Economics, Management and Entrepreneurship
Prof. Pratap K. J. Mohapatra
Department of Industrial Engineering & Management
Indian Institute of Technology – Kharagpur

Lecture 43
Inventory Management (Contd.)

Good morning. Welcome to the 43rd lecture on Economics, Management, and entrepreneurship. In our last lecture, we were discussing inventory management. Today, we shall continue to discuss the same topic. First, let us recall that towards the end of the lecture we were discussing about how to control inventory in the simplest of the situation where we assume demand to be continuous, constant and known and supply lead time is = 0. We can show this in the form of a diagram such as this.

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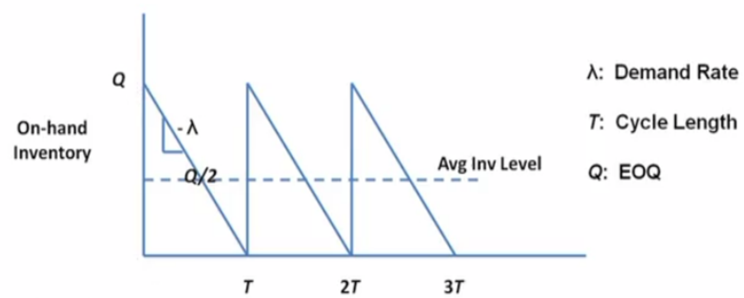

$$Q = \lambda T$$

$$TC/cycle = K + CQ + IC(\frac{Q}{2})T$$

$$TC/Year = \frac{1}{T}[K + CQ + IC(\frac{Q}{2})T] = \frac{K}{T} + C\lambda + \frac{1}{2}IC\lambda T = \frac{K}{T} + C\lambda + \frac{1}{2}h\lambda T$$

where $h = IC$ (Rs/unit/year)

In this diagram, the x-axis is time and the y-axis is the on the hand inventory. If we have a maximum amount of inventory held as Q and lambda is the demand rate and T is the cycle length, then after T time, this Q will be depleted to 0, inventory will be depleted to 0 and then we place an order for Q items and since the supply lead time = 0. It is immediately obtained in the stock and then because of the continuous demand, it continuously falls and this continues in a cyclical manner.

This looks like saw tooth and therefore this diagram is known as saw-tooth diagram. The cost associated with such an inventory placement policy are 2, one is the procurement cost K+CQ and then the inventory holding cost. The average inventory held is Q/2 and multiplied by T is the inventory held in the T cycle time and IC is the inventory holding cost, IC per unit multiplied by this. So this is rupees per unit per year and this comes to K+CQ+ICQ/2*T per cycle.

Per year, it will be 1/T that makes it K/T+C lambda+1/2 h lambda T.

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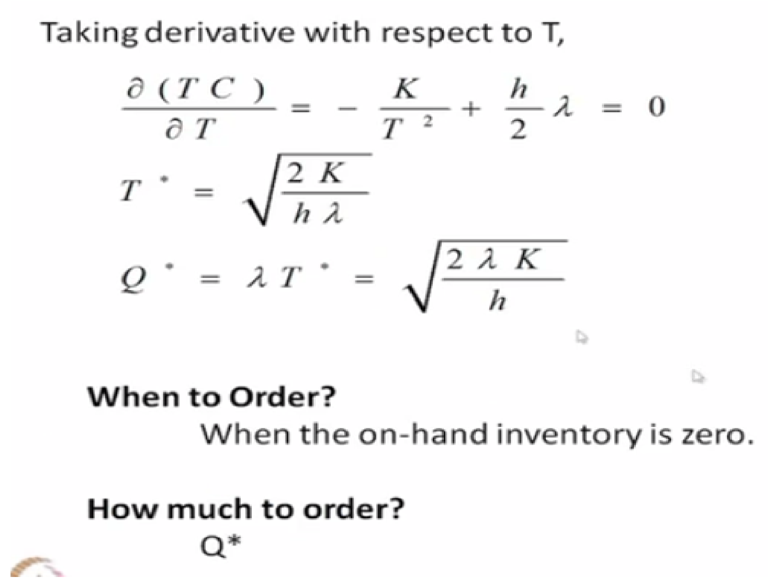

Once we have such an expression containing T, we differentiate this with respect to T, we get the optimal value of T* that minimizes the value of total cost and since we know that Q* is nothing but lambda T*, we can get this expression, which is root over 2 lambda K/h. In the numerator, we have the demand rate lambda. If demand is more, then Q has to be more, but if and if the fixed requirement cost or ordering cost is high, then Q* should also be high, but if the inventory holding cost is more, inventory should be, the amount ordered should be less.

This looks like, this contains a square root, therefore this formula is called square root formula. It is also called Wilson’s lot-size formula. So from here we answer 2 questions when to order, we order when the on-hand inventory is 0 and how much to order, we order Q*, which is = 2 lambda K/h.

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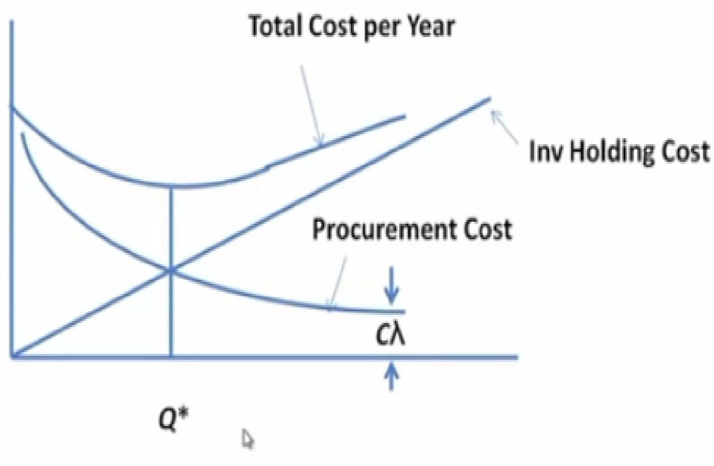

Now here we are showing how the cost vary with the amount of Q. As Q increases, the inventory holding cost increases, but the procurement cost reduces. The total cost takes a minimum at the value Q*.

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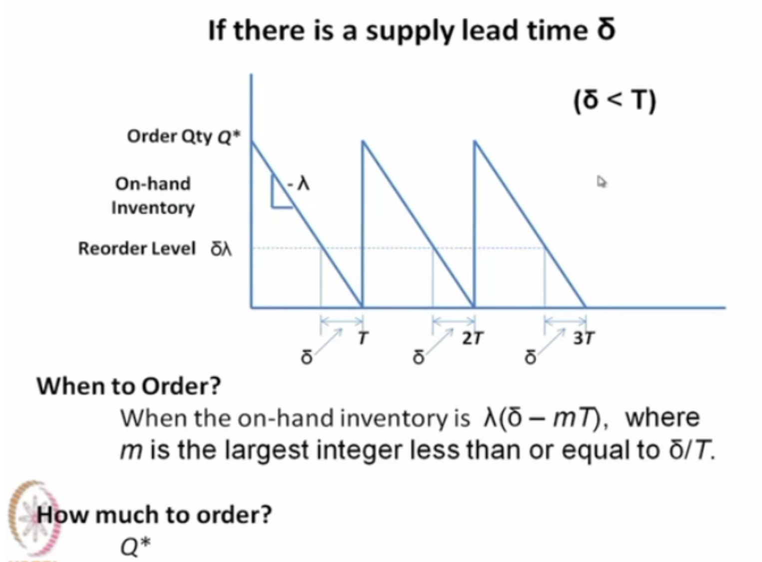

Now all these we had discussed in our last class. We would like to take up more realistic cases of lead time not constant to start with and then we shall relax many of our assumptions. We shall take up cases when demand is not known, is not constant, but is probabilistic, which are also considered the case of lead time being probabilistic. To start with, we shall consider the case of deterministic demand, but the lead time is not 0, but < T, the cycle time.

So it is very simple, almost all other remains the same, Q*, 2 lambda K, root over 2 lambda K/h, but when to order is delta time before the inventory comes to 0. That means when the inventory position is delta multiplied by lambda. That is the re-order level. We call it the re-order level. At this point, when the inventory comes down to the level of delta * lambda, this amount places an order and after delta time, which is called the lead time, the amount is obtained in the stores.

Continue to do it to this, once again it falls to the level delta*lambda+ an order for Q and the amount will come at this point. So this is the case, but sometimes the lead time can be > T.

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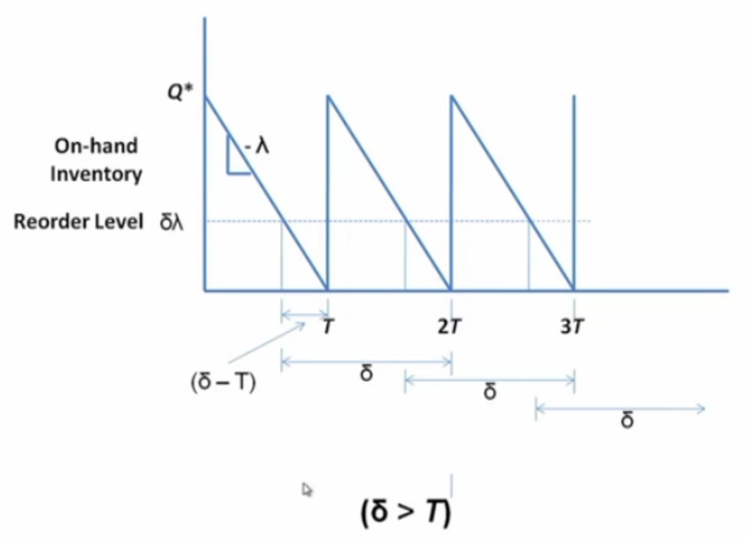

Say for example, here we are considering lead time as big as this. So we still order, what we do. We consider lambda delta – mT, where m is the largest integer <= lambda yT. So in this case, delta yT is 1. something, so m as 1, consider this, that means the re-order level is still delta*lambda, but the quantity arrives only after delta time. It arrives here. So when this has arrived, because of demand or replacement order that was placed sometime at this point.

When it comes down to the level of delta into lambda+ and order for Q and that will come after delta time again, that means this is due to a demand or due to replacement order that was placed at this point of time and similarly it continues. So this is the case when delta is > the cycle time T.

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Example:

The annual demand of an item is 1,000 units/year. The order cost is Rs. 10 per order. The holding charge is 20 %/year. The unit cost is Rs. 10 per unit. The supply lead time is 50 days. Find the economic order quantity and the reorder point.

Given: λ = 1,000 units/year, K = 10 Rs/order, I = 0.20/year, C = 10 Rs/unit, δ = 50 days.

$$Q^* = \sqrt{\frac{2\lambda K}{h}} = \sqrt{\frac{2(1000)10}{(0.20)(10)}} = 100 \text{ units}$$

$T = Q^*/\lambda$ = 100/1000 = 0.10 year = 36.5 days

δ/T = 1.37.

m = the largest integer less than or equal to δ/T = 1.

ROP = $\lambda(\delta - mT)$ = (1000)(13.5)/(365) = 37 units

We take an example to illustrate this case. Let us say that the annual demand is 1000 units per year. The order cost is rupees 10 per order. So this is K, K is 10 rupees per order, lambda is 1000 units per year, holding charge, which is I*Ch is 20% per year, so we take 0.2 per year. The unit cost of the item is rupees 10 per unit and the supply lid time is 50 days. Now here, we straight away use our how much to order is Q*, which is given by root over 2 lambda K by h.

We put the values, we get this as 100 units. Now when to order will be determined by first of knowing the cycle length. We know the supply lead time is 50. The cycle length is nothing but Q*/lambda. That comes to 0.10 year, which is something like 36.5 days, but the supply lead time is 50 days, so delta/cycle time is 1.37 that is 50/36.5 and the largest integer <= to this value is 1. Therefore, re-order point, ROP is lambda*delta*mT=1000*this quantity, which comes to 37 units.

Now because this is 37 units, when the inventory level comes down to 37 units, we place an order for 100 units. So we thus know when to order, when the inventory position comes down to 37 units and how much to order is 100 units. This is the economic lot size.

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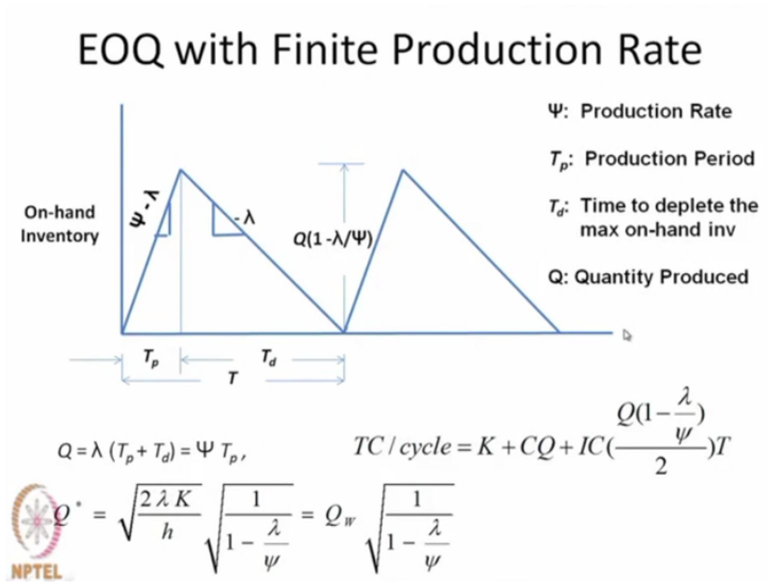

Now we take up another case, we say that the order is replaced by the factory itself by producing the amount that is ordered. So this is a case of finished product inventory. So in the finished product inventory case, we still assume that the demand from the customer is constant and is known whereas the supply is not instantaneous. It does not take place in one lot, but it is continuously produced and is available at the stores.

Because it is produced at the factory. We are plotting that case here. We are plotting on-hand inventory against time. We are saying that the production facility is used for different types of purposes. For this particular product, whenever there is an order, it produces for some time, we are calling it Tp, the production period. So it is producing certain amount for this product and then the facility is used for some other product.

Because we are considering only this particular product. We are assuming that Tp is the production time and suppose the size of the product rate during this period also, the demand of lambda is taking place, therefore the rate at which the inventory rises is the production rate – demand rate and at this point, the total quantity demanded for production for replenishment is available and therefore no more production takes place for this particular item.

From this time onwards, the on-hand inventory reduces as per the demand rate lambda. So the slope is –lambda here and then it comes to 0. At this point, once again the order is placed for production and once again the inventory rises and it falls. Therefore, it still in this case also we can see cycle, but it is not a saw-tooth type of a curve. It has a rising straight line here, first phase for Tp time and for Td time, Td time is the time to deplete the maximum on-hand inventory to 0.

So we see that psi-lambda*Tp=Td*lambda. If Q is the amount that was ordered, then the maximum inventory that we shall hold is Q*1-lambda/psi. So this is the maximum amount of inventory. Once again, the total cost per cycle is the fixed order cost and that for us is the setup cost. For every product, for every order that is received for a different product, the machine has to be set up. So that, there is a setup cost involved.

So for this particular case where production is involved, we call it the setup cost. K+CQ + same thing IC*average inventory held, the average inventory is the maximum inventory/2. Maximum inventory is this Q*1-lambda/psi*T and proceeding as we have done earlier, we can find that Q*, the optimal value of Q that minimizes this total cost per year is 2 lambda K/h root over multiplied by another factor and this 2 lambda K/h root over is nothing but the Wilson's lot-size.

We are writing W here to indicate that this is Wilson's lot-size. So Qw, which is root over 2 lambda K/h multiplied by this factor 1/1-lambda y psi. That is the amount to be ordered. So when to order is when an inventory comes to 0 and how much to order is Q*=Wilson's lot-size formula *1/1-lambda/psi root over.

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Example:

The annual demand of an item is 1,000 units/year. The order cost is Rs. 10 per order. The holding charge is 20 %/year. The unit cost is Rs. 10 per unit. The production rate is 1,500 units/year. Find the economic order quantity and the reorder point.

Given: λ = 1,000 units/year, K = 10 Rs/order, I = 0.20/year, C = 10 Rs/unit, Ψ = 1,500 units/year.

$$Q_w = 100 \text{ units}$$

$$Q^* = Q_w \sqrt{\frac{1}{1-\frac{\lambda}{\psi}}} = 100 \sqrt{\frac{1}{1-\frac{1000}{1500}}} = 176 \text{ units}$$

Cycle time = Q^*/λ = 176/1000 = 0.176 year = 64.24 days

Production run time = $T_p = Q^*/\Psi$ = (176/1500)(365) = 42.80 days

T_d = 64.24 – 42.80 = 21.44 days

We take an example. Similar example as before, the annual demand is 1000 units. The setup cost is 10 rupees per year, holding charge is 20% per year, the unit cost is rupees 10, now we have added the production rate as 1500 units per year. So we have already found this is the same case as before excepting that the production rate is 1500 units per year. So Wilson's lot-size is nothing but 100 and Q* is nothing but Qw * this, that is 100/this. That comes to 1076 units.

That means we should order for 1076 units and from there we can determine the cycle time, which is the total quantity consumed in that cycle time. therefore, this divided by the demand rate that comes to 64.24 days and production run time, we can find that as 42.80 days. The remaining time where there is no production and the demand is existing and therefore the inventory level comes from the maximum value to 0 level is 21.44 days.

So we have considered 2 cases, the simplest case of demand is constant and the instantaneous supply of order and the second case we have taken when the supply is not instantaneous, but is continuous, because it is produced in house. Now there are many such variations of this economic order quantity formula, but we will not consider all those cases in any standard text book, you can get them.

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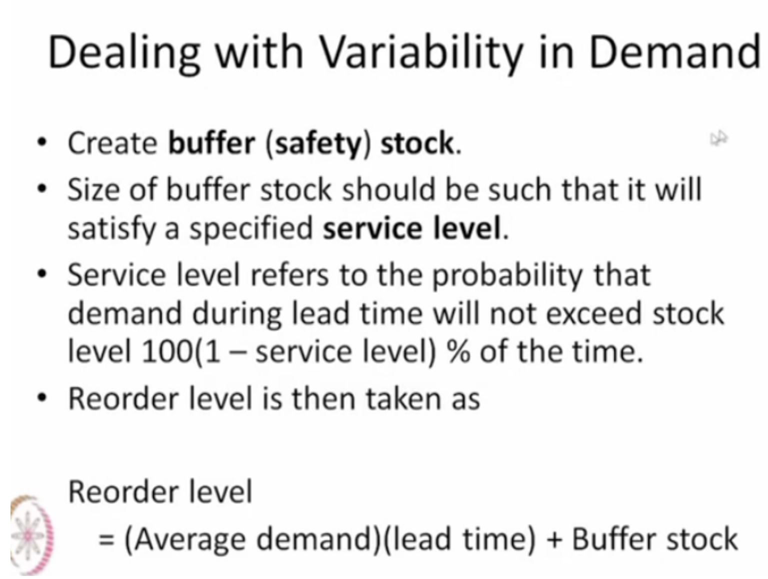

We are now considering the case when there is a variability in the demand and when the lead time is also variable. Now when there is a variability in demand, naturally you will have to keep little more stock because there is much greater variation in the demand. That means around the mean value of the demand, there is a variation. Therefore, whereas earlier, you could predict because demand was constant, lead time was constant.

Therefore, you could know that during the lead time, if lambda is constant and the lead time delta is constant, then delta*lambda is the stock that you should have, but now that you know that there is a variation in the demand, you must keep more than this amount. This amount that you should keep to account for the maximum demand that can occur during the lead time beyond the expected demand during the lead time is called the buffer stock or the safety stock.

So in dealing with the variability and demand, we create buffer stock. How much buffer stock or safety stock? we should keep is basically a policy of the management. Management specifies the service level. A service level basically says that this is the probability that the demand during lead time will not exceed the stock level so much percentage of the time. So if it says that it does not expect 5% of the time, during a particular cycle, then it means that the management policy is that the service level is 95%.

If instead the management policy is that at any time, the maximum demand during the lead time should not exceed the inventory by more than 1%, which means that its service level is 99%. So this is how the service level is defined and the re-order level then is. First of all, this is the expected demand during the lead time, assuming lead time is constant and even though demand is varying, we take the average value.

Average demand/the lead time is the expected demand during the lead time. Remember that we are assuming lead time to be constant, but demand variable. On top of this, in addition to this, we have to store buffer stock and the amount of buffer stock is the function of the service level that the management decides. It is like this.

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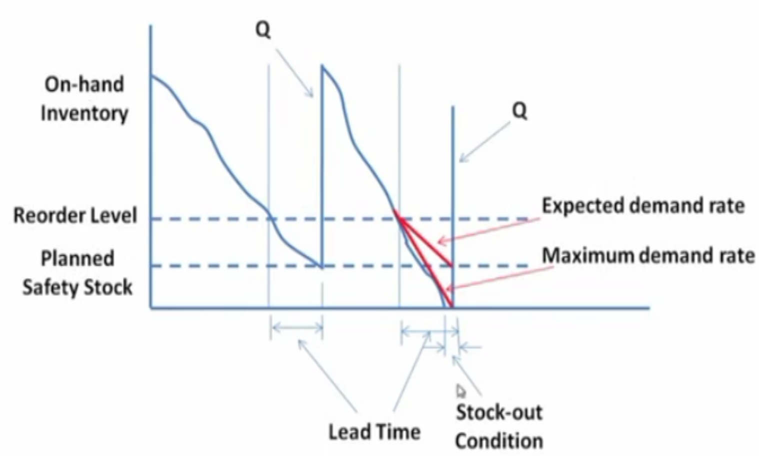

Variable Demand and Constant Lead Time

That you see this is the time when we are placing an order for replenishment. On-hand inventory is falling and when reaches a re-order level, we place an order for an amount Q and it arrives after a constant time. Now you see that the way it is reducing is not a straight line, meaning that the demand rate is not constant. It is varying and it is expected that one would like that the inventory should not come to 0 during this lead. Now here is the situation that it is not 0.

But in the second cycle, you will see that when we get the replenishment and then once again it falls, it is possible that before the next replenishment takes place, which occurs after this lead time, the same time the inventory has come down to 0. At this small time length is the period for which we have a struck out condition. So what is normally done is that the re-order level is just not the expected demand. Expected demand is basically the average demand* the lead time.

Probably only this much is the expected demand, but on top of this, we in fact should think of a maximum demand that can occur during this time. We should create a little more stock and that is called the planned safety stock. That depends what I said on the service level. So this is a case variable demand, as you can see, variable demand and constant lead time.

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Variable Demand and Constant Lead Time

We assume

λ is the daily demand rate (random variable) and follows normal distribution with mean of μ and variance σ^2:

$$\lambda \sim N(\mu_D, \sigma^2)$$

Then the total demand during the Lead Time λ_L is a sum of L random variables λ_i:

$$\lambda_L = \sum_{i=1}^{L} \lambda_i \sim N(\mu_D L, L\sigma^2)$$

where λ_i is the demand on the ith day.

Now how we handle using probability theory. If lambda is the daily demand rate, which is considered a random variable, and follows a normal distribution with mean mu and variance sigma square, so we have to use probability theory concepts. So if you do not have a good background in probability theory, then it will be difficult for you to understand this and we are not discussing the concepts of probability theory.

I request you to go through any book on probability theory and this is very simple introductory material in any book on probability and statistics. Here we are assuming that the lambda is a random variable that follows normal distribution, with mean mu of demand d and variance sigma square. Then the total demand during the lead time lambda l is a sum of l random variables, because it is let us say the daily demand rate.

So everyday lambda is varying, so if l consists of so many days, so it is a sum of l number of random variables and the variance, the mean of such a random variable lambda l, which is the sum of l random variables is l*mu d. The mean mu d+mu d+mu d+.. l times and variances are also added, sigma square + sigma square + sigma square … l times. Because there are l number of such random variables that are added.

We are trying to define the sum of l number of random variables lambda i, so means are added and variances are also added. Therefore, lambda l follows normal distribution with mean mu d*l and variance l*lambda square where lambda i is the demand on the i-th day.

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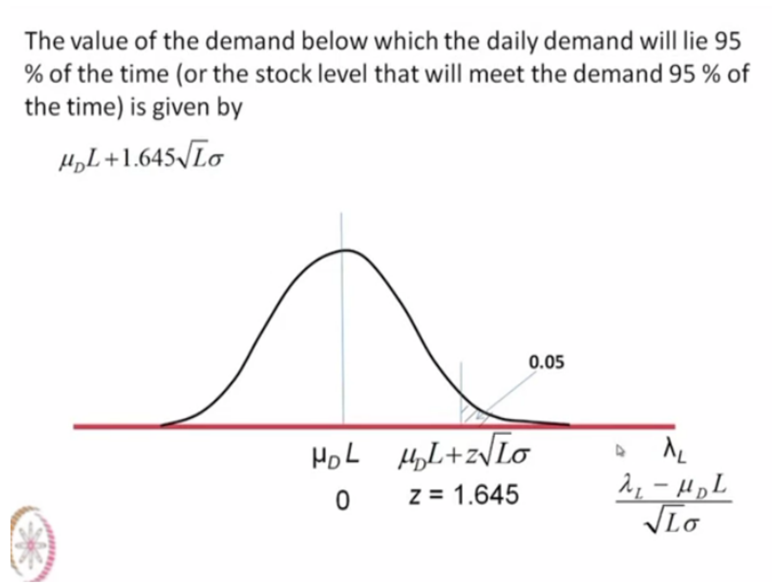

So basically, we are saying that this is the normal distribution whose mean is mu d*l and we are plotting lambda ld sum of, that means the total demand during the lead time and if the service level is 95% that means we are interested to keep a service level here such that the maximum demand will exceed this value only 5% of the time. So the area under the normal distribution is 0.05. If the area under the normal distribution is 0.05, then by how much if the mean is mu d*l and if the variance is root over l.

This is the standard deviation root over l*sigma*z, the standard normal random variable from the normal distribution table, we can find the value of z as 1.645. So normally, normal distribution tables are available in the appendix of any book on probability theory and one can find out corresponding to 0.05, what is the value of z and the value of z is 1.645 and such a distribution whose means is this and the variance is lambda l, can be standardized by a standard normal random variable, which is lambda l-the mean/standard deviation, which is root over l sigma.

Such a standard normal random variable has a mean 0 and a variance 1. So this value of lambda l=mu dl+z * this or for this the value is z=1.645 for 0.05. Now this requires a little bit of knowledge of, as I said, probability theory, which is quite simple. Now how do we then calculate the buffer stock.

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Buffer stock

= Maximum demand during lead time that can be met by the buffer stock at the specified service level
- Expected demand during lead time

$= [\mu_D L + z(\sqrt{L}\sigma)] - \mu_D L$

$= z(\sqrt{L}\sigma)$

For 95 % service level,

Buffer stock = $(1.645)(\sqrt{L}\sigma)$

z is called the **safety factor**.

Buffer stock then is equal to the maximum demand during the lead time that can be made by the buffer stock at the specified service level – the expected demand during the lead time. So for us, this is the maximum demand at the specified service level that corresponds to the value of z – the expected demand during the lead time is mu d*l. So this quantity z*root over l*sigma is called the buffer stock. If 95% is the service level, buffer stock=1.645*the standard deviation, which is the root over of the variance. Normally z is called the safety factor.

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Example:

The average usage rate of an item is 100 units/day. The standard deviation of the daily demand is 5 units/day. The lead time is 9 days and the firm desires a service level of 95 %. Find the buffer stock and the reorder level.

Given

μ_D = 100 units/day, σ = 5 units/day, L = 9 days,
Service level = 95 %.

Buffer stock = $z(\sqrt{L}\sigma)$ = 1.945($\sqrt{9}$)(5) = 29.4 units

It means that by keeping a buffer stock of 29.4 units, there will be 1 stock out in 20 (5 % = 100(1 - .95) % replenishment cycles.

Reorder Level = $\mu_D L + z(\sqrt{L}\sigma)$ = (100)(9) + 29.4 =929.4 units

We take an example to illustrate this case. The average usage rate of an item is 100 units per day. The standard deviation of the daily demand is 5 units per day. The lead time is 9 days. The firm desires a service level of 95%, find the buffer stock and the re-order level. So what are given are the mean value of the demand, which is 100 units per day, variance is given as 5 units per day, lead time is given as 9 days. Service level is given as 95%.

Buffer stock is then corresponding to 5%, which is 1-service level from the normal distribution table, z is 1.645, I am sorry, this is not correct. This is 1.645, now this calculation may be wrong, but you make the calculations. So buffer stock=29.4 units. It means that by keeping a buffer stock of 29.4 units, there will be one stock out in 20 replenishment cycles, because 5% is the stock out in one replenishment cycle.

Therefore, in 20 replenishment cycles there will be one stock out. This is the meaning of the service level. The re-order level is the expected value of the demand during the time + the buffer stock. The expected demand during the lead time is given by the average usage rate 100 multiplied by the lead time 9 + buffer stock, which is 29.4. Therefore, the value of the re-order level is 929.4.

It means when the inventory position comes down to 929.4 units, you place an order for Q and you will then ensure a service level of 95% meaning that there will be a 5% chance of stock out, meaning that there will be one stock out in 20 such replenishment cycles.

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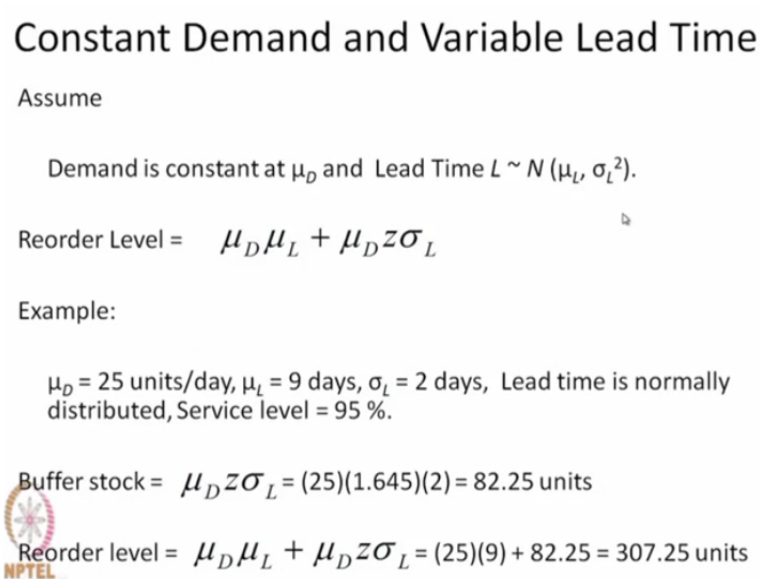

Now we consider the case, when the lead time is variable, but the demand is constant. So here demand is constant at mu d, but the lead time l follows normal distribution with mean mu l and variance sigma square l. Here the re-order level is as before. The expected value of the demand during the lead time, which is mu d*mu l + a buffer stock created because the lead time itself is changing and that is z to sigma l* mu d, mu d is the mean value of the demand constant value of course * sigma l.

We take an example, suppose the demand is 25 units per day and l has a mean value of 9 days, but it has a standard deviation of 2 days, then the lead time is normally distributed and suppose that the service level is 95%, then the buffer stock is just applied this one mu d z sigma l, z is 1.645 from normal distribution table, sigma l is 2 days and mu d is 25 units, that comes to 82.25 units. The re-order level is the expected demand that is 25*9, which is 225+82.75 that makes it 307.25 units. So this is the case of constant demand and variable lead time.

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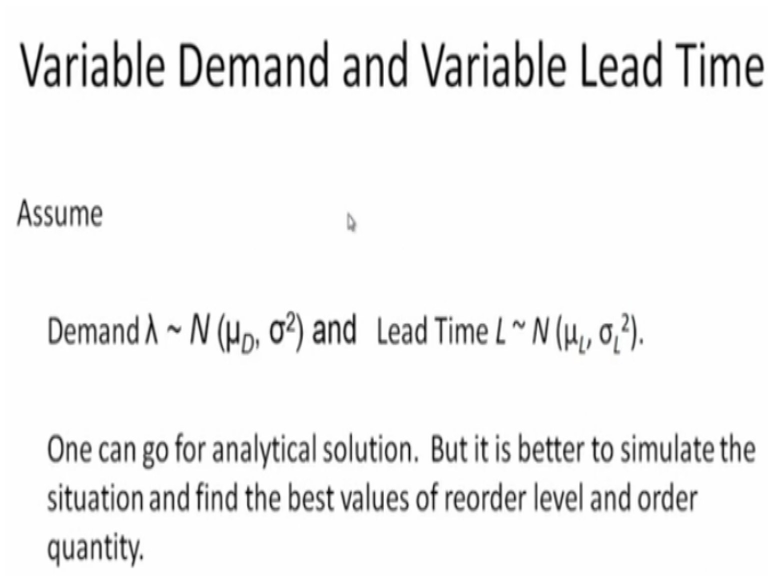

Now we consider the more general case of variable demand and variable lead time. Now this is more difficult and one can of course go for analytical solution, however one can also go for simulation because it is a little more complex case. That is what we are writing here. The demand follows normal distribution with mean mu d and variance sigma square and the lead time l also follows a normal distribution with mean mu l and variance sigma square l.

One can go for analytical solution, but one can also go for simulation. Now although we have assumed that the demand and the lead time vary according to normal distribution, they may not actually follow normal distribution. They may follow any other distribution, then the cases are even more difficult to handle. The natural solution for such cases is by resorting to simulation. Now in these cases, we have considered the independent demand case.

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Service Level and Number of Shortages

- 100(1 – service level)
  = Pr(lead time demand > buffer stock)
- Thus, a 95 % service level means that
  Lead time demand exceeds buffer stock 5 % of the time.
- The chance of stock out is only once per cycle.
- If the order quantity is Q, 20 times a year, then the expected number of shortages is (0.05)(20) = 1 per year.
- If the order quantity is 2Q, 10 times a year, then the expected number of shortages is (0.05)(10) = 0.5 per year or 1 in 2 years.

Before we go for the dependent demand case, we will give some more insights into the service level. As already, I have told the chance of stock out service level 100*1-service level is the probability that the lead time demand is > buffer stock. Thus, 95% service level means, lead time demand exceed buffer stock 5% of the time. Thus the chance is 5% stock out in one cycle.

If the order quantity is Q and is given 20 times a year, then the expected number of shortages is 0.05*20, which is 1 per cycle. If the order quantity is TQ and is given 10 times a year, then the expected number of shortages is 0.05*10, which is 0.5 per year or 1 in 2 years.

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Expected Shortage

- It has been shown that the **ratio of expected quantity short to the standard deviation of the lead-time demand** is a function of the safety factor as follows:

Service Level	z	Exp Shortage/σ_D
99.9 %	3.090	0.00028
99.5 %	2.576	0.00158
99.0 %	2.326	0.00441
97.5 %	1.960	0.00945
95.0 %	1.645	0.02089

- If service level is 95 %, and σ_D= 9,
  Exp shortage per order = (0.02089)(9) = 0.189 unit/cycle
  Exp shortage per year = (0.189)(number of orders per year)
  Shortage cost per year = (Exp shortage per year)(Unit shortage cost)

Sometimes we are also interested in finding out the numbers of x, not times the inventory is out of stock, but the number of units, which are short. This is the expected shortage. Now without deriving the formula, we are showing results that have been arrived at by various researchers. They have shown that the ratio of expected quantity short, number of items short to the standard deviation of the lead time demand is a function of the safety factor z.

They have given tables such as this under the assumptions of normal distribution of the lead time. So z is the safety factor. The ratio expected number of units short by the standard deviation of the demand is given by this for different service levels. Thus if the service level is 95% that the management has set and if sigma d, the demand standard deviation is 9, then the expected shortage for 95% is 0.02089 multiplied by sigma d, which is 9 is the expected shortage for order.

That comes to 0.189 unit per cycle, so expected shortage per year is 0.189*number of orders per year, so one can thus find out the actual number of units short in a year, the expected number of units short per year. So if the number of orders per year is 10, then 0.189*10 is 1.89, those units are short. So one can thus find out the shortest cost, which the expected shortage per year multiplied by the unit shortage cost.

In addition to our 2 costs that we have considered for finding out the economic order quantity namely the ordering cost and the inventory hold cost, we can also consider the shortage cost. To find out the shortage cost, we have to find out first the number of units short and then we must have an estimate of the cost of shortage of one unit and that depends on 2 conditions.

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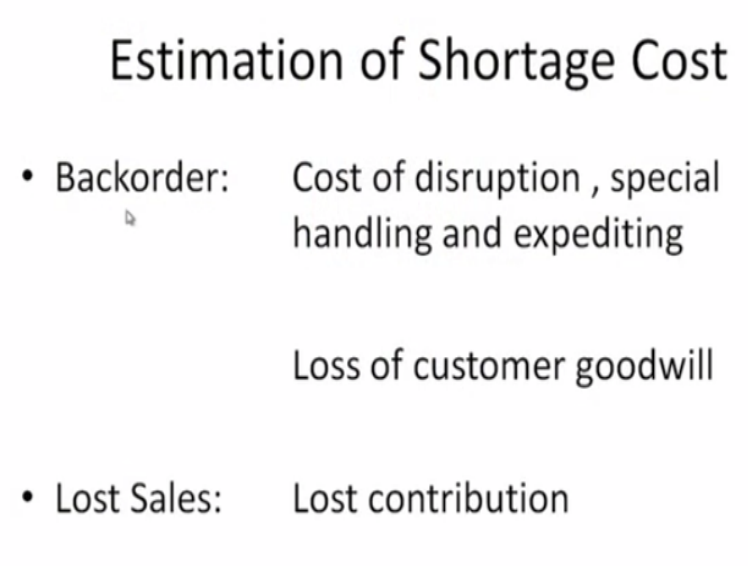

Whether it is a case of backorder, if certain unit is short, then there is possibility that the demand that is coming is held back or order backlog, that means the order is received and is not canceled. It is held, because the customer can wait for some time and we will fulfill the order at a later point of time. This is a case of order backlog or backorder case and the other case is the lost scales case, in which the order is received, but because you do not have enough inventory, you cannot fulfill the order and therefore it is lost.

The customer does not wait. In any case, for both the situations there are costs involved. The backorder cost in the first case and the lost sales cost and the backorder cost is that there is a cost of disruption, you have to specially handle this request or the backorder and go for expediting it. There is a loss of customer goodwill and in the lost sales case, there is a total loss because of the contribution of the revenue is completely lost.

So one has to estimate the unit shortage cost multiply that with the expected number of units short to find out the shortage cost per year and following the other 2, one has to also estimate the ordering cost and the inventory holding cost, add to that the expected shortage cost and then follow the same procedure to find out the economic order quantity. So friends we have considered till now the case of independent demand.

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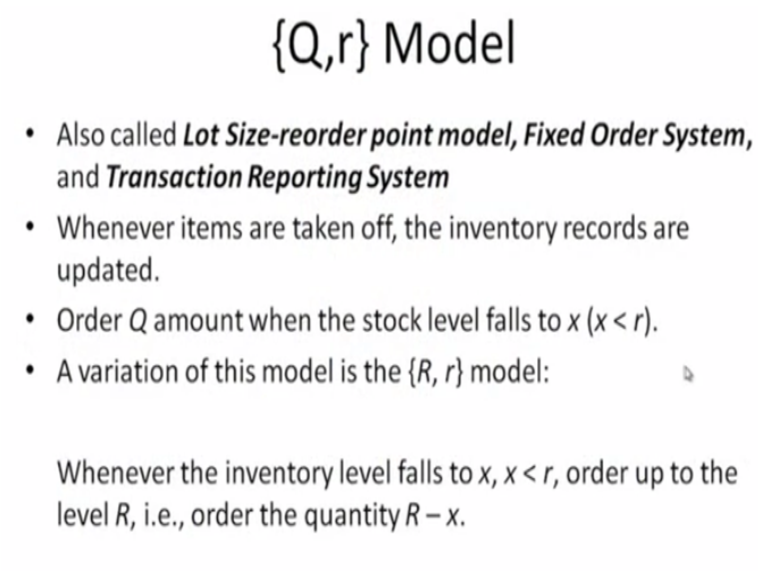

Now even before that let me introduce we can find out Q, but how to actually operate the inventory. There are different models for that. The first model is called the transaction reporting system or fixed order system, also known as Q, r model. Q is the fixed amount of order or the lot size Q. Whenever items are taken off, the inventory records are updated immediately, that is why it is called transaction reporting system.

Order Q amount, the fixed order or the lot size order, when the stock level falls to x, x<r, that means when the inventory position falls to < r, +an order for Q and a variation of this model is the r, r model, where whenever the inventory level falls to x, x<r, order up to level r, not an exact quantity Q, but up to level r, so these are the 2 variations.

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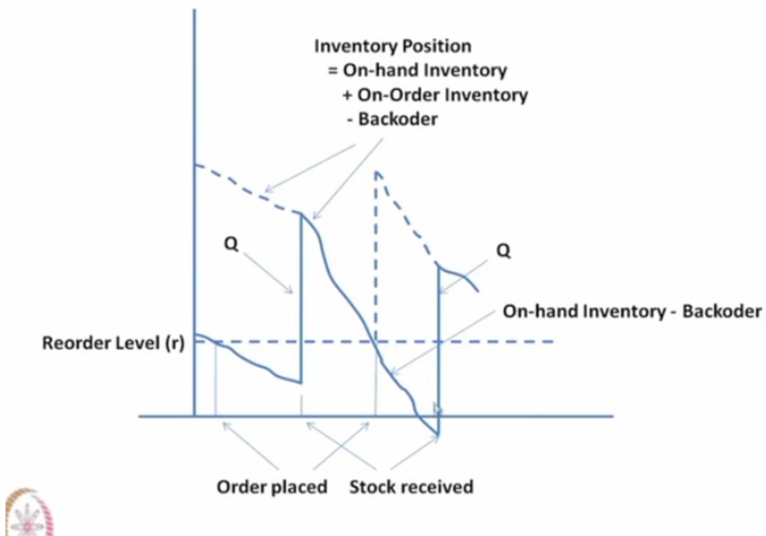

The Q-r Model

So this we are plotting here. This is the inventory and this is the time axis, this is the re-order level calculated the way we have already decided, r and this is a Q, r model. So inventory is falling. This is the formula in the inventory line. As the inventory is falling somewhere here, it reaches the re-order level + and order Q and their lead time is this, after that it comes. The replenishment arrives after the lead time. By that time, the inventory is depleting.

So when the inventory comes, it is here. Once again, the inventory starts falling, at some time the inventory falls to r or <r. We place an order for Q. So this dotted line says the inventory position, which is the on-hand inventory + the on-order inventory – the backorder. So there is no backorder here, so it is 0. On-hand inventory + on order inventory, so this arrival occurs because of a prior order that was placed sometime back.

So the total inventory position on hand + on order assuming no backorder. So it was falling like this. At this point, the order arrives. Again it falls, again here we have placed an order, therefore the on-hand + on-order becomes this. By that time, the actual on-hand inventory position reduces. Therefore, the inventory position also reduces and after lead time, which is not fixed. So orders are placed here and here and stocks are received against the order here and here. So these are the lead times, this and this.

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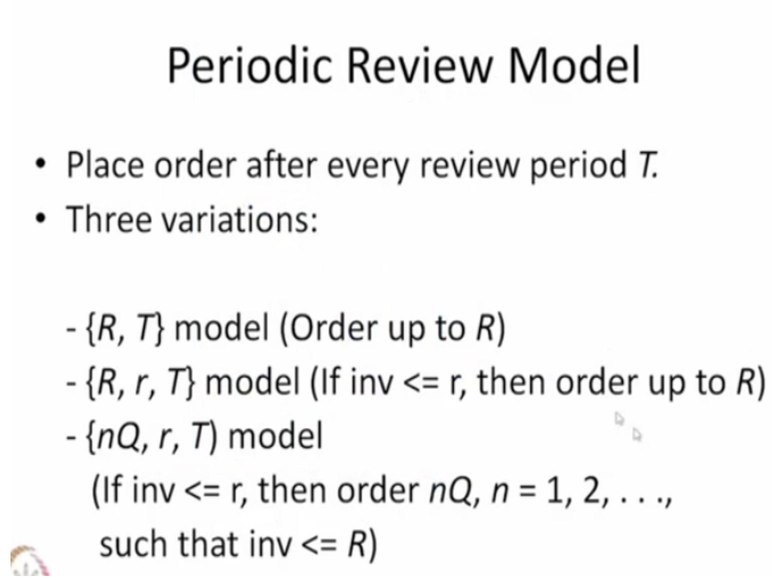

This is the Q, r model. Compare that to another type of a model, which is called periodic review model. Here we place order after every review period T and there are quite a large number of variations r, T model that is order of 2r, r, r, T model if inventory is <= r, then order of 2r or n, Q, r, T model order n, constant times Q, n can be 1 or 2 such that inventory is <= r. So there are many variations, we show them in this graph.

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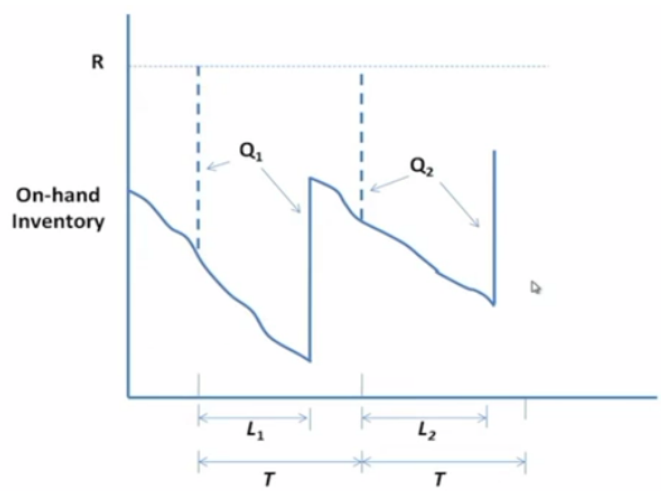

R-T Model

This is on-hand inventory. This is the r, T model and this is the T at every T, we place an order. So at this point, we place an order for Q item as well as we should have drawn a line like this, dotted line like that, because then it would have shown the inventory position. Now at this point, we place an order of 2r and then it comes after l1 period and then at this point, we once again place an order. You can see that the order quantity is different.

Here the order quantity is Q1 that comes here. Here the order quantity is Q2 because we are all the time placing an order up to r. So here it is different, so what is constant here is the T, the length of the period that is constant, but amount order is placed is different, so this is another way of also operating the inventory. It has been seen that the average inventory held in a Q, r model is more than that in the periodic review model.

Because in the Q, r model we have to keep a buffer stock and that makes the value of the average inventory held more than the amount that is held in a periodic review model here. So we have considered the inventory situation in great detail so far, but all those cases we have considered are for independent demand case. We shall end our lecture by discussing on the dependent demand case.

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The best way to consider a dependent demand case is by considering MRP, the materials requirement planning. First of all, what is this case of dependent demand. It is ideal for companies assembling end items from components in batch manufacturing processes. That means if you have to assemble some units or certain components, and if the number of assemblies to be produced is known, then the number of components is automatically known.

So one has to therefore have 3 things master production schedule, bill of material and inventory file.

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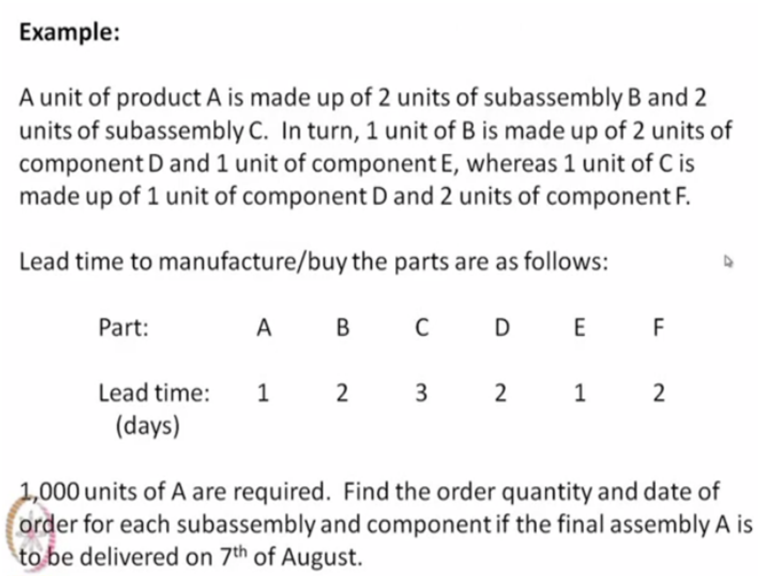

We are given an example to illustrate what we are saying. A unit of product A is made up of 2 units of subassembly B and 2 units of subassembly C. in turn, one unit of B is made up of 2 units of component B and one unit of component E. Whereas one unit of C is made up of one unit of component D and 2 units of component F. Now basically they are saying that the final assembly is made up of 2 subassemblies.

Each subassembly is in turn made up of 2 components each and the lead time to manufacture or buy the parts are this. 1000 units of A, the final assembly are required. Find the order quantity and the date of order for each subassembly and component, if the final assembly A is to be delivered on 7th of August. You can see that the demand for various items B, C, D, E, F are all defined, once the demand for A is known and the demand for A is known, 1000 units.

Therefore, the demand for B, C, D, E, F are known. This is the case of dependent demand. Now, these relationships that A is made up of 2 sub-assemblies, each subassembly is made up of 2 components, can be shown in the form of a product structure diagram.

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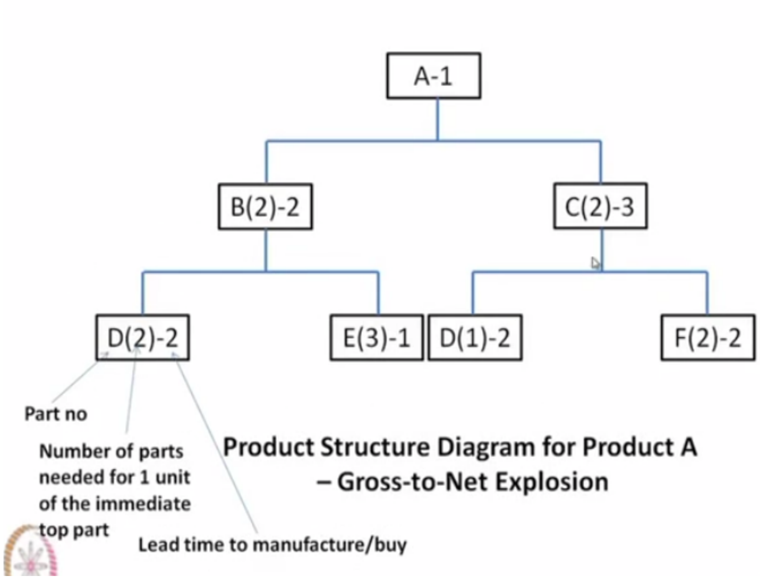

A tree structure, A consists of B and C, B consists of D and E and C consists of D and F. This within parenthesis says that one unit of A requires 2 units of B and 2 units of C. One unit of B requires 2 units of D and 3 units of E. One unit of C requires 1 unit of D and 2 units of F. Also written here is some numbers, 1, 2, 2, 1, 2, 3, and 2. They are all lead times. Once the work starts, it will take one day or one week, in our case one day.

Once we place an order for B, it will be available after 2 days, etc.

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		1	2	3	4	5	6	7	Lead Time
A	Reqd Date							1000	1 wk
	Order placement						1000		
B	Reqd Date						2000		2 wk
	Order placement				2000				
C	Reqd Date						2000		3 wk
	Order placement			2000					
D	Reqd Date			2000	4000				2 wk
	Order placement	2000	4000						
E	Reqd Date				6000				1 wk
	Order placement			6000					
F	Reqd Date			4000					2 wk
	Order placement	4000							

Now we show this in this form A, B, C, D, E, F are the parts, final assembly, 2 subassemblies and different parts. We need to have a 1000 units and it has a lead time of 1 week, therefore order for item A must be placed one day before that is on 6th of August. Now to make 1000 units of A, we need 2000 units of B and 2000 units of C, 2 items of B and 2 items of C for every unit of A and B’s lead time is 2 weeks, C’s lead time is 3 weeks.

Therefore, there is a lead time offset, order must be placed for B 2 days in advance and 3 days in advance for C. So for C again the required date for D and E are mentioned, and like this one proceeds backward, so one has to proceed this way and proceed that way and we finally get to know that on day 1, order must be placed for F and for D, this many items. On day 2, order should be placed for D. On day 3 order should be placed for E, 6000 items and for C 2000 items.

Day 4, order should be placed for 2000 items for B. On day 6, order should be placed for A 1000 items. This is a very simple example of materials requirement planning.

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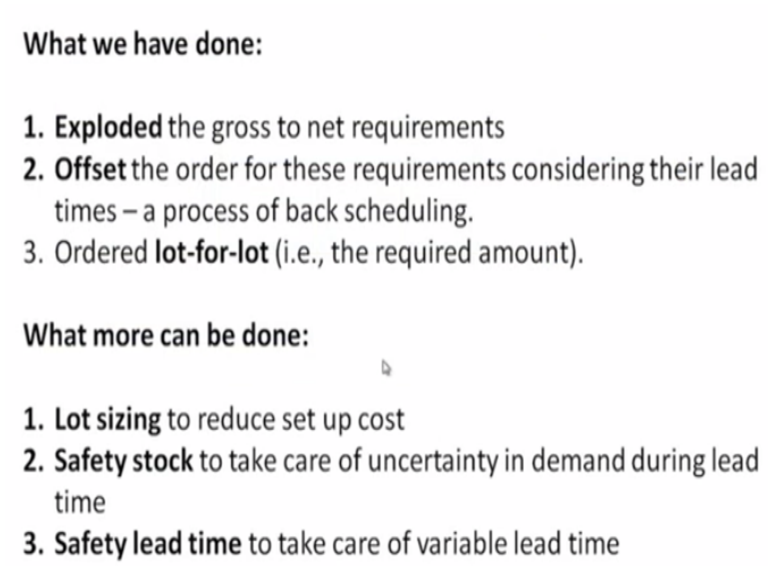

So what we have done so far is that we have exploded our gross requirements and to net requirements, offset the order for these requirements considering their lead times, which is also known as back scheduling and we have ordered for lot-for-lot, which the lot size is not constant. What can be done is lot sizing to reduce setup cost. One can also have a safety stock to take care of uncertainty in demand during lead time.

One can also give little more lead time as a safety, just one or 2 days more. So friends, inventory control is quite extensive. There are a large number of topics. We have covered some topics, particularly the deterministic demand and lead time cases. Today, we consider the probabilistic lead time and probabilistic demand cases. We introduce the concept of buffer stock. We also said that in practice, either fixed order system is followed or fixed period system is followed.

Towards the end of the lecture, we introduced the dependent demand case and we will take up one more topic on inventory management in our next class before going to a new topic and that topic will be supply chain management. Thank you very much.