

## **Security Analysis and Portfolio Management**

**Prof. J. Mahakud**

**Department of Humanities and Social Sciences**

**Indian Institute of Technology, Kharagpur**

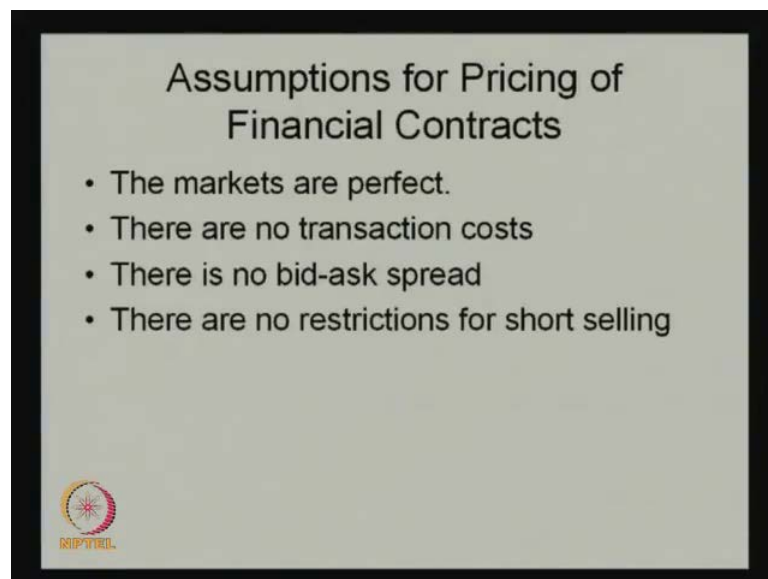
**Module No. # 01**

**Lecture No. # 38**

**Derivatives - II**

In the previous class, we discussed about the different derivatives instruments what generally we use for investment in the market. So, today we will be discussing about that, how generally the pricing or the valuation of those derivatives take place and how this pricing is done by the investor before going to take this decision, whether he should invest in this particular instruments or not.

(Refer Slide Time: 00:48)

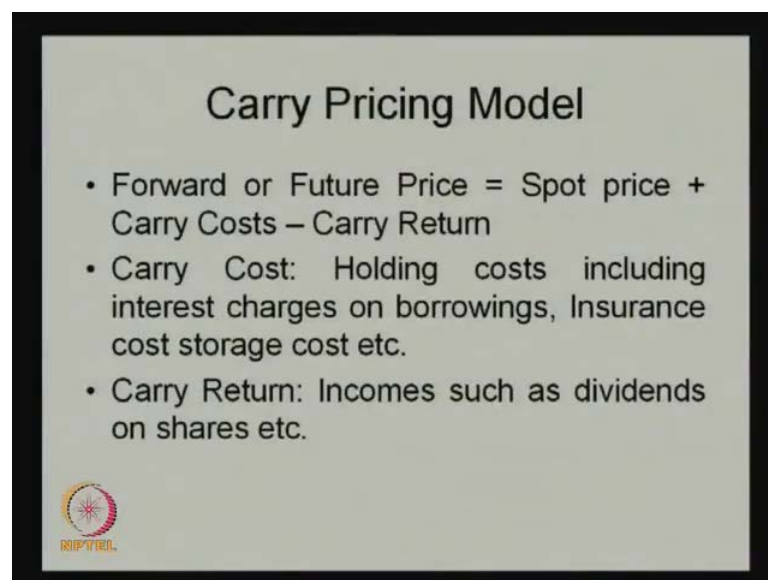


So, before going to talk about the pricing of these financial contracts with respect to the derivatives, there are certain assumptions always we take, before the pricing part. This assumptions are basically we can take that the markets are perfect; that means, that should not be any information gap between the different stakeholders of this particular market or a different participants in this particular market.

And there are no transaction cost involved in this, there is no bid-ask spread; that means, always bid price is equal to the ask price; that means, we can say the market is information point of with market is totally efficient and there are no restrictions for short selling; that means, the short selling is allowed.


So, more or less the market should operate in a particular environment, where the market is basically highly perfect and every investor has the same type of information or same level of the information by which, by using this information, they can go for the valuation of this particular derivative instrument easily. That is the basic philosophy or the basic assumptions what generally always we take, whenever you talk about the valuation of the derivatives in the financial market.

(Refer Slide Time: 02:12)



**Carry Pricing Model**

- Forward or Future Price = Spot price + Carry Costs – Carry Return
- Carry Cost: Holding costs including interest charges on borrowings, Insurance cost storage cost etc.
- Carry Return: Incomes such as dividends on shares etc.



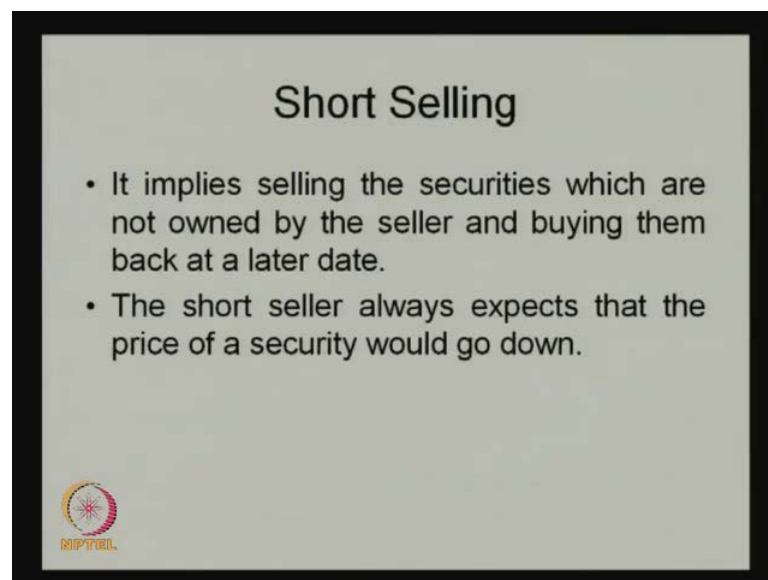
Then, which are the models we use? Basically there are different models we use for valuation of this forward or the future price. Generally if you talk about this concept of these particular different models.

Let us see, let us start with the pricing of the forward or the future then we will go add with these options. How these are all the future price determined? According to the carry pricing model, there is a model we call it carry pricing, the carry pricing model says that, the future price is equal to this spot price plus the carry cost minus the carry return. Then that the question will arise, what exactly the carry cost is. The carry cost is nothing but, it

is holding costs including the interest charges on borrowings, insurance cost, and storage cost etcetera. That means, to carry the financial asset, to keep this financial particular asset **on which** from which the particular pricing of the derivatives has been derived, whatever cost we are incurring, that is basically defined as the carry cost.

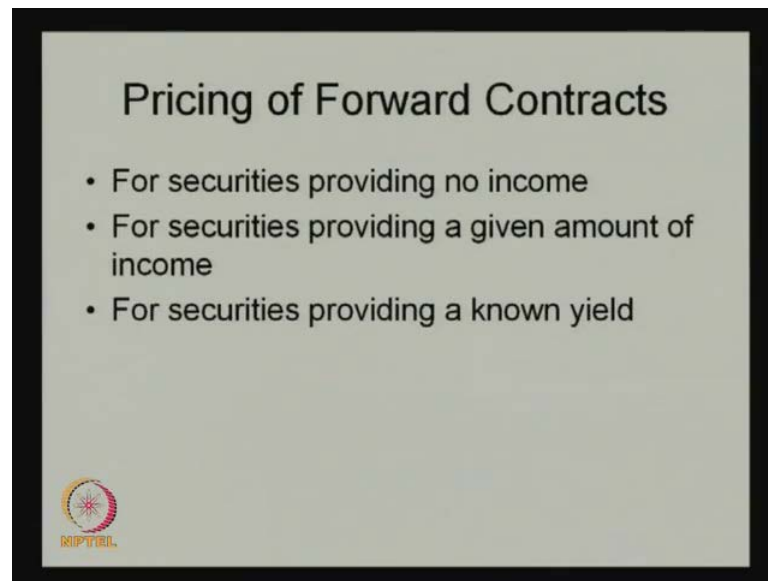
Then, what is the carry return? The carry return is whatever income we are deriving from this particular asset for example, you say that **if you** it is equity, then we get the dividends and as well as if you **talk about the we** talk about the bonds, then we always think about the coupons.

(Refer Slide Time: 03:42)



Then before going to discuss, let us see what exactly the short selling is, before we have that is assumption what we have taken. Short selling is basically implies, selling the securities which are not owned by the seller and buying them back on a later date. So, that is why the short seller always expects that, the price of the security should go down, which basically against this normal pricing, the philosophy of this particular investor, whenever we expect that the price should go up.

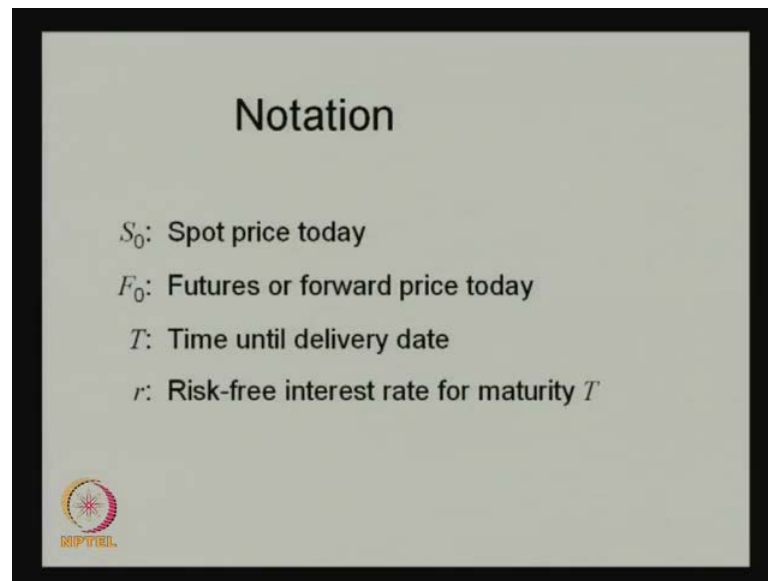
(Refer Slide Time: 04:08)



So, the whenever we talk about the pricing part of this forward contracts, let us see that how generally there are different ways the pricing is done. One is for securities providing no income and another ways for securities providing given amount of the income and for the securities providing a known yield. Yield means, we talk about that what is a return you are going to get from these particular instruments, what you are using for the financial market for the investment.

So, therefore, there are three ways or three conditions on which, we should derive this pricing of that particular forward contract at a particular time. So, one is for securities providing no income, for securities providing a given amount of the income and for securities where we know this, what kind of yield we are getting.

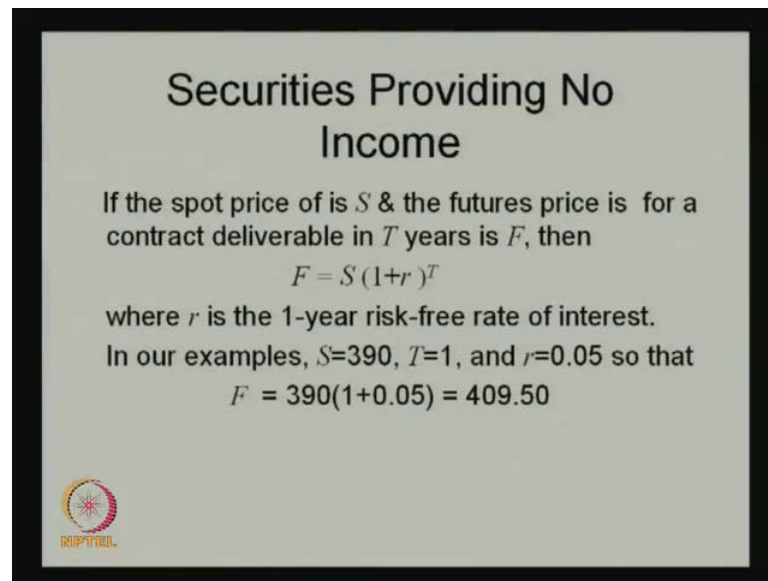
(Refer Slide Time: 05:10)



So, before that, we are using certain notations or using certain abbreviations for this developing this model. So, let us see, the notations which we always use whenever we go for the pricing of the securities or the pricing of the financial contracts in terms of the derivatives. So, one is already we have use that, we have a notation called the spot price.

So, that is noted as  $S_0$  in the current period or another one is the future, what we are deriving over a estimating the future or the forward price today, what your anticipating. And  $T$  is basically time, until the delivery date and we have  $r$  which basically, the risk free rate of interest or the risk free rate **risk free rate of interest** at the time of maturity for maturity  $T$  **or maturity  $T$**  what we have taken into account here.

(Refer Slide Time: 06:41)




**Securities Providing No  
Income**

If the spot price of is  $S$  & the futures price is for a contract deliverable in  $T$  years is  $F$ , then

$$F = S(1+r)^T$$

where  $r$  is the 1-year risk-free rate of interest.  
In our examples,  $S=390$ ,  $T=1$ , and  $r=0.05$  so that

$$F = 390(1+0.05) = 409.50$$


Then one by one let us see that, how this particular pricing is done? Let if the spot price is  $S$  or we have taken it is  $S_0$  and the future price is for a contract delivery in  $T$   $S$  is  $F$ , which is you have taken at  $F_0$ . Then how this future price is determined? It is basically your  $F_0$  is equal to  $S$  into  $1$  plus  $r$  to the power  $T$ .

That means, what we have taken that the future price, at least we should calculate on the basis of the minimum return what we can expect from the market at that particular time and that minimum return is nothing but, that this is the risk free rate of interest, what we can derive in that particular time.

So, here if you see an example, let your  $S_0$  is basically  $390$  and  $T$  is equal to year  $1$  and your  $r$  is equal to  $5$  percent that means  $0.05$ . So, then obviously, your  $F$  will be your  $390$  into  $1$  plus  $0.05$ , then obviously, it will be if it is more than the period will be more than  $1$  then you could have this particular notation here, then it will be  $409.5$ . That means we are expecting that, if the securities provide no income that means, the minimum income is whatever rate of interest we are getting in terms of the risk free rate.

(Refer Slide Time: 08:19)

**When Interest Rates are Measured with Continuous Compounding**

$$F_0 = S_0 e^{rT}$$

This equation relates the forward price and the spot price for any investment asset that provides no income and has no storage cost

If  $F_0 > S_0 e^{rT}$  Investor may buy the asset by borrowing an amount equal to  $S_0$  for a period of  $T$  at the risk free rate, and take a short position in forward contract. At the time of maturity, the assets will be delivered for a price of  $F$  and amount borrowed will be repaid by paying an amount equal to  $S_0 e^{rT}$  and the deal would result in a net profit of  $F_0 - S_0 e^{rT}$

If  $F_0 < S_0 e^{rT}$  Investor would short the assets, invest the proceeds for the time period  $T$  at an interest rate  $r$  and long a forward contract. When the contract matures the asset would be purchased for a price of  $F$  and the short position in the asset would be closed out. This would result in a profit of  $S_0 e^{rT} - F_0$

Accordingly we can decide that what should be the price of that particular future in a particular time period, but this is the simple interest rate what we have taken into account. But, for example, the interest rates are measured with continuous compounding, continuous compounding what basically happens in the banking sector or any other financial sector.

Then what will happen? Instead of using this  $S_0$ , we have taken in the simple case we have taken  $F_0$  is equal to  $S_0$  into  $1 + r$  to the power  $T$ , but whenever we take this compounding interest rate, then it should be  $F_0$  is equal to  $S_0$  into  $e$  to the power  $rT$ .

What this equation basically reflects? This equation basically reflects that it relates the forward price and the spot price for any investment asset, that provides no income and has no storage cost that already we know. So, if your  $F_0$  or the future price today is greater than this spot price into  $e$  to the power  $rT$ , then investor may buy the asset by borrowing an amount equal to  $S_0$  for that period  $T$ , at a risk free rate and take a short position in forward contract.

So, at the time of maturity, the assets will be delivered for a price of  $F$  and the amount borrowed will be repaid by paying an amount equal to  $S_0 e$  to the power  $rT$ , then the deal would result in the net profit, which will be  $F_0$  minus  $S_0 e$  to the power  $rT$ . That

means this is your predicted value and if this is your actual value, then if you know that this actual value will be greater than the predicted value, then what the investor should do? The investor should buy the asset may be you have to borrow the money and take the short position in the forward contract.

So, at the time of this, if your  $F_0$  is greater than this, then he will make the profit of the total profit he can earn that is your  $F_0$  minus  $S_0 e^{rT}$  that is the logic what this particular investor is trying to use. So, if  $F_0$  is less than this  $S_0 e^{rT}$ , then the investor would short the assets, invest the proceeds for the time period  $T$  at an interest rate  $r$  and long a forward contract, when the contract matures the asset would be purchased for a price of  $F$  and the short position in the asset would be closed out, this would result in a profit of  $S_0 e^{rT}$  minus  $F_0$ .

So, generally this is the logic what we use in the equity market, the same kind of logic is used here also, we are comparing between the actual price and the expected price and if you assume that, the actual price will be less than expected price or the actual price will be more than the expected price, accordingly the investor takes the position in the market and from that he can earn some profit.

So, in second case basically, if your  $F_0$  is less than  $S_0 e^{rT}$ , then what will happen? The total profit will be **total profit will be**  $S_0 e^{rT}$  minus  $F_0$ . So, when on investment asset provides a known income, basically we know that this is the asset we use in the market in terms of the preference shares, then what happens? Because preference share if you already know this, what do you mean by the preference shares, preference share is basically provides the fixed amount of the income at a regular interval or in a specific time period.

So, therefore, what we can say? That preference share is more or less the characteristics are also close to the debt kind of instruments than the equity instruments. That is why if the particular investors who own the preference share of a company they do not have any kind of what we can say, the voting rights or the ownership on this particular company.




(Refer Slide Time: 12:43)

When an Investment Asset Provides a  
Known Income (preference Share)

$$F_0 = (S_0 - I)e^{rT}$$

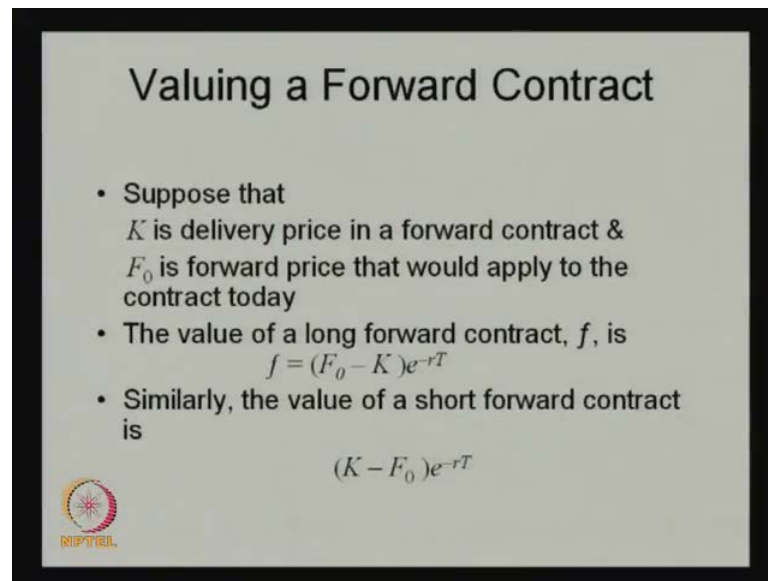
where  $I$  is the present value of the income



If you know this known income, then how the pricing is done? The pricing is done in this way, here your  $F_0$  is equal to  $S_0 - I$ ,  $e$  to the power  $rT$ , what this  $I$  represents?  $I$  basically represent the present value of the income present value of the income. So, if the future instrument is derived from a particular asset on which or from which we know that, how much return we are going to get or if the fixed amount of the return in a regular interval we are expecting, then this particular valuation can be done in this way.


Then another condition or another situation is that, when an investment asset provides a known yield, then what will happen in this case? In this case, it will be same because it is the rate of interest what we are getting. So, that is why your  $F_0$  is basically  $S_0 e$  to the power  $r$  minus  $qT$  and **what** this  $r$  represents? The risk free rate already you know, the  $q$  represents in this case basically, the average yield  $q$  is equal to the average yield what you are getting, the average yield during the life of the contract; during the life of the contract expressed again with this continuous compounding, that already we have taken into account. So, therefore, these are the three conditions on which, how generally the pricing of the future or pricing of the forward can be taken place.

(Refer Slide Time: 14:42)



**Valuing a Forward Contract**

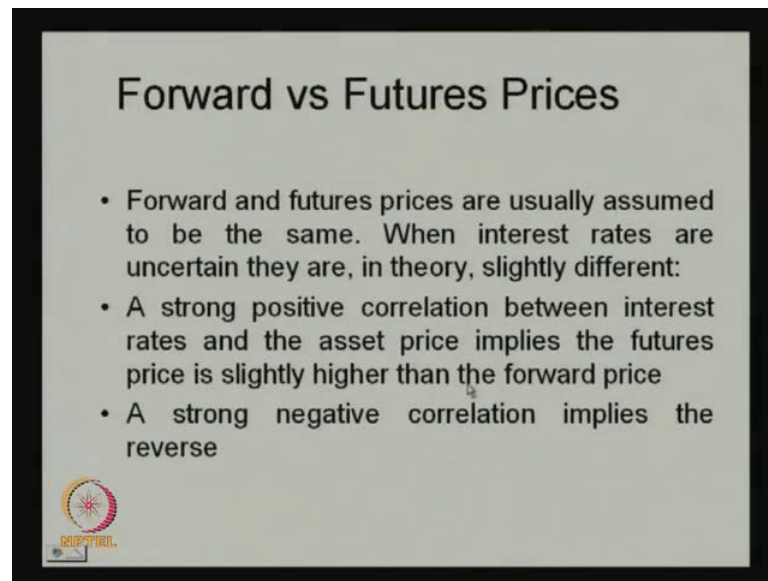
- Suppose that  $K$  is delivery price in a forward contract &  $F_0$  is forward price that would apply to the contract today
- The value of a long forward contract,  $f$ , is  
$$f = (F_0 - K)e^{-rT}$$
- Similarly, the value of a short forward contract is  
$$(K - F_0)e^{-rT}$$



So, already we have taken into account, so in general if you talk about how the valuation of the forward contract is done, suppose that  $K$  is the deliver price,  $k$  is the deliver price in a forward contract and  $F_0$  is forward price that would apply to the contract today. Then the value of a long forward contract, long forward contract in the sense, we can talking about the buyer contract  $f$ , let you denote it as  $f$  then how it is done it is  $f$  is equal to your  $F_0$ , which is the future forward price that would apply to the contract today minus  $K$ , which is the delivery price in the forward contract into  $e$  to the power minus  $r$   $T$ .

So, like that if it is the value of a short forward contract, then it will be  $K$  minus  $F_0$  it is just opposite  $K$  minus  $F_0$   $E$  to the power minus  $R$   $T$ . This is the way the value of the forward contract done, for this long forward contract and the short forward contract.

(Refer Slide Time: 15:50)



### Forward vs Futures Prices

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
- A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
- A strong negative correlation implies the reverse

INSTITUTIONAL

Generally, if you take in the market, the forward and future prices are usually assume to be the same. When interest rates are uncertain, they are in theory slightly different; that means, if interest rates are uncertain in the market then we can say there is some difference in the price of the future and forward, unless most of the time usually forward and future prices are usually assume to be same.

There is a strong positive correlation between interest rates and the asset prices that implies the future price is slightly higher than the forward price and a strong negative correlation implies the reverse. If you find there is a positive correlation between interest rate and the asset price, then we can imply that the future price will be little bit higher than the forward price, what you can derive from this particular formula.

And if the strong negative correlation you are observing, then which will imply the reverse situation in that particular time period. So, this is the way generally we can differentiate between the forward prices and the future prices. Then another is instrument always we use or always the derivatives instruments are based on this or this is a price through which, this price of other instrument have been derived like financial derivatives have been derived, one is that is the stock index.

(Refer Slide Time: 17:22)



**Stock Index**

- Can be viewed as an investment asset paying a dividend yield
- The futures price and spot price relationship is therefore

$$F_0 = S_0 e^{(r-q)T}$$

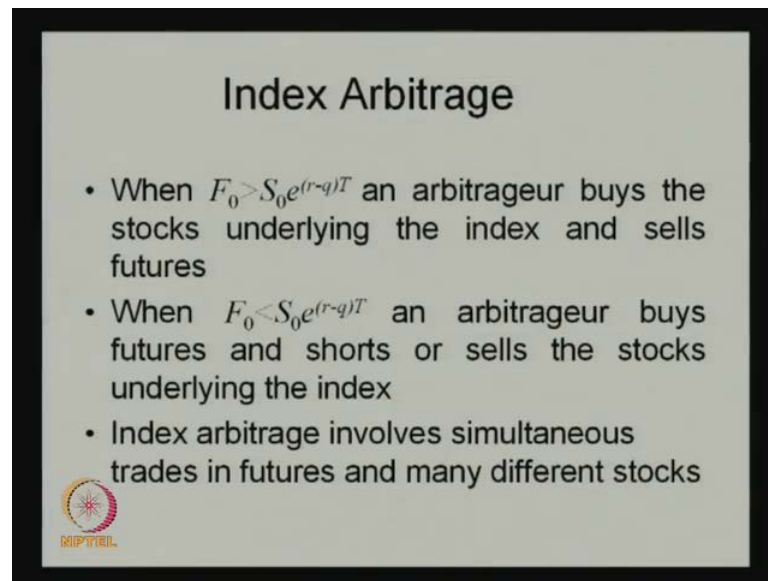
where  $q$  is the dividend yield on the portfolio represented by the index



So, the stock index can be viewed as an investment asset, which paying dividend yield because every stock basically price the yield, that is why we are expecting that thing. Then in this case, what is the relationship the future price and spot price we can have? The relationship between future price and spot price basically, your  $S_0$  the same we know that  $e$  to the power  $r$  minus  $q$  is basically the average yield into  **$e$  to the power**  $e$  to the power  $r$  minus  $q$  into  $T$ .


And already you know that,  $q$  is the dividend yield on the portfolio represented by the index. Though so  **$q$  is basically**,  $q$  is basically the dividend yield, dividend yield on the portfolio, because index is basically portfolio that is why the  $q$  is the dividend yield on the portfolio. Because once it is the stock index, we are expecting that some kind of yield from these, some kind of return from these, then the formula for this dividend yield or the known yield will be used here for the valuation of this forward the future contract.

(Refer Slide Time: 18:42)



**Index Arbitrage**

- When  $F_0 > S_0 e^{(r-q)T}$  an arbitrageur buys the stocks underlying the index and sells futures
- When  $F_0 < S_0 e^{(r-q)T}$  an arbitrageur buys futures and shorts or sells the stocks underlying the index
- Index arbitrage involves simultaneous trades in futures and many different stocks



Then there is certain situation, where the index arbitrageur opportunity will be available in the market. So, how it is basically used? How generally the arbitrageur opportunity will be possible in the market? So, here there are certain conditions through which the investor or we can say the arbitrageur can decide that, whether the arbitrageur opportunity can be prevailed in the market or he can take certain advantage out of this particular opportunities or not.

So, here in this case, already we have seen that, how the valuation of the future contracts which is based on the some stock index. So, in this case, if you use this kind of strategy or this kind of formula on which we can conclude that, whether is there any kind of index arbitrageur opportunity is existing in the market or not. So, how it will be possible?

So, that is why if you say that, your  $F_0$  will be greater than here  $S_0 e^{(r-q)T}$ , and what does it mean? Already we know that, this is your calculated value and this is the actual value what we are deriving and this is the value what you are expecting from this, then if this value will be greater than this, then what will happen then on arbitrageur buys the stocks underlying the index and sells the future.

Obviously, you will buy this stock because this value will be more in this case and sells this particular future in that particular time. So, that is why he can get the profit like your

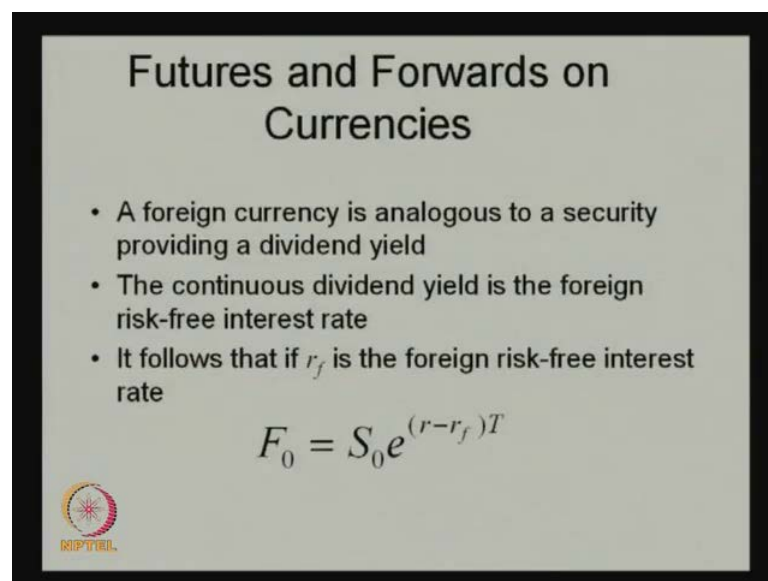
$S_0 e^{(r-q)t} - F_0$ . So, this much will be the profit he can earn in that particular time, but if it will just reverse; that means there is some arbitrage opportunity, if there is a difference between the expected price and the actual price.

But, when  $F_0$  is less than the  $S_0 e^{(r-q)T}$ , then what will happen? These arbitrageurs will by the future, you will buy this and you will sell this and in that case what you will do? An arbitrageur buys the future and shorts or the sells the stocks underlying the index, **if he has the money, sorry** he owns the stocks, and he will sell this stock unless with generally he always sells the stocks underlying the index.

So, the profit he can earn that is your  $F_0$  minus  $S_0 e^{(r-q)T}$ . So, this is the profit basically he will earn if this is your first condition, this is your second condition. So, this is the way the arbitrage opportunity can be used by the investor to earn the extraordinary profit.


So, that is why basically the index arbitrary involves simultaneous trades in future in many different stocks. So, you have to take the different position at a particular time, both in the stock market particularly, the spot market and as well as the future market. So, if you take the different position both in the spot market and the future market, then what will happen? The arbitrage opportunity can be prevail by the investor to earn some extraordinary profit.

(Refer Slide Time: 22:26)



**Futures and Forwards on Currencies**

- A foreign currency is analogous to a security providing a dividend yield
- The continuous dividend yield is the foreign risk-free interest rate
- It follows that if  $r_f$  is the foreign risk-free interest rate

$$F_0 = S_0 e^{(r-r_f)T}$$


Then we have some futures and forwards on currencies, because there are current currency is also they analogous to security providing the dividend yield and the continuous dividend yield is the foreign risk-free rate of interest. So, it follows that, if  $r^F$  is the foreign risk-free rate of interest, we can use the same formula wherein the beginning we have said that,  $F_0$  is equal to  $S_0 e^{(r^F - r)T}$ . So, this is the valuation of the future, if the future is derived from the foreign currency.

Then futures on the consumption assets, so consumption assets means we can say that, it depends on the storage cost and as well as the there are certain variables which play the significant role, that is your storage cost that is your asset value etcetera. Then what we can see here, then **the value will be the** future value should be  $F_0$  should be  $S_0 e^{(r + u)T}$ , what this  $u$  represents?  $u$  is the storage cost, storage cost per unit. If this is your storage cost per unit then, in the same strategy we can apply, in the same way we can say that, let your  $F_0$  is greater than or equal to  $S_0 e^{(r + u)T}$  then what we can say that, what kind of position the investor should take and if your  $F_0$  is greater than or equal to  $S_0 e^{(r + u)T}$ .

So, in certain cases, whether the investor should invest in this particular asset or the investor should invest in this particular future, already we have seen that if there is a difference between these two, then there is an arbitrageur opportunity which can be used by the investor and accordingly, he should take the position in the same way whatever way we have discussed just now. So, this is about your future and option pricing **sorry** future and forward or the future pricing and then we can move to the option pricing, which is also very important concept or important instrument we use always in the financial market.

(Refer Slide Time: 24:57)

Notation	
• $c$ : European call option price	• $C$ : American Call option price
• $p$ : European put option price	• $P$ : American Put option price
• $S_0$ : Stock price today	• $S_T$ : Stock price at option maturity
• $K$ : Strike price	• $D$ : Present value of dividends during option's life
• $T$ : Life of option	• $r$ : Risk-free rate for maturity $T$ with cont comp
• $\sigma$ : Volatility of stock price	



So, like your forward and future, before talking about the options, we use some notation; small  $c$  here we will be using which will be denoted as the European call option, capital  $C$  will be denoted as American call option, you know what is the difference between these two, it can be exercised at anytime, but European option can be exercised only that particular date maturity date.

So,  $P$  is equal to small  $p$  is European put option and capital  $P$  is basically American put option  $S_0$  is stock price today,  $S_T$  is equal to stock price at option maturity,  $K$  is equal to your strike price,  $T$  is equal to the life of the option,  $D$  is equal to the present value of dividends during the option's life, then your  $\sigma$  volatility of the stock price and  $r$  is equal to risk free rate for maturity  $T$  with a continuous compound.

So, these are the notation will be using for deriving this pricing of the options. So, then there are certain variables which basically affect the option prices, the certain variables are like this, you have your let this is your already we know that, this small  $c$  basically talks about the European call option and this is your European put option, this is your American call option, this is your American put option (Refer Slide Time: 26:12).

There are certain factors which effect this option pricing, that is your stock price, strike price, then the maturity term to maturity, then your variation of the stock price, then your



$r$  is equal to your rate of interest, then already we have seen that what this  $d$  represents, this is the present value of the dividends during the option's life.

So, these are 1 2 3 4 5 6 factors which always have or play the significant role for determination of the option pricing in the real world situation. So, whenever we derive this option pricing, either it is for European option or the American option, we always think about we study this 6 factors, how this 6 factors are behaving in that particular time, accordingly we say that how this pricing will be done for this particular option.

So, this strike price if it is more, it will have a positive impact on the call option, but it will have a negative impact on the put option for both the cases, like your if this is your strike price, this is your current stock price, if current stock price is more, than the call option will be more, but the put option will be less.

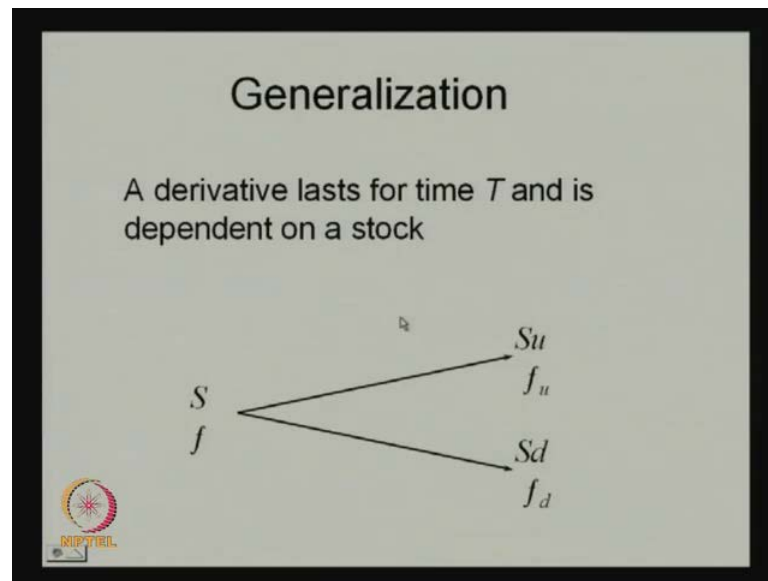
If strike price will be more, then the option premium will be automatically less that is why it will have a negative relationship, but the put option case it will be positive. It is the same for American option, but term to maturity because it is only matured at the time of the maturity. So, there is no such kind of a relationship we want to you want to establish here.

But C and P if it is American call option, American put option, the larger the term to maturity, then the price of both call and put option will be also higher. If the stock market is more volatile, the price is more volatile than we are expecting this option premium for both call option and put option will be more.

And the risk free rate of interest will be more then the premium on call option will be also more, but the rate of interest for is more than option premium for the put option will be less. Then if it is the present value of the dividends, if it will be higher than the call premium, call option premium will be less, but the put option premium will be higher it is same for both American option and the European option.

So, this is the summary table, what we are deriving from for valuation of option prices and particularly these are the factors which are responsible for deciding how much how much should be the value of the option at a particular time.

(Refer Slide Time: 29:09)



So, there are certain ways through which this option pricing is done, there are different methods we will be using here two methods, the methods of option pricing. So, the methods are basically one method is your binomial tree model and another method where what we are use basically the black model.

So, there are two methods popularly used for pricing of the options. So, here first, we will talk about this binomial tree, and then we go to the black model. So, what basically the binomial tree model talks about. So, the let you have the derivative which the term to maturity is a last for the time  $T$  by the depended on the stock. So, if this is your stock and this is the current stock price, this is the future price, then what will happen either this or the different probability always they talk about (Refer Slide Time: 30:00).

Either it can go up to this or it can the future price also to this or it can go down by  $d$  amount or  $f$  also can go to  $d$  amount. So, these are the probabilistic function we can draw from here and from this function, we can decide this how much should be the price of this particular option. So, how it is done? Let consider a portfolio, that is long the delta shares and short one derivative; that means, is buying delta share, delta amount of the shares and selling one derivatives in that case.

Then what will happen that basically, either it can go up to  $S u$ ; that means, this is the change into delta minus your  $f$  of  $u$  and this is you will get  $s d$  delta minus  $f$  of  $d$  this


much profit he can earn. So, the portfolio is riskless, the portfolio will be riskless when you are change in the upper  $S_u$  means, it is the increasing  $d$  means it is declining, then change in the upper side multiplied by the delta to the shares minus  $f_u$  should be equal to this change in the lower side into delta minus the  $f_d$ .

Or we can say, the delta is basically your  $f_u$  minus  $f_d$  divided by your  $S_u$  minus  $S_d$ . If this condition is prevailed, then we can say the portfolio is riskless; the portfolio is riskless if you consider that, your delta should be in this change in the future change in the upper part minus change in the future price of the lower part divided by the  $S_u$  minus  $S_d$  and  $S$  represents the stock price only.

(Refer Slide Time: 32:22)

### Generalization (continued)

- Value of the portfolio at time  $T$  is  
 $S_u \Delta - f_u$
- Value of the portfolio today is  
 $(S_u \Delta - f_u)e^{-rT}$
- Another expression for the portfolio value today is  $S \Delta - f$
- Hence  
$$f = S \Delta - (S_u \Delta - f_u)e^{-rT}$$



In this case, what basically we have seen that value of the option at time  $T$  is basically nothing but, either it is  $S_u \Delta - F_u$  or it is value of the option. Today value of the option at time  $t$  is  $S_u \Delta - f_u$  because it has increase further, then the value of the portfolio today is if you want to **you want** go for discounting it using your valuation formula, then it will be  $S_u \Delta - F_u$  to the power into  $e$  to the power minus  $rT$ .

So, another one is another expression for the portfolio value today is generally  $S \Delta - F$ . So,  $F$  is nothing but,  $S \Delta - S_u \Delta - F_u$  then  $e$  to the power minus  $rT$ . So, the future premium, the future price is basically calculated the option premium is calculated by change in this stock price minus this particular value of this

particular future in that particular direction on which the probabilistic function and which this particular value changes.

So, if you put this delta substituting that equation, then what you will get? Your  $f$  into in that previous equation, where your delta is equal to what your  $f$  of  $u$  minus  $f$  of  $d$  divided by  $S_u$  minus  $S_d$  then what we will find? Then  $P$  is equal to  $f$  of  $U$  plus  $1$  minus  $P$  into  $f$  of  $d$  to the power  $e$  to the power minus  $r T$ , which basically shows that the probability of the  $p$  represents the probability, where the  $P$  is equal to basically  $e$  to the power  $r T$  minus  $d$  then  $d$  minus  $d$ .

$E$  to the power  $r T$  minus  $d$  divided by  $u$  minus  $d$  this is the way basically this pricing is taken place. So, how generally if you take a numerically example, then it will be more clear then how basically it takes place. Let your  $u$  is basically your  $1.1$ , your  $d$  is equal to  $0.9$ , that means,  $u$  and  $d$  that means the stock price. Let it was basically this way, it was let  $10$  rupees it can go up to  $11$  or it can go up to  $9$ .

This is the way generally you can derive this. So, then  $u$  is equal to your  $1.1$ ,  $d$  is equal to either it will increase by  $1$  unit or it will decrease by  $1$  unit. So, let  $u$  is equal to  $1.1$ ,  $d$  is equal to  $0.9$ , then  $r$  is equal to your  $0.12$ , then  $T$  is equal to your  $0.25$ , then  $f$  of  $u$  is equal to  $1$ , then  $f$  of  $d$  is equal to  $0$ . So, if it will increase by  $100$  percent, then it will not decrease.

Then  $P$  is equal to your  $e$  to the power  $r T$ ,  $r$  means your  $0.12$  into  $T$  basically is nothing but,  $0.25$  minus  $0.09$  divided by  $1.1$  minus  $0.9$ . This is your  $f$  of  $u$  minus  $f$  of  $d$  that will be  $0.652$ . The then your  $f$  is equal to basically  $e$  to the power  $0.12$  into  $0.25$  into  $0$  in to  $0.25$  into  $0.6523$  into  $1$  plus  $0.100$  minus this  $0.347$  into zero that will give you  $0.63$ .

So, when we are valuing an option in terms of the underlying stock, the expected return on the stock is irrelevant; basically we are deciding, what should be this particular value in that particular time or how this probabilistic function how much, what is the percentage of probability of increasing, what is the probability of decreasing of this particular stock price, that is basically we will decide how much should be the particular price in that particular time period.

(Refer Slide Time: 37:02)

### Risk-Neutral Valuation

- The variables  $p$  and  $(1-p)$  can be interpreted as the risk-neutral probabilities of up and down movements
- Expected payoff from the option =  $p f_u + (1-p) f_d$
- The value of a derivative is its expected payoff in a risk-neutral world discounted at the risk-free rate


$S$   
 $f$

$p$

$S_u$   
 $f_u$

$(1-p)$

$S_d$   
 $f_d$




So, that is why we call it the risk neutral evaluation. So, the variables  $p$  and  $1 - p$  can be interpreted as the risk neutral probabilities of up and down movements and the expected payoff from the option is probability of upping of this particular price and probability of downing. If the probability up a price will up, if probability of up probability of up is  $p$ , then the probability of down is  $1 - p$ . So, that is why the expected payoff from this payoff is equal to payoff from the option, from the option is basically  $p$  into  $f$  of  $u$  plus  $1 - p$  into  $f$  of  $d$ .

(Refer Slide Time: 38:37)

### Risk-Neutral Valuation Cont...

- $E(S_t) = pS_0u + (1-p)S_0d$
- $E(S_t) = S_0 e^{rt}$
- Stock price grows on average at the risk free rate
- In risk neutral world all individuals are indifferent to risk



So, the value of a derivative basically the value of a derivative is its expected payoff in a risk neutral world, discounts it at the risk free rate. So, let this same thing you can take  $s$  and  $f$ . So, the probability of up is  $p$  and probability of down is  $1 - p$ , so it can go to  $S_u$ ,  $f_u$  it can go to  $S_d$  and  $f_d$ . So, therefore, what you can say that the expected price of the stock at period  $t$  is basically  $p$  into  $S_0$  into  $u$  plus  $1 - p$  into  $S_0$  into  $d$ . So, the expected price of the stock  $e$  into  $S_t$  is equal to  $S_0 e$  to the power  $r T$ .

So, what we can assume here, what we can see or what we can say that, the stock price grows on average at the risk free rate. We are expecting the price will grow, the price will grow at a risk free rate and in risk neutral world, and all individuals are indifferent to risk, which is a very unrealistic situation. So, in the risk neutral world or the people are not very much concerned about the risk, they are totally indifferent about the risk apatite indifference about the risk, what they are going to face, if they are going to invest in certain financial assets in a particular time period. So, that is what the investor always wants to see.

(Refer Slide Time: 39:52)

### Original Example Revisited

- Since  $p$  is a risk-neutral probability  
 $20e^{0.12 \cdot 0.25} = 22p + 18(1 - p)$ ;  $p = 0.6523$
- Alternatively, we can use the formula  

$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \cdot 0.25} - 0.9}{1.1 - 0.9} = 0.6523$$

So, if you go back to **if you** see the example in this case, let you have a stock price your example **one of the example we can say what we can see in this case sorry** what we can say in this case that, your  $s$  of  $f$  and the probability we are taking into account that is your  $p$ , which is up it is  $1 - p$  then,  $S_u$  is equal to 22,  $f_u$  is equal to 1, then  $S_d$  is equal to

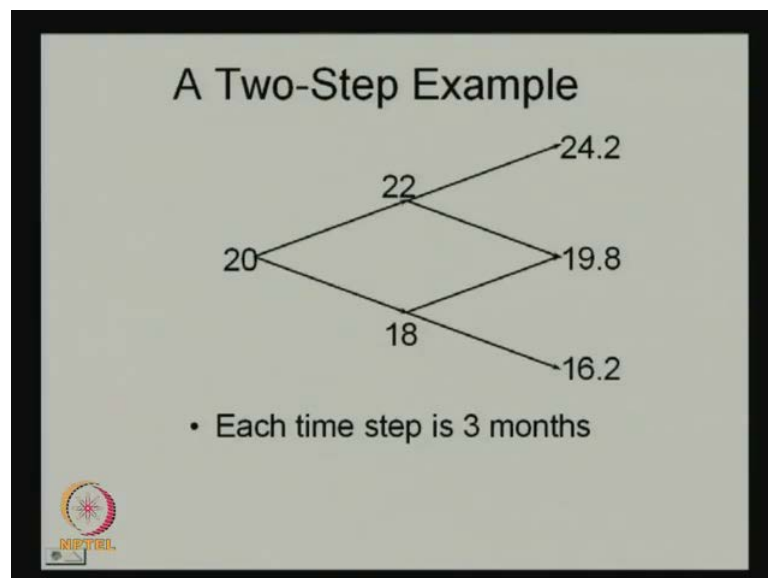
18, there is a probability that it will go up to 21 it can go down to 18. So, the f of d is equal to 0.

So, the P is equal to price is since p is a risk neutral probability, then we have 20, e to the power 0.12 into 0.25, this is the value already we have taken into account in the previous example, then it will be 22 p plus 18 into 1 minus p. So, the p is equal to already 0.6523 then, we can also use this formula p is equal to  $r T \text{ minus } d \text{ divided by } u \text{ minus } d$ , which will be e to the power 0.12 into 0.25 minus 0.9 divided by 1.1 minus 0.9 equal to 0.6523.

So, if you want go for the valuation of this, then the probability of up is 0.6523. So, this will be your 0.6523 and this will be your 0.347. So, then the value of the option is then the value of option is basically e to the power 0.12 minus 0.12 into 0.25 into 0.6523 into 1 plus 0.3477 into 0 that will be 0.63. So, the value of the option in this case will be 0.633.

So, basic objective is you want to derive the probability of up and the probability of down. So, if these two probabilities function we can get then, the value of the option can be calculated using this rate of interest and other time period etcetera to calculate.

(Refer Slide Time: 42:51)

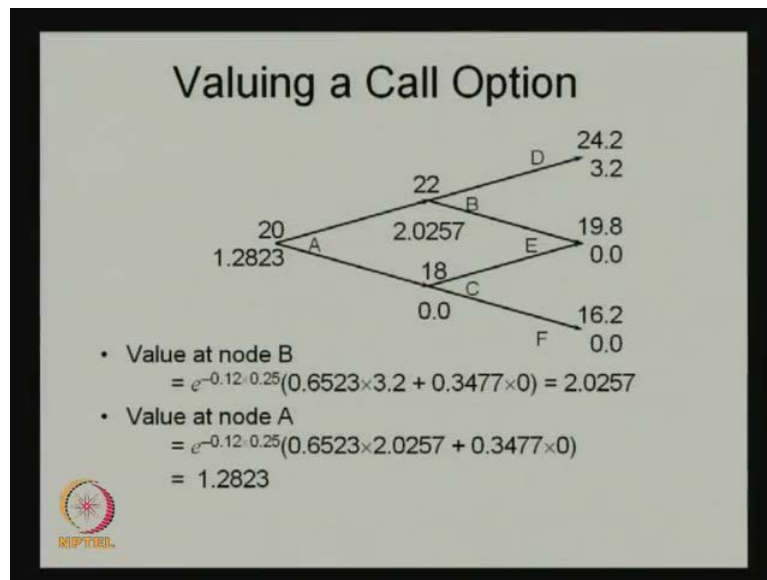


But, there is a another case here, we have taken that the price can go up to this, price can go up to this, but if there are two steps, let the price was originally 20 it can go up to 22,

it can up to 18 go down to 18. So, again it can go up to 24.2 or it can again go down to 16.2 or it can again down for both the cases, it can go down 19.8 or it can go up to 19.8.

So, each time let the step is for 3 months, each time you assume that each time this step is step is equal to 3 months. Then how we can calculate this probabilistic function in this case? So, there are different nodes basically we can see from this and from there we have to find out the value, let we have taken a valuing of a call option. So, the value at node b basically go back to we have taken into account the probabilistic same probability, then the value at node B, this is your A, this is your B, this is C, this is D, this is E, this is your F ( Refer Slide Time: 44:00).

(Refer Slide Time: 43:52)



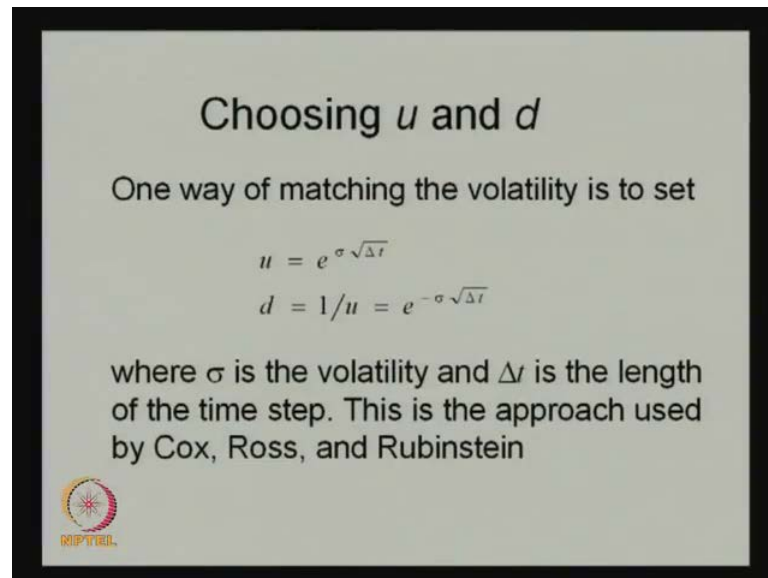
So, the value at node B, let that means, you are talking about here, the value at node B what will be the value, that we know already that e to the power minus 0.12 into 0.25 into 0.6523 into 3.2 because here already it is given, this 3.2, it is 0.0, this is also 0.0, the f of u, f of d is given, f is given 1.2823 is equal 2.0257 0.0, 3 plus 0.3477 into 0 it will be 2.0257.

So, value at node A will be e to the power minus 0.12 into 0.25 into 0.6523 into 2.0257 power 1 plus that means, it will come down here, that is why **that is why** 0.3477 into 0 1.2823. So, this is the way or the different steps we can see that, how the value will



fluctuate how this stock price will fluctuate and accordingly we can say that, how the valuation of this particular options can take place.

(Refer Slide Time: 45:59)




**Choosing  $u$  and  $d$**

One way of matching the volatility is to set

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = 1/u = e^{-\sigma\sqrt{\Delta t}}$$

where  $\sigma$  is the volatility and  $\Delta t$  is the length of the time step. This is the approach used by Cox, Ross, and Rubinstein



Then how basically we can choose this  $u$  and  $d$ , whenever we have use this  $u$  and  $d$  extensively, but how this way what is the way through which, this  $u$  and  $d$  can be calculated. So, one way of matching the volatility is to set is basically we know that, how we can measure this  $u$  is nothing but,  $e$  to the power  $\sigma$  into root of the  $\Delta T$  and  $d$  is equal to basically  $1/u$  that is equal to  $e$  to the power minus  $\sigma$  your  $\Delta T$ .

And what is this  $\sigma$ ?  $\sigma$  is basically the volatility of the stock price, volatility of the stock price,  $\Delta T$  is basically the length of **length of** the time step and this approach is basically used by Cox roes and Rubinstein. So, to know that what should be this  $u$  and what should be **what should be the**  $u$  and what should be the  $G d$  is basically, we can use this approach on which this  $e$  and  $d$  can be calculated.

(Refer Slide Time: 47:24)

### The Probability of an Up Move


$$p = \frac{a-d}{u-d}$$

$a = e^{r\Delta t}$  for a nondividend paying stock

$a = e^{(r-q)\Delta t}$  for a stock index where  $q$  is the dividend yield on the index

$a = e^{(r-r_f)\Delta t}$  for a currency where  $r_f$  is the foreign risk-free rate

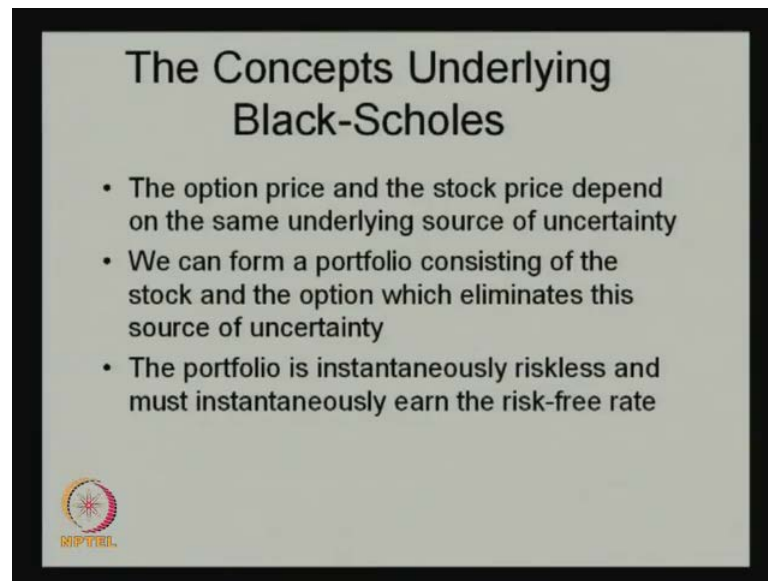
$a = 1$  for a futures contract



So, the probability of an up move and down move if you want to know, how it can be calculated, the  $P$  is equal to the probability of up move and this is we already know, this is  $a$  minus  $d$  divided by your  $u$  minus  $d$ ,  $a$  is already we can denote  $e$  to the power  $r$  delta  $T$  for a non dividend paying stock **no dividend paying stock**. And **your** if you want to dividend paying stocks, then it will be  $a$  is equal to  $e$  to the power  $r$  minus into delta  $T$  for a dividend paying stock index.


And if it is basically the currency, then it is  $e$  to the power  $r$  minus  $r_f$ , it is the risk free rate of the foreign currency into delta  $T$ , your foreign currency and  $a$  is equal to 1 for a future contract. So, this is the way the probability of the up that what is the probability that the price can go up that can be decided. So, here already once the up can be calculated, the down can be also calculated.

(Refer Slide Time: 49:00)



The Concepts Underlying Black-Scholes

- The option price and the stock price depend on the same underlying source of uncertainty
- We can form a portfolio consisting of the stock and the option which eliminates this source of uncertainty
- The portfolio is instantaneously riskless and must instantaneously earn the risk-free rate

 IIM PATNA

So, the another method, which is very much popular in the financial literature for valuation of the options that is your black-scholes model. So, the black-scholes model is has the lot of popularity for the valuation of the option, which will be largely used by the investor and as well as the policy makers to do this for valuation of the options.

So, how this particular options or the valuation of valuation of the options is taking place? Here, the option price and the stock price depend on the same underlying source of uncertainty, we can form a portfolio consisting of the stock and the option, which eliminates the source of uncertainty, that is the basically the concept what was trying to say and the portfolio is instantaneously riskless and must instantaneously earn the risk free rate.


So, this is the situation on which we can go for the valuation of the options at a particular time and this is that is what the black-scholiast trying to say.

(Refer Slide Time: 50:16)

### The Black-Scholes Formulas

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$
$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

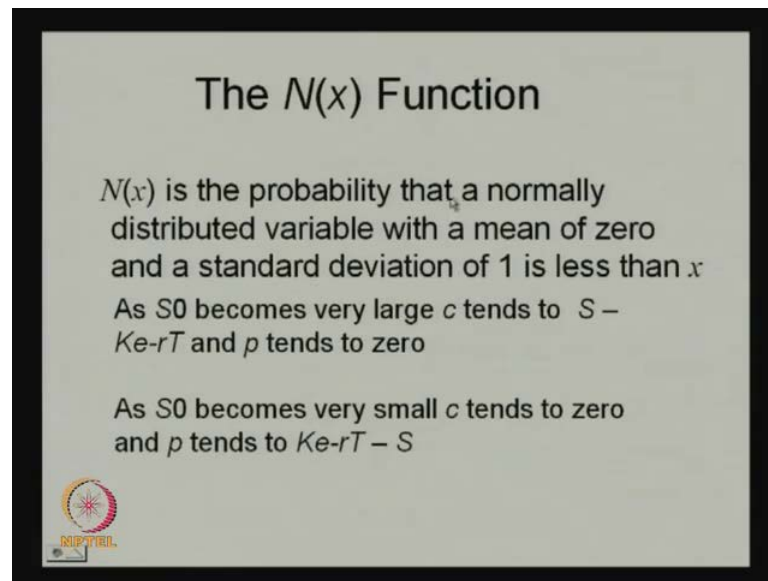
where  $d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$


But, it is little bit complex in that way, because black-scholes was taken certain assumptions to derive this formula and they are basically this is a log normal distribution he has followed.  $c$  represents your call option,  $p$  represents the pool option,  $S_0$  represents the stock price,  $K$  represents the strike price and  $N d_1 d_2$  basically is the function, what the normality assumption he has taken.

So, here  $S_0$  into  $d_1$  minus  $K e$  to the power minus  $r T d_2$  and  $K e$  to the power minus  $r T$  into  $N$  minus  $d_2$  minus  $S_0 N$  minus  $d_1$  and he has derived that how this  $d_1$  and  $d_2$  is calculated and accordingly, if you find out this and you can use this, how the black-scholed option pricing model will be used.

(Refer Slide Time: 51:06)



So, how this function is derived in this case? So,  $N(x)$  is basically the probability that a normally distributed variable with a mean of zero and a standard deviation of 1 is less than  $x$  as  $S_0$  becomes very large,  $c$  tends to  $S - Ke^{-rT}$  and  $p$  tends to zero. As  $S_0$  becomes very small,  $c$  tends to zero and  $p$  tends to  $Ke^{-rT} - S$ , it is basically  $Ke^{-rT} - S$ ,  $p$  tends to  $Ke^{-rT} - S$ , it should be  $Ke^{-rT} - S$ .

And  $p$  tends to 0 this is basically nothing but, it is the  $S_0$  and this is also here  $S_0$  and this is also the  $Ke^{-rT}$ , this is also the  $Ke^{-rT}$ , this is not the this thing (Refer Slide Time: 52:25). So, what we can say here? Always the black-scholes model has its own significance, it is basically what we can say that, this is what the black-scholes was trying to say.

But, the basic problem with this black-scholes model is that, it does not talk about the simplicity and sometimes, we face the problem because of this a normality assumption what the black-scholes was taken.

But, still it has its own popularity to be used. So, that is why you would I have just given the introduction of that and it can be used to we can put this values and in this formula then find out the option pricing, but before that we can see that, whether the particular distribution follows a normal distribution or not, where the means should be equal to 0

and the variance of the standard deviation should be equal to 1, that is what the black-scholed was trying to say.

And this is about the option pricing and there are different strategy what the people use in the market for this, which is beyond this particular course. What generally basically we always because we talk more about the other financial assets, but what we can say that, derivates is also one of the basic instruments or one way we can say major instruments, which are **which are** used by the investor regularly to maximize the return in the particular time period and we use the different strategy to get that.

So, this is about only the brief review of or the brief concepts, which are really used in the market for the use of the derivatives and largely we can say that, after discussing this it is also very much important to know that, how this whenever we make the portfolio and we start the investment, how my investment is performing and what kind of performance measure, I should use to know that whether my investment is doing well or not and what kind of performance measure we should use.

So, that we can also see or we should see before going into the market and or whether is there any kind of revision is required that is the enough what about portfolio whatever **we have** we have made. So, for that we should discuss about something related to some concept related to a portfolio performance evaluation that we will be discussing in the next class, thank you.