

Security Analysis and Portfolio Management

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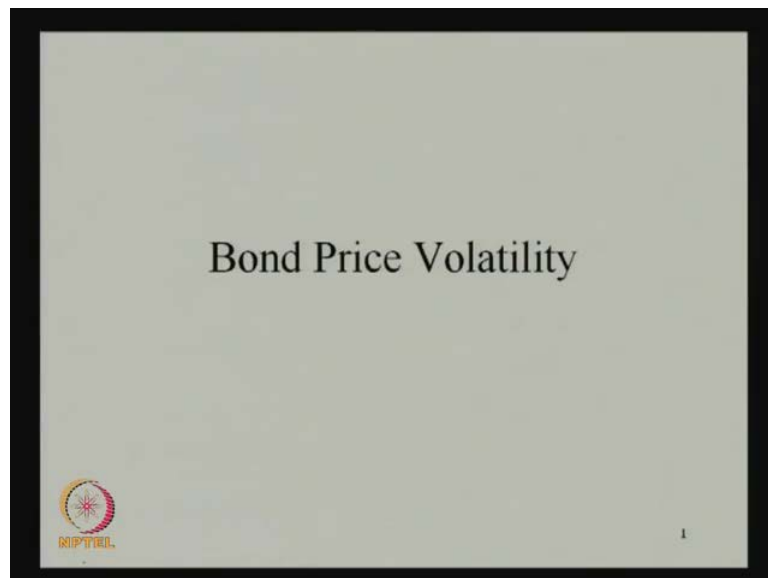
Indian Institute of Technology, Kharagpur

Lecture No. # 34

Bond Price Volatility

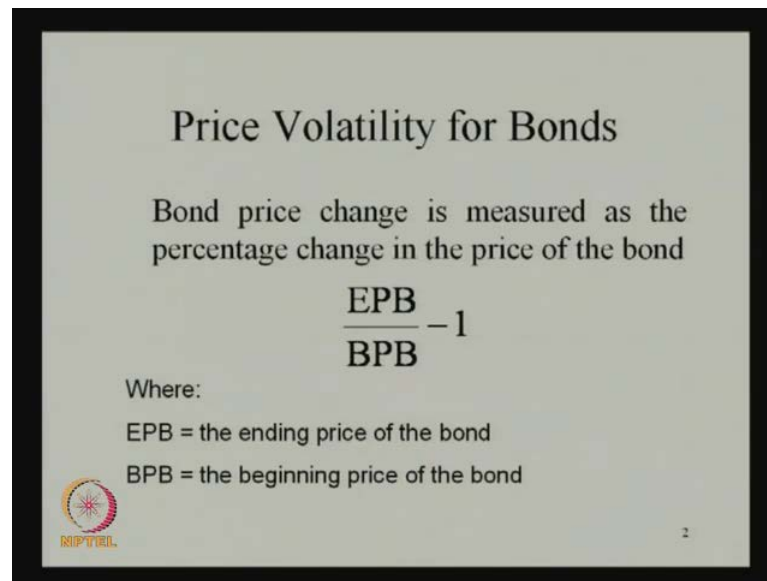
In the previous **class**, we discussed about the term structure interest rate, which is the main factor which affects the bond pricing.

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And today, we will be discussing about that why this bond price basically becomes volatile or we in general, we call it the bond price volatility and which are the factors which affect this bond price volatility; these are two objectives, what we are going to discuss today, and how also we can measure this different kind of instruments through which this price risk of the bond can be managed. in this case, if you see that, first of all you have to see that, which are the factors which affect this bond price volatility or the fluctuation in the bond pricing in the market.

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Price Volatility for Bonds

Bond price change is measured as the percentage change in the price of the bond

$$\frac{EPB}{BPB} - 1$$

Where:

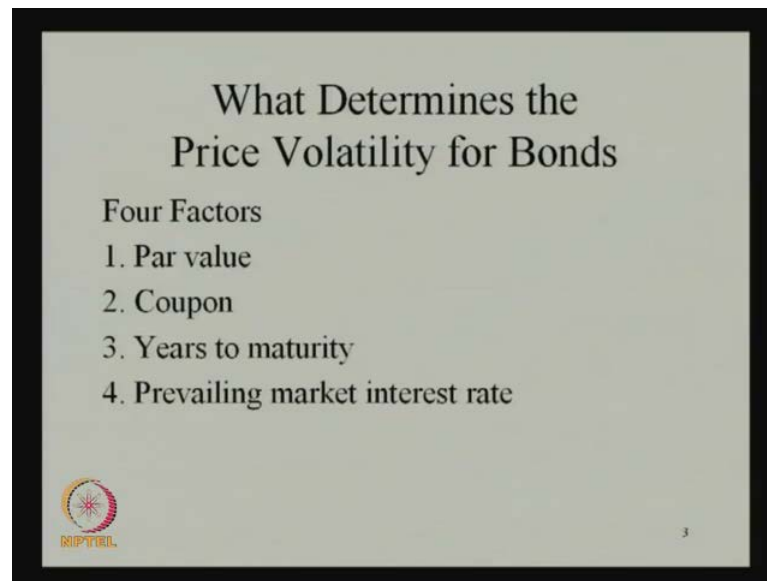
- EPB = the ending price of the bond
- BPB = the beginning price of the bond

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After this thing, that what exactly this bond price volatility means, before that, then we can go ahead with other things; Bond price change or bond price volatility is basically measured as the percentage change in the price of the bond. Basically, how it is measured that, it is basically this EPB by BPB minus 1, where the EPB represents the ending price of the bond, and BPB is the beginning price of the bond.

So, it is basically this ending price divided by beginning price minus 1, which gives you the percentage change in the price of the bond for a particular period of time. So, that is the way through which the bond price can be measured.


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**What Determines the
Price Volatility for Bonds**

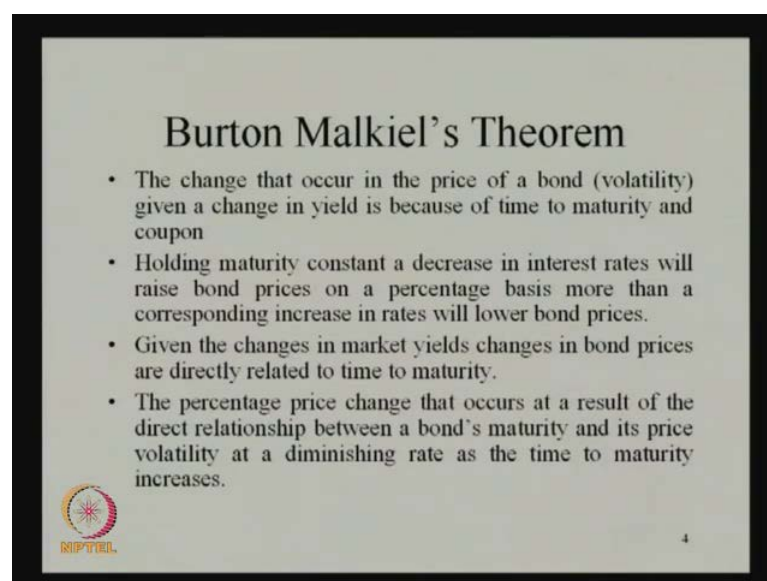
Four Factors

1. Par value
2. Coupon
3. Years to maturity
4. Prevailing market interest rate

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Then which are the factors or what determines this bond price volatility; if you see there are basically four factors, which affect the bond pricing, one is your par value of the bond, already we know what is the par value, then we have the coupon, then we have years to maturity, then we have the prevailing market interest rate. So, these are the different four factors, which affect the volatility of the bond pricing in a particular time in a market.

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Burton Malkiel's Theorem

- The change that occur in the price of a bond (volatility) given a change in yield is because of time to maturity and coupon
- Holding maturity constant a decrease in interest rates will raise bond prices on a percentage basis more than a corresponding increase in rates will lower bond prices.
- Given the changes in market yields changes in bond prices are directly related to time to maturity.
- The percentage price change that occurs at a result of the direct relationship between a bond's maturity and its price volatility at a diminishing rate as the time to maturity increases.

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So, how it basically works; in this case, if you go through these theorems given by the Burton Malkiel's, then it will be clearer. What Burton malkiel's say that the changes that occur in the price of a bond, what basically we mean, it volatility given a change in yield is because of time to maturity and the coupon. In 1962, when the malkiel has explained these things in a theorem basis, but how this different or what the different characteristics of the bond price volatility, and which are the factors which affect the bond price volatility.

Then another reason is holding maturity constant, it decrease in interest rates will raise bond price on a percentage basis more than a corresponding increase in rates will lower bond prices, given the changes in market yields changes in the bond prices are directly related to time to maturity. And the percentage price change that occurs as a result of the direct relationship between a bond's maturity and its price volatility, at a diminishing rate as the time to maturity increases; these are the different theorems what malkiel has given.

You see this example, it will be more clear what we have seen that, your maturity, your coupon, your yield, these are the different factors, which affect the bond pricing, and there are certain observation, what malkiel has found that also, we can also see through this example.

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Impact of Maturity on Bond Price Volatility

P. Value of an 8% (Coupon) Bond, Par value = Rs 1000/-

YTM	1Yr		10Yrs		20Yrs		30Yrs	
	7%	10%	7%	10%	7%	10%	7%	10%
P.V of Interest	75	73	569	498	858	686	1005	757
P.V. of Principal	934	907	505	377	257	142	132	54
Total value	1009	980	1074	875	1115	828	1137	811
% Change in total value of Bond		-2.9%		-18.5%		-25.7%		-28.7%

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Let us have, let us see how we can observe the maturity affects the bond pricing, the impact of maturity on bond price volatility, **impact of maturity on bond price volatility**, let us be calculating the present value, present value of an 8 percent coupon bond, coupon present value of an 8 percent coupon bond, and par value is here, **the par value is**, par value is equal to 1000.

In this case, let us take this as your yield to maturity, let this be your P V of interest, let this be your P V of principal of the bond pricing, let this be your total value, this is your percentage change in total value of the bond; this is the case, then how we can prove these theorems. Let us have 1 year term to maturity, then you have 10 years term to maturity, then you have 20 years term to maturity, then you have the 30 years term to maturity; you have the different bonds which are available to you, then you have to calculate this, that how this thing affects your pricing part. **If you**, this yield to maturity, let in each case, we have two different yields to maturity, we can take let the yield to maturity is 7 percent, this is 10 percent, this is 7 percent, this is 10 percent, 7 percent, 10 percent, 7 percent and 10 percent.

Then how much interest you can get in 1 year for a 1000 rupees bond, you calculate in 1000 rupees coupon is 80, then 80 rupees, and the present value of interest in this 1 year is 75 with 10 percent the present value will be 73; it is 8 percent coupon, that is why it is discounted at 7 percent 80 rupees has been discounted as 7 percent, here it is 80 rupees has been discounted 10 percent. So, already we know that interest rate has an inverse relationship with the price of the bond. So, it will be 569, this will be your 498, because these are all the 10 years, that means, 800 the coupon your discounting at 7 percent, here the 8 hundred has been discounted, at 10 percent in the 20 years, like that if you discount, it you will get 858, this will be your 686, this will be your 1005, this will be your 757.

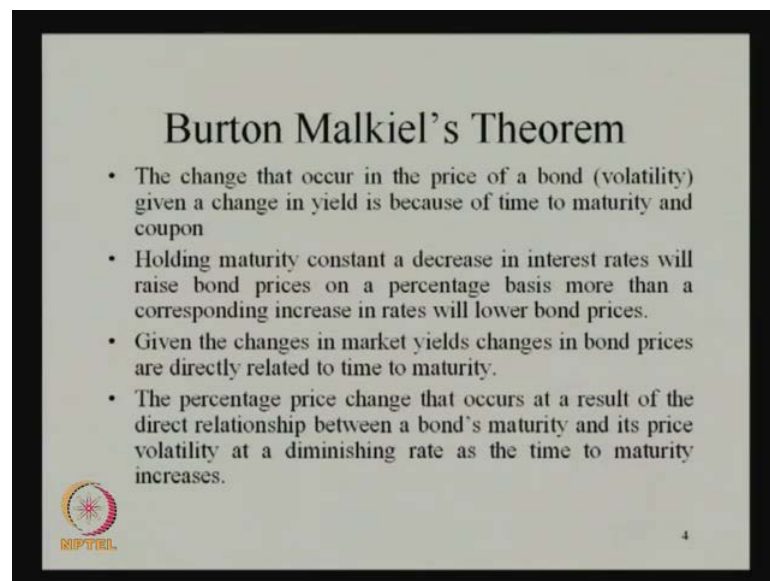
If you calculate the present value of the principal, principal amount is your 1000 rupees, that the par value what you have, if you discount it, it will be 93 for the 7 percent, it is 907 for 1 year case, it is 10 percent; 505 for 10 years if you discount it 377 discount it at 10 percent for 10 years, for 20 years, it is 257, 142 or 10 percent, here it is 132, and here it is 54.

If you calculate the total value this plus, this it will give you 1009, this will be 9080, this will be your 1074, this will be your 875, this will be your 1115, this will be your 828, this will be your 1137; this will be your 811.

So, then if you see that what is the change, whenever the yield to maturity has been changed from 7 percent to 10 percent, what we have observed the change is minus 2.9 percent, here it is minus 18.52 percent, here it is minus 25.7 percent, here it is minus 28.7 percent.

So, what generally we have seen here that given the changes in the market yields, changes in bond prices are directly related to time to maturity. So, **it will be**, once your term to maturity changing, your bond price volatility also changing, and it is increasing whatever way we have observed here.

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Burton Malkiel's Theorem

- The change that occur in the price of a bond (volatility) given a change in yield is because of time to maturity and coupon
- Holding maturity constant a decrease in interest rates will raise bond prices on a percentage basis more than a corresponding increase in rates will lower bond prices.
- Given the changes in market yields changes in bond prices are directly related to time to maturity.
- The percentage price change that occurs at a result of the direct relationship between a bond's maturity and its price volatility at a diminishing rate as the time to maturity increases.

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So, what generally we have seen that, whenever the market yield changes the bond prices are directly related to the time to the maturity, and another observation if you see here, what we have seen here the 40 point, if you see the percentage price change, that occurs as a result of the direct relationship between the bond's maturity and its price volatility at a diminishing rate as the time to maturity increases.

So, if you go on measuring this, you see it is from 2 minus 2.9 to minus 18.5, here it is around difference of 16 percent, but here it is around 7 percent, but here again it is again

decline to 3 percent. So, there is a diminishing rate, we will observe as we go on increasing the term to maturity. This is the theorem, what we can derive on the basis of the Burton Malkiel's concept.

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Effect of Coupons on Bond Price Volatility
P. Value of 20 Yr. Bond, Par Value = Rs 1000/-

YTM	0%		3%		8%		12%	
	7%	10%	7%	10%	7%	10%	7%	10%
PV of Int.	0	0	322	257	858	686	1287	1030
PV of Principal	257	142	257	142	257	142	257	142
Total Value of Bond	257	142	579	399	1115	828	1544	1172
% Change in Total Value			-44.7%	-31.1%	-25.7%			-24.1%

Then we have, we can observe that affect of change in coupon, affect of coupon on the bond price volatility. If you observe here we have taken a bond at the same the present value of a 20 years bond, where the par value is 1000 rupees.

In this case, if you observe this thing, what we will find, again you can take here YTM your present value of interest, your P V of principal, your total value of bond, then we have percentage change in total value. Then, if we see that the coupons from the beginning, let it was 0 percent, then it is let three percent, then 8 percent, then 11 percent, these are the different coupons, and the YTM, already we know we have taken this at 7 percent and 10 percent.

So, if you see here, it is 7 percent or 10 percent, say again 7 percent, 10 percent, 7 percent, 10 percent, 7 percent and 10 percent. If you see here, it is 00, there is no coupon, the present value of the principal will be for 0 percent coupon for 20 years; we have taken the 20 years term to maturity.

So, if the 20 years, if you see that, it will be 257, it will be 142 ,everywhere it will be same 257, 142, 257, 142, 257, 142, because we are only taking the 20 years bond, where

the rate of interest say 7 percent and 10 percent respectively. So, what is the total value here 257, it is 142, but here if you take 3 percent coupon, it will be 322 the present value, it will be 257, this will be 858, this will be 686, it is 1287, this is 1030, and this is total is 579, 399, this is 1115, this is 828, this is 1544, this is 1172. So, then, if you see this change it is minus 44.7 percent, this will be minus 31.1 percent, 25.7 percent minus 24.1 percent, what it means he is saying that holding maturity constant, a decrease in interest rates will raise the bond prices on a percentage basis more than a corresponding, increase in rates will lower the bond prices, that means, and another thing if you see other things equal the bond price fluctuation and bond coupon rates are inversely related.

If you keeping all these things constant, when your coupon is changing, your bond price volatility fluctuations are also changing. So, that is why equal bond rate fluctuate being equal, bond fluctuations and bond coupon rates are inversely related. So, we have an inverse relationship, we can observe the coupon is increasing the volatility is decreasing. So, that is the observation what we get from these at what the malkiel theorem was talking about.

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Impact of Yield on Bond Price Volatility
PV of 20yr bond, Coupon = 4%, Par value = 1000/-

	Lower Yield		Intermediate Yield		High Yield	
	3%	4%	6%	8%	9%	12%
YTM	3%	4%	6%	8%	9%	12%
PV of Int.	602	547	462	396	370	301
PV of Principal	582	453	307	208	175	97
Total value	1164	1000	769	604	545	398
% change		-14.1		-21.5		-27

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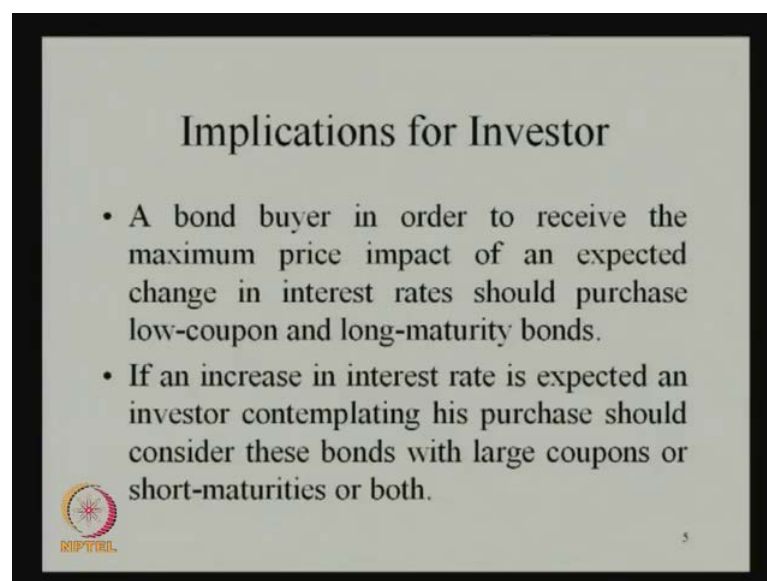
Then, another part is remaining that is on yield, **yield** impact of yield on bond price volatility, impact of yield on bond price volatility, where again we take the P V of 20 years bond coupon is 4 percent par value is 1000 rupees, and like that you can take your YTM, your view of P V of interest, your P V of principal, this is your total value, this is

your percentage change of the total value. Let you have taken the lower yield, then you have the intermediate yield high yield, then you can write that also high yield you can take this 3, but we can observe here, let your 3 percent was then it is increased to 4 percent, 6.28 percent, this is your that 9 percent to 11 percent.

So, the present value of the interest will be 602, this 547, in the 6 percent, it will be 462, it is 396, 370, and this is 301, then present value of principal 562, 453, this will be your 307, this will be your 208, this will be your 175, this will be 197, then this will be total will be 1164, this will be 1000, this will be 769, this will be 604, this will be your 545, this will be 398, then percentage change is 14.1, this is your minus 21.5, this is your minus 27.


So, what we observed here that holding maturity constant your maturity is 20 years, here holding maturity constant a decrease in interest rates will raise bond prices on a percentage basis more than a corresponding increase in rates will lower the bond prices. So, once we observe these things what we can say that, these are different ways, through which the bond price volatility works or if you increase your interest rate by 1 percent, whatever price volatility you can observe, **that will be less than the**, if you decrease your increase interest rate by 1 percent, that means, it will be more reactive towards the declining trend than the increasing trend.

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Implications for Investor

- A bond buyer in order to receive the maximum price impact of an expected change in interest rates should purchase low-coupon and long-maturity bonds.
- If an increase in interest rate is expected an investor contemplating his purchase should consider these bonds with large coupons or short-maturities or both.

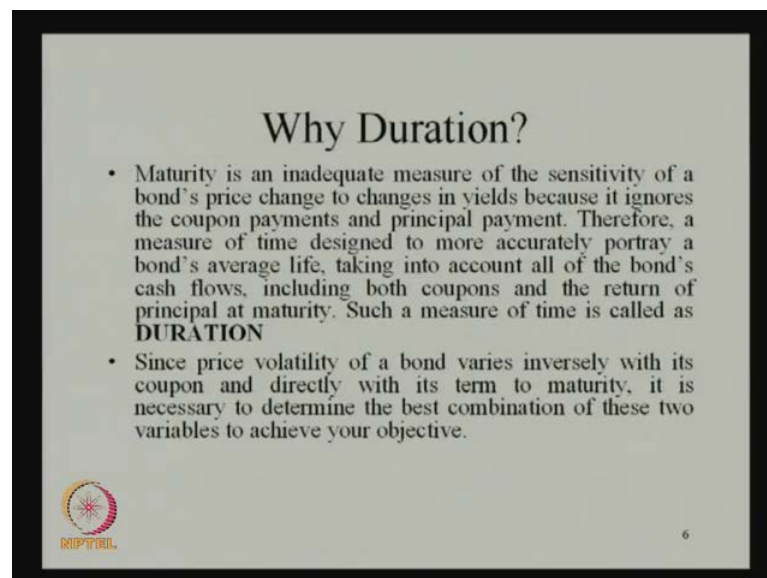
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Then, these are the different Malkiel's theorem, what we have observed from these, what kind of implications the bond investor can get.

The bond investor can get a bond buyer in order to receive the maximum price impact of an expected change in interest rates should purchase a low coupon and long maturity bonds, because these are highly sensitive towards this interest rate changes.

If an increase in interest rate is expected an investor is contemplating his purchase should consider these bonds with large coupons or short maturity or both, that means, these are the different implications through which the investor can use their anticipation or their expectations level to take the decision in the market.

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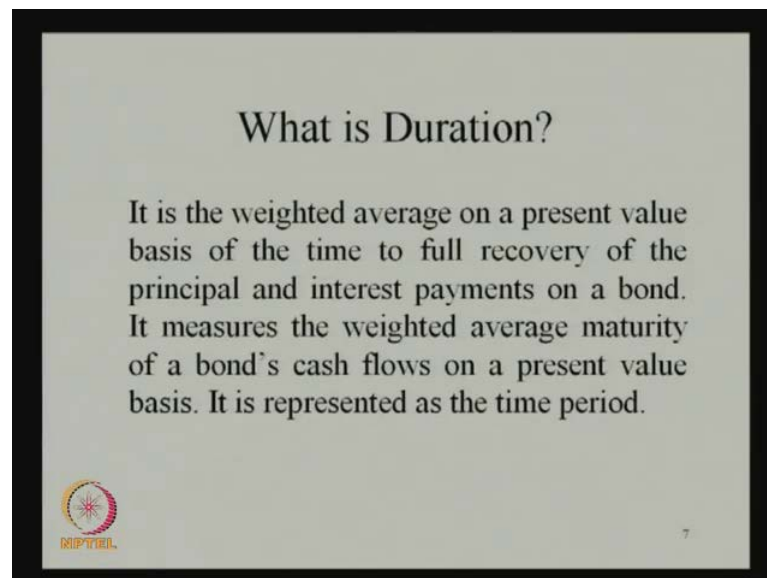
What we have observed that, if your bond price is fluctuating because of change in interest rate, change in coupon, change in yield, change in term to maturity etcetera, then what we can say that definitely those things are very much link to or sensitive to this interest rate risks, and solve the also the price risk which is basically happening because of the change in interest rate.

So, in this case what we can see there are different concepts, we use to minimize that interest rate risk in the bond pricing; in this context 1 is your duration, so, that is why, if you ask yourself why duration? The duration is used, because maturity is an inadequate measure of the sensitivity of a bond's price change to changes in yields, because it

ignores the coupon payments and principal payments. Therefore, a measure of time designed to more accurately portray a bond's average life taking into account all of the bond's cash flows including both coupons, and the return of principal at maturity such a measure basically is defined as the duration.

So, here, that means, already what I told that duration is the concept, which is used by the investor to minimize the risk in the market. Since price volatility of a bond varies inversely with a coupon and directly with its term to maturity, it is a necessary to determine the best combination of these two variables to achieve the objective. Therefore, the measure of the duration is basically takes into account, the both coupon and as well as the term to maturity, by which the present value, whenever we taken to account, then what will happens that it basically minimizes the interest rate risk in the market. So, a composite measure considering both coupon and maturity would beneficial for the investor.

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So, that is why what is maturity, **sorry** what is duration? **Duration** is basically, it is the weighted average on a present value basis of the time to full recovery of the principal and interest payments on a bond, it measures the weighted average maturity of a bond's cash flow on a present value basis; it is represented as the time period.

The representation of the durations time period, what is calculated on the basis of the weighted average on a present value of the cash flows, what we are expecting from a bond in a particular period of time; so, if you observe that, how it is measure.

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Macaulay Duration

Par value = ₹ 1000/- Assume, the mkt. int. rate = 8%, Coupon = 4%

Year (1)	Cash flow (2)	PV of Cash flow (3)	PV as % of Price (4)	(1) x (4)
1	40	37.04	0.0506	0.0506
2	40	34.29	0.0469	0.0938
3	40	31.75	0.0434	0.1302
4	40	29.4	0.0402	0.1608
5	40	27.22	0.0372	0.1860
6	40	25.21	0.0345	0.2070
7	40	23.34	0.0319	0.2233
8	40	21.61	0.0295	0.2360
9	40	20.01	0.0274	0.2466
10	1040	481.73	0.6585	6.585
		Total = 731.58	1.00	

So, this duration is basically defined as the Macaulay duration, we call it in the name of Macaulay, we call it the Macaulay duration, how is as measure, assume the market interest rate the market interest rate is 8 percent. And how it is measured, let this is your year, this is your cash flow, this is your present value of the cash flow, this is your present value as percentage of price of the bond, and this is 1, this is 2, this is 3 column, this is 4 th column, this will be your 1 into 4. What that basically, shows let there is a 10 years term to maturity 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, then what you can observe here, then the coupon is 8, basically the market interest rate is 8 percent.

Assuming market interest rate is 8 percent, and each year what is the coupon, if you observe here, another attach will be given to you, **the**, it is basically 4 coupon is, coupon is equal to 4 percent, coupon is equal to 4 percent. Then, each year you will have a cash flow 40 rupees, because **the** let the par value is equal to 1000, so then, you will yield 40 rupees, this is 40, this is 40, this is 40, this is 40, this is 40, this is 40, 40, 40, and the 10 years, if the term to maturity, and then, in this year the cash flow will be 1040, because it includes the principal also.

Then the present value of the cash flow, if you want to calculate from the table, then you can calculate it is 37.04, 34.09, this is 30, 31.75, and if you go on increasing your term to maturity, then automatically the present value of the cash flow will be declining, this is already you know plus 29.4.

This is your 27.22, this is your 25.21, this is your 23.34, this is your 21.61, this is your 20, 20.01, this will be your 481.73; the total will be 731.58, the total value. Then, if you calculate the present value as a percentage of the price, how we can calculate this, what is the percentage of the price, that is, we have to calculate that what is this percentage of the total cash flow present value as a percentage of the cash flow from these, then it will be basically, let present value here the 37.0 is a 0.0506.

Then, you have this is your 0.0469, 0.0434, this will be your 0.031, this will be your 0.0345, 0.0.295, this will be your 0.0274, it is 0.6528. So, total will be 1. **Sorry, 0 5 0 6 0 4 6 9 0 4 3 4.** So, this will be as well as 0 .6585. So, this will be your 0.274, 0274. So, here there is a mistake, **and this will be,** after this it will be 0.0402, this will be 0372, this will be your 0345, this will be 20, 0319, then this will be 0219, this will be your 0274, and it will be 1.00.

Then if you multiply with a 1 to 2 this, then you can get the value of this 0.0506, this will be 0.0938, this will be your 0.1302, 0.1603, 0.1860, 0.2070, 0.2233, then 0.2360, then 0.2466, 6.585, 8.1193 or it is approximately 8.12, the duration of the 10 years term to maturity the coupon is 4 percent, interest rate is 8 percent, it will be equal to the basically this duration will be 8012, and it is represent, in fact, years.

So, like that, if you change your coupon, let if your coupon will be now 8 percent, instead of 4 percent, if you make it 8 percent, then what will happen automatically the duration will go down?

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The Duration Measure

$$D = \frac{\sum_{t=1}^n \frac{C_t(t)}{(1+i)^t}}{\sum_{t=1}^n \frac{C_t}{(1+i)^t}} = \frac{\sum_{t=1}^n t \times PV(C_t)}{\text{price}}$$

Developed by Frederick R. Macaulay, 1938

Where:

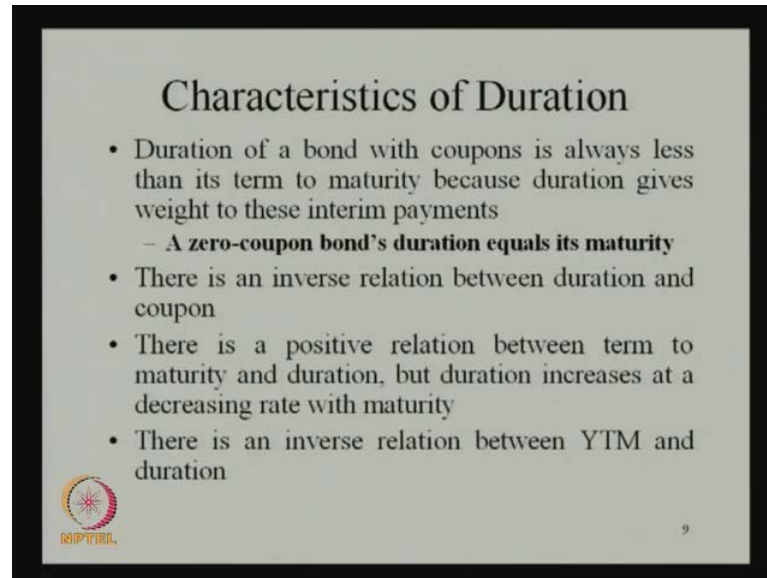
- t = time period in which the coupon or principal payment occurs
- C_t = interest or principal payment that occurs in period t
- i = yield to maturity on the bond

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So, this is the observation what this malkiel's, that is why it is written here, the duration is nothing but at the coupon, your discount in the coupon here divided by CT1 into present value of the coupon, and multiplied by the T the time period divided by price level, and that will give you the value of the duration of this particular bond.


So, **so**, this is your coupons, what you can get it, this divided by the price of the bond, that particular time market price of the bond, that will give you the duration in that, where this period T is equal to time period, in which the coupon or principal payment occurs, and the interest CT represents the interest of the principal payment, that occurs in period T i is equal to basically the yield to maturity on this particular bond.

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Characteristics of Duration

- Duration of a bond with coupons is always less than its term to maturity because duration gives weight to these interim payments
 - A **zero-coupon bond's duration equals its maturity**
- There is an inverse relation between duration and coupon
- There is a positive relation between term to maturity and duration, but duration increases at a decreasing rate with maturity
- There is an inverse relation between YTM and duration

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So, if you observe that which the different characteristics of duration are, duration of a bond with coupons is always less than its term to maturity, because duration gives weight to these interim payments, what the term to maturity does not give.

That is why we have certain risk can be minimized say through duration, which is not possible, in the case of term to maturity, and another thing is a 0 coupon bond's duration equal its maturity; that means, if there is no coupon involved in that, then the duration of that particular bond which exactly equal to its maturity level. There is an inverse relation between duration and the coupon, what already now we told it, that if it coupon will be more than the value of the bond will be or we can say the duration of the particular bond will be less.

There is a positive relation between terms to maturity and duration, but duration increases as a decreasing rate with maturity. There is a positive relation between term to maturity and duration, because the term to maturity will be more, the duration also will be more, but duration increases at a decreasing rate with maturity. There is an inverse relationship between yield to maturity, and the duration, if yield to maturity will be more then the value of the duration also goes down, because if you discount it with yield to maturity of the cash flows, what we are getting from there that basically declines.

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**Modified Duration and Bond Price
Volatility**


An adjusted measure of duration can be used to approximate the price volatility of a bond

$$\text{modified duration} = \frac{\text{Macaulay duration}}{1 + \frac{\text{YTM}}{m}}$$

Where:

m = number of payments a year

YTM = nominal YTM

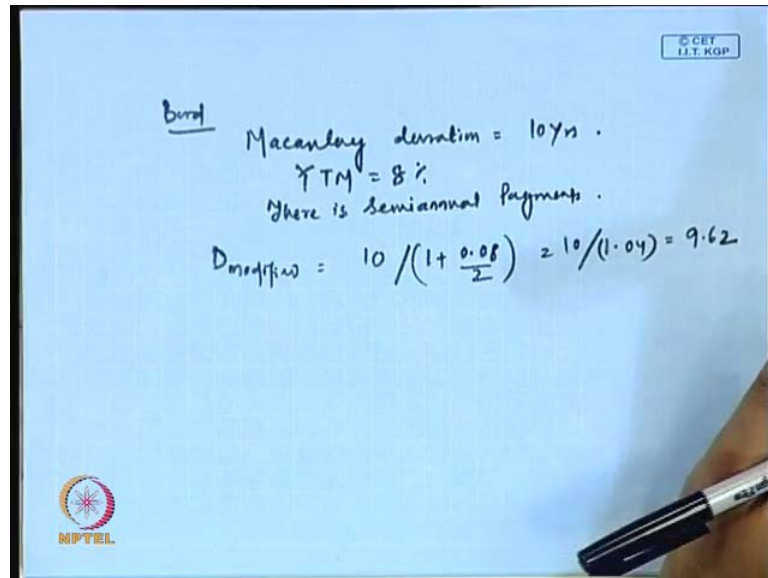


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So, in this context, we have another theme also or that we use in the financial market or the bond market. So, what generally we see that, we call it the adjusted, adjusted measure of duration or the modified duration, what this modified duration is basically is.

The modified duration is basically measured as the Macaulay duration, what we have measured divided by the 1 plus yield to maturity divided by the number of payments in a year, for example, if it is a semiannual payment, in this case, for example, a bond, there is a bond, **bond, where the Macaulay duration is**, Macaulay duration is 10 years, and your yield to maturity at 8 percent, and there is semiannual, there is semiannual payments of interest.

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Bond
Macaulay duration = 10 yrs.
 $Y_{TM} = 8\%$
There is semiannual payments.
 $D_{modified} = 10 / \left(1 + \frac{0.08}{2}\right) = 10 / (1.04) = 9.62$

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Then what will happen, the modified duration will be d , modified will be 10 by 1 plus 0.08 by 2, that will give you 10 by 1.04, then this will be a 9.62. So, the modified duration basically is nothing but, **it is the**, we adjusted measure of a duration which is adjusted towards the number of payments in a particular year.

So, what generally we can see that, whenever the modified duration or the price of a particular bond will vary with proportionally with the modified duration for a small change in the data, what it basically shows that, that bond price movements will vary proportionally with modified duration for small changes in the yield, if there is a small change in the yield, then the modified duration also will change, and it proportionally it will change with the bond price movements.

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Modified Duration and Bond Price Volatility

- Bond price movements will vary proportionally with modified duration for small changes in yields
- An estimate of the percentage change in bond prices equals the change in yield time modified duration

$$\frac{\Delta P}{P} \times 100 = -D_{\text{mod}} \times \Delta i$$

Where:

- ΔP = change in price for the bond
- P = beginning price for the bond
- D_{mod} = the modified duration of the bond
- Δi = yield change in basis points divided by 100

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So, if you want to estimate of a percentage change in bond prices, which basically equals the change in yield time the modified duration. So, if you want to know, what is the percentage change in bond price, you can multiply the modified duration with change in yield, that basically will give this, what exactly the price has been changed.

So, here, **if you**, it is measured as basically delta P by P into 100 is equal to the modified duration multiplied by delta i, delta P is the change in price of the bond beginning, P is equal to the beginning price for the bond, it is the D modified duration, delta i represents the yield change in basis points which divided by the 100.

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Bond
Macaulay duration = 10 yrs.
YTM = 8%
There is semiannual payments.
 $D_{modified} = 10 / \left(1 + \frac{0.08}{2}\right) = 10 / (1.04) = 9.62$

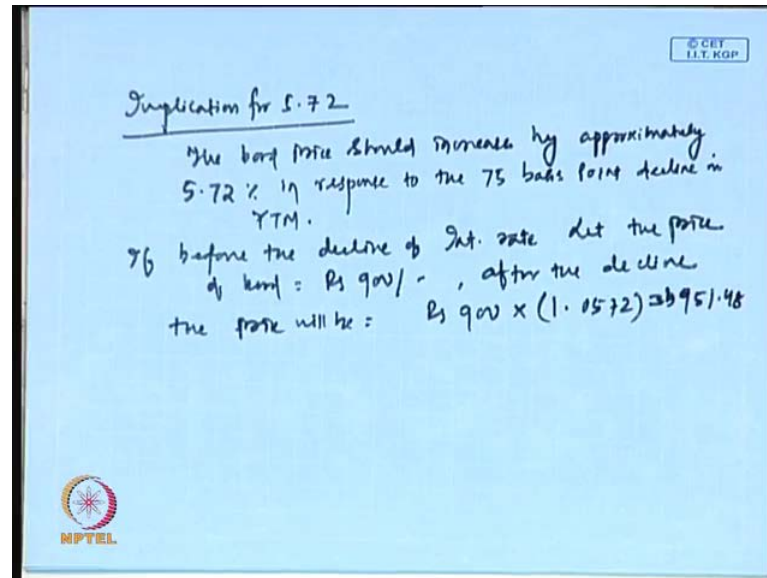
Example
Bw, $D = 8$ yrs. $i = 10\% = 0.10$
Let YTM has decline by 75 basis point
(10% to 9.25%)
 $D_{modified} = 8 / \left(1 + \frac{0.10}{2}\right) = 8 / 1.05 = 7.62$
% change in the price of the bond = $-7.62 \times \frac{-75}{100}$
 $= 5.72 = (-7.62)(-0.75)$

Let if you observe here, for example, consider a bond, where the, example if you see consider a bond, where the D is the Macaulay duration is 8 years, and your interest rate is equal to 10 percent, 0.10. Let the YTM, YTM has declined to decline by 75 basis point, then, that means, it has declined from 10 percent to 9.25 percent, then how you can calculate the modified duration.

So, the D modified basically nothing but if that 8 divided by 1 plus 0, if you want to calculate the modified duration 0.10 by 2, that will be 8 by 1.05, that will be 7.62. So, the modified duration of the particular bond is a 667.62, but whenever the I want to calculate the percentage change in the price of the bond, then percent change, change in the price of the bond, if you want to calculate directly, it is basically, that means, minus 7.62 multiplied by the minus 75 by 75 basis point divided by the 100, that will give you minus 7.62 into minus 0.75, that we that will be basically give you value of 5.72.

So, this indicates that the price should increase by approximately 7.72 percent in the response to the 75 percent basis point of the decline of the yield to maturity. So, therefore, if the price of a bond, before the decline in interest rate was 900, then the price after the decline will be after the decline in interest rate should be 900 multiplied by 1.05.

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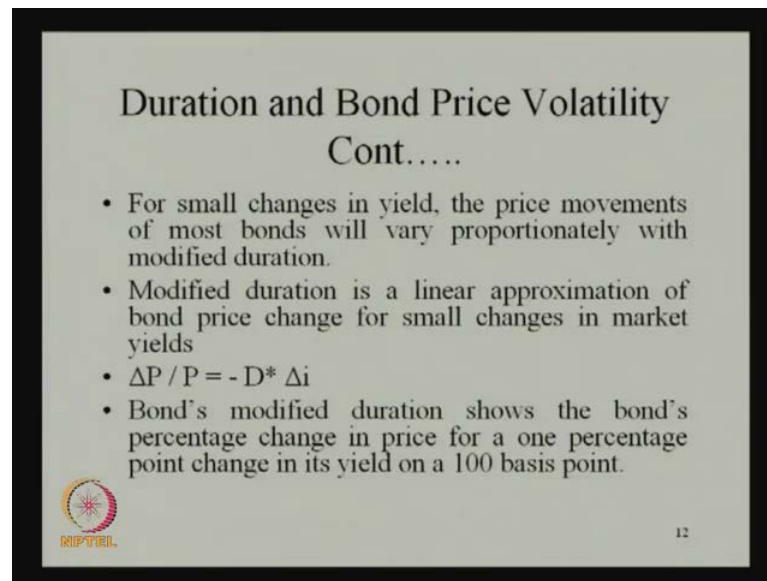


Let what it basically this 5.72 implies, the 75.72 basically implies that the bond price implications, implication for 5.72. What it basically shows that the bond price should increase, bond price should increase by approximately 5.72 percent in response to the 75 basis point declined in YTM.

If before the decline, **before the decline** of interest rate, let the price of bond is 900 rupees, then after the, **after the** decline, the price will be 900 multiplied by 1.0572, which give you 951.48, that means, the value of the bond will be increasing.

So, that is why it basically establishes the relationship between the value of the bond and the change in interest rates, and if you know the value of the modified duration, then you can measure that what is the percentage change in the price volatility or percentage change in the price of this particular bond.

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The slide is titled "Duration and Bond Price Volatility Cont...." and contains the following text:

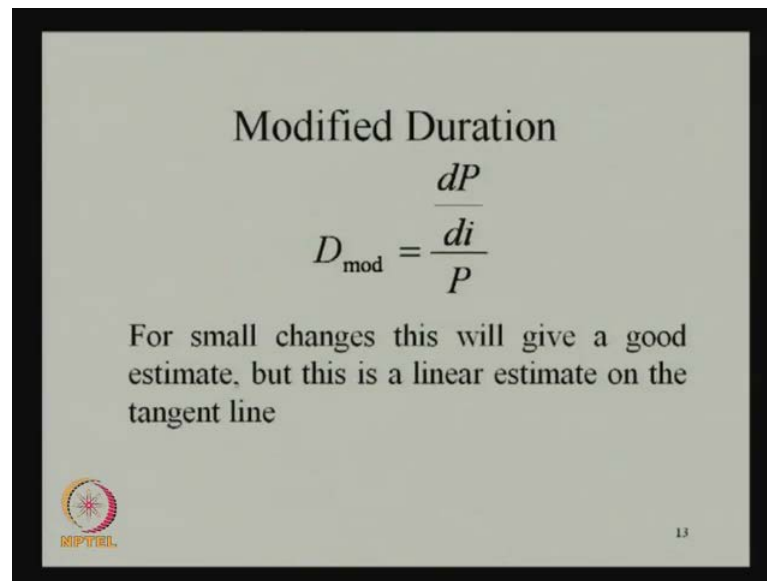
- For small changes in yield, the price movements of most bonds will vary proportionately with modified duration.
- Modified duration is a linear approximation of bond price change for small changes in market yields
- $\Delta P / P = - D^* \Delta i$
- Bond's modified duration shows the bond's percentage change in price for a one percentage point change in its yield on a 100 basis point.

In the bottom left corner, there is a circular logo with a sunburst design and the text "RIIPYTEL" below it. In the bottom right corner, the number "12" is displayed.

Then accordingly you can also measure that, what exactly the modified duration is, and how it is used to calculate the change in the price. So, then what we have seen that for small changes in yield, the price movements of most bonds will vary proportionately with modified duration, what we have observed now from this example, that is why the modified duration is a linear approximation of bond price change for small changes in the market yields. So, if you want to explain basically, what exactly this modified duration is.

That the modified duration is nothing but it is a linear approximation of bond price change for small changes in the market yields, that is why, you call it delta P by P is equal to minus D modified duration into delta i. So, the bond's modified duration shows the bond's percentage change in the price for a 1 percent point change in the yield, and a 100 basis point; so, that basically this defined as the modified duration.


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Modified Duration

$$D_{\text{mod}} = \frac{\frac{dP}{di}}{P}$$

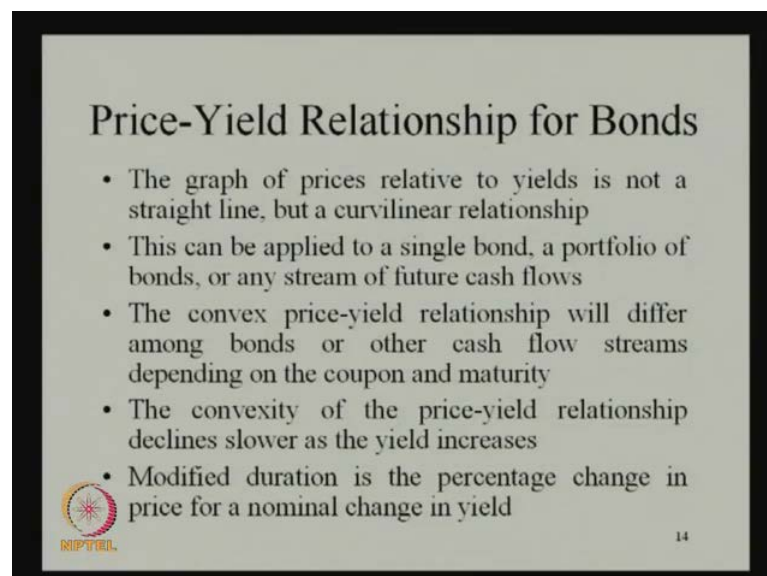
For small changes this will give a good estimate, but this is a linear estimate on the tangent line



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So, if you want to exactly see that how this modified duration is measured, it is nothing but the positive derivative of the change in the price of this particular bond; it is this dP by di in by P , that means, for small changes, this will give you good estimate, but this is a linear estimate on the tangent line.

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Price-Yield Relationship for Bonds

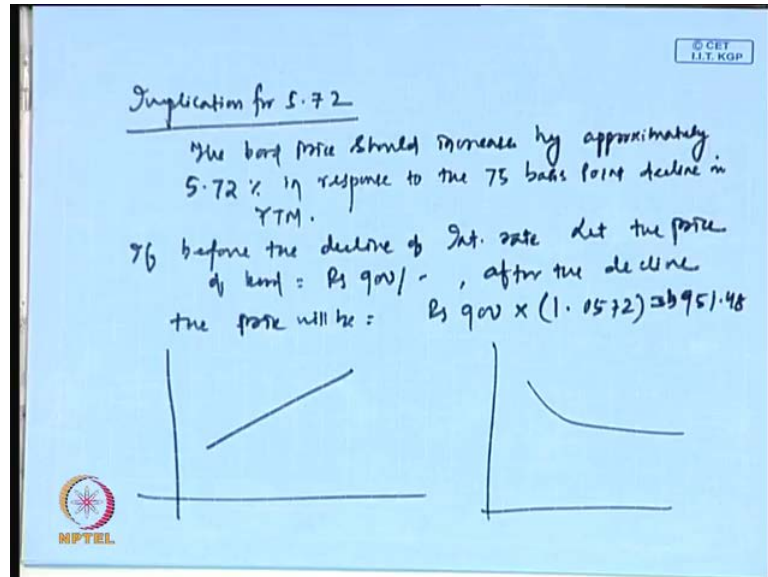
- The graph of prices relative to yields is not a straight line, but a curvilinear relationship
- This can be applied to a single bond, a portfolio of bonds, or any stream of future cash flows
- The convex price-yield relationship will differ among bonds or other cash flow streams depending on the coupon and maturity
- The convexity of the price-yield relationship declines slower as the yield increases
- Modified duration is the percentage change in price for a nominal change in yield



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The order derivative of the change in the price of the bond, it basically gives you the modified duration. So, the graph of price relative to yield is not a straight line, but it is a curvilinear relationship, already you know that, what is the, how the yield price relationship shows, how it have the curve looks like. So, therefore, this can be applied to a single bond a portfolio of bonds or any stream of the future cash flows.

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Basically, this relationship is not a straight line, it is basically this curve.

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Price-Yield Relationship for Bonds

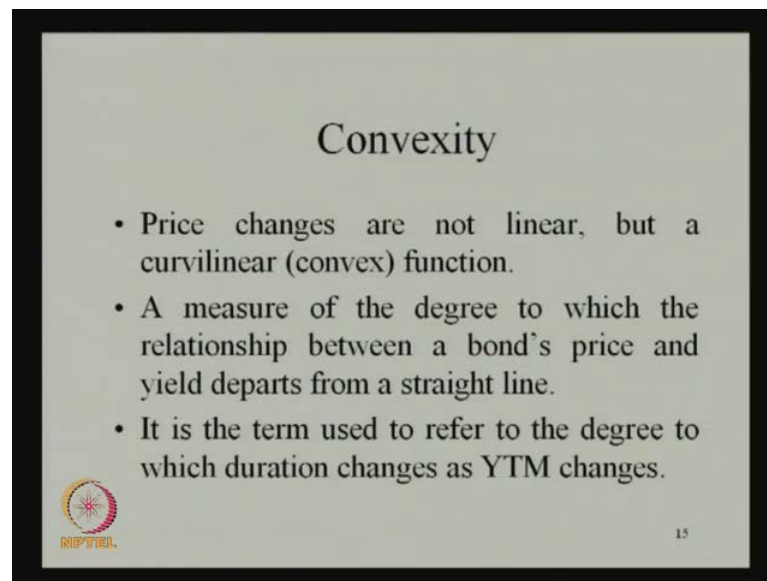
- The graph of prices relative to yields is not a straight line, but a curvilinear relationship
- This can be applied to a single bond, a portfolio of bonds, or any stream of future cash flows
- The convex price-yield relationship will differ among bonds or other cash flow streams depending on the coupon and maturity
- The convexity of the price-yield relationship declines slower as the yield increases
- Modified duration is the percentage change in price for a nominal change in yield

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So, if you want to show this, then how this particular curve will look like, and why this curve basically looks like this. So, that is the question always we have in our mind. So, in this context, we should know that what it exactly means, it means, basically this we **know the**, or we can infer from these, that are this theoretical considerations, basically conclude that, there is a convex relationship between these two.

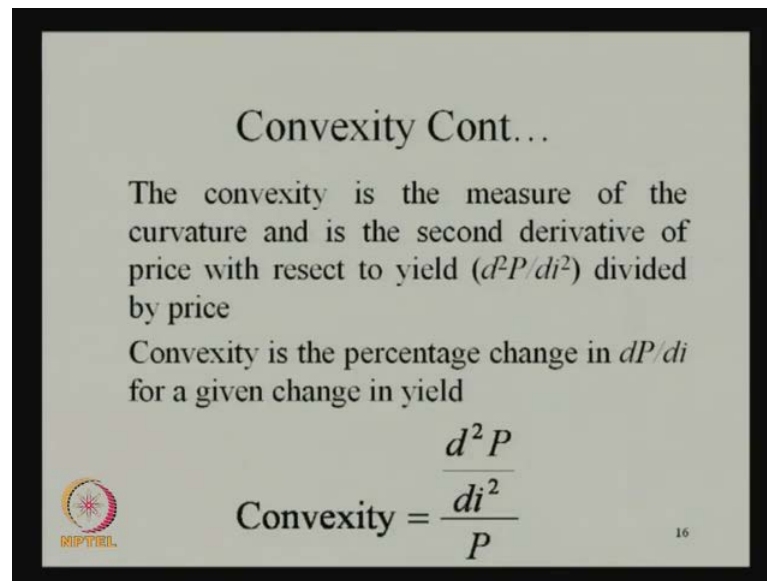
Therefore, the convex price yield relationship basically will differ among bonds and other cash flow streams, depending on the coupon and maturity. So, the convexity of the price yield relationship declines slow slower as the yield increases, if your yield will increase interest rate will increase the convexity of the price yield relationship declines. So, modified duration is the percentage change in price for a nominal change yield that already we have discussed.

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So, if you assume that, the price change are not linear or price changes are not linear, but a curvilinear function or the convex function, then in this case a measure of the degree to which the relationship between a bond's price, and the yield departs from a straight line, generally it is defined as the convexity. So, it is the term used to refer to degree to which the duration changes, as the yield to maturity changes; if your yield to maturity or the interest rate changes, how this duration also changes accordingly, that basically explains the concept of convexity, in the bond price volatility case.

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The slide is titled "Convexity Cont...". It contains the following text: "The convexity is the measure of the curvature and is the second derivative of price with respect to yield (d^2P/di^2) divided by price". Below this, it says: "Convexity is the percentage change in dP/di for a given change in yield". The formula for convexity is given as
$$\text{Convexity} = \frac{d^2 P}{di^2 P}$$
. In the bottom left corner, there is a logo for "RIIPITEL" and in the bottom right corner, the number "16" is displayed.

So, what basically the convexity, then the convexity is nothing but it is the measure of the curvature, and it is the second derivative of the price with respect to yield divided by the price of the bond. So, convexity is the percentage change in dP/di change in the price of this particular bond for a given change in the yield. So, that is why how the convexity is defined, the convexity is nothing but it is defined as basically $d^2 P/di^2$ by P , it is a second order derivative of change in the price due to change in the yield.

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Determinants of Convexity

- Inverse relationship between coupon and convexity
- Direct relationship between maturity and convexity
- Inverse relationship between yield and convexity

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So, which are those factors or the determinants, which are basically measure this convexity. There is inverse relationship between coupon and convexity keeping your yield to maturity constant. There is a direct relationship between maturity and convexity keeping your yield and coupon constant, and there is an inverse relationship between yield and convexity, keeping your coupon and maturity constant.

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$$\text{Convexity} = \frac{\frac{d^2 P}{di^2}}{P}$$

Price-yield curve is more convex at its lower yield segment.

NPTEL

That the price yield curve is more convex as it lowers yields segment; that means, the price yield curve price yield curve is more convex, **is more convex** at its lower yield

segment, **lower yields segment**. So, this is the definition of the convexity, and how this convexity can be measured.

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Measurement of Convexity

$$\text{Convexity} = \frac{d^2p}{di^2} \cdot \frac{1}{P}$$

$$\frac{d^2p}{di^2} = \frac{1}{(1+i)^2} \left[\sum_{t=1}^n \frac{Cf_t}{(1+i)} (t^2+t) \right]$$

$$= \frac{d^2p/di^2}{\text{PV of Cash flow}} = \frac{d^2p/di^2}{\text{Price}}$$

So, if you want to compute or measure this convexity, how we can measure it, so, let we call it the measurement of the measure of the convexity. How the convexity can be measured, the convexity basically it looks little bit complex, but it can be broken into different steps. So, how generally the convexity, let already we know, that you know the convexity is nothing but it is this d square p by your d i square by p.

So, if you want to get this derivative d square p by d i square, it is nothing but it is 1 by 1 plus i square, then you take t is equal to 1 to n, it is your cash flow t to measure your price 1 plus i into t square plus t, if you calculate this derivatives, we can get this. And what this is particular thing **is means basically measures, this what basically here**, if you want to measure it the convexity is nothing but it is the it is basically d square p by d i square divided by the present value of the cash flow, which is nothing but the d square p by d i square by the price.

Then, this part we know that d square p by d i square is equal to this, then what is the convexity, the convexity, if you want to measure, then how it can be measured, this is the way it can be measured, let you take example to measure the convexity, how this example basically look like.

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Example: 3 Yr. Bond, 12% Coupon, 9% YTM

Year	Cf _t	PV of cashflow	t ² +t	3x4
1	2	3	4	
1	120	110.09	2	220.18
2	120	101	6	606.00
3	120	92.66	12	1111.92
3	1000	772.2	12	9266.4
		<u>Price = 1075.95</u>		<u>11204.5</u>

$$\frac{1}{(1+i)^2} = \frac{1}{(1.09)^2} = 0.84$$

$$\text{Convexity} = \frac{11204.5 \times 0.84}{1075.95} = \frac{9411.78}{1075.95} = 8.75$$

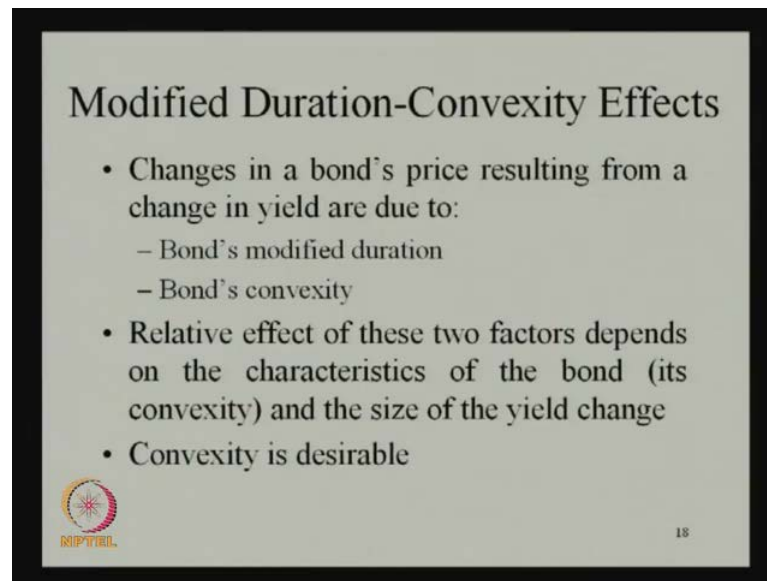
Let there is three year bond, there is three year bond, your 12 percent is your coupon, then your 9 percent, your yield to maturity, then let this is your year, this is your cash flow, this is your present value P V of cash flow lend, this is your let t square plus t, this is 1, 2, 3, 4.

Then, let this is your value is 3 into 4, then this will at 1, 2, 3, 3, let this is your 120, 3 years bond 120, 120, 120, and end of the day difference value will be 1000. Then, the present value will be your 100.09 at the rate of 9 percent, this is your 101, this is your 92.66, this is your 7700, and 72.2, then your price is basically 1075.95.

T square t, t is nothing, but the time a 1, then 1, 1, 1 square plus 1 is equal to 2, this will be 6, this will be 12, this will be 12, then if you multiply these two, you will find 220.18, this will be 606.00, then your 111.92, then 9266.1, and finally, the value will be 11204.5.


So, then you can calculate 1 by your 1 plus i square, that is your 1 by 1.09, that will be 0.84, then the value will be 11204.5 into 0.84, that will give you 9411.78. Then, the convexity is equal to 9411.78 divided by 1075.95 that will be 8.75; this is the way the convexity has been measured.

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Modified Duration-Convexity Effects

- Changes in a bond's price resulting from a change in yield are due to:
 - Bond's modified duration
 - Bond's convexity
- Relative effect of these two factors depends on the characteristics of the bond (its convexity) and the size of the yield change
- Convexity is desirable

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Then, how these effects basically we can calculate, if you observe that changes in bond's price resulting from a change in yield are basically due to the bond's modified duration and bond's convexity that already we have observed.

And the relative effects of these two factors basically depends on the characteristics of the bond; that means, it is convexity, it is the coupon, it is the yield, etcetera, and the size of the yield change, how the change in the yield basically happens, what is the total change in the yield.

And we can conclude that or we can say that convexity desirable to get the return from the market or to maximize the benefits from the bond investment. So, in this context what we can say that, whenever we talk about the measurement of the convexity, which basically talks about the risk measure of this bond price volatility or to if you say that volatility of the bond price is nothing but the bond price risk.

And for that how we can measure those kind of risk that basically is defined by the duration or the modified duration and the convexity, and this is the way, generally, we can measure the bond price volatility, and these particular concepts basically more used in the bond investment strategy in the financial market, and how basically it will be used, and how this investor basically maximize the return by using those techniques or those

estimations, that we will be discussing in the discussion on the bond portfolio strategy in the next sessions. Thank you.