Course on Decision Modeling
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Lecture No 28
Variance Reduction and Simulation Software

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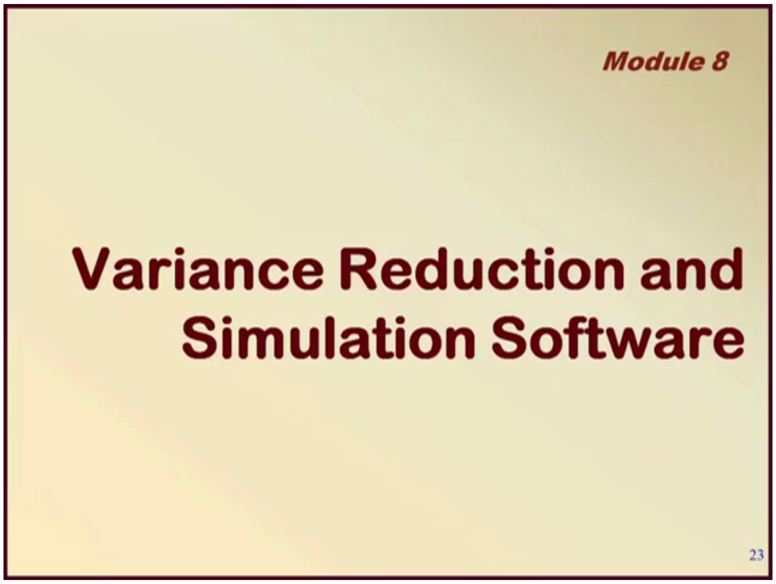

Now in this section we are going to discuss the output analysis specifically the variance reduction technique and thereafter we shall look a simulation software because please remember the simulation that we have shown they are very rudimentary and that kind of simulation is not sufficient for proper simulation, if you have to do we have to have a software. Now there are different kinds of software in the market we are not going to discuss about all of them, just one software arena that we are going to discuss a little bit and something about that particularly how a giving network can actually be simulated that we shall see also in that particular section.

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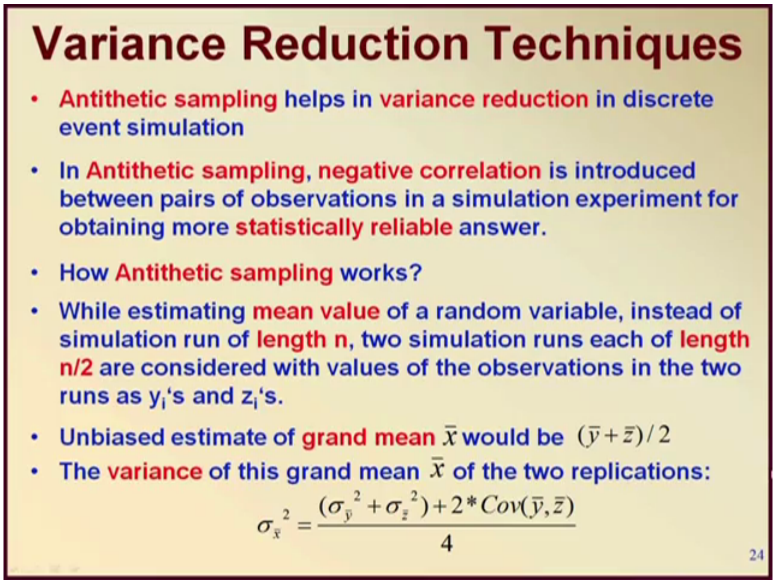

So let us start with the variance reduction technique you see, we have seen first of all the length of simulation run very much dependent on these particular value that is the variance, right? So now you see the method that we have variance reduction that is known as Antithetic sampling, what is antithetic sampling? In antithetic sampling we are basically introduced a negative correlation between pairs of observation in a simulation experiment for obtaining more statistically reliable answer.

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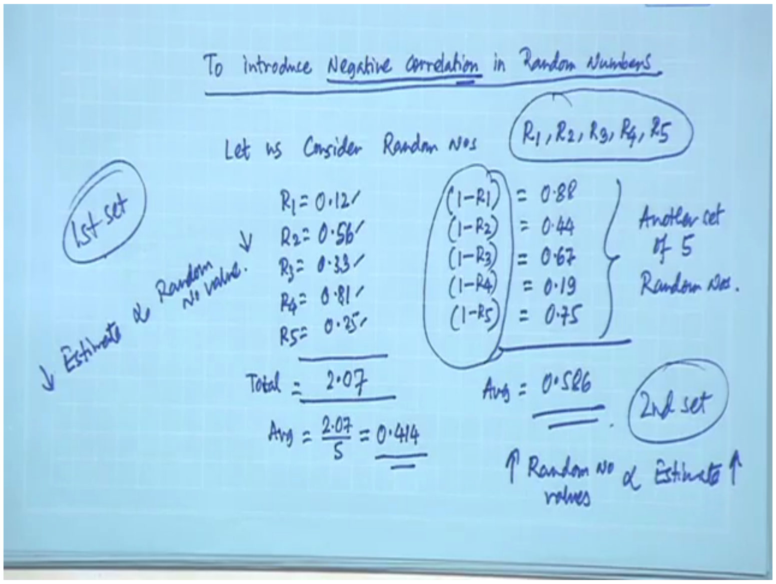

So what is really we are doing? We are really putting a negative correlation between the set of random numbers. How it is possible to really put some other random numbers which are negatively correlated? Let us look at one example, you know how the negative correlation, to introduce negative correlation in random numbers. You see let us take, suppose we require 5 random numbers and we assume these 5 random numbers chosen are R1, R2, R3, R4, R5. Now it so happened that these random numbers, suppose they are 2 digits only read us consider, R1 is equal to 0.12, R2 is 0.56, R3 is 0.33, R4 is 0.81 and R5 is 0.25. So arbitrarily some 5 numbers I have taken.

Now let us see what is your total? 8, 11, 12, 5, 17, 1, 2, 7, 10, 18, 20 their totally us 2.07, so what is their average? Average will be 2.07 by 5, 27057 by 5, so it will be 0.414. So look here we have taken 5 random numbers their average is 0.414. Interestingly this average is not 0.5 because if you take 5 random numbers or even more random numbers ideally their average should be around 0.5.

So this random numbers are on the slightly lower side, right? So suppose they are estimating something and suppose these their estimate is directly dependent on the random numbers than the estimate will be slightly on the lower side because see suppose we estimate it is directly promotional to the random number value, now since random number value is slightly on the lower side estimate will be slightly on the lower side.

Now assume another set of 5 random numbers 1 minus R1, 1 minus R2, 1 minus R3, 1 minus R4 and 1 minus R5, what are these numbers? 0.88, 0.44, 0.67, 0.19 and 0.75, so these are the 5 numbers. What, see they are just complimentary numbers to each of the 5 random numbers that we have chosen, what are they? They are another set of 5 random numbers. So it is not required to really make the total, we can directly write what is their average?

Their average will be 0.586, how do I know? Because the average of 2 should be 1 they are some or all 1, alright? So what is this average? So you see these random number values are on the slightly higher side because above 0.5, so they are on the higher side. So the estimate will be on the higher side. So compare these, suppose we do one set of experiment with these random numbers, another set of experiment with these random numbers then these 2 values the estimates at the first set and estimates at the second set.

This is the first set and this is second set, the estimate in the first set and estimated the second set will be negatively correlated, right? So this is how we can introduce negative correlation in random numbers, right? So this point is very important, how can I introduce negative correlation in random numbers?

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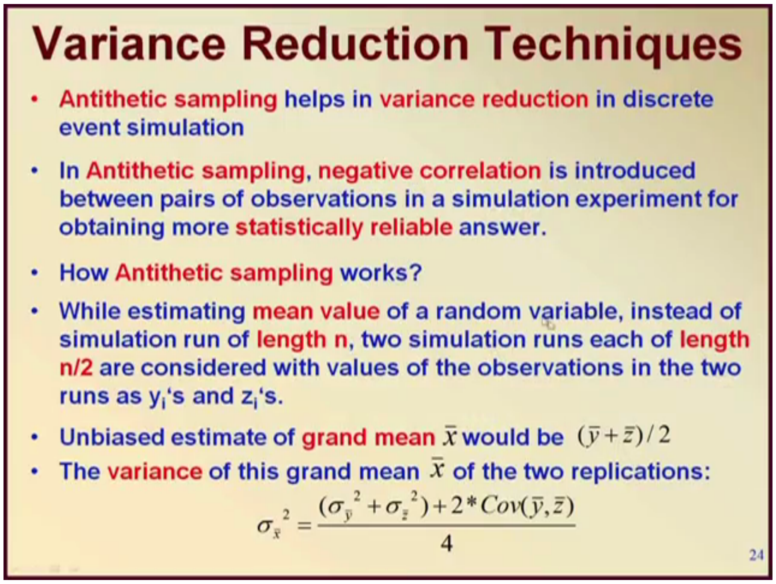

So let's go back, so in Antithetic sampling negative correlation is introduced between pairs of observation in a simulation experiment for planning more statistically reliable answer, why it works? So while estimating the mean value of a random variable, instead of simulation run of length n. Suppose we run to simulation runs of length n by 2 are considered with values of observations in the 2 runs as yi's and zi's, right?

Then the unbiased estimate of the grand mean x bar would be y bar plus z bar by 2 and the variance of this grand mean x bar of the 2 replications will be Sigma x bar square is Sigma wide bar square plus Sigma z bar square means 2 star covariance y bar z bar by 4. So look at these equations very carefully what have we done? We had the original run of length of simulation n say n is 100 and you know what was the estimate?

The Estimate was x bar instead of that, that particular experiment we are now dividing the simulation into 2 sets of 50 each. One said the mean is y bar another said the mean is z bar, so what will be the estimate of grand mean? Obviously average of the 2, so that's why you wrote the grand mean estimate is x bar is y bar plus said bar by 2 and what about the estimate of the variance?

If the original variance was Sigma x bar square then the variance of the grand mean will be Sigma y bar square plus Sigma z bar square plus 2 star covariance y bar z bar by 4. So in this relation, look at this relation very carefully. If you really estimate the relation, it is very much dependent on the covariance of y and z but if why y and z are negatively correlated that the covariance value will be negative and therefore you know this Sigma x square estimate will become lower, right? So because of negative covariance here, so that is the essential idea of the variance reduction techniques.

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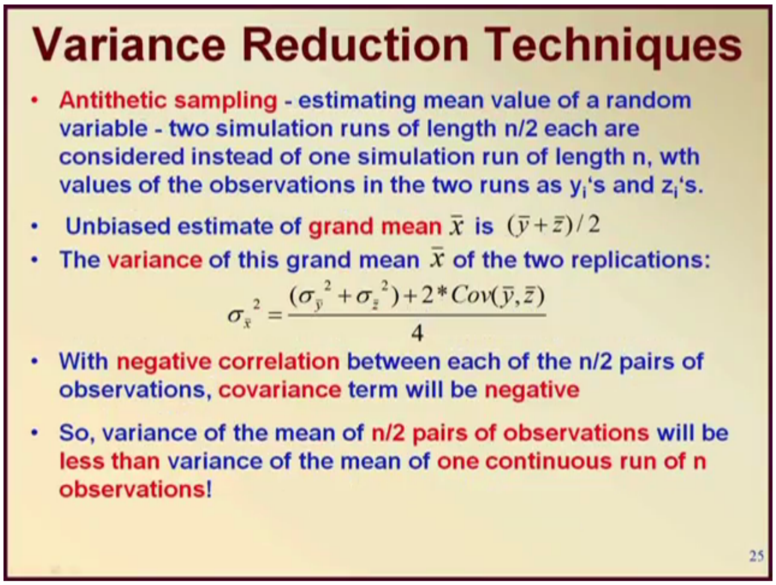

Let's see what happens? So you know estimating that is what we are doing really. So the variance of the mean of n by 2, you know by pairs of observation would be less than the variance of the mean I know of the one continuous run of n observation. So with negative correlation between each of the n by 2 pairs of observations covariance term will be negative, right?

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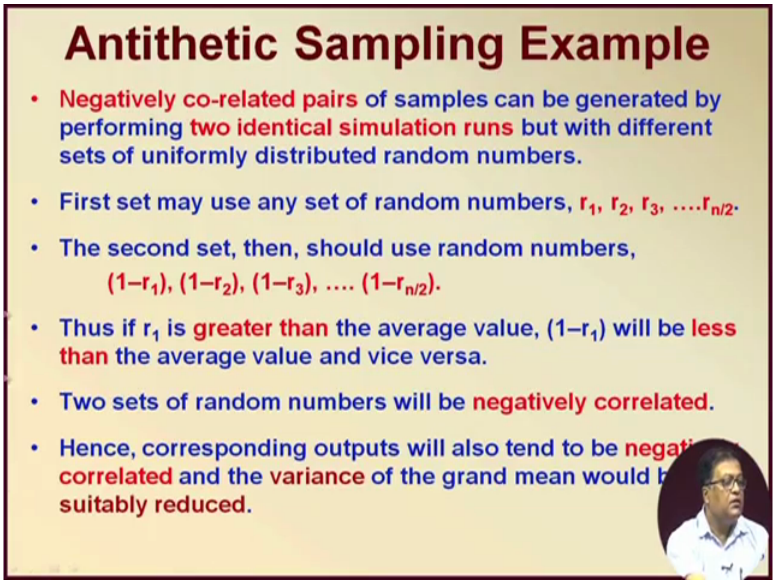

Why it is happening? Because we are running, you know 2 sets of runs each with n by 2 and if this negative correlation remains again how this negative correlation is possible? Let us look at these examples that is I have already given the negative correlated pairs of samples can be generated by performing 2 identical simulation runs but with different sets of uniformly distributed random numbers.

In the first set we use r1, r2 to rn by 2 and in the second set we use 1 minus r1, 1 minus r2, 1 minus r3 etc thus if r1 is greater than average value 1 minus r1 will be less than average value and vice versa, right? Then 2 sets of random numbers will be negatively correlated hence corresponding outputs will also tend to be negatively correlated and the variance of the grand mean would be suitably reduced. What is the advantage of the variance reduced? The variance reduced then we require less length of simulation run to estimate the same value that is the advantage.

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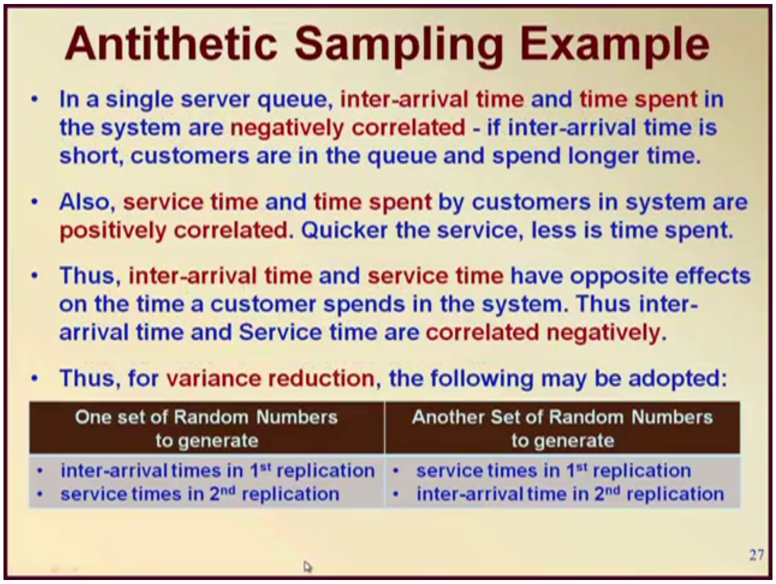

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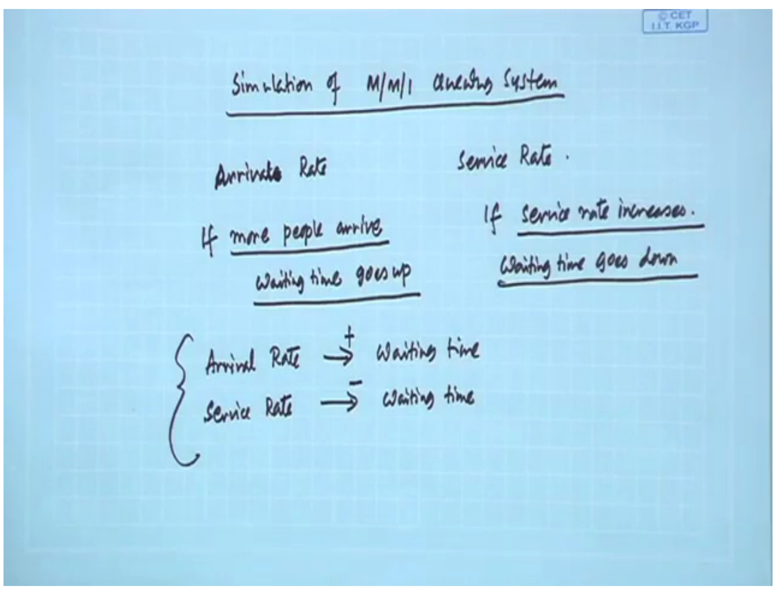

So here is one more example, in another example let us say look at this particular thing. So simulation of MM1 queuing system, now in any queuing system there is 2 most important considerations one is arrival rate another is service rate. If more people arrive waiting time will go up or go down, more arrivals in a given situation waiting time goes up.

So if more people arrive waiting time goes up, if service rate increases, waiting time goes down this is very clear. If you have better service waiting time will go down, so what is the relationship between arrival rate to waiting time? We can put it plus, more arrival time more waiting time. Service rate you can put a minus there is more service time less will be waiting time.

So if you compare the 2 then look here the arrival rate and service rate they are reversely correlated. If more is the arrival rate than service rate should be increased even more. If arrival rate is less service rate could be less but suppose the opposite happens arrival rate increases but service rate reduced waiting time will increase even more. So their effect on waiting time is exactly opposite that is a point I wanted to make.

So you know let us look here the inter-arrival time and time spent in the system are negatively correlated. So you know if more is the arrival time if this is the opposite way this is written inter-arrival time, so if inter-arrival time is short customers are in the queue and spend longer time also service time and time spent are positively correlated. Quicker the service less is the time spent, already we have seen that.

So does inter-arrival time and service time have opposite effects on the time customer spends on the system? Thus inter-arrival time and service time are correlated negatively, right? So for variance reduction the following may be adopted, what we can do? Suppose we generate one set of random numbers to generate inter-arrival times in first replication and service times in second replication in the second run.

Whereas another set of random numbers to generate service time in the first replication and inter-arrival time in second replication. So look here what happens? The same random numbers which are used for arrival time in the first run they are used to generate service time. So what happens? The 2 replications will there be having negative correlation and if they are negatively correlated then what will happen?

The overall variance will go down, if the variance goes down then what will be the advantage? Then you know we have either we go for less number of simulation run or we will have better results, right? So that is the thing that either Sigma will reduce or we can with the same Sigma second run with less number of simulations. So in order to that is the essential idea of antithetic sampling that they are helping us in reducing the variance in the simulations.

So with that said us look at a little bit more a little bit more on the verification and validation part of simulation models also. The both the verification and validation the essential requirements, right? Again we know that verification means the model should be built right or in other words the specifications model should be the first model that we have the computational model that we have developed finally out of the conceptual model they should be consistent.

But on the other side the validation basically focuses on whether the proper system is developed or not. Basically the correlation is not with the specifications that is obtained but with the real system and if the decision-makers of the system whether they are happy with the system that is developed.

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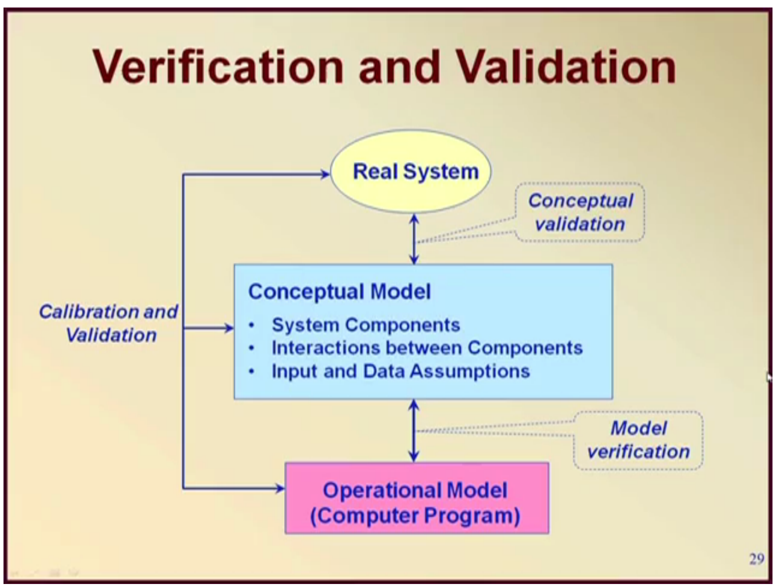

So here is a very low comparison that when we have the real system and we have a conceptual model, first we must do a conceptual validation, right? How? Really top experts and let them see whether the model does what is required? So the conceptual model including system component, interaction between components and input and data assumptions, really should see whether is at par with the operational model, means whether the specs are at par or not, that's all model verification.

And finally that computer program on the operational model should be calibrated and validated with the real system. So that is how the entire verification and validation system should work particularly in the simulation framework. with that let's go to the power next thing that is about you know the simulation language and as I say one particular simulation language that we shall see an essentially today's world one of the very importance of that is known as the arena simulation software.

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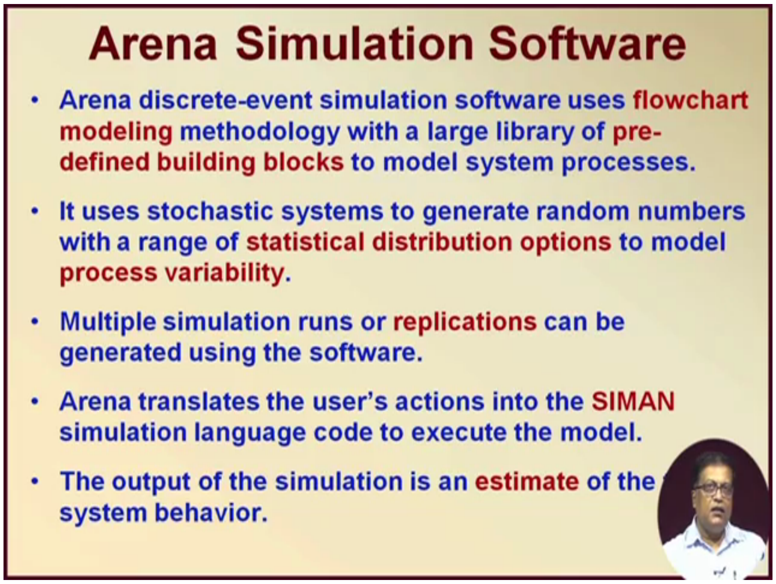

It is discrete event simulation software and mainly use the flowchart modeling methodology, so what is the advantage? That you really don't have to write any equations, suppose you have a set of what you call a queuing network where a set of you know queuing systems they are **after** one after the other and from 1 block the thing goes to the next block and there is certain arrival and service patterns then using some predefined building blocks of the arena software you can model that system.

And after modeling you can put appropriate values and the distribution that you have chosen you can actually simulate a lot of variations and really come up with very interesting results not only that it also has facility for carrying out replications, generating a number of replications using the software and finally arena translates the users actions into a simulation language called SIMAN, right? That SIMAN language usually that executes the model, right? And finally the output of the simulation is an estimate of the system behaviour. so essentially the arena software really works on a large number of predefined building blocks.

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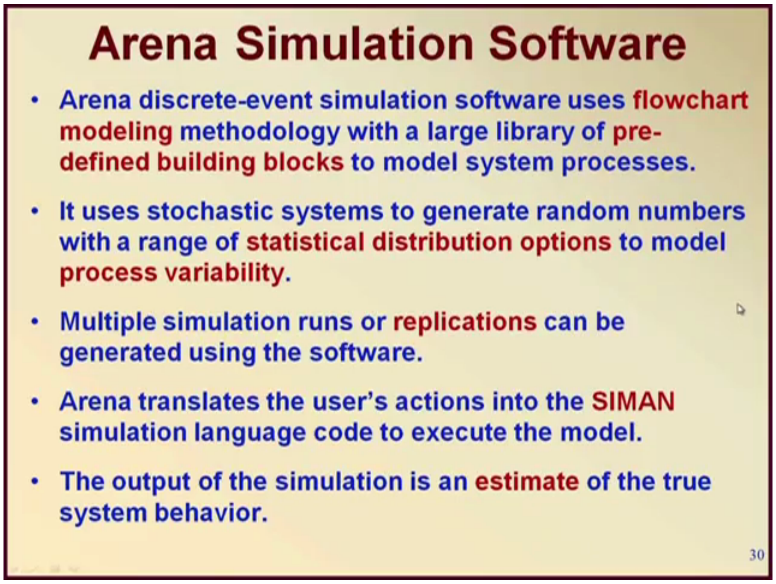

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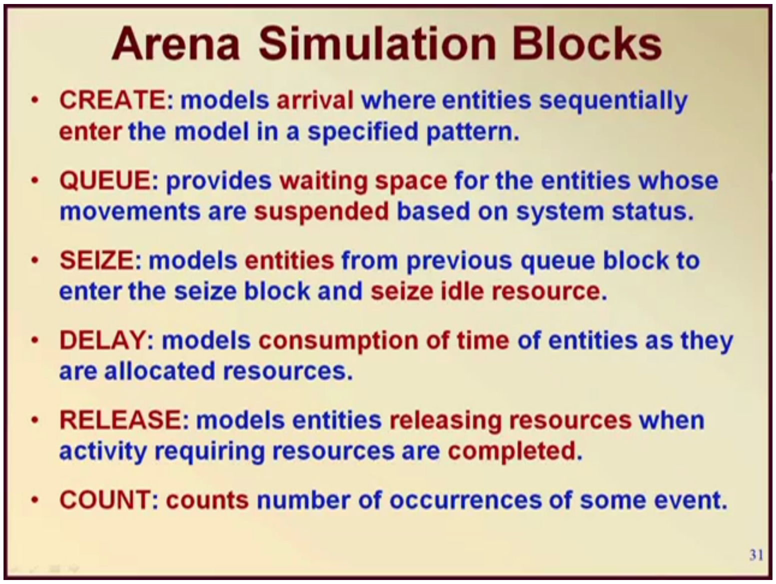

So what are those predefined building blocks? Let us see them you know here are some of these building blocks the very first building block is called a CREATE, right? The CREATE actually models the arrivals where entities sequentially enter the model in a specified pattern, right? So what happens in a CREATE block that in a create block you really specify that what are the entities? How many entities are entering? And these and what are the distribution that is coming up and things of that sort.

And they will enter one after another in a sequential pattern. The second building block is called a QUEUE. The QUEUE provides awaiting space for the entities of means are suspended based on the system's status why are they suspended? Because if the system's status is busy that means that these units cannot go to the next unit, what are the next unit, where service is being provided, then they wait in a queue and that is the essentially called a QUEUE or awaiting space.

The third one is called a SEIZE, so modern entities from previous queue block to enter the SEIZE block and seize an ideal resource. So you know the way it is put that first it seizes, that means it actually captures the given resource, here resource is the serving facility. So there is a service facility the units are waiting in the QUEUE block and from queue block it goes to the SEIZE block and at least goes to SEIZE block than it actually gets that particular resource.

And after getting resource then they should include a DELAY block, what delay block thus? It models consumption of time of entities as they are allocated resources. So after the seize block then it goes to the delay block where the consumption of time takes place that means the time it spends in the resource. Obviously this time will be generated by a statistical distribution. Next one is the release block.

After the delay then that particular service will be over and these entities will now release the resources because activity requiring resources are complete means the service is over and that you say something like a count that counts should really look into the number of occurrences of some event and that will be counted. Dispose, it provides a map and is for modeling the departure of entities from the system.

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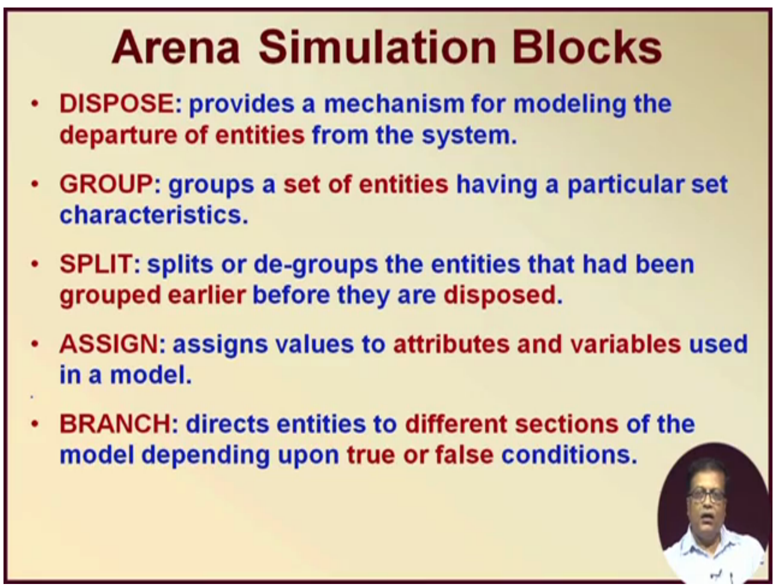

So you know usual simple simulations, we have not specifically looked at dispose because moment the release takes place we assume dispose has taken place it may not be happening, right? So therefore be a specifically dispose requirements also. Then there is a command called GROUP it group says set of entities having a particular set characteristics then there could be SPLIT, a split is for De-grouping.

So like we have looked we also have split, so from the group you know that splits or degroup are entities that have been grouped earlier before they are disposed. Then Assign, assign values to attributes and variables used a model. What is Branch? It directs entities to different sections of the model depending upon the true or false condition. So supposing there are 2 alternate parts to be taken for 2 different sequences for 2 different you know conditions and based on whether that condition is true or false the entities takes different parts.

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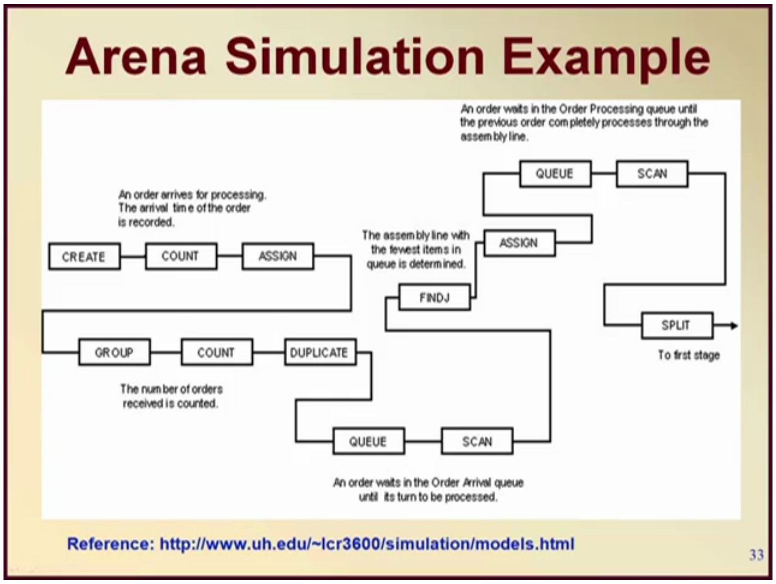

So here is an example I have taken, the references also given. you know you can see this particular situation first of all an order is arriving, so it's an order processing system arriving for processing the arrival time of the order is recorded. So at the very first CREATE is given then COUNT is given an assign resources assign and then they are GROUPED and then COUNT and then DUPLICATE the number of orders received is counted then it joins a queue then order waits in the order arrivals you until it's turned to be processed.

Then there is a scan and then find J the assembly line with the fewest items in the queue is determined then assign and then finally QUEUE, SCAN and finally SPLIT. So one stage is over, right? So like this we carry out simulations stop here, the next section will take up continuous simulations, so thank you very much.