Course on Decision Modeling
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Lecture 21
Module 5
Introduction to Simulation

Today we shall begin a new topic that is on Simulation.

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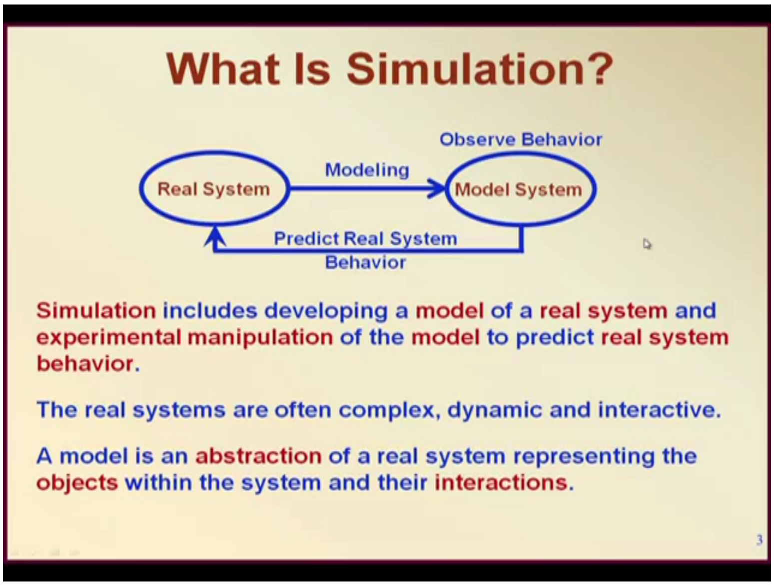

So first of all let us have an introduction to the simulation and as an introduction first of all what is simulation? So if you look at this particular diagram then one can understand that suppose we have a real system and we want to know the behaviour of this particular real system, what could be a real system? It could be a queuing system, or it could be an actual you know supply chain system, or some inventory management system, or some marketing system. So whatever that real system may be because it is happening in a real world any kind of changes that you make to that system could be costly.

So if you really want to know about the behaviour of particular system and you make changes in the real system the results may not be likeable, what we can do then? We can create a model system out of that real system and observe behaviour of the model system rather than the real system. So what is happening we want to know about the real world it is costly to do experiment on the real world, so we make a model system and observe that model system and from those observance we predict the behaviour of the real system this is called simulation.

So as you can really understand the simulation includes first of all developing a model then of a real system and then experimental manipulation of the model to predict real system behaviour. The real systems are often complex, dynamic and interactive, right. So but what is a model, model is an abstraction of that real system representing objects within system and their interactions.

So really speaking you see suppose we really want to make model it is not necessary that we really try to mimic all parts of the real system behaviour really speaking something like suppose you know you want to simulate the hand behaviour of persons and you make a model which is basically a Robo and you know it need not really that Robo need not really do all functionalities of a human being because your objective here is really to predict the behaviour of the hand movements so that Robo may actually look like a hand.

So why I give this example of physical simulation? Basically that model need not mimic the entire real system behaviour one must set an objective and with regard to that particular objective all the aspects of reality should be modelled so that is essential requirement of simulation.

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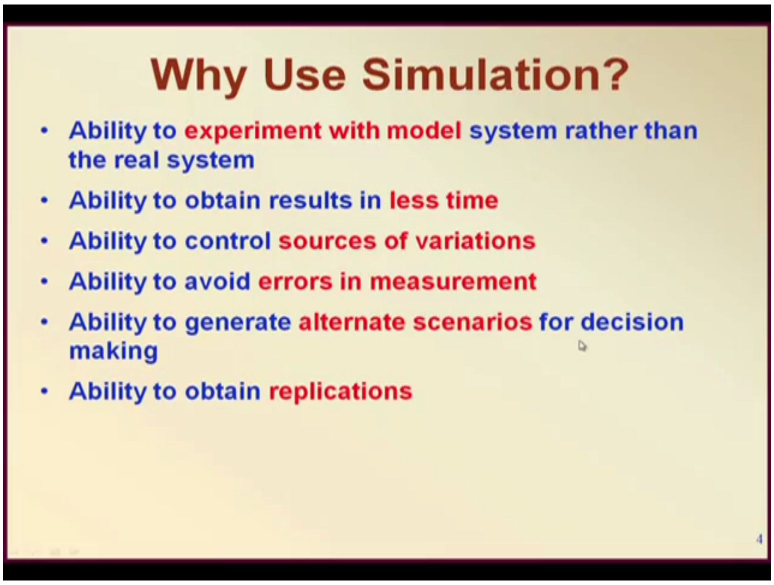

First of all then the question is that why use simulation? You know when we can do, what else we can do if we do not do simulation, we want to restrict our discussions specifically to the mathematical models and that to apply to business situations, right to societal systems. So something like as I gave example the queuing systems, supply chain systems, inventory management systems, marketing systems and so on so forth.

In all of these particular systems you see sometimes the equations could be so complex and interactive and as an alternative to simulation if you really want an analytical solution, right you want a close form analytical solution and you want to solve. Say for example in one such simulation which is called continuous simulation we are getting a set of differential equations, right a reality could be as high as may be 20 or even 22 order differential equations. Imagine solving a 20 or 22 order differential equations how difficult it could be, right.

It may not be always possible to obtain a close form analytical solution easily under those situations the only answer is simulation, why? Because it really you know tries to calculate the variables based on the you know one step you calculate the next step is a function of the previous step that is in continuous simulation and in discrete-event simulation you apply or use the random numbers, right.

So we will discuss all of them in detail here are some more reasons, right. How simulation can help? You can see first of all you can experiment with model system so there is no need to really disturb the real system, ability to obtain results in less time, then you can control the source of variation how by generating alternative scenarios. So you can avoid errors of measurements and you can ability to generate alternate scenarios that is what I already told and finally you can obtain replications.

What are replications? I will discuss in detail because replications are something which is essential for any simulation experiment, any simulation experiment that you do you have to repeat that experiment with a different set of random numbers because when you use a set of random numbers to do a simulation experiment you get one result but how much we can trust that result because you know that result that is probably not even a mean value it is just one observance out of a random experiment. If you do another experiment you will get another result. So that experiment is called a replication, right. So we will see that in detail.

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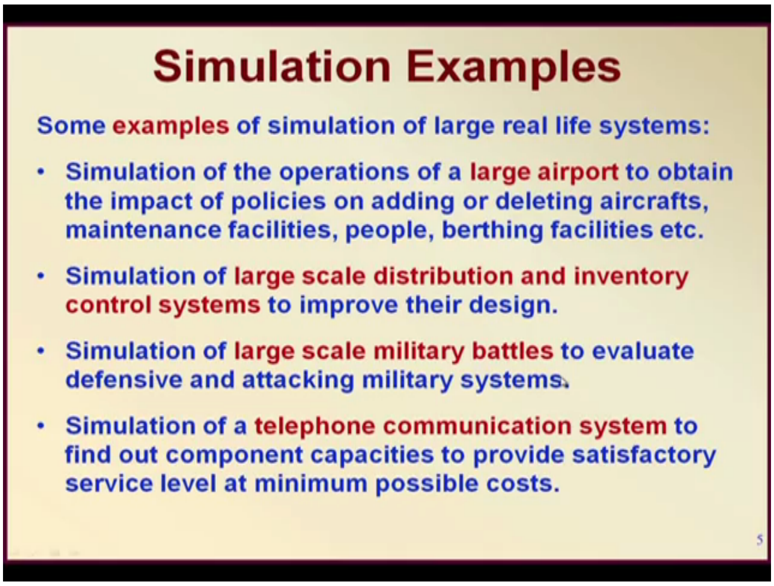

Here are some examples the first example is about a may be a large airport, so what happens in a large airport, right. So many flights some are really landing, some are taking off, then you know loading cargo. So there are hundreds and thousands of activities that are going on in a large airport. So how do you do facility management out there? Suppose you want you know really to evaluate whether the space and facilities that are available for that given airport is it sufficient or we need to augment services.

Here you can make use of simulation, right you can see that you know you can experiment on. Suppose you give more facilities and then see what are the advantages or disadvantages but let me tell you those kind of simulation model development would really require and involved and sustained effort. It is not something like one day you sit and do some modelling on paper and you know you can simulate a large airport, you need you know team, you need software and you know the team should really first of all do data collection and actual modelling so it will take time it takes time and it takes effort, anyhow.

Simulation can also be done for a large scale distribution and inventory control systems of large companies or large service systems. You know this large scale military battles this is another a very good example of simulation where you can actually simulate the defensive and attacking strategies, right like area fire and so on so forth. So lot of such very interesting simulation modelling can be done for military applications. Telephone communication system could be another example like that there are hundreds and thousands of examples but let us go into the thing directly.

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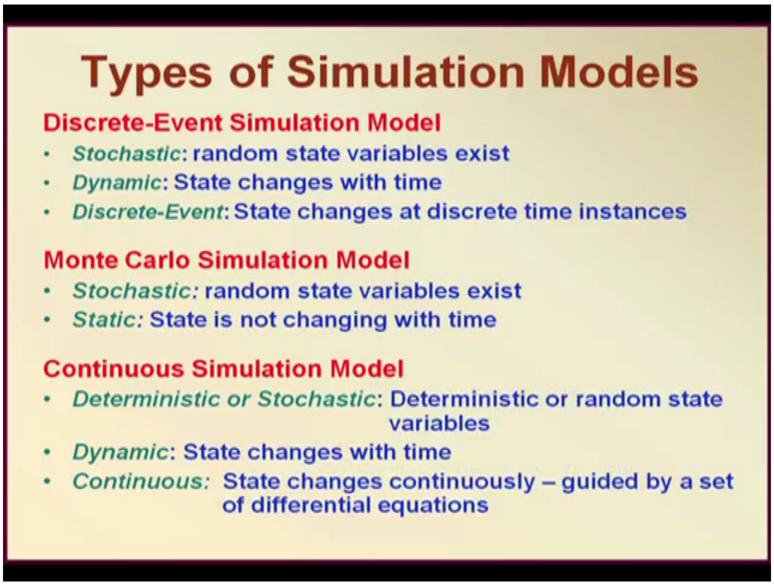

Mostly in this particular lecture on simulation we shall discuss three kind of simulation models what are they? The first one is a discrete-event simulation model. What happens in discrete-event simulation model? You know we have a random state variables means the state variable can take random values. What is the state variable? A state variable is a variable which represents the state of a system at a given point of time.

Let us take a very good example of a discrete-event simulation model is the queuing system. So we have already discussed queuing system in our previous set of lectures. What happens in a queuing system? The customers arrive they join a queue then they get service and leave the particular queuing system. So what really happens in such a thing what are some discrete-events that you can think off? That people arrive and then join queue, so it is an event.

The other event the people are in queue and then they join the service, the other the service is over and the person leaves the system. So these are some events that happens. Now obviously those events are guided by some probability distributions so those probability distributions again can throughout any value which value will it really we should take for a given point of times.

Suppose we begin the simulation t equal to 0, now the system is empty nobody is there and you know nobody is in service, nobody is in queue. So we have first use random number and generate a particular customer. What happens the customer comes and directly goes into the service because there is no queue. So what has happened we have started the simulation. Now let us say another customer is generated before the first persons service is over. So when the second person is generated the second person will now join the queue.

So you see really what is happening we are able to as if you know mimicking the real thing with the help of random numbers and generating a queuing system if you do it for let us say 100 minutes then we have a set of data number of people have come, number of people have joined the queue some has gone into the service, some has gone out of the service. So if we keep keeping the data now we can really calculate what was the average waiting time of people in the system, what was the average waiting time of people in the queue, what was the length or number of people in the system, number of people in the queue so all these parameters can actually be obtained. So that is usually called a discrete-event simulation system.

So we have random state variables so a state variable could be the number of people in the system, right so that is the state variable. Why it is random? Because random events are happening, people are arriving may be based on a random process, people are also getting service on a random process. So that is where a stochastic variable a stochastic random state variables exists. It is dynamic because the state changes with time, right.

And finally discrete-event because state changes at discrete time instances at a given point of time a new customer arrives, at a given point of time person leaves the queue and join service, etc. A special class or problem where things are a little different not there no discrete-event is really occurring but we are still using random numbers we still have random state variables and you know we can actually generate what is known as you know some some information or something about a given system.

An example I will give in detail later but at this point let us just think of suppose we have to estimate the value of pi, right. We know the area of circle is you know pi r square. So since area of a circle is pi r square can we really use this fact? Can we generate a circle and and inscribe it you know inside a square and then estimate how many points are falling within a circle out of the different points which are generated out of the square.

So you see if the area of the circle is pi r square and area of you know that particular square within which it is inscribe has got another area called A, right. So pi r square by A would be the ratio of the number of points that falls within the circle out of the total points. So from there probably you can estimate a value of pi. So look here we will give that example in detail later while we discuss Monte Carlo simulation but interesting point to note here is that you know we are really not having any event as such all that we are doing is estimating the value of pi, is it alright.

So under these kind of situations you know the kind of simulation that we do are called a special class of simulation called Monte Carlo simulation, right we will discuss this in detail later on. A third type of simulation models really speaking you can even tell it is an alternate way of simulating the discrete-event simulation model which are called continuous simulation models.

What happens there? We have a deterministic or random state variables the state variables could be probabilistic which again changes with time or even they could be deterministic and they may be guided by a state of set of differential equations, right there is a continuous state change on the basis of a continuous function, right. Even if you take a very simple example like change of population so there is a certain number of population P and change of population ddt of population is can be defined probably as a difference of rates that is birth rate minus death rate, right.

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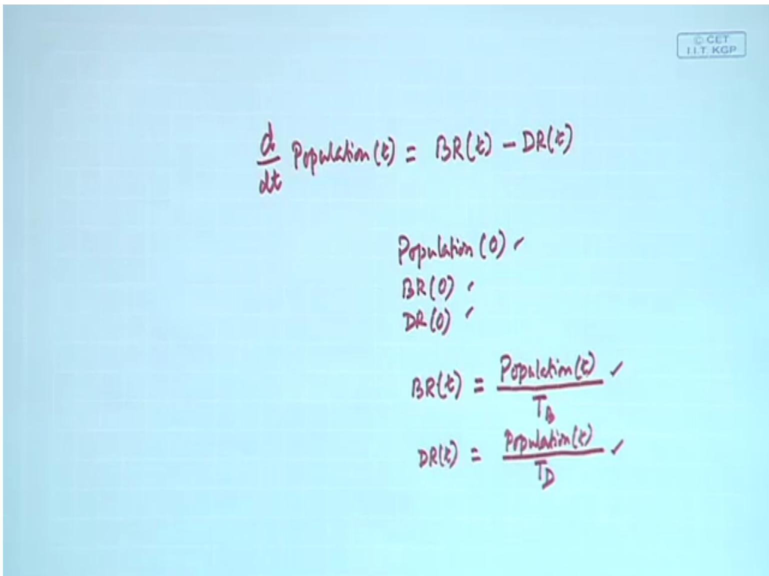

So we can have a kind of equation something like ddt of population which is itself a function of t equal to birth rate BR(t) minus DR(t). So this is a you know differential equation so suppose if we can find out a initial value what are those some initial values initial values could be like we can have the population 0 that is at t equal to 0, we also have a birth rate equal to 0, we also have a death rate at 0.0.

So if we have all these values with us and if we have a set of relationship suppose we have this relationship this BR(t) equal to population t by a time say T TB and DR(t) equal to population at a given point of time by another time constant TD. So what happens you know if we have a population values using those population values we can calculate the birth rate and death rate, right and using this particular differential equation we can find out the next value of population, right and we can continue a finding these values and you know obtain a may be population versus time plot. So the kind of simulation is called continuous simulation.

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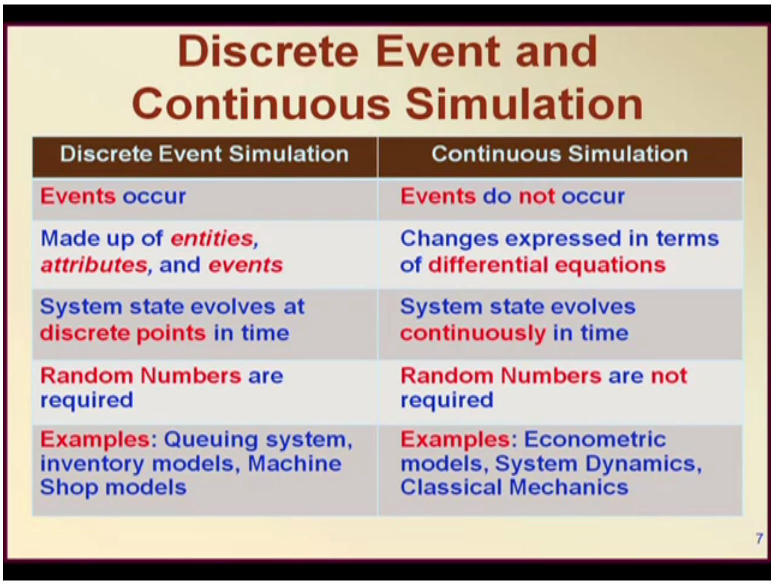

How do they compare. Let us look at the comparison between a discrete-event simulation and continuous simulation. In discrete-event simulation the events occur, in continuous simulation the events do not occur, right. Here we are assuming the state variable is a function of several other functions several other you know other variables something like rate variables like birth rate, death rate, etc and but did not really happening in terms of events.

Discrete-event simulation events are occurring, right people are arriving, people are joining the service and people are going out of the system so they are like events. Discrete-event simulation made up of entities, attributes and events. Entities the people who are arriving, the queuing system and attributes the properties about entities, right so like people who are coming what are they are attributes they are attributes may be they arrive in a Poisson process. The queuing system it actually serves based on an exponential distribution.

So all these variables are you know they are all available in discrete-event simulation. In continuous simulation the changes are expressed in terms of differential equations. In discrete-event simulation system state evolves at discrete points of time when events occur. But continuous simulation the systems are evolving continuously. So in order to generate the events we need random numbers in discrete-event simulation and in continuous simulation random numbers are not required because we have a set of differential equations.

Here are some examples queuing system, inventory models, machine shop models. Here on the other side continuous simulation econometric models, system dynamics we will discuss about system dynamics how it happens, classical mechanics we know those mechanical equations. So these are some examples of discrete-event versus continuous simulation.

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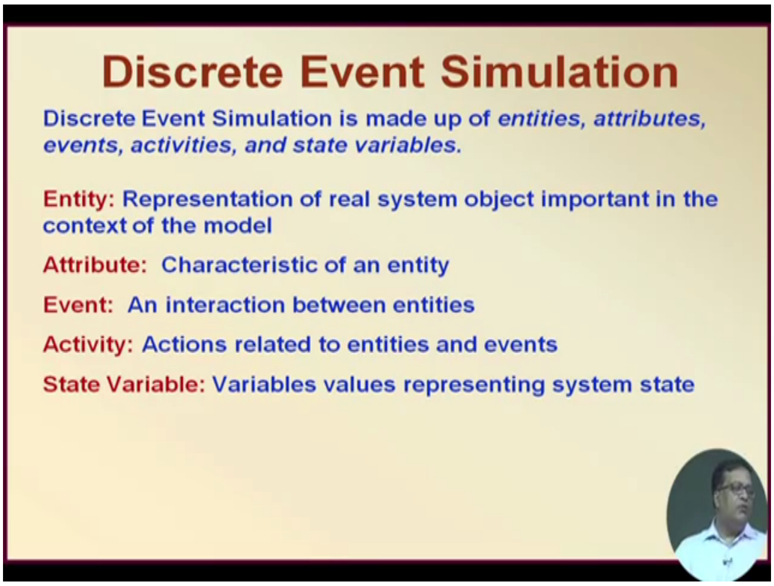

Here is some more details about an discrete-event simulation that is what we shall begin with you know you have entities, attributes, events, activity and state variables, right these are some of the parameters which we must obtain before event we start a discrete-event simulation process, right. So what are those things? Now first of all entities are representations of real system object important in the context of the model so what are some entities that are important in the context.

Attributes some characteristics of that entity, event and interaction between entities like when something happens, right and activities are something which are related to entities and finally the state variable variable values representing system state.

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So here are some examples, right. So what happens in production system? So production system there are machines the attributes speed, capacity, breakdown rate, activities welding, cutting, turning. Events breakdown, state variables status of machines busy, idle or down, right. for inventory system the entities are warehouse, attribute is capacity, activities withdrawing, purchasing, event is demand, state variable inventory levels, backlog demands.

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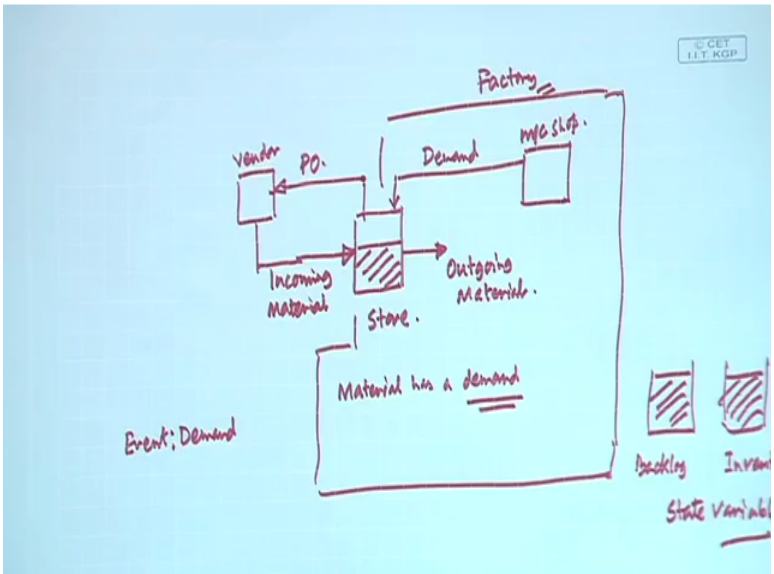

Let us look at it at a slightly more detail. See when we talk about let us say an inventory system what happens let us look at so supposing this is an inventory system basically is a kind of store. So in store what is happening what is happening is first of all the material is incoming so incoming material, right. So incoming material and then here we have the outgoing material outgoing material.

So really how under what condition some incoming material comes in, so we know that first of all the material has a demand, demand by whom? Demand is because this store is not something which is independent the store is a part of a bigger system that is that may be the entire factory, right. Now within the factory you know may be say let us say here is some machine shop and the machine shop requires certain material so it places the demand.

So as the demand is placed, right as the demand is placed then you know this warehouse takes appropriate measures and may be here is some vendor the vendor is asked or vendor is placed some purchase order, right and on the basis of that purchase order the vendor is sending materials, is it alright. So you can see that a purchase order is to be generated when the receives purchase order takes a time, gives a input material and then the store quantity going on.

See when if you are really trying to simulate such a system then you might have to generate all of these particular thing. So like when we say an event is a demand, what is that demand? Demand is the demand is raised by the machine shop as the demand is raised then what happens purchase order goes up. So you know the backlog goes up the backlog means the material is demanded so if some supposing we have two state variables you can assume them as some sort of drum one can be called as store inventory means the material that we have and backlog so the backlog goes up so backlog goes up why? Because demand is raised and when material is supplied then inventory goes up and then when the material is given to the issued to the machine shop then backlog goes down.

So all of these keeps happening in a factory like environment and if you are really trying to simulate then we have to take care of all of these facts. So these are like our some state variables is it alright anyhow so these are the beginning points that we have.

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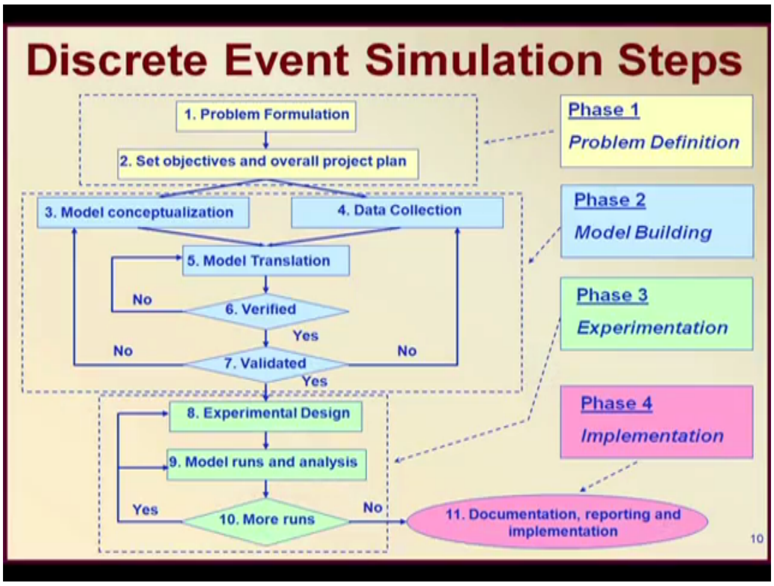

Now this is a important diagram let us look at this particular diagram about a states of discrete-event simulation. You see on the other side we have there are 4 distinct phases, what are those 4 distinct phases. In the phase 1 we have problem definition, right two important components of problem definition are first of all formulating the problem and set objectives and overall project plan.

So you see we know very well that without objective simulation has no meaning, even if you are simulating a queuing system the objective could be that trying to find out may be the parameters such as average waiting time in queue, why we want to know the average waiting time? Because you know the you know the queuing costs that one side is the service levels, other side is the waiting cost, right so we need a trade-off. So in order to have the trade-off first of all we must know how much is the waiting time, right in the queuing system. So if you really want to know the queuing system then we have to first of all have the problem formulation and have the objectives in place.

Then in the second phase we have the model building so what is that model building? We have to first conceptualize the model. Look at this diagram I have drawn, so the kind of diagram I might have drawn here is a kind of concept model I was trying to build of an inventory system. What is that concept model? Machine shop raises demand as the from factory.

As the machine shop raises demand the store raises PO, the vendor gives incoming material vendor gives incoming material inventory level goes up, as the inventory level goes up the outgoing material is given to machine shop, then inventory goes down and backlog also goes down. So all these things are you know when you are just drawing trying to understand thing that can be called as the conceptual model. But after that you have to collect data, why you have to collect data? Because you have to understand the model that you have made whether is a correct model or a wrong model.

So you need to really translate that model into may be a set of mathematical equations initially it is just a diagram, concepts, ideas certain things written on paper but you have to construct a mathematical model out of it. And once the mathematical model is developed it should be verified and validated is it alright you should verify the model and validate the model.

After that comes the experimental design that is a experimentation, what is experimental design? We have to know what kind of random numbers we must have, how do we carry out the experiments so all those things and run the model and have more runs so that the sufficient results are obtained.

And at the final at the stage of implementation make documentation, report and implement. Like in queuing model if we find out the waiting cost then the document will tell that these are the waiting cost comments on that whether these waiting cost are high or waiting cost are rather low and depending on that the service facility has to be augmented suppose it is to be augmented so then it should be at implementation advice, right and the company or the queuing system owners can actually implement this suggestions, right. So we stop here we shall go more into the mathematics part of it in our next lecture, so thank you very much.