

Economics, Management and Entrepreneurship
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Lecture – 23
Comparison of Alternatives (Contd.)

Good morning. Welcome to the 23 lecture on Economics, Management and Entrepreneurship.
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Comparison of Alternatives

Methods for making comparison:

- Present-Worth Comparison
- Equivalent Annual Cost Comparison
- Internal Rate of Return (IRR) Comparison



If you recall the last 2 lectures we discussed about time value of money and comparison of alternatives. (00:41) for 2 more lectures, including this particular lecture. First remember that we had to define the rate of return. We defined minimum attractive rate of return and then we developed 6 interest formula where we related the first cost to a series of equal payments and to the final sum. So, in a sense we defined 3 quantities p .

Which we said are defined as principal sum to we paid or received at the present and then an equal payment of A rupees per interest period and F the final sum. We defined 6 interest formula one was related to finding out the present worth of either the final sum F or equal payment in series A then the compound amount factor of either the principal sum P or equal payment in series A at the end of the interest periods N .

We call that F or we found out the equivalents of a series of payments or receipts. In equal

payment manner throughout the project period we called it either sinking fund factor or capital recovery factor depending on whether we are trying to find out A even F, or A even P. In a sense what we are trying to do if we are given F and we are required to find P this is present worth.

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<u>Six</u>	P F :	Present Worth (of a Single Payment)
	F P :	Compound Amount (of a -do-)
	P A :	Present Worth (of Equal Payment Series)
	A P :	Capital Recovery Factor
	F A :	Compound Amount Factor (of Equal Payment Series)
	A F :	Sinking Fund Factor

Find P even if of a single payment. The opposite is given P, find F? So, we are trying to find out the compound amount factor of a single payment. We can find out P even A we call it present worth of equal payment series. The opposite is finding out we call it find A given P, we have invested the P call it capital recovery factor. It is not necessary to write equal payment series when only capital recovery factor is enough.

Because there are 2 present worth, it is required to qualify it by saying whether the present worth is for a single payment or for an unequal payment in series. That is why we need to write this. But in this case it is always equal payment in series so this is only capital recovery factor. One can also find F even A that means if there is equal payment in series find F we call that compound amount factor.

And since there are 2 compound amount factor qualify it by saying of equal payment series. Lastly, A given F which is called the sinking fund factor and it is not necessary to qualify it. But of course this is for equal payment series. So, these are the six fundamental factors that we shall be using. Now, we had used this to make comparison among economic alternatives and in that

we had discussed 3 different methods.

One was if you recall the present worth comparison method. This is what I had written here. This was taken up in the last time equivalent annual cost comparison method and internal rate of return comparison method these 3 are very basic comparison methods that are based on consideration of time value of money. We had taken the present worth cost comparison.

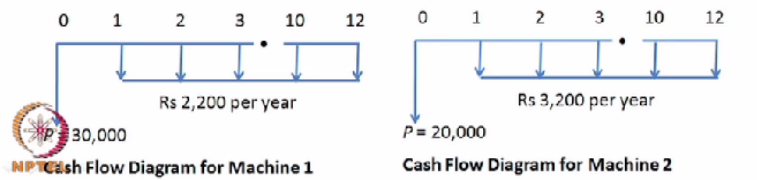
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Example

A machine can be purchased at a cost of Rs.30,000/-. It requires maintenance expenses of Rs. 2,200/- per year.

A similar machine of a different make can also be purchased at a lower cost of Rs. 20,000/-, but it requires a higher maintenance cost of Rs. 3,200/- per year. The estimated life of the machine is 12 years with no salvage value at the end of the 12-year period.

If the minimum attractive rate of return is 15% per year, which alternative is better?



We had taken an example where we had considered machine 1 and machine 2. The company can either buy machine 1 with higher investment cost but lower maintenance expenses. Compared to another one which has lower initial investment but a higher maintenance expense and they had the same period 12 years of life, no salvage value. So, this is the cash flow diagram for the 2 situations.

So, what basically present worth cost comparison does is to find out the present worth of this cash flow that means find out the equivalent amount at this point. And similarly for this cash flow and then find out whichever is minimum and that is preferred alternate.

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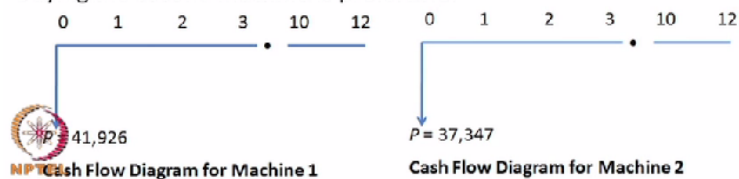
Present-Worth Cost-Comparison Method

The present worth of each of the two sets of cash flows is:

$$\begin{aligned} PW_1 &= 30,000 + (2,200) (P|A, 15, 12) \\ &= 30,000 + (2,200) (5.421) = \text{Rs. } 41,926/- \end{aligned}$$

$$\begin{aligned} PW_2 &= 20,000 + (3,200) (P|A, 15, 12) = 20,000 + (3,200) (5.421) \\ &= \text{Rs. } 37,347/- \end{aligned}$$

Buying the second machine is preferable.



So, in this particular case we found out that the present worth of machine 1 was higher at 41,926 rupees compared to the machine 2 which was only 37,000 since it is a payment from the company. Machine 2 is less costly and therefore preferred. So, this is a direct application of present of worth cost comparison. Observe that in both these cases the number of years for which we are making the comparison is the same.

If the number of years varies then the present worth cost comparison method cannot be applied
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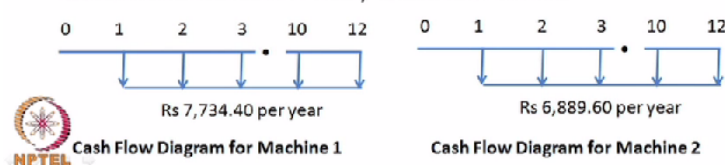
Equivalent Annual Cost Comparison Method

The equivalent annual costs of the expenses associated with machine 1 and machine 2 are as follows:

$$\begin{aligned} EAC_1 &= 30,000 [A|P, 15, 12] + 2,200 \\ &= 30,000 [0.18448] + 2,200 = \text{Rs. } 7,734.40 \text{ per year} \end{aligned}$$

$$\begin{aligned} EAC_2 &= 20,000 [A|P, 15, 12] + 3,200 \\ &= \text{Rs. } 6,889.60 \text{ per year.} \end{aligned}$$

Machine 2 is an economically better alternative.



Then we also for the same problem used the equivalent annual comparison. So, basically in this method we find out the equivalent annuity for both the opposites. Now, in this found out 30,000

was the initial value P we calculated the equivalent A, the capital recovery factor multiplied that with the value P which was 30,000 added to that already existing annuity the maintenance expenses of 2200 that give a value of 7,734.40 paise.

And the similar thing we did for machine 2 and that resulted in 6,889.60 paise of the 2 this is lower and therefore I machine 2 is less costly and therefore preferred. You can see that it is possible both in present worth cost comparison method and in the equivalent annual cost comparison method we are getting results that are similar.

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Internal Rate of Return (IRR) Method

- The first cost of investment on a project is usually quite high.
- Cash receipts take place throughout the life of the project.
- Arithmetic sum of the receipts (i.e. at zero rate of return) is usually higher than the cost of investment, but the time value of money reduces the value of the future returns.
- Higher the rate of return, lower is the present worth of the future returns.
- That rate of return at which the present worth of returns equals the present cost of investment is known as the **internal rate of return**.

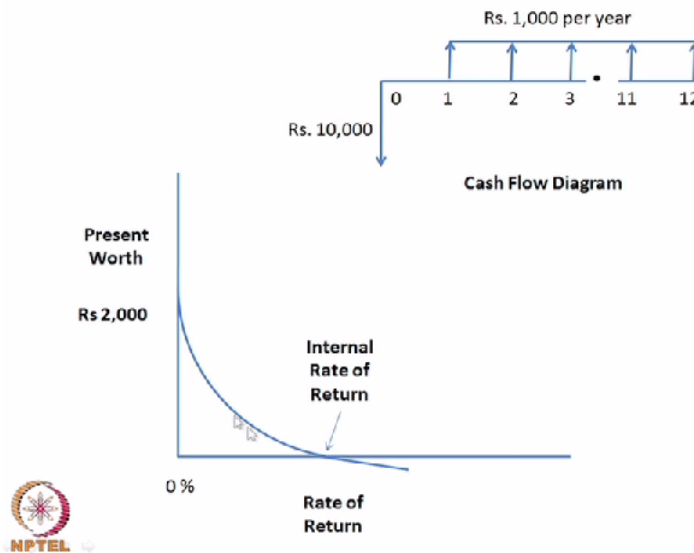


Now, before we enter the last lecture we were talking about the internal rate of return method. By internal rate of return method here it is defined it is the rate of return at which the present worth of all cash flows = present worth of returns = the present cost of investment. That is present cost of all cash flows = 0. So, it is that rate of return at which the present worth of all cash flows inflows and outflows together = 0. Now, remember that if the rate of return = 0.

Then it is expected that for a project to be viable total net revenue must be greater than the investment. Total net revenue means revenues – expenses because the rate of return is 0. It is the arithmetic sum of all the revenues – the expense that is the net revenue and that must be positive. And this must be higher than the investment made initially. Otherwise project is not at all viable. Now, if however, rate of return is positive which is always.

So, because of discounting the value at the present time of all the future revenues will be less. Therefore there will be a point at which the discounted value of all future returns or future net revenues will be = initial investment. And if the discount rate is even higher it is possible that it was negative. This is what is shown in this diagram.

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Here is the case the same case but of course what we had is that we had converted the case of 2 machines to a single machine.

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We first convert the problem in to a problem of a single alternative.

Differential Cash Flows

Time	Cash Flows		
	Machine 1	Machine 2	M/c 1 – M/c 2
0	-30,000	-20,000	-10,000
1	-2,200	-3,200	1,000
2	-2,200	-3,200	1,000
.	.	.	.
.	.	.	.
.	.	.	.
12	-2,200	-3,200	1,000

Something like a differential machine. Differential machine means incremental investment. Now,

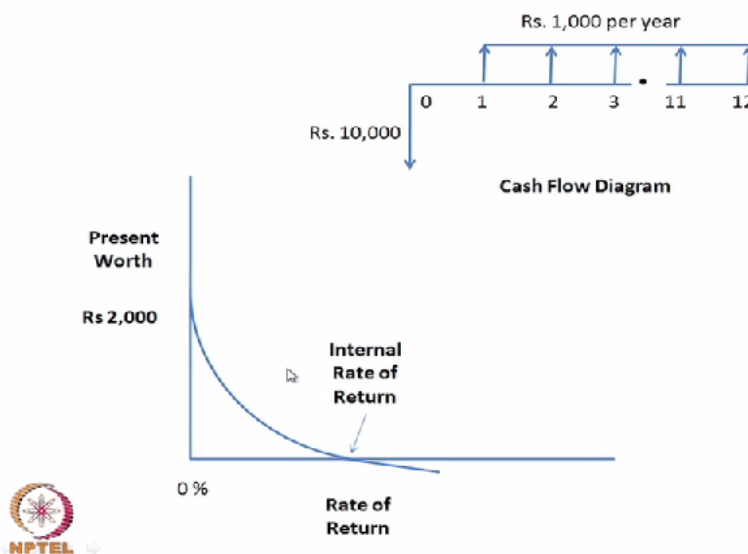
look at this. This is machine 1 that is more costly 30,000 rupees we have put negative sign to indicate that it is the cash outflow all these are negative because they are all outflows. These are maintenance expenses. These are also the maintenance expenses and the initial investment for machine 2.

Now, if we subtract machine 1 –machine 2 it means this is an incremental investment on machine 1. Suppose if a company decides to go for machine 1 then whether any additional investment would be better if we subtract this to this. If this is viable it means that machine 2 is economically more viable then machine 1. So, we are considering incremental investment for a differential project converting 2 projects to a single project.

This is how the internal rate of return method has to be applied. It is applicable to a single project. So, this indicates the differential project that implies additional investment over and above the most costly equipment or machine. So, subtracting this we get this is -10,000 subtracting this we get 1,000, 1,000 each. Now, if you see these are positive so if we add this for 12 months this becomes 12,000 and the investment is 10,000.

Therefore, if the rate of return is 0 then this additional investment is worth making. So, that is what is shown here.

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The cash flow diagram for the differential project is shown here. These are all the revenues, net revenue which is positive therefore the arrows are upwards in the cash flow diagram. And this is the initial investment of 10,000 as I was saying if MARR or the rate of return. If the rate of return is =0 then the discounted values of all this is nothing but 12,000, $12,000 - 10,000 = 2,000$. So, when the rate of return is =0% then the present worth is 2,000.

Now, as the value of rate of return increases then the present worth value will come down because discounted values of this future cash flows will be lower and lower. So, at some point it will be =10,000 and if the discount rate is even higher then it may become negative. We are defining that the rate of return at which the discounted value or the present worth of all the future cash flows = the capital investment, the initial investment that is the internal rate of return.

So, how to find out this particular value? If one uses the formula it will contain powers, R to the power N. So, in this case there will be R to the power 12. So, these values are difficult to solve analytically one has to therefor go for approximate solution to find the value of R. Now, interest tables at this point are very useful. Because interest tables give for different values of rate of return and N, values of the factors.

We are interested to find out the present worth of a series of equal payments that is to find P even A. R and N, R is not known but N is known as 12. So, what we can do? We can look at different interested tables for a particular value of A, N and under the column B even A. So, doing that we found that when $r=2\%$ then the value of the present worth of equal payment series present worth factor for 2% and 12 years is 10.575.

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The present worth of this differential cash flow is given by

$$PW = -10,000 + 1,000 (P|A, r, 12)$$

At the internal rate of return r^* , this present worth is zero:

$$(P|A, r^*, 12) = (10,000)/1,000 = 10$$

From the 2 % and 3 % interest tables

$$(P|A, 2, 12) = 10.575, \quad (P|A, 3, 12) = 9.954$$

Applying the method of linear interpolation, $r^* = 2.93\%$ per year.

$$IRR = 2.93\%, \quad MARR = 15\%, \quad \text{and } IRR < MARR.$$



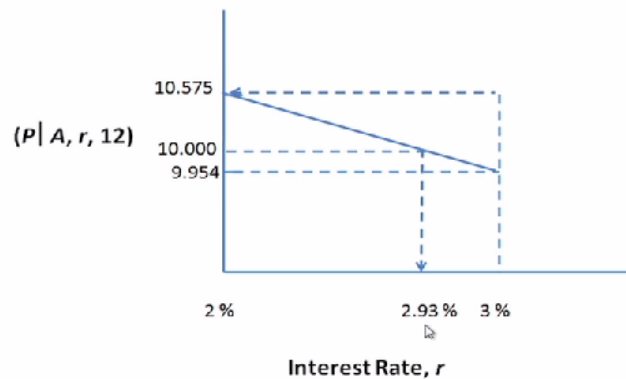
The differential project (incremental investment) is economically not feasible. Hence, **Machine 2 is preferred to Machine 1.**

Firstly, how this 10 comes? Because the present worth of the total cash flow is $-10,000 + 1,000 *$ the equal payment series present worth factor. The value of course of r is not known and therefore if or the internal rate of return r star this present worth is 0. Then we will put this as $=0$ and therefore this quantity will be obtained as $10,000/1,000$ which is $=10$. So, we have to find the value of r square where this factor takes the value 10.

So, that is what I am trying to tell you. That go to the interest tables for different r , find out for what value of r star or r P/A is close to 10? When we do that we find that for 2% it is 10.575 and for 3% it is 9.954. For other values of interest rates 4, 5, 0, 15 etcetera they are vastly small, very small. Because you can see at 3 is 9. So, as this becomes 4, or 5 or 7 or 10 or 15 the values will slowly go down.

We are interested to find values that are little more or less than 10, close to 10. So, the value of r at which this relationship holds is definitely between 2% to 3%. Now, if one actually tries method of linear interpolation then one can find that the particular value of r star this ratio is $=10$ is 2.93% that is what is shown here.

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For 2% interest rate the value was 10.575 the value of the factor = payment series, present worth factor and for 3% the value was 9.954. We are interested to know the value of r at which the factor takes a value 10. So, basically it is comparing similar triangles. This one triangle and this is another triangle. So, this divide by this is equal to this divided by this. This is $r-2$ so one can find out r which comes to 2.93%.

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For a project to be economically feasible, the present worth of the cash flows at the minimum attractive rate of return (MARR) must be positive.

IRR > MARR: Project is economically viable.

IRR < MARR: Project is economically not viable.

We can apply the IRR method to judge the feasibility of a *single* project.



We consider a project to be economically viable or feasible, if the present worth of cash flows at the minimum attractive rate of return must be positive. That means if the internal rate of return is greater than MARR than the project is economically viable. If however the internal rate of return is less than the minimum attractive rate of return the project is economically not viable. Now, for

any company MARR is normally known. In this particular case MARR was given as.

What was the value given? It was given as 15%, r was 15% that was MARR value. Whereas internal rate of return is coming as only 2.93% so naturally internal rate of return is less than the minimum attractive rate of return. It means that the incremental project, incremental investment is not good. That means machine 1 – machine 2 is not good. That means machine 2- machine 1 is good. This means that machine is 2 is preferred to machine 1.

Look at this logic. Logic is that machine 1 – machine 2 is not economically feasible. Therefore what is economically feasible is machine 2- machine 1. It means that machine 2 preferred to machine 1. This is the conclusion that we get therefore if we have a single project it is easy to judge through IRR whether the particular project is to be accepted. And how is it to be done? We find out first of the IRR and then see whether IRR is greater than minimum attractive rate of return MARR.

It is then that project is better to take up. And when we are making a comparison then between 2 projects then we have to find out or take the difference. Take the difference in such a manner that the initial investment is negative. So, that is why we had machine 1- machine 2. So, that this is the initial investment and these are the returns. So, we had chosen machine 2 to be subtracted from machine 1.

Therefore this becomes negative this is how it is to be chosen and then find out IRR. If IRR is higher than MARR, then this initial investment is worth making else initial investment is not worth making. In this particular case IRR is much lower than MARR indicating that this investment in machine 1 to the order of 10,000 is not worth making. Machine 2 is better. This machine 2 is called the base project.

Minimum investment project is called the base project and we are comparing whether any higher investment is possible. Now that we have taken this.

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Example:

Consider two projects, A and B, with the following data:

A: Initial investment:	Rs. 60,000
Annual net cash inflow:	Rs 22,000
Number of years:	4
B: Initial investment:	Rs. 73,000
Annual net cash inflow:	Rs 26,225
Number of years:	4

Find IRR and PW for each project.



Now, consider 2 projects, A and B, with the following data. In this example we will show that if the internal rate of return is applied to individual projects while making comparison it can give wrong results. Take this case initial investment of project A is 60,000 the annual net cash flow is 22,000. Numbers of years 4. Initial investment is 73,000 and the annual net cash flow is 26,225 and that also as in the same numbers of years as that of project A which is 4 years.

Now, we can apply present worth. First comparison method we can also apply internal rate of return method. Let us apply both and see the results.

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	A	B	$\Delta(B - A)$
Cap Investment	-60,000	-73,000	-13,000
Ri - Ei	22,000	26,225	4,225
n	4	4	4
IRR	17.3 %	16.3 %	11.4 %
PW (10 %)	9,738	10,13`	Positive

IRR can thus lead to an ***inconsistent ranking problem*** when comparison is made on the basis of IRR of the individual projects.

One has to work, instead, with the differential project with an incremental investment.



This is tabulated here. This is the capital investment I am sorry this row has to be here. That is

okay. Capital investment in proposal, project 1 is 60,000 and the net return meaning the revenue – the expense is 22,000. For machine B the values are 73 and 26, 4 years each. Now, if you do not consider the differential project purely on the bases of these data.

Suppose we apply IRR for project A and for project B the way we had applied it in the earlier example that the value of IRR is obtained at 17.3% for project A and 16.3% for project B. So, purely if we consider or apply IRR in this way and make a comparison between 2 different projects then we will be led to believe that machine A is preferred to machine B because it has a higher IRR.

But if we apply present worth factor, present worth cost comparison method out 10% MARR rate of return. We find that project B has got a higher value. This is 10,130 compared to 9,738. So, you can see that machine B is preferred. This is positive. Machine B is preferred to machine A on the basis of the present worth cost comparison method. And since they have equal periods interest are equal, periods numbers of year are equal.

Present worth cost comparison method is best applied there and therefor this result given by present worth cost comparison method gives the consistent result, correct result. Whereas internal rate of return method gives an inconsistent result, inconsistent with the present worth method and it is called inconsistent ranking problem. Because it ranks project A higher then project B which is not really the case.

There for IRR should not be applied in this way to make comparison between 2 different projects. The right approach to apply IRR is to find out the difference purely based on the cost of investment, initial cost of investment A should have been preferred. Why should I go for initial an additional investment in B. So, that the additional investment -13,000 incremental investment and the incremental revenues.

Whether this is economically viable that I should judge from the internal rate of return. So, apply internal rate of return method on these cash flows and in this case the value of IRR comes as 11.4% which is higher than MARR of 10%. It means that this additional investment project that

needs this additional investment with these revenues is worthwhile is economically viable and therefore go for B rather than (C) (29:23) for A. So, this is the right approach for internal rate of return method.

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Use of IRR for Comparing Multiple Projects

Suppose there are 4 projects A, B, C, and D.

Steps:

1. Rank order the projects on the basis of increasing initial capital investments. Assume the order is: B, C, A, D
2. Find IRR for each project.
3. If B's IRR > MARR, then B is the **base project**. Otherwise B is to be discarded and C is the base project, if C has an IRR > MARR. Assume C satisfies this condition. Hence, C is the base project.
4. Now consider $\Delta(A - C)$. Find its IRR. If IRR > MARR, then A is preferred to C. Now take A as the base project. Instead IRR < MARR, then C continues as the base project. Assume A remains as the base project.
5. With the new base project, test for $\Delta(A - D)$.



Now, we sometimes encounter problems of multiple projects and suppose that one needs to use or one is required to use the internal rate of return method when there are multiple alternatives or multiple projects then how to apply it that is what we have shown here in this particular slide. Suppose that there are 4 projects A, B, C, and D. So, first what we do? We rank order them that means in increasing order of their initial capital investment first of all put them.

Let B be the least cost of initial investment then higher cost is C and the highest is D. So, let the sequence of A, B, C, D be B, C, A, D when they are arranged in the increasing order of their initial capital investment. So, the right approach is compare B with C and whatever is preferred compare that with A. Whatever is preferred compare that with D basically one has to proceed in a sequentially manner.

So, what is first of all done first find the IRR for each project and in particular we are interested in B. Because that is the least cost but we have to see whether IRR of B is > MARR. If IRR of B is itself is not greater than the MARR then we discard B and then written only C, A, D that is what I have written here. If however IRR of B is higher than that of C then B is called the base

project and then we compare the differential project C-B.

Whether on additional investment over B is justified. And if this differential project higher than MARR then B is discarded and C is continued as the base project. Then we have C, A, D. Now compare A-C the differential project A-C once again find whether the IRR of this differential project is higher than MARR if it is higher than A is the base project C is rejected or discarded. And then finally compare A and D.

So, this sequential process is applied if we have to apply IRR. I think this is clear.

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Relative Advantages of the Methods

- The ***present-worth method*** is the simplest and the most preferred. But it is applicable to ***co-terminus projects***, i.e. to projects which have the same duration, and which start and end together at the same time points.
- The ***equivalent annual cost method*** can be applied directly to projects with unequal number of periods.
- ***Internal-rate-of-return method*** is applicable to single projects.



Now, we go to discuss the relative advantages of the methods. Firstly, let us understand that the present worth method is the most preferred, easily understandable and then it is very popular in the industry. It discounts all the future revenues and costs to the present and then compare it with the initial investment. However, when we make a comparison between 2 projects the number of years for which the project has a life they have to be the same.

If they are different we cannot apply present worth cost comparison method if we apply that then it will give us wrong results. Say one project is for 4 years another is for 6 years so naturally they will give different results and they are not comparable. Therefore, present worth cost comparison method in spite of its simplicity has this difficulty that it can be applied only when the number of years or the 2 projects is the same.

Now, the second method that we tool up was equivalent and all cost comparison method. This difficulty that we had with the present worth cost comparison method is surmounted in the equivalent and all cost comparison method. Meaning that if 2 projects have different periods and therefor different types of cash flows of course where present worth cost comparison method cannot be applied we can very well apply the equivalent and all cost comparison method.

Because there we are trying to find out in a year what is the cost? We (()) (34:53) So, whether it is 4 years or 6 years it does not matter we just see whether the equivalent and all cost for each project whichever is lower cost that is what is preferred. If it is revenue whichever is higher is better. So, unequal times or unequal periods if we are pressing for project then we select the equivalent and all cost comparison method.

So, here we find out A given P or A given F that means the capital recovery and the sinking fund factors are to be used in this case. Because the purpose is to find the value of A. Now, lastly the internal rate of return method. Internal rate of return method is good when we have a single project neither present worth cost comparison method nor the equivalent and all cost comparison method can compete with internal rate of returning method.

IRR can compare with the MARR value if the internal rate of return method is higher than MARR that project is very viable that if it is less it is not so viable. But if we have to apply IRR to comparison amongst alternatives then it is a very round about process. So, what we do is take the project with the minimum initial investment as the base project and then compare it with the next best project,

That means we take a differential project and see whether this additional investment is justified so here again we require equal numbers of years to apply IRR when we compare 2 different projects. So, these are the pros and cons of the 3 methods. However, there are more to it as we shall discuss.

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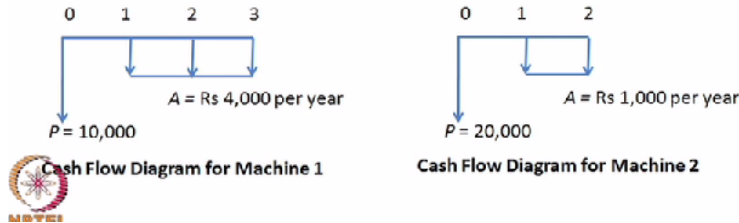
The present-worth method can be applied also to projects that are not co-terminus. For that, one makes a **repeatability assumption**: the two projects are **repeated** so that they become co-terminus.

Assume the following values for the two projects:

$$P_1 = \text{Rs. } 10,000/-, A_1 = \text{Rs. } 4,000/-, n_1 = 3 \text{ years}$$

$$P_2 = \text{Rs. } 20,000/-, A_2 = \text{Rs. } 1,000/-, n_2 = 2 \text{ years}$$

$$r = 10 \text{ percent per year}$$



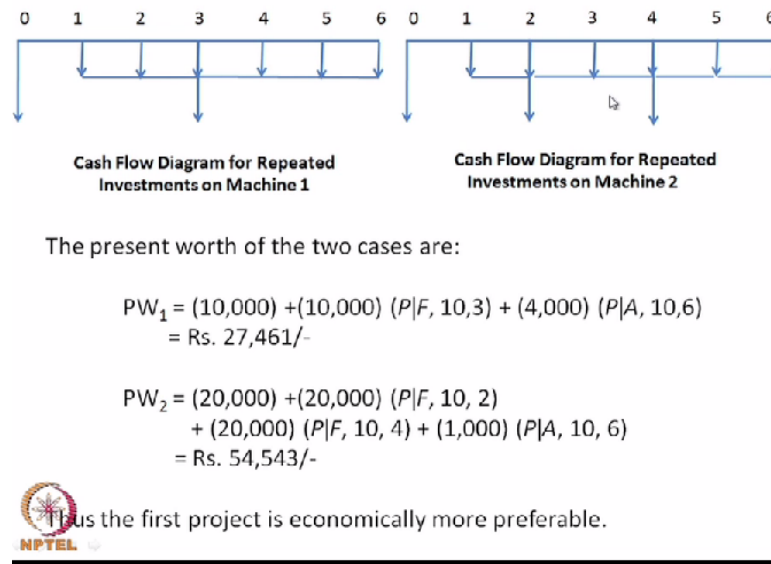
Suppose that we have to use present-worth method because that is the simplest for comparing between 2 projects. Here are 2 projects one project is for a 3 year duration another project is for a 2 year duration and the initial investment here is 10,000 and the annual expenses are 4, 000 per year in the first project. In the second project it is 20, 000 and 1,000 rupees. The annual expenses every year.

Now, you can see arithmetically they are equal $4,000 * 3$ is $12,000 + 10, 000$, $22, 000$ and this is $1,000 * 2$ is $2000 + 20, 22,000$. So, arithmetically they mean the same thing however there are differences. So, how to use present worth for these cases 2 case or this case. Because the periods or unequal we cannot straight away apply it. We can use 2 approaches to solve this problem if we have to use present worth.

One is to make a repeatability assumption, that means we will assume that once the project is terminated a similar project will start immediately. So, a project with 3 years duration as soon as third year is complete a similar project is taken up with similar cash flows for next 3 years and likewise for a 2 year project once 2 years are over once again the similar project is taken up and after 4 year still another is taken up making it 6 year.

That means 3 times the second project is taken up and first project is taken up 2 times. If that happens then both have the same period 6 years. That is what I have shown here.

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Project A as soon as this is complete then immediately another project is taken up for 3 years so this is the initial investment for project 1 when it was taken up for the first time and at the end of the third year again it was taken up. So that twice the same project was taken up total span of the 2 project becomes 6 and in this case project 2 was taken up here for 2 years as soon as it was over once again it was taken up.

As soon as the 4th year was over still once again project 3 was taken up. So, the total span for this case was 6 years. Now foreseeable we have made both the spans of the project equal. Once we have this we can apply the present worth cost comparison method. So, this was 10,000 + 10,000 that is find out the present worth of a future sum F even 10% interest rates and 3 years that is why I have written down single payment present worth factor.

This one I am discounting to the present this value was also 10,000 and these payments were 4,000 each for 6 years. So, I found out 4,000 * this is equal payment series, present worth factor for 6 years and interest rate of 10. So, from the table I can find out the values to through this calculation and the value came to 27,461 and for this particular case this was 20,000 remains. This 20,000 has to be discounted to the present.

So this is single payment present worth factor 2 years this one is single payment, present worth

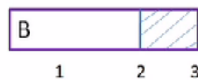
factor for 4-years. So, that is what we have done $20,000 \times$ single payment present worth factor for 2 years single payment present worth factors for 4 years multiplied by 20,000 + these annual payments were 1,000 rupees this continued for 6 years. So, this is equal payment series, present worth factor 6 years that thing came to 54,543.

So, this was much higher compared to this. Since they are all cost or expenses or cash outflows we will say that project 1 is preferred to project or machine 1 is preferred to machine 2. So, if we have to apply present worth cost comparison method one way is to make what is called (()) (43:00) assumptions same project is repeated after it is terminated. There is yet another approach to use the present worth cost comparison method.

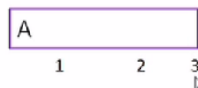
Which is also used and that is called Co-Terminated assumption.

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Co-Terminated Assumption



- Re-investment of cash flows at MARR at the end of 2 years.
- Find the Final Worth at the end of 1 year.
- Multiply that by $(F/P, r, 1)$ to find FW at the end of 3 years.



- Find the Final Worth at the end of 3 years.



Take the same example this was a case of project A has a 3-year duration and project 2 completes here. So, this is an extra thing. So, we were unable to make a comparison between project A and B unless a repeatability assumption was made. So, what we can say that find out the final sum or final worth at this point. Final worth means find out the compound amount factor, use the compound amount factor everytime for all cash flows.

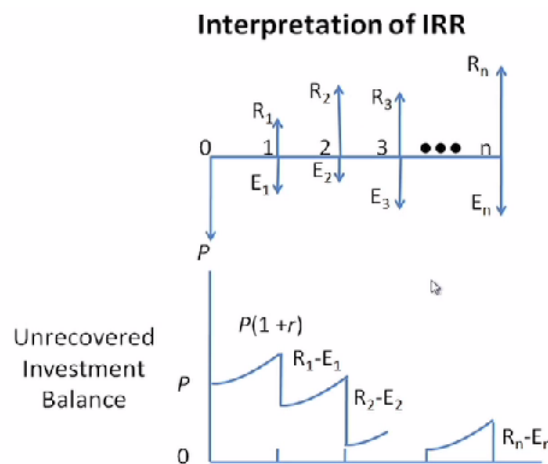
To find out the value at the terminal time period 3 that is the final worth just as we were using


PW as present worth if we convert all our cash flows to the final time period we will call that final worth. So, find out the final worth of all cash flows at time point 3. And similarly find out the final worth of all cash flows at .2 and then assume that at this point the amount that came as the final worth is reinvested at an MARR for 1 more year.

So that it becomes equal to 3 and then find the final sum. So, what we have done we first find out the final worth of the cash flows at time period 2 and then multiply that with $1+R$ that is all. Where R is MARR for 1 year multiply that by F even $P, r, 1$ which is nothing but just $1+r$ to find final worth at the end of 3 year. And compare that final worth with this final worth whichever is higher or whichever is lower because they are all cost in our case take that.

So, this is another useful method of using present worth cost comparisons when the time periods are unequal.

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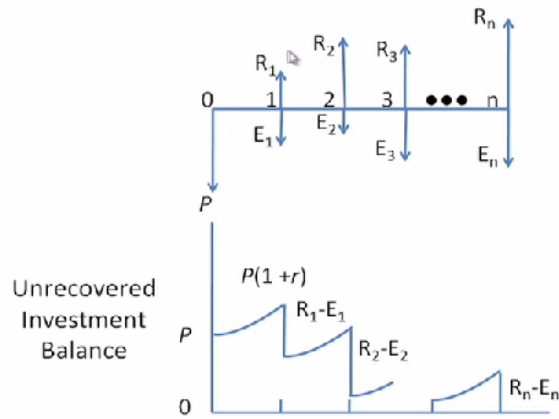



 Unrecovered investment balance is re-invested. At the end of n periods, the reinvested investment balance becomes zero.

Now, we are still in IRR because internal rate of return is very much talked about at particularly in industry they are interested to find out IRR. We are trying to give this particular slide an explanation of why the word internal? Why we are calling it internal? What is the meaning of word internal in the IRR internal rate of return? Now you see take this particular cash flow.

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Interpretation of IRR



 Unrecovered investment balance is re-invested. At the end of n periods, the reinvested investment balance becomes zero.

To illustrate that I have taken this cash flow I have assumed that there is an initial investment of P and for every year 1 through n there are certain revenues and certain expenses. So R_1 and E_1 are revenue and expense in time period 1. For 2 it is R_2 E_2 etcetera. R_n E_n now what basically internal rate of return means that P is invested at that R , IRR let us say R is that internal rate of return then its value becomes $P \cdot (1+r)$ after 1 year.

And then it is reduced by $R_1 - E_1$ so the unrecovered investment becomes $P \cdot (1+r) - (R_1 - E_1)$ within parenthesis $R_1 - E_1$. So, whatever we have invested after 1 year its compounded amount is $P \cdot (1+r)$ but it is reduced by the net return that comes to me. From here once again it is as if it is invested. So, it is an internal investment that we are thinking it is as if it internally invested once again for another year resulting in whatever amount this was $\cdot (1+r)$.

But then it is less the net revenue which is $R_2 - E_2$ (48:51) at this point and this continues till at some point the value is such that when we subtract $R_n - E_n$ the net unrecovered investment balance becomes 0. This is the basic meaning or internal rate of return that means that when we talk about internal rate of return it has an implication that the fund generated or fund invested is internally reinvested.

Such that after sometime at a particular value of R the net cash flow or present worth of net unrecovered investment balance is 0. Now, in our next lecture we shall study about the external

rate of return. If this is internal rate of return what is external rate of return. We shall also study about the interest rates that change from period to period and if compounding takes place more frequently then only once in a year.

So these cases are quite interesting and also they are quite realistic. That is the reason we shall spend some more time on this time value of money and on comparison amongst alternatives before we actually take up new topics of managerial economics. Thank you.