Security Analysis and Portfolio Management

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Module No. # 01
Lecture No. # 28
Other Optimal Portfolio Selection Models

In the previous class we discussed about the Markowitz portfolio theory or how the Markowitz can define this optimal portfolio and what was the conclusion from there that we should decide this portfolio on the basis of the diversification. And the basis of diversification is basically, the correlation between the different assets or the return of the different assets then finally, what we have seen that whenever the efficient frontier will be tangent to its indifference curve of the investor. There only we can decide that this is the point or this is the portfolio on which this investor can maximize the return.

And here two things basically we observe that this optimal portfolio should be an efficient portfolio and as well as it should be this portfolio, where with the given amount of the risk, the return can be maximized or with the given amount of the return, the risk can be minimized. So, after this we have said that how this, whenever this Markowitz theory has talked about the decision of choosing the assets and how we can choose the different assets for the optimal portfolio. But they could not answer that how this particular assets will be decided and as well as, also how the allocation of the forms to the different assets can be given. So, to answer those questions there are other theories which have been developed and what we call it the other portfolios selection models and here the one of the model is basically the Sharpe's optimization model, then another one is basically, whatever we have that is on Lagrange multiplier model.

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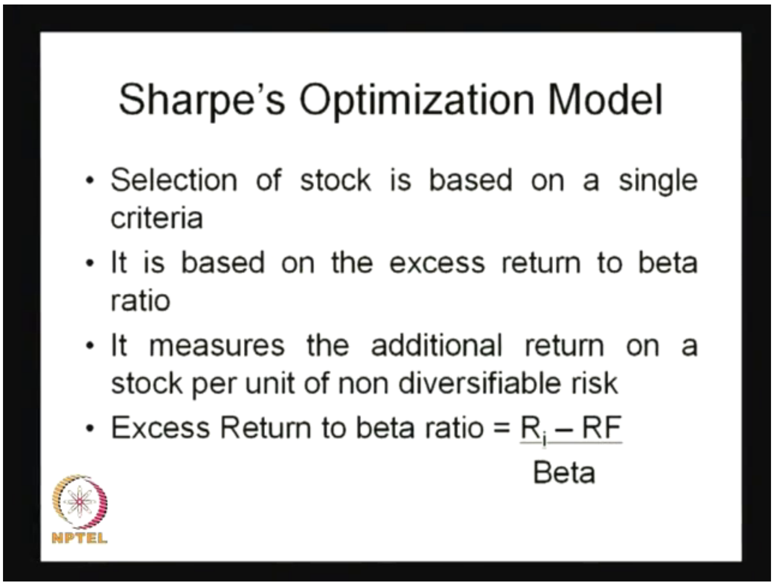

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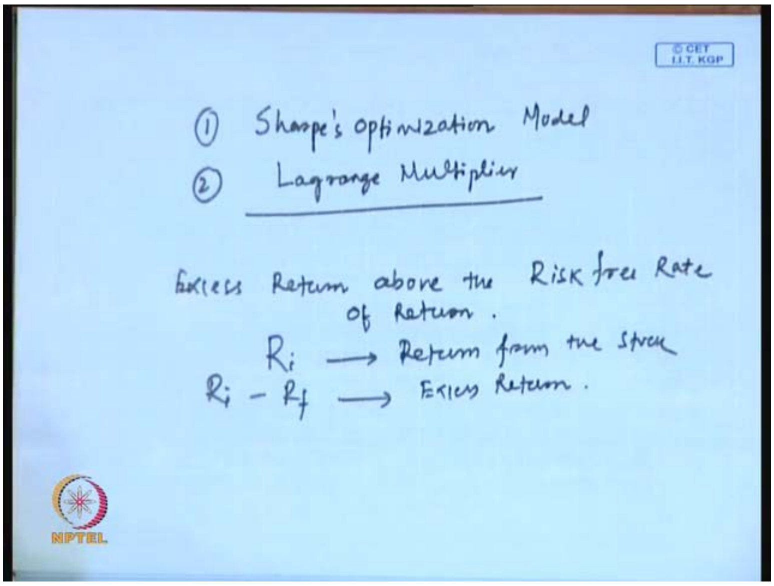

So, we have two models. So, one is your Sharpe's optimization model. One your Sharpe's optimization model and another one is your Lagrange multiplier method through which we can decide also, how to allocate the funds. So, there are two methods basically has been discussed by the other theories which gives this idea, how this portfolio can be constructed. According to Sharpe, what basically Sharpe was trying to explain? Sharpe said that the selection of the stock is based on a single criteria, instead of choosing that whether the correlation is negative or correlation will be below this level or that level, what this Markowitz theory was discussing.

Here the Sharpe has given a definite criteria and what is that criteria on which this selection of the stock can be made. And what is this criteria he has given, that the selection of the stock should be based on the excess return to the beta ratio. He said that, what do you mean by the excess return? He said the excess return is the excess return what you are getting, the excess return above the risk free rate of return. The excess return above the risk free rate of return. So, that means, if your Ri is equal to your return from the asset, return from the stock, then your Ri minus Rf, Rf basically your risk free rate of return, which gives you the excess return. So, according to Sharpe the excess return is basically, the additional return on a stock per unit of non-diversifiable risk. Once allowed you have seen that the risk can be categorized into two types. One is your diversified risk and another one is the non-diversified risk. The particular risk which cannot be diversified because of that the excess return the investor should get.

And this excess return basically is calculated by deducting the risk free rate of return from the return what you are getting from the asset. So, here already what you have seen that he said the excess return to beta ratio, the excess return to beta ratio is basically your Ri. Your Ri means, your return from this asset minus Rf is equal to your risk free rate divided by the beta. Beta already we have explained. So, in extensive manner beta is nothing but it is the market risk or the systematic risk what that particular company of a particular stock is going to face in the market.

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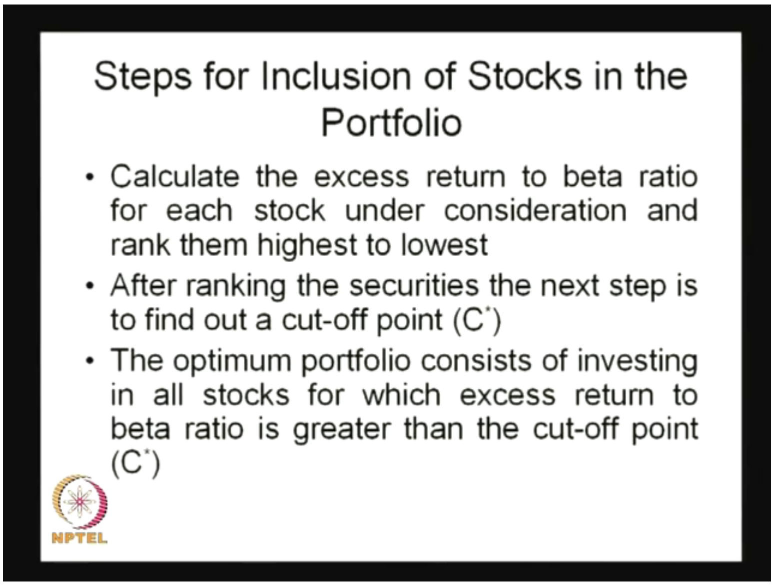

So, in this context what basically, the steps he has given. That he said that first of all, you calculate the excess return to beta ratio for each stock under consideration and rank them highest to lowest. And after ranking the securities, the next step is to find out a cutoff point. And the optimum portfolio consists of investing in all stocks for which the excess return to beta ratio is greater than the cutoff points. That means, for example, you have a universal of stocks. Let for example, you have 100 stocks in your kitty or 100 stocks are available and out of the 100 stocks you have to decide that which are the stocks should be taken into consideration.

And according to Markowitz, what he said that we should take those stocks, which are negatively correlated or they do not move in the same direction. But here, what we have seen that here, what the Sharpe is trying to explain? Sharpe said we have to take those stocks where the excess return to beta ratio is more than the cutoff rate. And what exactly the cutoff rate, we will be explaining that thing and what are the logic behind of this cutoff rate that also we will explain here. So, that is why it is one, we can say this more deterministic in nature and whenever they said that this cutoff rate, what this cutoff rate is basically, the cutoff rate what this Sharpe has explained.

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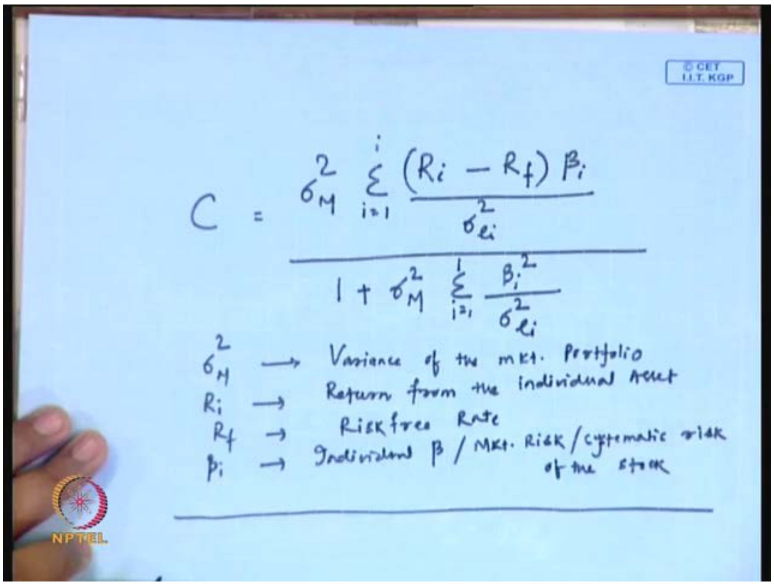

Sharpe said that the cutoff rate is explained in this way. This is the variance of the market portfolio. Then, you have the different assets whatever you have taken. Then, this is your return from the different asset, individual asset minus the risk free rate of return into beta, individual beta of the stocks. Then the variance of ei, that means, that is basically the unsystematic risk, then whole divided by 1 plus the variance of the market then i equal to 1 into i, then beta square divided by the variance of the unsystematic risk. So, here if you want to explain it basically, this represents the variance of the market portfolio and your Ri represents the return from the individual asset, return from the individual asset. Rf represents your risk free rate, beta i represents the individual beta or the market risk or the systematic risk of the stock.

So, this is the part basically, the logic behind of this cutoff rate what this Sharpe has explained that whenever we calculate the cutoff rate, we should consider the cutoff rate by taking in to account three types of the risk. What are those risk? One risk he says that the total risk what your market is facing and the unsystematic risk what this particular individual stock is facing and as well as the systematic risk of that particular individual stock is facing. That means, if you can accommodate or you can adjust all the three stocks or all the three, sorry not all the three stocks, it is basically all the three types of risk what basically we always face in the market.

One is your market risk, another one is your market risk in the sense with reference to that particular individual stock, in another particular risk is the variance of the market. That means, how the market is fluctuating then, as well as the undiversifiable risk, non diversifiable risk what these particular company cannot diversify. So, if all the three types of risk will be taken into consideration then only, we can say that what should be the cutoff point of that particular stock and whether the stocks should be incorporated into our portfolio or not. So, that is decided basically on the basis of all types risk, what the particular stock is facing in a particular time period and therefore, what Sharpe has explains, Sharpe has identified there are three types of risk what particular stock cannot avoid.

So, that is why we should incorporate all the stocks, risk into our consideration. Therefore, he has taken this unsystematic risk, he has taken the systematic risk of this particular individual asset and as well as also he has taken the variance of the market, that is why the three types of the risk will be taken. And as well as, he has also incorporated the risk free trade because already what you have seen, that whenever you are taken any extra risk that is why the return should be like that will be more accordingly or in comparison to risk free rate of return which is prevailing in the market in that particular time. Therefore what we can see here, Markowitz has sorry the Sharpe has explained that this is the way the cutoff should be calculated and looking into this cutoff basically we should decide those stocks and we should incorporate those stocks in our portfolio where the excess return to beta ratio will be more than the cutoff rate.

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Example

Security Number	Mean Return (%)	Beta (β_i)	Unsystematic Risk (σ^2_{ei})	Risk Free Return (%)	Market Return (%)	Excess Return	Excess Return to Beta	Rank
1	20	1.2	20	8	25	12	10	3
2	14	1.0	30	8	25	6	6	6
3	12	2.0	40	8	25	4	2	9
4	16	0.9	20	8	25	8	8.89	5
5	24	1.1	15	8	25	16	14.54	1
6	18	1.1	50	8	25	10	9.09	4
7	19	0.8	16	8	25	11	13.75	2
8	13	1.3	25	8	25	5	3.85	7
[illegible]	11	1.4	30	8	25	3	2.14	8
[illegible]	9	1.6	10	8	25	1	0.63	10

So, if you see this example then, it will be more clear. We have a the let we in our universe we have the ten assets or the ten stocks, 1 2 3 4 5 6 7 8 9 10. These are the security numbers or number of the stocks which are available. These are the returns in a particular month or particular time period which is a average return whatever we are getting from this different securities. First is 20 percent, 14 percent, 12 percent, 16, 20, 4, 18, like that. Then this is these are the beta what already we know how to calculate the beta. So, this is the beta calculated for individual stocks. Then, this is your unsystematic risk what this particular stock is facing. The unsystematic risk is nothing but the systematic risk, we have to deduct the systematic risk from the total variance of the particular stock. So, if you have the total risk, total risk is composition of the systematic and unsystematic risk.

So, if you have the total risk and from there we have to, if you deduct the systematic risk then the unsystematic risk will be there. So, this will be 20, 30, 40 percent of the different assets. Then, this is a risk free rate of return which will be same for all the assets because the risk free rate of return is related to the market only, that is why it will be 8, 8, 8 only, 8 percent we have taken assumption. The market return in that type is also same for all the assets which is 25 percent. Then, the we have, it is not the market return variance of the market. Actually, variance of the market return or the risk of the market return. Then, the excess return whatever we have that is already your return mean return minus the risk free rate of return, 20 minus 8 is 12. Remember, this is not the market return, it is basically variance of the market return. you please note it down it is variance of the market return. And this is your excess return which is 20 percent minus 8 percent is 12 percent, 14 percent minus 8 percent is 6 percent, like that.

This is your excess return what this particular assets are getting. This is your excess return to beta, which is a basically we have taken the beta is 1.2, the excess return is 12, 12 by 1.2,10 like that 6 by 1 is equal to 6, 4 by 2 is equal to 2 like that, this is your excess return to beta ratio. Then, accordingly whenever you have calculated the excess return to beta ratio as per the steps given by the Sharpe theory or Sharpe's model, what we have given that we have given the different ranks on the basis of the excess return to beta what we are getting. So, that is why this is the highest excess return to beta, that is why we have given the rank 1, next is 13.75 which is 2 then, it is a 3 is a 10, 4 is a 9.09, like that we have given the ranks to different assets whatever we have in our total universe of the assets. So, then after calculating this we have to use this cutoff rate formula, what already I have given here. That this is basically your cutoff rate formula which is given and we have to calculate the Cutoff rate from there.

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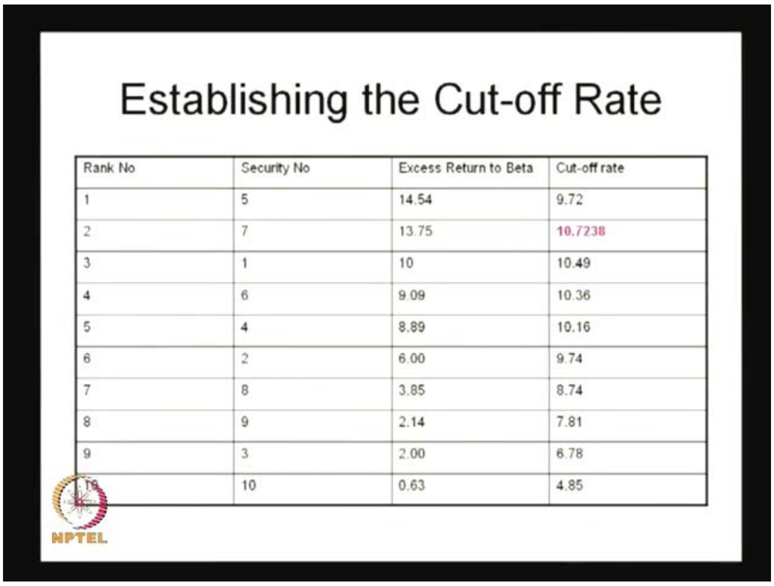

Establishing the Cut-off Rate

Rank No	Security No	Excess Return to Beta	Cut-off rate
1	5	14.54	9.72
2	7	13.75	10.7238
3	1	10	10.49
4	6	9.09	10.36
5	4	8.89	10.16
6	2	6.00	9.74
7	8	3.85	8.74
8	9	2.14	7.81
9	3	2.00	6.78
10	10	0.63	4.85

So, here if you observe that these are the rank numbers 1 2 3 4 5 6 7 8 9. These are the ranks which is given. So, security number 5 has got the rank one, security number 7 have got rank 2, security number 1 has got rank 3, like that. So, we have given the different ranks to the different security and here the excess return to beta ratio is arranged also accordingly on the basis of the ranks 14.54, 13.75, 10, 9.09, 8.89, 6, like that. Then from there what generally putting this all the values whatever we have, we have calculated the cutoff rate. So, this your 9.72 is the cutoff rate for stock 1, 10.7 2 3 8 is the stock 2, 10.49 is for the stock 3, 10.36 for stock 4, like that. Then, we have 4.85 for the stock 10. So finally, what we have seen that already we have our condition is we will incorporate those stocks in our portfolio where this excess return to beta ratio. So, here these are the different cutoff rates, whatever we have.

So, first of all we have to see how this cutoff rate, where the cutoff rate is more than the excess return to beta. So, first case is 9.72, excess return to beta is 14.54. Second case, it is 10.72 but excess return to where beta is 13.75. So, therefore, what we have seen then, third case it is you see a 10.49 but excess return to beta is 10. Like that cutoff rate is 10.36 but excess return to beta is 9.09. So, like that if you go on then, you have seen that in all the cases the cutoff rate is more than the excess return to beta except the rank 1 and 2 securities, like security number 5 and security number 7. The stock number 5 or stock number 7 or the security number 5 or security number 7 these are only the two stocks which are available here which have the more return than, excess return to beta is a more than the cutoff rate. So, as per the theory these two stocks should be taken into consideration to make the portfolio. So, now it is clear that whenever according to Sharpe theory or according to the Sharpe model, what we have seen that the stock number 5, stock 5 and 7, that means, the stocks 5 and 7 should be included in the portfolio.

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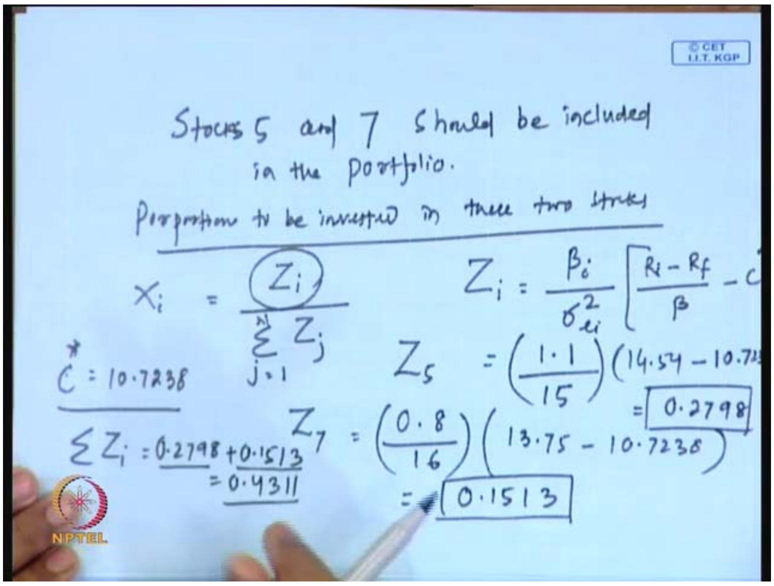

If stocks 5 and 7 should be included in portfolio then, the next question will arise how to allocate our funds between these two. Now Sharpe has answered the first question of making the portfolio or the construction of the portfolio that how many securities are required to make my portfolio or make my optimal portfolio. But the next question is once this number of assets or number of securities have been decided that these are the different assets should be taken into account to construct the portfolio. Then the next question arises, how this different fund allocation can be made or in the simpler term we can say, if you have 100 rupees that 50 rupees, how this 50 rupees can be allotted between these two stocks because in our example what we have seen there are two stocks in your portfolio which can maximize your return.

So, then for these what Sharpe has given this there are different ways to which this Sharpe has given the formula, Sharpe has given the logic that how this allocation of the funds can be made. Then, what Sharpe said that how, what is the proportion should be included? Proportion to be invested in these two assets, two stocks. So, how he has decided? He said that you calculate this proportion from here in this way he said Xi is equal to Zi into j is equal to 1 to N and Zj. And what is this Zi? Zi you have to calculate in this way which is your individual beta divided by the means ratio of systematic risk to unsystematic risk and then your excess return and divided excess return to beta minus the cutoff rate.

So, this is your excess return to beta minus cutoff rate. So, first case it is already we have seen that beta value already you know, that for first the security number 5 and you take the example of the security number 5, security number 5 the beta is 1.1 and your unsystematic risk is 15. And your risk free return minus excess return to beta ratio is 14.54 and your cutoff rate basically, what we have seen the maximum cutoff rate what you have decided here, that is the maximum cutoff rate 10.7238. So, this will be minus that mean this is the optimal cutoff rate whatever we have. So, we have taken minus 10.7238, which is generally defined as C star. So, your C star which is the cutoff rate which will 10.7238, that is why 14.54 minus.

So, these are the different basically these are the different C's 9.729, 10.7238, 10.49, 10.6, etc., but like that this is the rate which this cutoff rate will be maximum. So, that is why we call it, the optimal cutoff rate and after that what we have observed that always the cutoff rate is more than the excess return to beta ratio. That is why we have taken this way and we have calculated 0.2798. So, like that it is your Z5. So, then you have calculate your Z7 which is your beta is 0.8 divided by 16, which is your unsystematic risk. Then, it will be your 13.75 minus 10.7238. Remember always from the excess return to beta ratio, whenever the proportion will be calculated, we have to deduct the cutoff, maximum cutoff rate from there. Then, this will be your, we have calculated 0.1513.

So, this is the value what we get. That means, the Z value has been calculated from here what is Z1 or Z5 is equal to 0.2798. I mean here your Z7 for the security 7, it is 0.1513. And from here, now you can calculate your different combinations or the proportion of the funds. Then, according to this formula then the summation of Zi is basically 0.2798 plus 0.1513. If you take this then, what how much you will get 0.4311. So, then you have to calculate that what is the proportion of funds for 5 then, what is the proportion of the funds you have to allocate for the 7.

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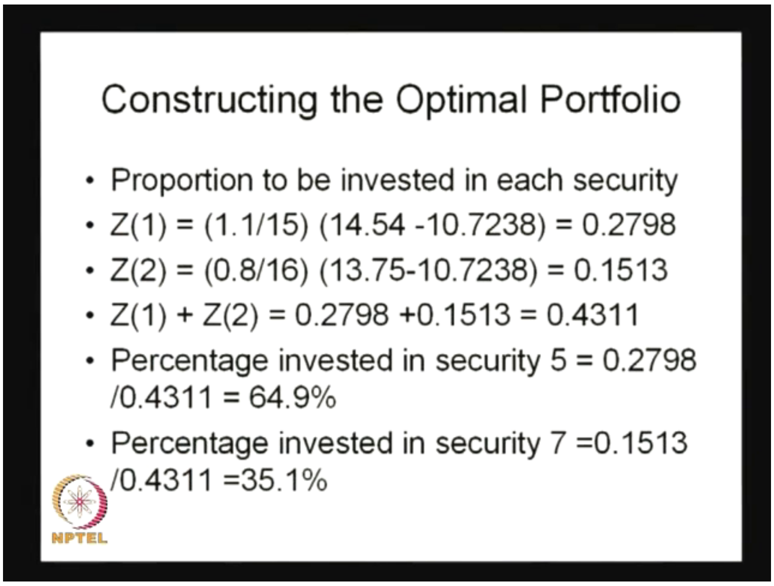

So, in this context what basically we should do? We should do that the percentage of invested in security 5 is the 0.2798 divided by 0.4311, this will be 64.9 percent, this will be 0.2798 divided by 0.4311 which will be 64.9 percent. And for others it will be already you know that this will be 0.53 and 0.1513 divided by 0.4311 this will be 35.1 percent. So, this is the way the allocation will be made and finally, what we have concluded we have concluded for security 5, they should allocate 64.9 percent funds and for this security 7, they should allocate 35.1 percent fund. So, like that the total fund allocation can be made. So, here if you observe one thing Sharpe's theory has given a complete answer for the optimal portfolio selection model, what this Markowitz theory could not give.

Because Markowitz theory has explain that how much assets or how much securities should be taken into consideration, should be taken to make the optimal portfolio but the Sharpe theory was trying to give the answer trying to explain that how much securities or how much stocks should be taken into consideration to make the optimal portfolio. Then as well as, the second question which is a major task for the allocation of the funds that also has been given by, that answer also has been given by the Sharpe theory. Sharpe has also explained that how the allocation of the funds can be made and what is the different proportion of the fund should be invested in the different security what we have taken into account. So, after considering this we have another theory what always also use that is your Lagrange multiplier theory.

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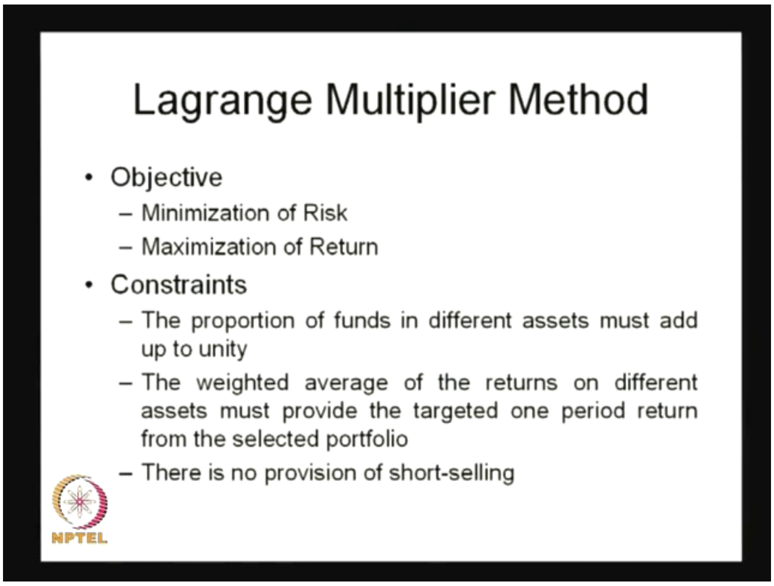

So, this is Lagrange multiplier method always you use to know that or to calculate the allocation of the funds because this is the major task always the investor faces. Even, if they can decide with certain criteria that what are the stocks should be taken into account or should be consider for the optimal portfolio. But sometimes they could not able to answer how the allocation of the funds can be made. Therefore, there are this Lagrange multiplier theory basically this differential equation method. We always use this particular method to allocate the funds this is basically nothing,, but a linear programming kind of problem here we have, if you observe here we have certain objective and in the case of optimal portfolio the objective can be either the minimization of the risk or the maximization of the return.

This is the two objectives, either of the objectives always the investors takes into account whenever they make their optimal portfolio selections. Then, in this context also like your other linear programming problem in this case whenever we talk about the different stocks into consideration or different assets into consideration, what we have seen that we also face certain constraints like your other problems. The constraints are like the proportion of funds in different asset must add up to unity. That means for example, you have if your proportion of the funds you have given let you have the three assets 1 2 3. Always they say according to them for example, the proportion of the vertex has been given for 1 is for X1 and this is X2, X3, that means, all the funds should be invested within this 3. That means, X1 plus X2 plus X3 should be equal to 1.

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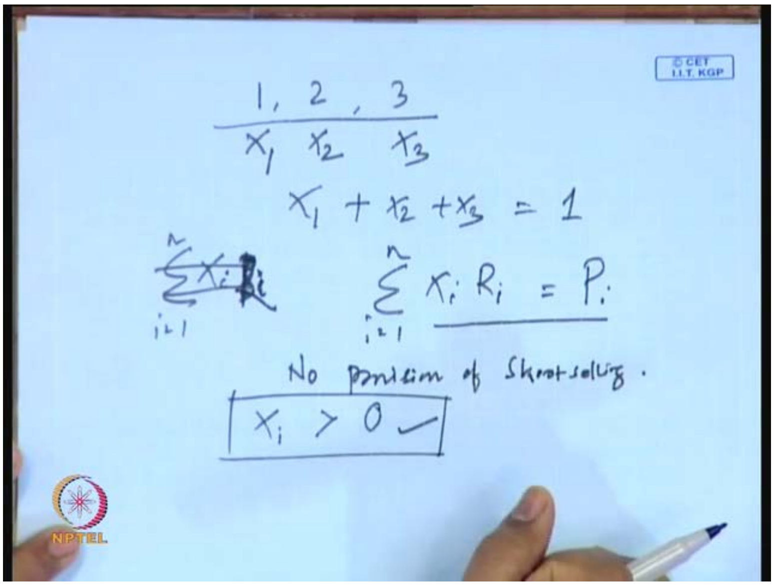

Then, another thing is the weighted average of the returns on investment assets, return weighted average of the return on different assets must provide the targeted one period return from selected portfolio. That means the proportion of the funds what you have calculated from here and as well as the different stocks whatever you have given. That means, this is basically if your return from the stock, return from the stock is represented by, if your return from the stock is represented by Ri. Then the total return multiplied by the vertex whatever we have given that should be equal to the portfolio return, that should be equal to the portfolio that is that is what this constraint is trying to explained.

Then another assumption also the constraint whatever we have there is no provision of, no provision of short selling short selling means already you must be knowing that without owning the stocks always you can invest in this. That means what you can say that there is no investment in the stock from the beginning you borrow it and you then investment; that means, the expectation of the people who go for the short selling always they expect that the price should go down because they have to pay less. That is why they said the proportion of the fund should be always greater than the 0. That means, there is no chance of going for the short selling. So, without owning this particular stock, we cannot invest it in the market. So, that is why this is also one of the very important assumption or important constraint we have to take in to account before going for this, using this Lagrange multiplier method for the allocation of the funds. So, that will be a more clear.

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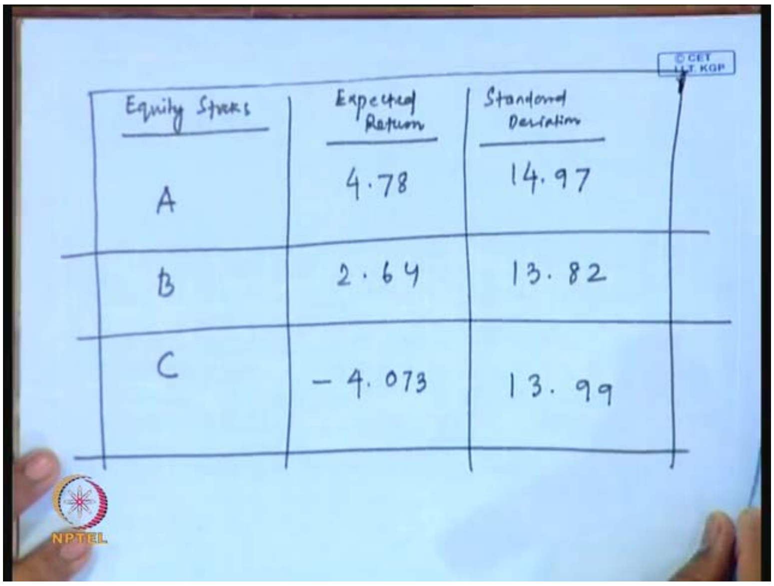

And if you now I will give you certain example, in this case let you have three stocks you have let you can make it like this is your equity stocks, then this is your expected return, this is your expected return and this your standard deviation of this different stocks, standard deviation of the different stocks. Let you have A, you have B, you have C. These are the different stocks, these are given to you, then let you have your expected return here is 4.78, here the 2.64, here it is minus 4.073. Let here the standard deviation for this stock is 14.97, here it is 13.82, 13.99. That means, in this case what generally we are trying to explain that you have already decided these are the stocks which according to the all the stocks which are available to you. These are the stocks which can give you the maximum or the these are the stocks should be considered in a portfolio.

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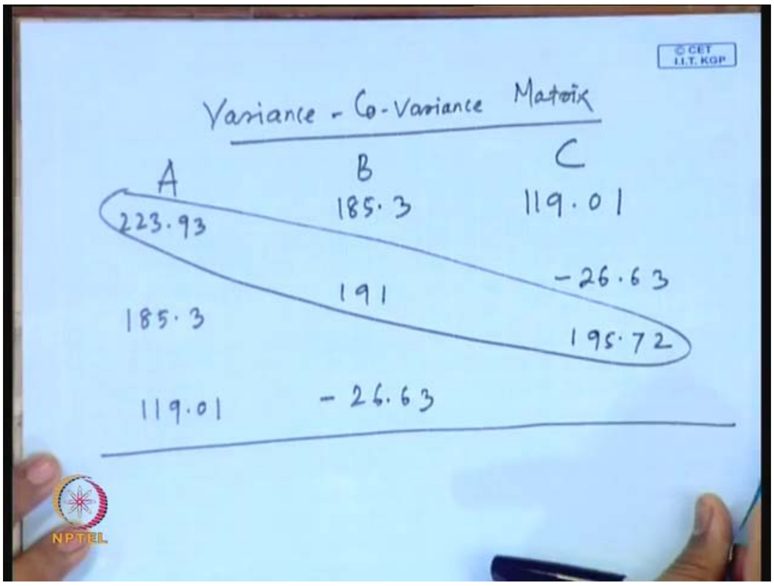

So, this is the data which is given to you and another data from there, it will be calculated. I will give you the variance covariance matrix, variance and covariance matrix of these three stocks because these are the matrix we require to calculate the risk of this particular portfolio. If this is your A, this is your B, this is your C then, this your variance is 223.93, for B the variance 191. Let this is your 195.72. This diagonals are always given as the variance. Then the covariance between A and B is 185.3. It is between A and C is 119.01, between B and A already we have given this that is 185.3, between B and C it is minus 26.63, between A and C the same figure here 119.01. And here it should be 192 and here also it will be minus 26.63. So, like that if your variance and covariance matrix will be given to you then, this is the data is given.

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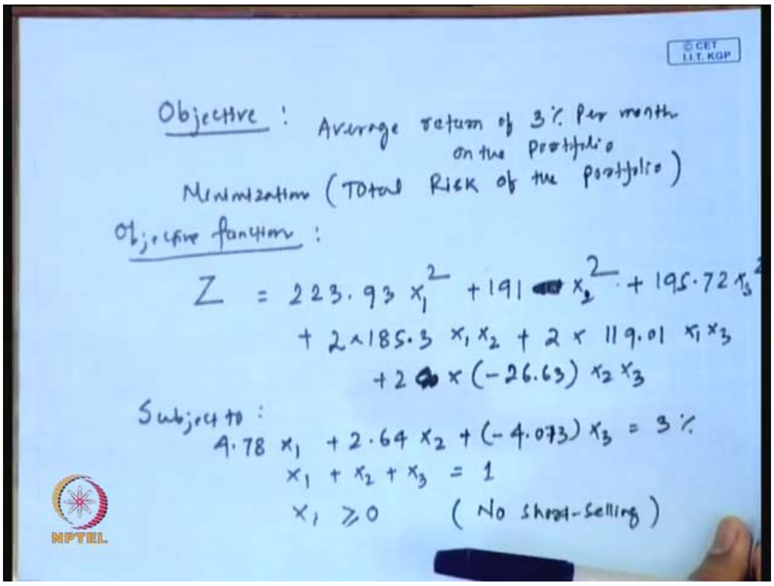

If this is the data is given and from here we have to your objective is, what is the objective? The objective is the investor wants to maintain or wants to get a average return of 3 percent per month. Let this is error per month data on the portfolio this is the objective. And this another thing is the objective is this and because of that what he wants to do? He wants to do the minimization of risk minimization of total risk of the portfolio, total risk of the portfolio should be minimized. So, because of that we can find out objective function. What is your objective function? The objective function will be here let is equal to Z. Z is equal to already you know that variance of the portfolio that is your 223.93 X one square because X1 is the vertex what you can give and plus 191.01, sorry 191 X2 square the variance plus 195.72 X3 square. Then plus 2 covariance, covariance means between X1 X2 is 185.3. 185.3 X1 X2. 2 covariance into X1 X2 plus 2 into covariance between X1 and X3, that is 119.01 X1 X3 plus 2 covariance between X2 and X3 is 2 into minus 26.63 into X2 into X3. And which are the constraint, which is given by subject to 4.78, already given the expected return from this one asset 4.78 X1 plus 2.64 X2 plus minus 4.073 which is the expected return from the stock C, X3 is equal to 3 percent.

This is one constraint another is already is, I have already told you X1 plus X2 plus X3 is equal to 1 then your Xi is greater than 0. That means, X1 X2 X3 all are positive, that means, we have to allocate the funds in such a way that either it is greater or equal to 0. We should not say that, it will be there is any short selling, no short selling. No short selling is allowed in this particular case. So, this is basically, you have now fixed your objective here. So, let your, if this is your objective and this is your constraints. And using getting this objective function and the constraint function now our objective is to find out your X1 X2 and X3. So, how we can get this X1 and X2 and X3.

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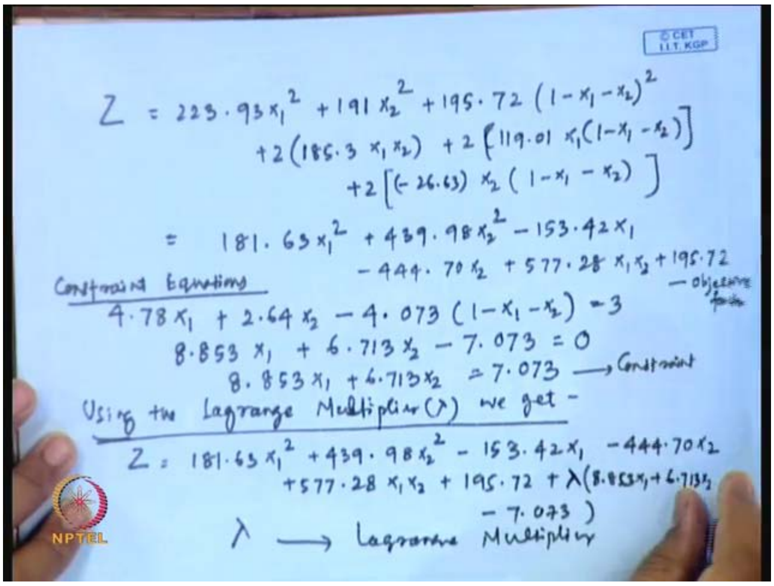

So, here what you can do that first of all we better we should convert there are only looking into this particular system of equations. It is very difficult to solve both X, all the three X1 and X2 and X3 at a time because that is why you should convert all the things into 2 variable case then, it will easy for solving this problem. So, let then your Z is equal to already we have mentioned 223.93 X1 square plus 191 X2 square plus 195.72 X3 square and instead of X3, we can write 1 minus X1 minus X2 square plus 2 185.3 X1 X2 plus 2 into 119.01 X1 then in terms of instead of writing X3 we can write 1 minus X1 minus X2. X2 plus 2 into minus 26.63 X2 into 1 minus X1 minus X2. That means, we have just deleted the X3 term from this equation because we have to convert everything in X1 and a X2.

And finally, what we have arrived? We have arrived in this function, if after the, if they were in simplification if you do this then, you will find 181.63 x 1 square plus 439.98 X2 square minus 153.42 X1 minus 444.70 X2 then plus 577.28 X1 and X2 plus 195.72. Then, this constraint equation if you want to also delete this X3 term from there then, you have to convert everything from again in the X1 and X2. Then, how this can be converted? Here we have equation is 4.78, this is your constraint equations, constraint equations. Then we have 4.78 X1 plus 2.64 X2 minus 4.073 1 minus X1 minus X2 is equal to 3 percent. Then, it will be automatically, if you after the simplification it will come to 8.853 X1 plus this 6.713 X2 minus 7.073 is equal to 0. Then, automatically it will be 8.853 X1 plus 6.713 X2 is equal to 7.073. So, this is your now, this is your objective function. This is your objective function. Then, this is your constraint.

Now, the two functions we got and from these two function what we have do? Now, we have to use this Lagrange multiplier method to find out the value of X1 and X2 and X3. Then, the using this Lagrange multiplier lambda we get, Lagrange multiplier we get, Z is equal to 181.63 X1 square plus 439.98 X2 square minus 153 4.42 X1 minus 444.70 X2 plus 577.28 X1 X2 plus 195.72 plus lambda into this constraint function is already we know that 8.853 X1 plus 6.713 X2 minus 7.073. So, this is your now the composite function what we get by inclusion of this lambda. Lambda is basically the Lagrange multiplier. Lambda is the Lagrange multiplier. Now, after this function if you want to, if you want to minimize it with respect to the different weightage then, the minimization function means we have to equate it with 0.

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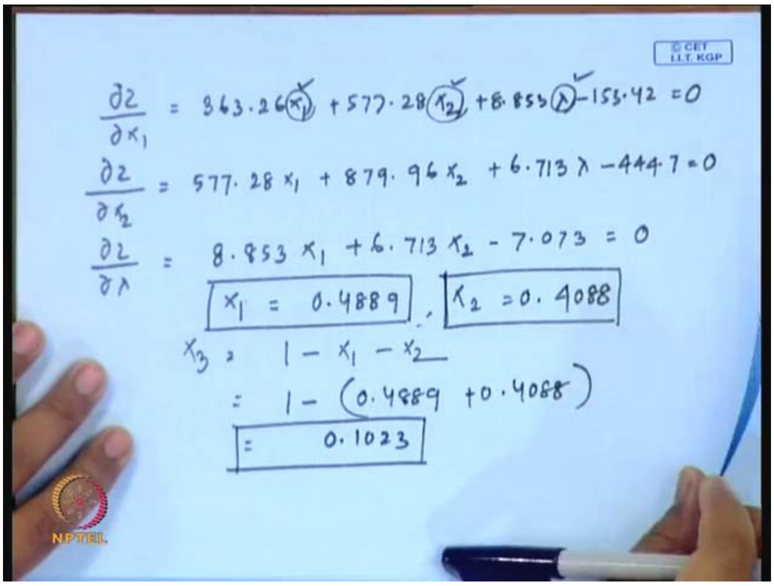

Then after equating with 0 then we can find out the different weightage of X1 X2 and X3. Then, in the first function let your the partial differentiation what we have to make, the first case we have to make it differentiate with respect to X1 then, it will be your

363.26 X1 plus 57 7.28 X2 plus 8.853 lambda minus 153.42 is equal to 0. Then you can take delta Z by delta X2 with respect to X2 we find 577.28 X1 plus 879.96 X2 plus 6.713 lambda minus 444.7 is equal to 0. Like that your delta Z, that means, with respect to lambda we can find 8.853 X1 plus 6.713 X2 minus 7.073 is equal to 0. So, then once we get this we can solve this three equations. Now, we have two variable, three variables what we have to lambda, we have to get it your X1, you have to calculate your X2 then, your lambda. From here this three functions or three equations, we can calculate your X1 X2and lambda. And then we can find from this. After this necessary simplifications what we find your X1 is equal to 0.4889 then, your X2 is equal to 0.4088.

So, now once from this equation you can find out your X1 and X2 then, your X3 is equal to basically 1 minus X1 minus X2. That means, it is 1 minus 0.4889 plus 0.4088, we can put in the bracket. So, this will be 0.1023. That means, overall what we can say that the 48.89 percent of the funds should be invested in security A. Then, 40.8 percent should be invested in security B and 10.23 percent of the security should be invested in security C. Then, the allocation of the funds in such a way that the X1 X2 X3, all will be invested in this particular portfolio and the allocation of the funds will be in this manner and basically this is the way, this Lagrange multiplier theory is trying to give this answer.

But here what we have seen? Now, if you want to observe two things what we can get from this the Sharpe's model and as well as this Lagrange multiplier method one basic observation what we find from here, that the Sharpe model was trying to explain this number of securities, which are the number of securities should be taken into consideration to make the optimal portfolio and as well as how the allocation of the funds can be made in the different securities. But here the two questions always comes to the mind. The basis of the cutoff trade what this Sharpe model has explained, that is not fully known or the logic the theoretical justification has not been explained by the Sharpe model.

How this particular formula has been given by the Sharpe, what is the theoretical justification behind that. That thing is little bit on explained by the Sharpe theory. This is number one. Number two what we have seen that sometimes Sharpe has assumed that this excess return to beta whenever we compare always we assume that, we can diversify the risk that is why only risk what we can face from this particular asset that is beta. But in the real world situation it is little bit difficult to diversify all the risk what we face from the market or all the risk, individual risk what the company faces of the particular stock faces. So, therefore, sometimes little bit theoretical in nature not to incorporate any kind of other risk apart from the beta whenever the excess return to beta ratio is calculated on that basis the rank of different securities have been given.

So, this is another problem always have the limitations what this Sharpe model says and coming back to your Lagrange multiplier theory, what this Lagrange multiplier theory was trying to explain? It explains the allocation of the funds or the Lagrange multiplier method is used to allocate the funds. But this theory again did not answer about the selection of the particular assets and which are to be considered in your portfolio and which are should not be considered for your portfolio. So, whenever we can see that maybe we can decide this different asset which are to be considered in the portfolio then we can use this Lagrange multiplier model method to know that how the allocation of the funds can be made.

But then, in this context if somebody or some industries is using this Lagrange multiplier method to calculate the different allocation of the funds then, this particular method is incomplete to answer to the two fundamental questions what this optimal portfolio theory was trying to rise. The two questions what is raised that what is the different number of assets should be allocated and number two question is basically how the allocation of the funds can be made. That’s why basically this is another problem with Lagrange multiplier theory. But on whole, if you compare between these three theories what we discussed in the previous class about the Markowitz optimal portfolio model.

Then also we discussed about this Lagrange multiplier theory today and also the Sharpe’s single index model or we can the Sharpe’s portfolio model. Then, what we have seen from all those three theories or what we can observe from these three theories that Sharpe theory is the only theory which could have the answer for two questions. What this optimal portfolio theory was trying to rise or say the two questions what is the number of assets are required, which can maximize your return or minimize your risk number one. Number two the question is that how this allocation of the funds can be made. This is number two which is complete answer has been given by the Sharpe theory or Sharpe model but only limitations was some kind of explanations what the Sharpe has given, it is not clear to the investor and what is the criteria on which this thing is decided.

Therefore, this is only this technical aspects what the Sharpe theory was trying to show, what still it is a better theory than the other theories what we have in over model. Then, in the recent period we have also the other aspects we can consider in our analysis that is the utility function and or the utility theory. People are now more concerned about this investor's sentiments or investor's perception about the stock market or investor's perception about the functioning of this particular stock. So, that particular part is not been taken care by any of the theories here. So, this is little bit what we can say this psychological factors subjective factors involved here. So, always the investor try to maximize the utility although the risk return framework is always used but within that particular framework always the investor should or investor is bothered about the sentiment or his appetite towards the utility level or the risk level.

So, therefore, what generally we can say here apart from those risk return factors what we are considering to decide the portfolio the other factors also should be consider to make the portfolio are the investor's moods, investor's sentiments, investor's utility function. So, if those factors will be taken into account then maybe we can have a complete model which can answer the optimal portfolio theories. But those factors are little bit difficult to measure because most of them are not quantifiable in nature. But still we can incorporate those theories or we can incorporate those factors into the consideration in our, we can take those factors into our consideration while making the optimal portfolio selection model into some extent. There are certain proxies whatever we have we can incorporate those things into our analysis and we have to see that when the investor's appetite towards the risk changes, when the investor's mood changes, when the investor's approach towards the market changes how the allocation of the funds is also changes. If the allocation of the fund changes according to the investor's mood and sentiments then, this theories may not be the complete theories which can explain this concept of optimal portfolio models into our analysis.

But still in our context what we are trying to say, that most of the theories are in our case what we can say after considering this optimal portfolio how generally this decision is taken by the investor or how what kind of strategy or what kind of policies the investor always follow, investors always follow to maximize this return within this risk return framework. That is considered as the different investment strategies in the portfolio management. So, that we will be discussing from the other coming sessions. Thank you.