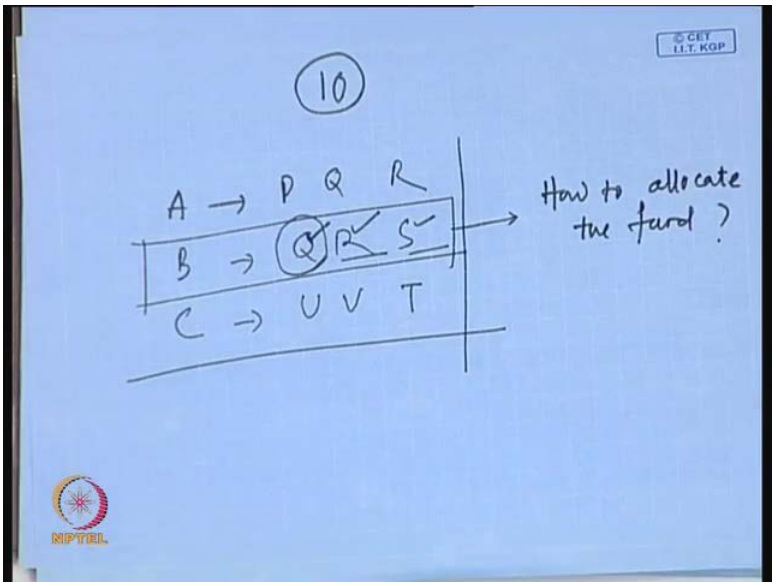


So, in this content, what are those basic issues that always comes to our mind, whenever we talk about the optimal portfolio selection models. The first issue comes to our mind is that, how should an investor go about selecting securities to purchase; that means, how the investor decides that which security he should purchase, which security he should not purchase, or in which security he should invest and in which security he should not invest. That means, what are those different ways or what are those different criteria he should follow to decide that what are those different assets he should include in his optimal portfolio?

And the second question comes that, how many rupees to invest in each security which is very important concept in the optimal portfolio selection process or the whole optimal portfolio selection process. What generally we hear we we are trying to do. For example, one investor has the hundred rupees and he has the different combination of the assets

and different combinations of the different securities, on which, he has made the different portfolios.

(Refer Slide Time: 04:11)

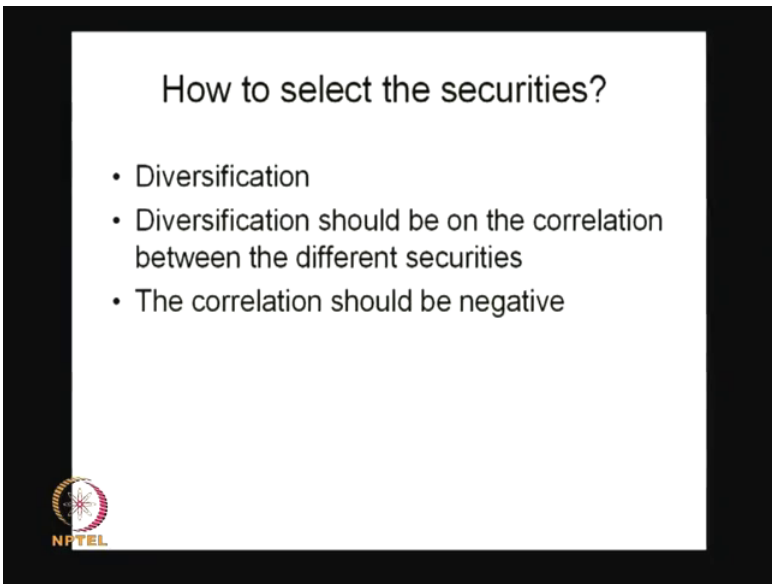


Let he has found that let B is the ABCDE; there are five portfolios he has constructed, and out of the five portfolios, he found that let whether he should invest a or he should invest b. And if for example, there are 10 assets in his, in his total kitty, and out of the 10 assets, that he found that A has a combination of portfolio; A has a combination of the P Q and R; then B also the combination of Q R S and C is a combination of the let U V and T. So, if these are the different combinations and as well as for example, he has decided that I will invest in this B combination which is Q R S; that means, Q is one asset and R is another asset and S is another asset.

But the basic question of the investor is how to allocate the fund; that means, for example, he has 100 rupees to invest, then how this 100 rupees can be divided between this two assets, and for example, whether he should invest all the 33, 33, 33 rupees in all the three assets and equally he will divide the money between the all the three assets or he should give 50 percent; he should invest or 50 rupees; he should invest on A Q and 25 on R and 25 on S. So, these are the things which are very important in the portfolio management process or the selection of optimal portfolio process, and the investor always wanted to reach or always wanted to know how he can allocate the fund in an effective manner to maximize his return.

So, that is another question always comes to our mind. So, the optimal portfolio selection model, so, we are trying to answer those questions in an effective manner, but first question that we raise that how should an investor go about selecting to versus those different assets; what he is trying to get or what he is trying to invest for to, to, maximize the return in the market.

(Refer Slide Time: 05:54)



Here, what already we have seen according to the Markowitz theory, the basic philosophy of deciding the securities is the diversification; that means, instead of using one particular security, you should have the combination of the securities, and if you have the combination of the securities, then you can diversify the total unsystematic risk what you are going to face, because already what we have discussed in the previous classes, that it is not possible to diversify the systematic risk, because it is applicable to for all the companies, because systematic risk is linked to the market risk basically.

But whenever you talk about the unsystematic risk, it is possible to diversify that risk or it is possible to reduce that risk, because these are unique in nature and those risk basically varies from different company to company. So, if you one particular company is facing one risk, the other company may not be facing that one, and as well as if you have the position in the different two companies at a time, then if one company is facing some risk, that risk can be neutralized by the other company, because that company is not expose to that kind of risk in that particular time.

So, this is the concept of diversification what Markowitz was trying to explain and as well as he also talked about diversification in the larger extent. Then the next question arises that how the diversification should be made and how generally we can decide that these are the different assets or these are the different issues on which or different securities on which we should invest in those companies.

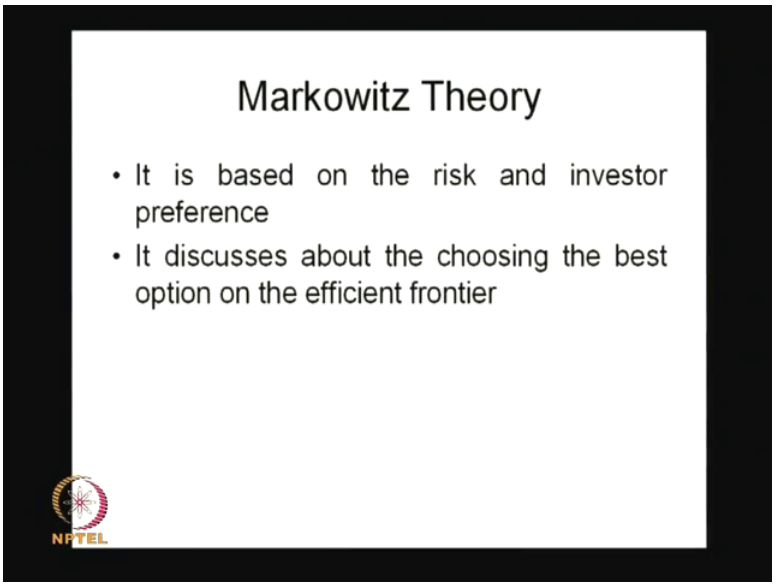
So, in this context, what generally Markowitz was trying to show or Markowitz has given this explanation that how generally we can make this diversification. He said the diversification should be made on the correlation on the basis of the correlation between the different securities; that means, if you already we have discussed that part, that means what Markowitz said that if you have the different combination of the security, then you should include those securities in your portfolio or you decide those securities on our portfolio based on the correlation between these two different assets.

And what should be the correlation? To answer that question, Markowitz had said: the correlation should be negative; that means, the correlation between the different assets should be negative, because if there is some problem is arising from the market and one particular asset is very much prone to that kind of risk. Then if he has another asset in his portfolio in his kitty, which is, which do not move in the same direction what the other asset is moving, then he can minimize the risk or the total risk can be minimized in that particular time.

Therefore, what Markowitz said that you should make the diversification and the criteria for choosing this asset for the diversification is to find out the correlation between the different assets, and what should be the correlation? And the correlation according to, the, him is the correlation would be perfectly negative.

It will have a very perfect negative correlated stocks. Then you will have the different where perfect diversification in the market, but sometimes it is not possible to find out very perfectly negatively correlated stocks, but still Markowitz said that it, it, it, should be the investment theorem, it should be the investment philosophy that always we should incorporate those stocks in our asset which are negatively correlated.

(Refer Slide Time: 09:17)



Then what Markowitz theory basically is talks about Markowitz theory basically based on the risk and investors preference and it discusses about the choosing the best option on the efficient frontier. Already I have introduced this concept of efficient frontier to you in the previous classes, and as well as also I told that the basic objective of the investor is to maximizes utility or the satisfaction level and what Markowitz was trying to explain? Markowitz was trying to explain that first of all there is a tradeoff between the risk and return, and if you can make a tradeoff between the risk and return within this tradeoff, you have the different efficient frontiers.

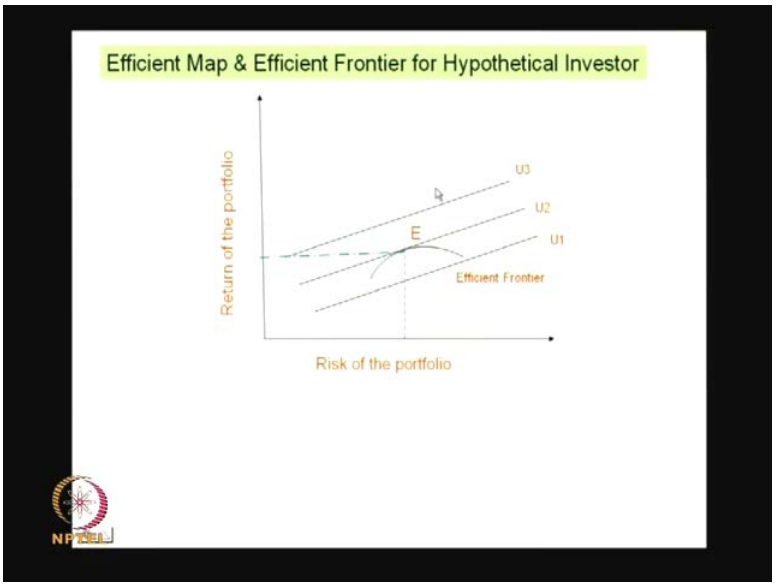
And you have the efficient frontier, whenever you have the efficient frontier and as well as you have the different preference cost which is, we define as the indifference cost. Then always the investor choose that particular portfolio which is the best portfolio on the basis of the risk control and tradeoff, and also it is in accordance with the satisfaction level, what the investor is trying to draw from that particular investment?

So, first of all what we can say that Markowitz was trying to show that always this particular investor's of portfolio choice basically depends on the risk and investor's preference, because this risk level, risk appetite, everything depends on this type of the investor or the individual investors and those risk basically varies from investor to investor. So, first of all whenever we talk about this the varied nature of the varying

nature of the investor's attitude towards the risk, then we can make the tradeoff between this or the relation between the risk and the investor's preference.

Then after establishing this, then what you can do? You just see that what kind of satisfaction the investor is getting, and once the satisfaction level is getting on the basis of the risk, what he has taken? Then, we can say that out of this, which is the best portfolio he should choose for the investment? So, that is basically the basic Markowitz theory and what Markowitz was trying to explain.

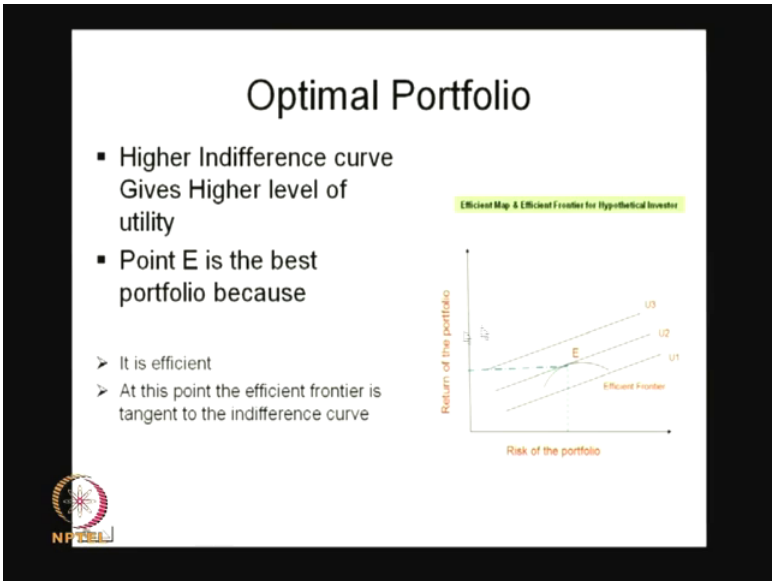
(Refer Slide Time: 11:25)



So, in this context, what this particular thing, already I have shown in the previous classes that this is basically your x axis represents the risk of the portfolio; then, y axis represents the return of the portfolio. So, this is your risk and this is your return and these are basically the different utility curves. Your u 1, u 2, u 3 these are the different utility curves and this is your efficient frontier, and what Markowitz is said that already you know that what do you mean by the efficient frontier, because the efficient frontier in each point basically each, it is the locus of all efficient portfolios or the and each efficient portfolio means either it is a given amount of the risk return is maximized or a given amount of the return the risk is minimized in all these particular points on the efficient frontier.

What Markowitz was trying to explain that always this, in this efficient frontier, somewhere the, wherever the efficient frontier will be tangent to this, any of the indifferent or indifference curves or the preference curves of the investor, there actually the maximization of the utility can be achieved.

(Refer Slide Time: 12:39)



So, here, if you observe these things, what, what is this optimal portfolio Markowitz was trying to explain? Markowitz was trying to explain this things - he said the or he has explained that already we also know from the previous classes that, higher indifference curve gives higher level of utility, that we know. Once you go above, then obviously your utility level will be more.

Then here, already I told you that the point e is the best optimal portfolio; the point e is the best optimal portfolio, because first of all it is on the efficient frontier line. Once it is on the efficient frontier line, then we can say that the particular portfolio is efficient number 1. Number 2 at this point, the efficient frontier is standing to the indifference curve; that means these are the different utility or the indifference curves where already we know that each point basically gives you the same utility level on the basis of the risk and return of the portfolio on the different combinations.

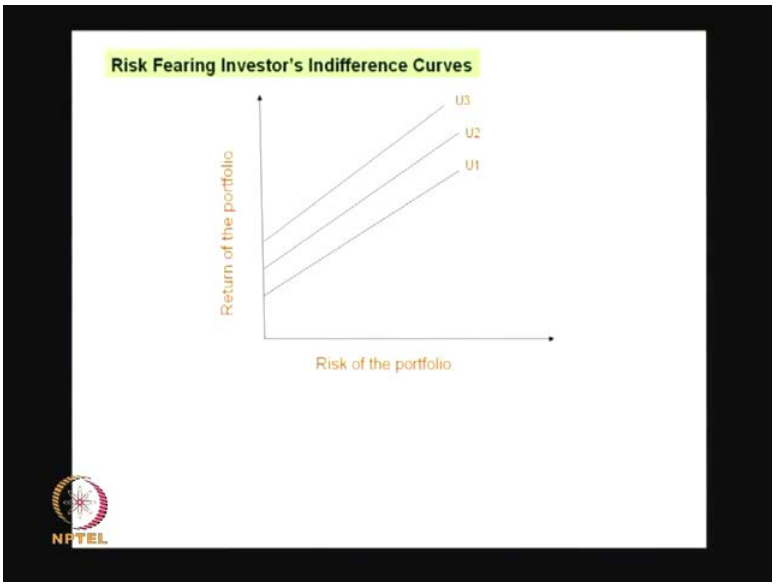
But here, what we have seen that this is the point where it is as well as also it particularly maximizes the utility and as well as also it is, it, it, is one of the efficient frontier, it is

one of the portfolio in the efficient frontiers. So, that is why once it is efficient and then it is the portfolio which is tangent to or it is this particular, particular, point where the efficient frontier is tangent to any of the indifference curves whatever we have. Then definitely, according to Markowitz, this e basically is the point where we can achieve our optimal portfolio.

But here, one question arises - does this indifference curve always look like this or is there any kind of different shapes or different type of characteristics? What we can observe from the indifference curves? That is the question always comes to mind of the academician and as well as practitioners, analyst and the investors, that how we can draw this indifference curves or the investor's preference curve. So, does this indifference curve is always like this, or on the basis of some characteristics or on the basis of the investor's characteristics or on the basis of something else, what are those factors which basically decide the shape of the indifference curve?

So, in this context to answer this questions, the analyst or the researcher who are trying to explain that the indifference, the shape of the indifference curve basically varies on the basis of, the type of the investor; that means, on the basis of the risk appetite of the investor, the, on the basis of the attitude of the investor towards the risk, this indifference curves or the preference curves of the investor's will be changed.

(Refer Slide Time: 15:22)



Then how it looks like? If you observe this thing, this is the curve which looks like little bit stiffer than the previous curve. What I have shown you that here, what you observe, that what we can see that, we call this risk fearing investor's indifference curve. Why it is risk fearing investor's indifference curve? Because to get the extra return, if you now see that, if you now draw this, is the line, where if this is the risk level and this is the return and this is once you will increase your risk level, then you are not going to take more risk in this case; that means, to get the extra return, the level of the risk what this investor is trying to gain or trying to take that basically will be very less.

That means this investor is basically is risk averse investor or he has more fairness towards the risk level; that means, the risk appetite of the investor is very less in this case. That is why little bit this indifference curves are more stiffer, but then, if you observe this curve, it is basically again for the risk fearing investor's but it is little bit flatter as compared to the previous curve. If it this particular curve basically talks about the less risk fearing investor's indifference curve, because to get some extra return, the level of the risk what this investor is trying to take is little bit higher than the previous case what just now we have discussed.

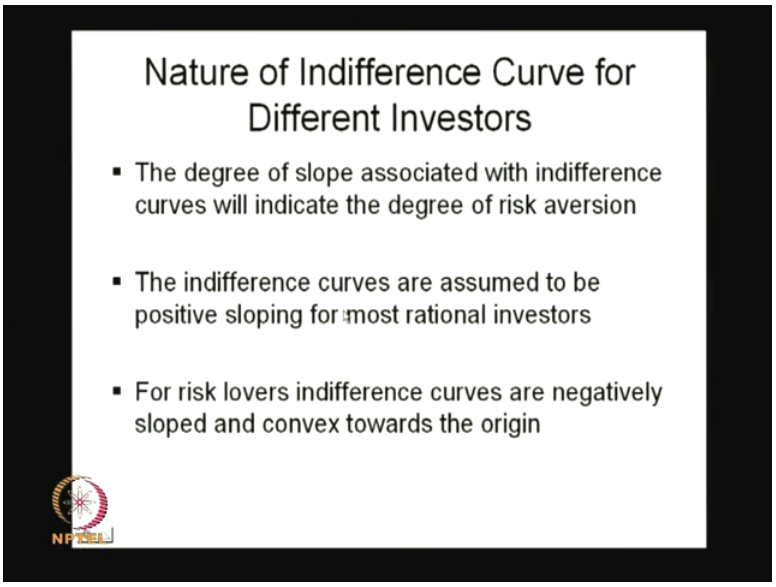
So, therefore, what we can see that once gradually this portfolios you have deciding or the investor's preference curves, you are making the shape of the investor's preference curve is basically depends on the risk appetite of the investor or the types of the investor,

and the same varies on the basis of or we can say that by looking the shape of the investor's preference curve, we can conclude that which kind of investor he is; whether he is a risk lover investor or whether he is a risk fearing fear investor or he is more fear of the risk level or he is a normal investor in gradual cases, in most of the cases, in the general theory, finance theory, always we assume that most of the investors are not the risk lover investors; most of the investors are basically the risk averse investor.

But as the rationality point of view, most of the cases, most of the investors cases always you will find this kind of indifference curve; that means, there not that much risk averse, but to take or to get more return, they can take certain amount of the risk in the market. So, that is basically for the risk, less risk fearing investor's indifference curve.

So, another indifference curve if you see, it is just opposite what we have drawn in the previous case. So, this is basically the risk lover investor. So, they can take more risk. Even if to get the same amount of the return, what the risk fearing investors are taking. So, here, if you see this is the risk level, this is the return, but once it will take more risk and the return also will be accordingly more. So, therefore, we say that, to get the extra amount of the return, those people are ready to take more risk in the market so that, that is the shape of the indifference curves for the risk lover investors.

(Refer Slide Time: 18:52)

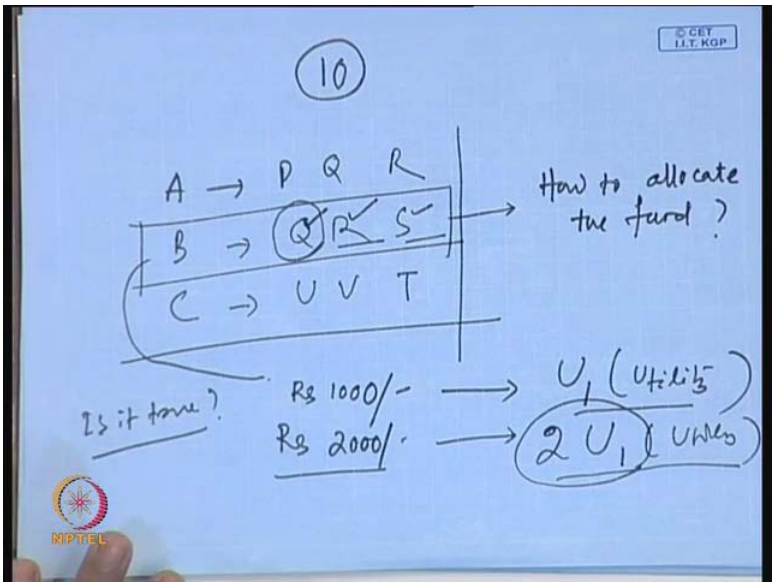


So, now, if you want to summarize that what we have seen from this different curves, that how this indifference curve should look like or how this particular indifference curve, basically the shape of the indifference curve varies from investor to investor on the basis of their risk appetite or on the basis of their fairness towards the risk. We see that the degree of slope associated with the indifference curve will indicate the degree of risk aversion; that means, if the curve is more flatter, then you can say that the investors are more, means less afraid of the risk, but if the curve is more stiffer, then we can say that the investors are more afraid of the risk or their appetite, risk appetite is very low.

The indifference curves are assumed to be positive sloping for most rational investors. Already I have shown you that most of the cases, the investors, the normal investor, the rational investor if you talk about, this those indifference curves are positively sloped. Why it is positively sloped? Because it is assumed that if you want to get more return or if you want achieve or you want to get more return from the market, then you have to take more risk. So, therefore, most of the cases, the investor's indifference curve or the investor's preference curves or positively sloped.

For risk lover indifference curve lovers, the indifference curves are negatively sloped and convex to the origin; that means, the risk lover for the same amount of the return, for example, if you talk about the risk lover investor, they always take more risk. Even if to get the same amount of the return, what the other investors take while getting or while investing in the market. So, therefore, what we can say that the indifference curves characteristics, their shape, their nature changes on the basis of the risk appetite of the investor and their slope basically varies on the basis of the degree of risk aversion of the investor towards that investment process.

(Refer Slide Time: 21:16)



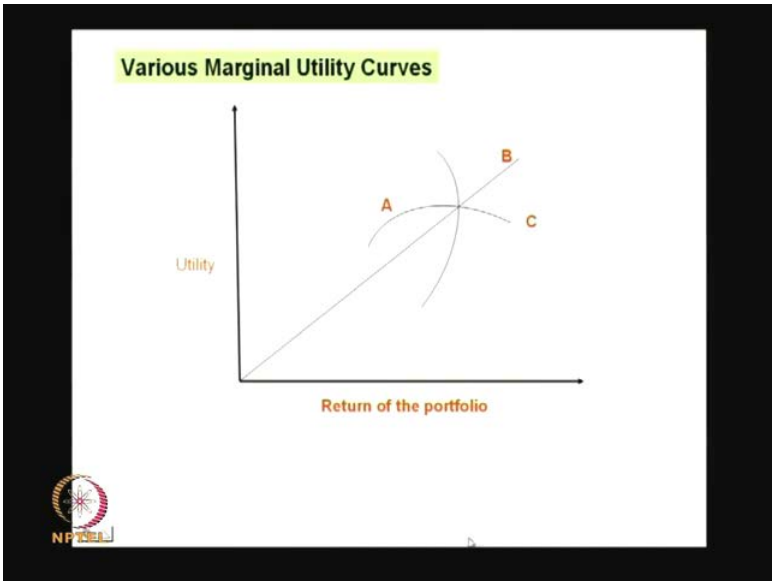
Then another question arises that is it possible always? Is it possible always to say that for example, we were investing 1,000 rupees in the market, if you are investing 1,000 rupees in the market, let you are getting the U 1 level of the utility. Whenever we are investing 1,000 rupees in the market in a portfolio, let this is the B portfolio and we are getting the U one level of utility. Let we have increase our investment up to 2,000 rupees. If we increase the investment up to 2,000 rupees, then what we should expect that the utilities level should increase by 2 times; that means, it should be 2 U 1; that means, it should be double. If the investment on that particular portfolio has been double, then the utility level of the investor also should be double; that means, it is the 2 U1 level of the utility, you should get from this market in that particular time.

But is it true? Is it true that if the acquired amount of the investment on a particular portfolio has been double, then is it possible or is it true to conclude that the utility level what you has deriving before from that investment will be, will be double from this particular investment by that particular investor. So, to answer this question, what we have to see? We have to see first that how this particular mechanism work or how this particular concept is really working in the financial market.

So, in this context, we have, we have to take the help of the marginal utility cost. What do you mean by this marginal utility cost? That means, whenever we add one extra unit of the income or extra unit of the investment in a particular portfolio, what is the extra

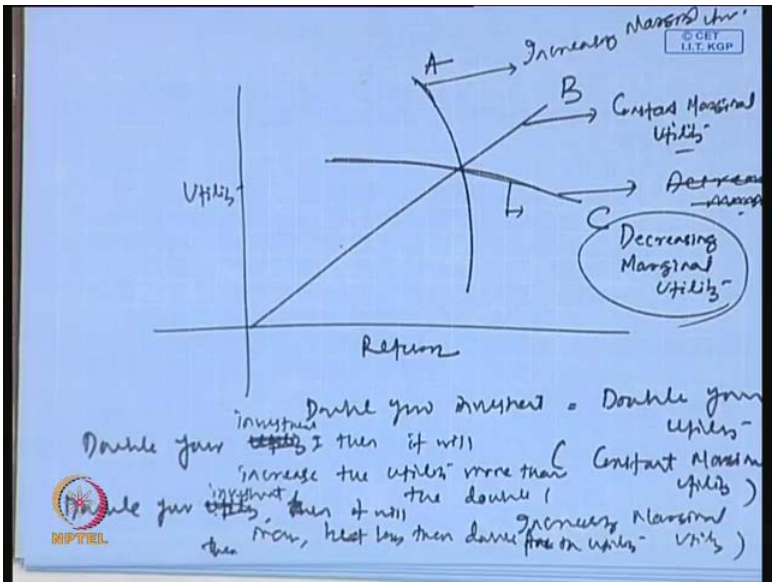
level of the utility or the satisfaction the investor is going to get? If you add one extra unit of money or extra unit of rupee or the dollar on that particular investment, then how the satisfaction or utility level of the investor will be changed, that basically is nothing but the marginal utility of this particular investor.

(Refer Slide Time: 23:41)



So, now, what you can see that you have the, now you are here, if you observe there are three curves, one curve is a straight line; another one you're a, which is, this is your B which is a straight line, and basically, what we have seen that another curve is A and another curve is C. What here, little bit you can see that this is your A curve and this is your C curve. There is some, basically what you can see? This is your A and this is your C. This should, this A should go from here to B. Here, what we can see that you just observe that one thing that whenever we talk about the A and C, it is little bit different because this is your A curve and this is your, this is your A curve and this is your C curve.

(Refer Slide Time: 25:04)



Then if you conclude from here, then what you can see, if you observe that this A, this is your A and this is your C and this is a particular increasing marginal utility curve and this is your decreasing marginal utility curve; that means, here A curve basically what here I am trying to explain that if this is your line and this is one curve and this is another curve and this is your A here and this is your C and this is your B, and here, this A basically, this A curve basically shows the increasing marginal, increasing marginal utility. This is your decreasing marginal utility, decreasing marginal utility, and this is your constant marginal utility.

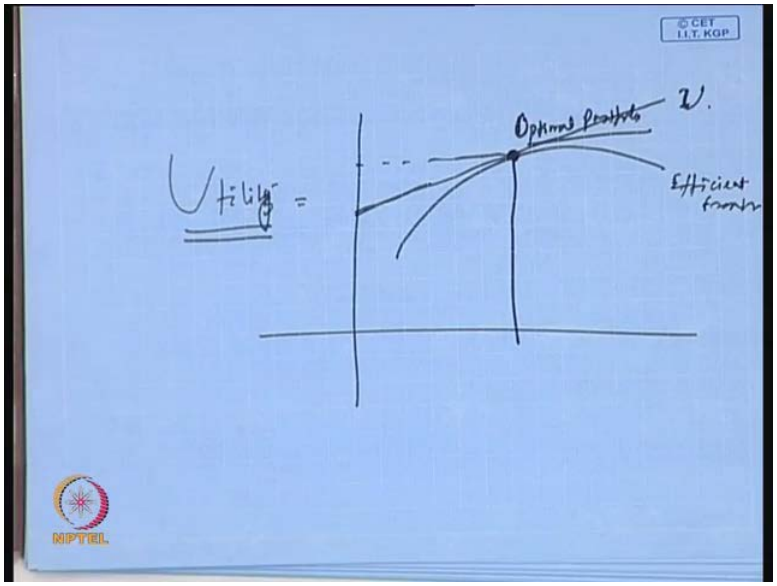
What it basically explains that if you take this is your return of the portfolio, this is your utility what you are deriving from here, what you can say that, for example, if you increase your, you double your investment, if it is exactly double your, it, it, will double your utility, then we call it the constant marginal utility. If you double your utility, double your utility, and it will increase, then it will increase the, increase the utility more than the double and we call it increasing marginal utility.

And if you double your utility and it will increase, but less than the double, the double your, double your investment, sorry, it is your double your investment; this is your double your investment; then, it will increase but less than double for the utility. Then it will be called the decreasing marginal utility.

So, here, what here we are trying to say that even if some times the indifference curves may not be a strict line or the indifference curves, it is not exactly the way what we are deriving, or whenever the accrued amount of the investment increases for an investment, it is not that always we should say that the same amount of the utility will be increased, by the, for the investor in market.

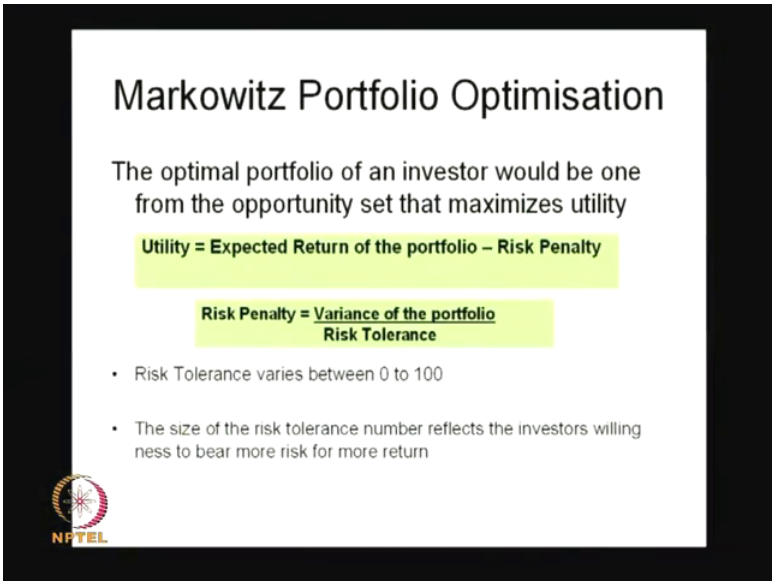
So, the utility level of the investor basically changes; the utility level of the investor basically changes on the basis of the investment of the investor in the market on the basis of the marginal utility of that particular concept, marginal utility of that particular investor. So, here, that is why we say that it is not possible always to say that it is just reciprocal the accrued amount whenever we will change, the same amount of the return or same amount of the utility will be increasing by the investor in that particular time. So, therefore, the basic theme of the Markowitz theory basically based on the utility.

(Refer Slide Time: 29:14)



So, Markowitz are basically given the importance of the concept of utility, and he said that whenever we talk about the optimal portfolio, here with the optimal portfolio means always the utility of the investor should be maximized. So, therefore, if you observe here, the optimal portfolio of an investor will be one from the opportunity set that maximizes the utility. Opportunity set means we are referring to efficient frontier. This is your efficient frontier, and according to him, the utility will be maximized whenever the indifference curves or the utility curves will be tangent to this efficient frontier. So, therefore, this is your optimal portfolio, this is your optimal portfolio.

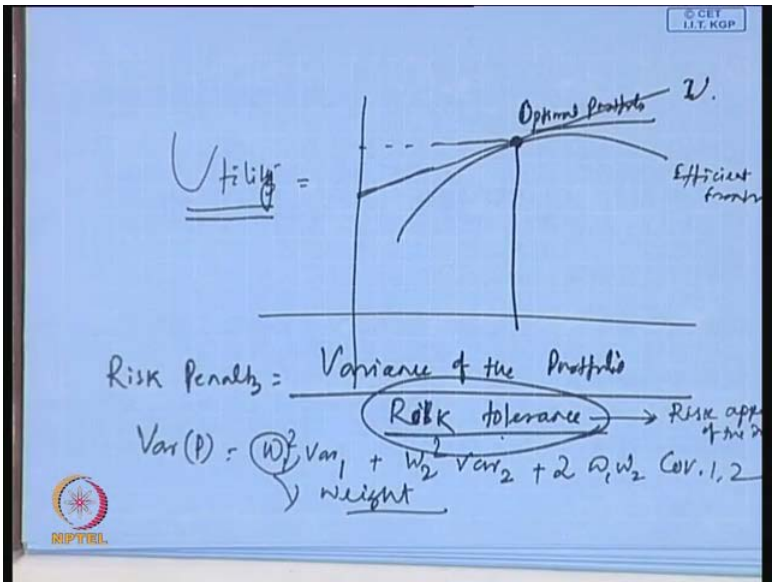
(Refer Slide Time: 30:33)



So that means the whole theme of the Markowitz theory is based on the utility function. So, therefore, Markowitz has tried to measure this utility, and how you can measure this utility? The utility is measured as the expected return of the portfolio what you are going to derive minus the risk penalty.

The concept of the risk penalty what Markowitz has introduced in this particular context, which basically talks about the investor's risk preference. The how much risk this particular investor is taking, and because of that, he should pay some penalty, and once that penalty will be adjusted to that particular expected return what you are going to derive from the investment, then what will happen? The extra amount what we will be getting that basically is the, what we can say that utility of what you will get from that particular investment.

(Refer Slide Time: 31:13)



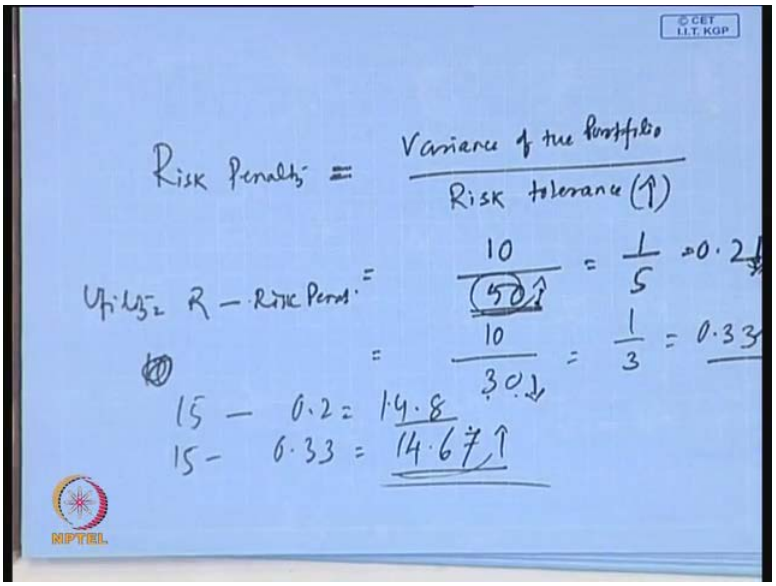
How we can measure this risk penalty? He said that the risk penalty is nothing but it is basically the variance of the portfolio, variance of the portfolio, divided by the risk tolerance. What do you mean by this risk tolerance that one how much risk the investor can tolerate; that means what is the risk appetite of the investor, and the variance of the portfolio already you know that the variance of the portfolio of the two assets basically nothing but it is the W 1 square. The variance of the return of asset 1 plus the W 2 square of the variance of asset 2 plus 2 W 1 W 2 the covariance of asset 1 and 2.

It is basically W means it is the weight age. What we are giving or the proportion of the total funds which are allotted to that particular asset divided by the risk tolerance. Risk tolerance is basically the risk appetite of the investor, risk appetite of the investor, but the question here is the risk tolerance limit is basically subjective in nature or we can say it is a more psychological variable than the quantifiable variable.

But according to Markowitz, the risk tolerance basically varies between 0 to 100; that means, if somebody has more tolerance limit, if somebody has more tolerance limits, then the risk penalty will be more; that means, the tolerance limit will be higher than the risk penalty will be more, and if the risk penalty will be more, then utility also will go down. The variance part if we make it constant, then the risk tolerance will increase, then this part will increase. When this part will increase, then total utility will be declining.

But the risk tolerance limit will be, the risk tolerance limit will be higher, the risk tolerance limit will be higher than the, sorry, the risk tolerance limit will be higher than the risk penalty will be more. Risk penalty will be more, then obviously the utility will be lower, but the risk, penalty, tolerance limit will be lower. Then what will happen? The risk penalty will be less. The risk penalty will be less because the denominator will be more than this particular value will be higher, but the denominator here if it will be lower, then this value will be lower. Then what will happen that the utility will be increasing.

(Refer Slide Time: 33:56)



That means what here we are trying to say for example, we say that the risk penalty, risk penalty is equal to the variance, variance, of the portfolio divided by the risk tolerance. Then if the tolerance limit will be more, then what will happen that the risk penalty will be, this will be more, then the variance of the portfolio, let the variance of the portfolio is 10. The risk tolerance limit is let 50, then it will be 1 by 5; that means 0.2, but there, let the risk tolerance limit will be 30, then it will be 1 by 3, then 0.33. Then here, what we can say that once the tolerance limit will be more, the risk penalty also, risk penalty reduces, but if the tolerance limit will be less, the risk penalty increases.

So, already we know that the utility is nothing but the return minus the risk penalty, return minus risk penalty. Then if the already we know that if this part will be more then the risk penalty will be less then; obviously, if the return will be 10, let the return will be

15, then 15 minus risk penalty here in this case will be 0.2, then it will be 14.8, but here, in this case will be 15 minus 0.33, that will be 14.67.

So, here, what we can say that in the first case, whenever the tolerance limit was higher, the utility will be lower; but the tolerance limit will be lower, then the utility will be higher. So, this is the basically the tradeoff between the tolerance and the utility what this Markowitz was trying to explain in his theory.

What basically the tolerance is that the size of the risk tolerance number reflects the investors willingness to bear more risk for more return; that means, to take more risk, to get more return, how much more risk the investor can take that basically your tolerance limit is trying to answer, and from the tolerance limit, basically we can calculate this risk penalty, and from this risk penalty, what we can say, you can calculate the utility function of this investor in that particular time.

(Refer Slide Time: 36:50)

Risk Return & Utility for Various mixes of Asset Classes

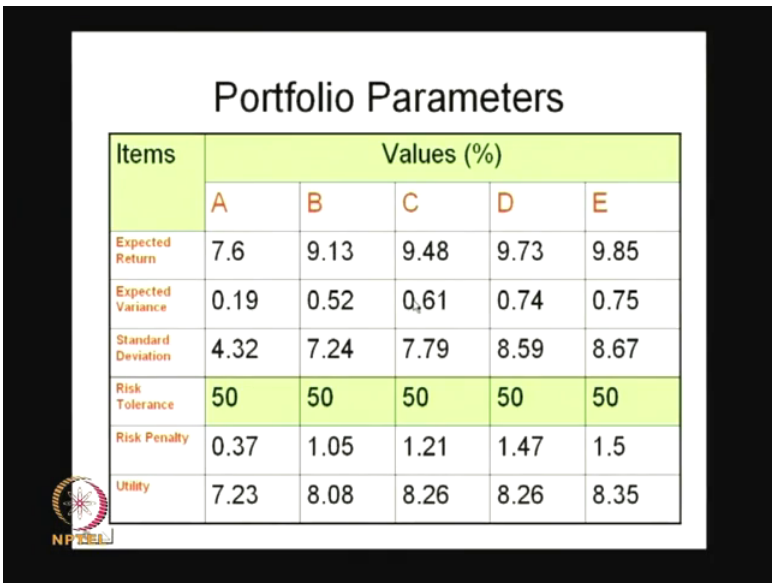
Assets	Portfolios				
	A	B	C	D	E
P			25%		16.7%
Q	50%	33.3%	25%	20%	16.7%
R				20%	16.7%
S		33.3%	25%	20%	16.7%
T				20%	16.7%
U	50%	33.4%	25%	20%	16.7%
Total	100%	100%	100%	100%	100%

So, if you see this example, it will be more clear for you. This is basically the hypothetical example what we have taken. Let we have different assets 1, 2, 3, 4, 5, 6 assets whatever we have and this is your different portfolios. We have made that is A, B, C, D, E and this portfolios has been basically constructed on the basis of the different proportions what we have taken from this different assets.

These are individual assets, and in the first case, if you see for portfolio A, we have invested this fifty percent of Q and 50 percent of U; the total cost investment is the 100 percent, and for the B, we are equally dividing more or less equally dividing Q S and U, that is your 33.3 percent, 33.3 percent and 33.4 percent, and for C, we are dividing this assets on the basis of the P Q S and U 25 percent, 25 percent of the total money has been invested in this particular four assets, and for D, we have, apart from the P, we have invested all those 20 20 percent in each of the assets equally. Then for E, we have also divided equally in the six assets which basically 16.7 percent for each assets, which consist of around say 100 percent.

So, this is the equal number of equal weightage has been given to all assets. So, here, we are not at all considering this P and all what is given has been giving to Q R S T U, and here, we have not considering T and R. We have taken only P Q S and U and we have given the, forty, 25 percent weightage to all assets, and for B, what we have taken? We have taken this Q S and U and which is 33.3 three percent of investment in each of the assets and which is basically nothing but we talk about the, equally it has been in divided between Q S and T equal weightage has been given and here also the equal weightage has been given half of the 50 percent of Q and 50 percent of the U. For example, once this data has been given to you for the for the different assets whatever we have and how the allocation has been made for those assets.

(Refer Slide Time: 39:20)



And then, if you go to the next table, what we can see here? We are assuming that this particular investor's risk tolerance limit is 50, the investor's risk tolerance limit is 50. If the investor's risk tolerance limit is 50 and the investor's the same portfolio what data we have taken in the previous slide, previous slide this is the data what we have taken, and from here, we have calculated the expected return of this particular asset, sorry, different expected return of the portfolio, and for a, expected risk of this particular portfolio and all are in the percentage star and the standard deviation, then the risk penalty and the utility, etcetera. Then once we have calculated, what we found that, let we found that in the previous slide, if you see this, these are the different allocation what you have made, but in the basis of the data, maybe all data is not giving to you here.

But for example, we take from the combination of A, we are getting a return of 7.6 percent. Already you know how we can calculate the return of the portfolio. Let you have invested, we have invested this 50 percent in Q and 50 percent in U, and once this 50 percent multiplied by the return of the Q plus the expected return of the Q plus the 50 percent multiplied by the return of the U, which is given you the 7.6 percent which is basically the expected return from the portfolio A.

And like that, from the portfolio B, we have this 33.3 percent multiplied by Q plus 33.3 percent multiplied by the S, the return from the S and 33.4 percent multiplied by the return from the U that basically is giving you 9.13 percent, and like that, C is the same way if you calculate the return for the C, it will be 9.48 percent, and if you calculate for the D, it will give you the 9.73 percent, and if you calculate the E, it will give the return of 9.85 percent.

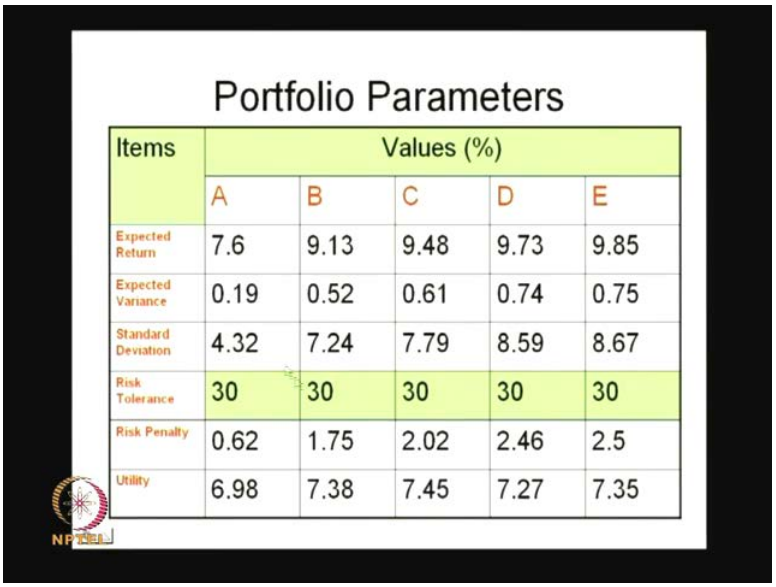
So, like that, if you calculate the expected variance, the variance will be here; it is 19 percent; it is the 0.19; it is 0.52, 0.61, 0.74; it is 0.75. The standard deviations are also calculated. Tolerance limit is here. Then what is the risk penalty? It is the expected variance divided by the risk tolerance. The expected variance is 0.19 divided by the risk tolerance that is 50, that is giving you 0.37, and here, it is 0.52 divided by 50; it is 1.05. Then here, it is 0.61 divided by 50, it is 1.21, and 0.74 divided by 50, it is 1.47; 0.75 by 50 is equal to 1.5.

Then, once you have calculated your risk penalty, what you have seen that what is the utility you can derive from this? The utility will be expected return minus the risk

penalty. Already what we have seen that it is 7.6 minus risk penalty that is 0.73, that is 7.23, 9.13, 3 minus 1.05, it is 8.08; 9.48 minus 1.21, it is 8.26; 9.73 minus 1.47, that is 8.26 minus; 9.65 minus 1.5, it is 8.35.

That means, from this table, if you observe after calculation of this items like expected return, expected variance, standard deviation, risk tolerance, risk penalty, utility risk tolerance, we have taken the 50 for this investor. Then we find that the utility level is maximized for the portfolio E which is 8.35. So, therefore, the investor should concentrate on the portfolio E which is giving the maximum level of the utility that is your 8.35 which is defined as the optimal portfolio.

(Refer Slide Time: 43:41)



But for example, if you change your, now the tolerance limit has been changed, let you keep all those, these are all automatically will be same for this investor, because from the different asset, the same amount of the risk or same amount of the return he can get, and now, the tolerance limit has gone down. For other another investor, let this is 30 is the tolerance limit. Then the risk penalty like that if you calculate, then the risk penalty will go up. If the risk penalty will go up, then automatically the utility level basically what we have observed that that has gone down.

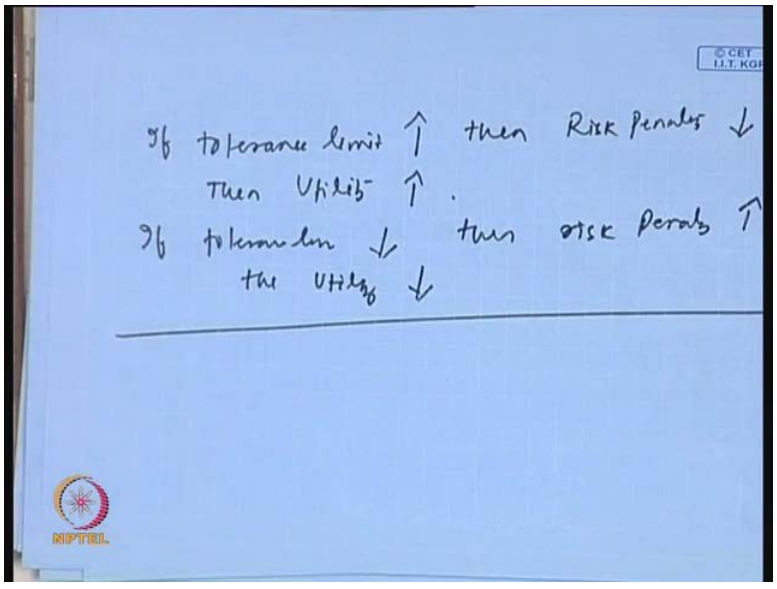
That means if here what we have observed that, this is the case whenever the risk penalty will be increasing, then utility will be decreasing. So, here, whenever the risk tolerance

limit is increasing, whenever tolerance limit is increasing, what we have observed that the risk penalty is increasing. The tolerance limit is increasing. Whenever the tolerance limit risk tolerance limit is increasing, then the risk penalty is increasing. What is this risk penalty in this case?

Whenever the tolerance limit, the risk penalty is, it is increasing. The risk penalty is declining if the risk penalty is declining, then what will happen that the utility will be declining. If the risk penalty will be declining, then the risk utility will be, it will, this will increase, then there utility will be decline. Then what here, we can observe that in this case, in the previous case, if you compare this two slides, then first case the risk penalty was 0.37, but next it has gone up to 0.62, because the tolerance limit has gone down.

So, like that it is, in the first case, it was 1.05 or second case, it has gone up to 1.75. So, in this case, what we have observed that the utility level also has declined for all type of portfolio, because the risk tolerance limit has gone down. The utility level what we have observed here the, now, the, what we can see that clearly what we have observed here that, once your tolerance level, if the tolerance limit increases, then risk penalty decreases.

(Refer Slide Time: 46:12)



The risk penalty decreases, then utility increases. If tolerance limit decreases, then risk penalty increases and utility decreases. So, this is the logic what we found from here, but here, if you observe that another observations from here that, once your tolerance limit has gone down, tolerance limit, risk tolerance limit has gone down, risk penalty has gone up. Then finally, there your utility has also gone down, but in the previous case, what we have seen that E is the optimal portfolio which is maximizing the utility, but in the next case, what we have observed that this portfolio has been changed. Now, C is the optimal portfolio which is maximizing the utility.

That means what here we can conclude that, even if the other variables remains constant, once your risk tolerance limit has been changed or risk tolerance limit has increased or decreased, then accordingly what we can see that, in this case, always we can observe that the optimal portfolio selection also changes; that means, here what the Markowitz was trying to show that risk tolerance limit is one of the very efficient, sorry, the important parameters to decide that which is the optimal portfolio for an investor, and if that risk tolerance limit increases or decreases in accordingly the optimal portfolio selection also changes.

So, another thing here what we can see that here, clearly what we have observed that in the previous case, the E was the optimal portfolio which was giving the utility of 8.45, but second case, whenever we change this risk tolerance limit to 30, we have seen that C is the optimal portfolio; that means, the investor who has the tolerance limit of 50 they can choose the portfolio E and the, the, investor who has the risk tolerance limit up to 30, they can choose the optimal portfolio like C because which is maximizing the utility for him.

But here, if you, one thing if you observe that, what this Markowitz is going to explain? The Markowitz theory is not that much has been developed in that direction that, how we can measure this risk tolerance limit or whether the risk tolerance limit is quantifiable variable or not or how we can measure this risk tolerance limit, and basically what already I told you that risk tolerance limit is psychological variable or it is more observatory or more kind of subjective variable than any kind of objectivity. So, in this context, what here we are trying to say that is it possible to really measure the risk tolerance limit of the investor, and if you can measure this risk tolerance limit of the investor, then the utility level can be measured.

So, again, this theory is also is talking about to the concept of utility, because utility is basically again in a is a subjective variable and as well as it is not possible always really to measure the utility of an investor depending upon the risk appetite in nature. So, if the risk level or on the basis of the different level of the risk it is not possible, always very accurately to measure the risk appetite of the investor and as well as also it is not possible to really know that what is the risk appetite, and accordingly, we cannot also come to conclusion that what is the tolerance limit. Then, how can we say that these are the different ways through which the utility maximization can be made. This is number 1 question always comes to the mind of the investor.

And second question comes to the mind that what this Markowitz theory did not explain that, how this allocation of the funds will be made. Basically, whenever we have given this allocation from the beginning, that these are the different allocation given to this, but how this allocation has been made? What is the different criteria on which the allocations should be made for this different investors or is there any kind of scientific logic that this kind of allocation will be possible for the investor for the various reasons. So, those explanations also is not explained by this Markowitz theory.

So, therefore, what we can say - maybe it is we can conclude in this way that, let this Markowitz theory is the theory which talks about how to decide the different or how many numbers of assets should be there in your portfolio and how this particular asset should be taken into account to make this optimal portfolio.

But it was not trying to say anything the allocation of the funds for this particular portfolio. Let your portfolio has five assets or ten assets, that ten assets how this funds will be allotted to them or how the allocation of the fund should be given to those assets, that has not been explained by the Markowitz theory, and number 1 number 2 the concept of the utility and the concept of the risk tolerance or the risk penalty is not clearly explained by the Markowitz theory, and because of that, it is not possible always to use that concept to determine to the optimal portfolio in a particular framework.

Then therefore, the other theories were trying to explain are basically this other theories like Markowitz theory or, sorry, the surface single index model or we can say this lagrange multiplier theory was has have given their importance to the concept of the

allocation of the funds and as well as the decision of the utility in this particular framework.

So, therefore, we will further see that how those theories has been better off than the Markowitz theory or the advancement of the Markowitz theory whose tries to fill up this gap to answer this fundamental questions which basically related to the allocation of the funds and as well as the concept of the utility. That we will be discussing in the next class. Thank you.