Decision Modeling
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Lecture 04
Value of Information

So in this section we shall really talk about value of information but before we really take up the value of information let us look at the sensitivity analysis problem which we had taken in our previous class.

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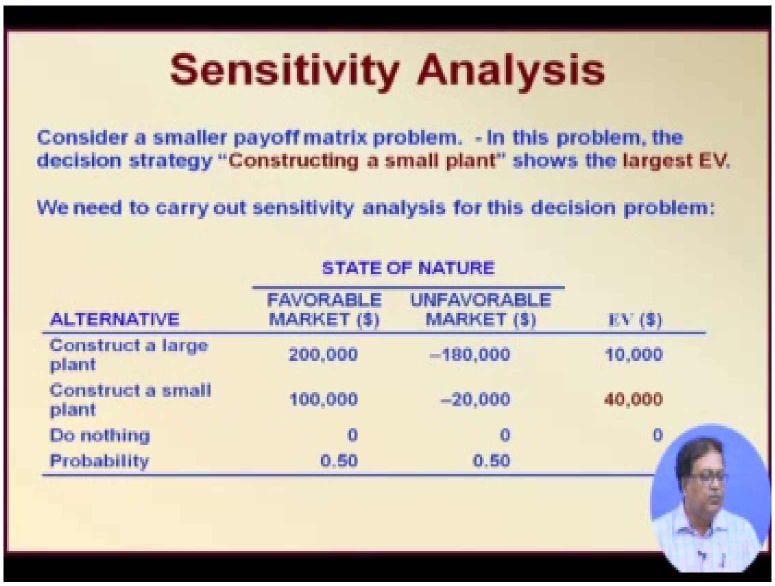

So the problem that we have taken up is, let me very quickly repeat, is that we have a decision situation where there are two options constructing a large plant and constructing a small plant, the expected value has come out to be higher in case of constructing a small plant. So we wanted a sensitivity of this decision for different probability situations.

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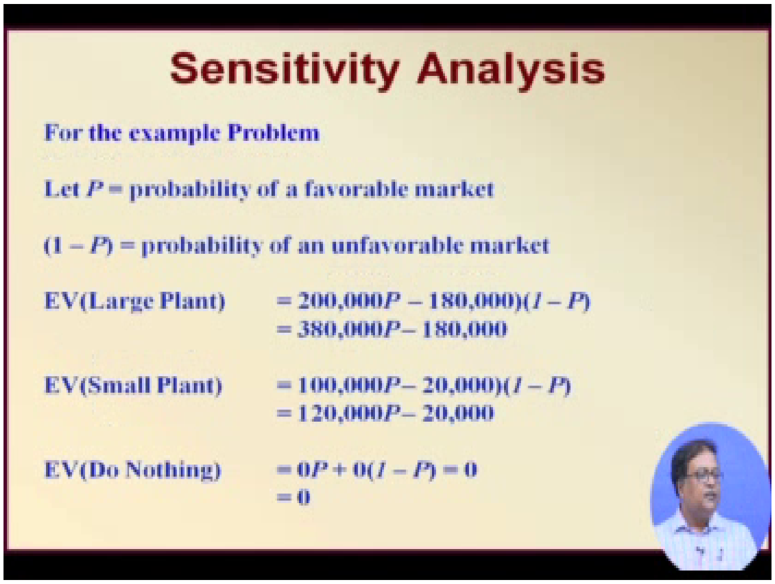

So what we do supposing let P as the probability of a favorable market and therefore one by P would be the probability of an unfavorable market,

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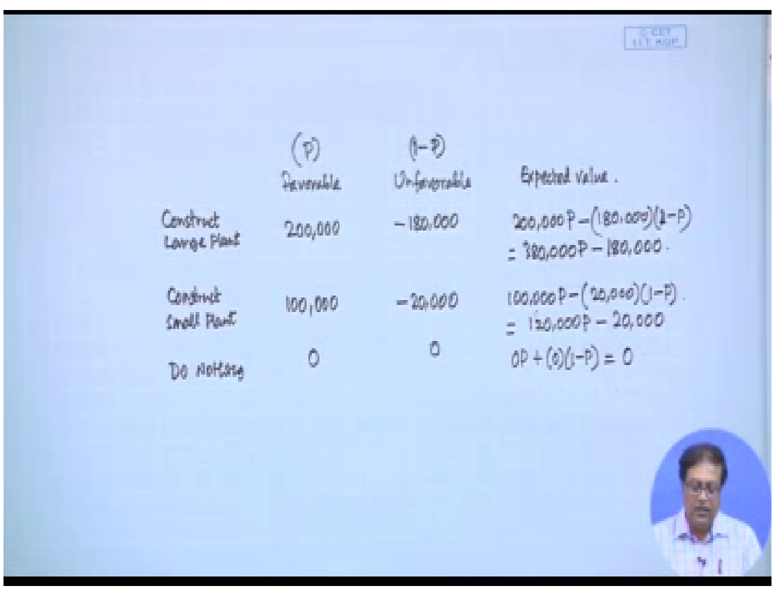

So let us do this problem, first of all these are the two decisions construct large plant and the second one was construct a small plant, right? So we have again two states of nature that is favorable and unfavorable right.

So supposing the probability is not uniform I mean not unique rather we say it is P and one minus P and these are our payoffs, say two hundred, hundred actually two hundred thousand and hundred thousand and here minus one eighty thousand and here minus twenty thousand, so these are our payoff, so you see what we have got here is that if you really look at the calculations then what would be the expected value.

The expected value would come out to be two hundred thousand P minus one eighty thousand into one minus P, because that is how is the expected value we compute, so if we evaluate this it will come out to be two hundred thousand that is three eighty thousand P minus one eighty thousand right, and similarly for the second one it will become hundred thousand into P plus basically minus so that minus we taken out, so twenty thousand into one minus P.

And it comes out to be one twenty thousand P minus twenty thousand right, so these are that what we get for the two and obviously there is a third also do nothing, for do nothing these are zero, so here it will be nothing but zero into P plus zero into one minus P equal to zero, so nothing to do here, so these are bases how the expected values that could be calculated in terms of the favorable probability P.

So one is coming to three eighty thousand P minus one eighty thousand and the second case is coming to one twenty thousand P minus twenty thousand.

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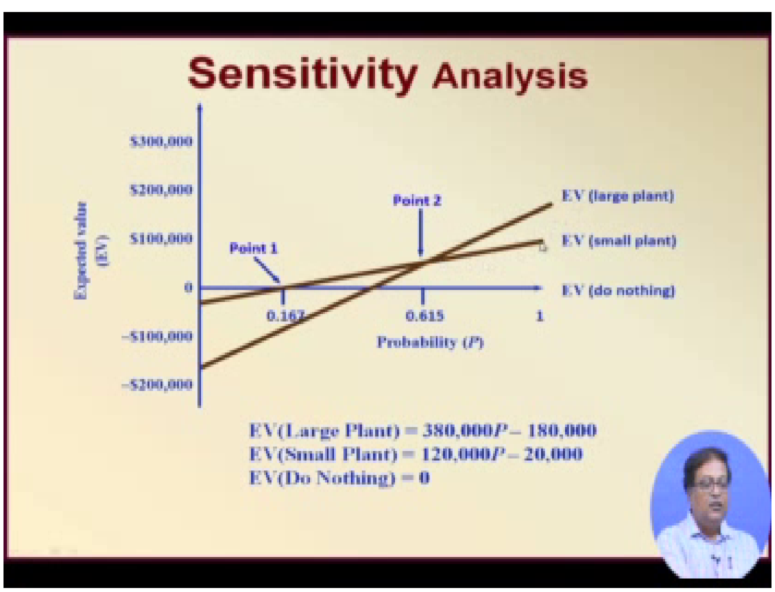

So let's look how they combine. So you see what you can do, you can actually plot them this so we have plot here, the EV large plant and EV small plant, look at this particular plot, so this side is expected value, so if you look at this first one is three eighty thousand P minus one eighty thousand.

So it starts with somewhere around minus one eighty thousand and this plot is because it has a larger slope right, it has a larger slope and it is actually going up that is called the EV large plant, on the other hand the EV small plant is having only minus twenty thousand to begin with but it has a smaller slope and those two points actually cut at a particular point, let's called point two, so that is point six one five right, so what is a significance of this particular graph.

You see what happens if the probability P equal to zero right, so you are expecting the small plant to be better and with essentially only minus twenty thousand will be its payoff and it will make even when the probability is point one six seven, so at a point one six seven probability the expected value for the small plant comes out to be zero right, and similar thing is coming out for the large plant at a slightly higher probability.

But at point six one five the expected value for the large plant becomes bigger than the expected value of the small plant, so what is you can do therefore, what you can do, you can divide this into two zones that if the probability is let us write it down,

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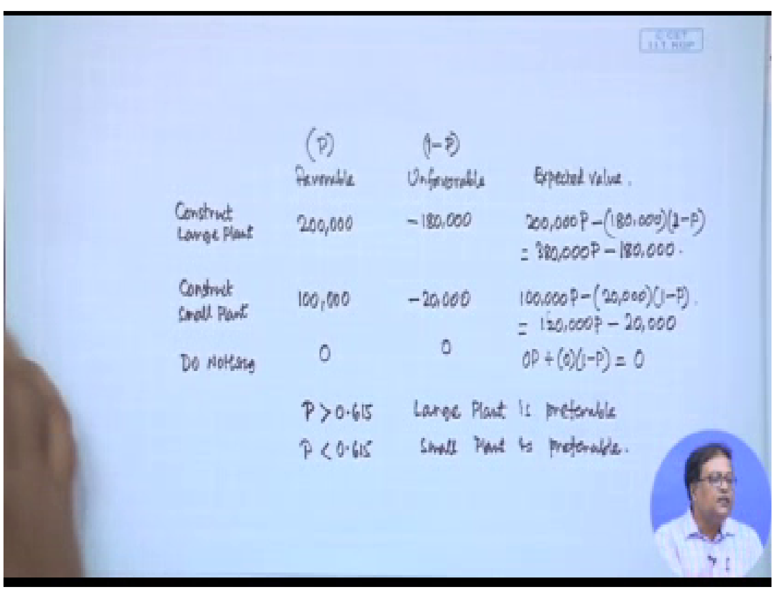

If the probability is above point six one five then large plant is preferable. If the probability is less than point six one five then the small plant is preferable right, so that is the significance of this sensitivity analysis.

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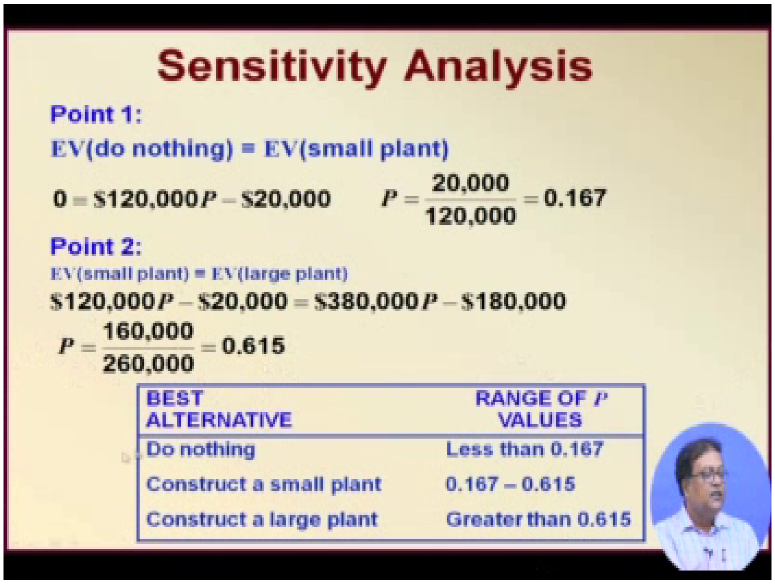

Let's look at the summary of this once again, so point one EV do nothing equal to EV small plant if you put, then if you put zero then we get P equal to point one six seven, so at point one six seven probability it is better to construct the small plant rather than do nothing and the point two is EV small plant equal to EV large plant if we equate then we get P equal to point six one five and then we get this particular decision matrix that best alternative is do nothing.

If the P is less than point one six seven, the construct a small plant if the probability is between point one six seven to point six one five and construct a large plant if the probability is greater than point six one five,

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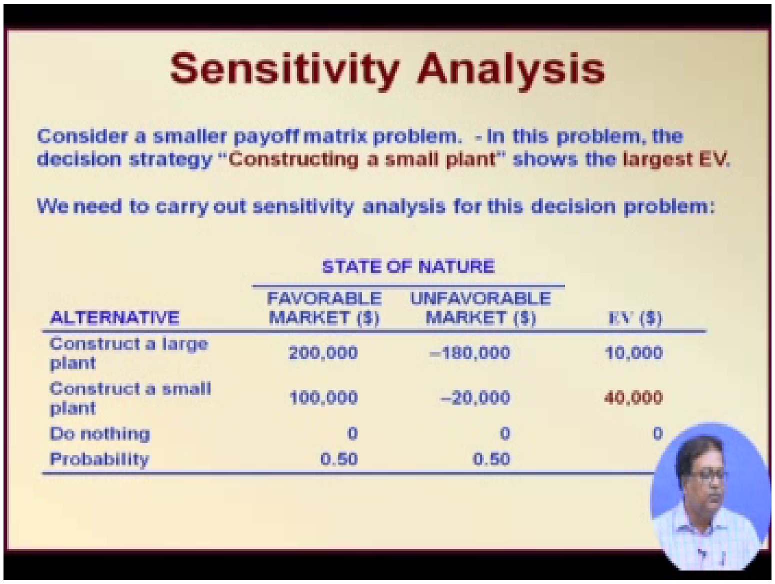

So this is actually important because the P was basically look at this thing the P is basically the probability of the favorable market right, so when the favorable market probability is less than point one six seven you take one decision.

When it is between point one six seven to point six one five you construct a small plant and which is greater than point six one five, it is better to construct a large plant right,

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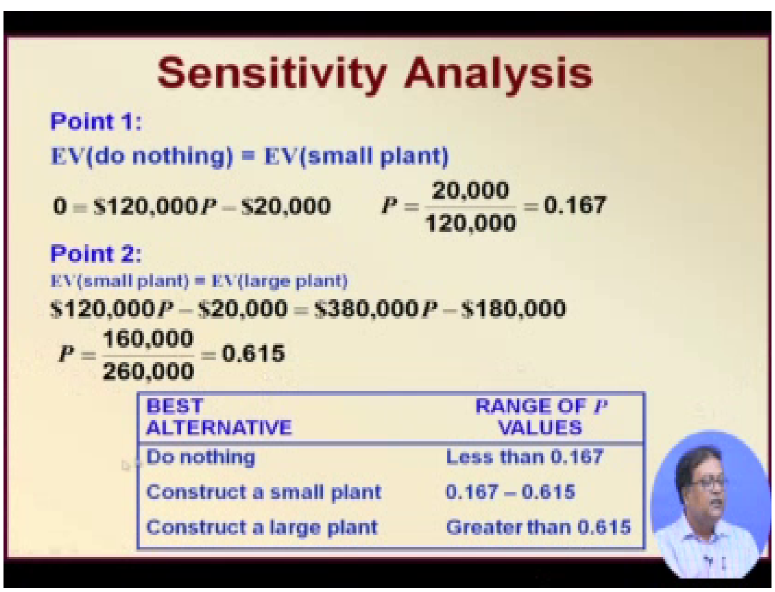

Now let's look at the concept which we initiated earlier that is about the value of information.

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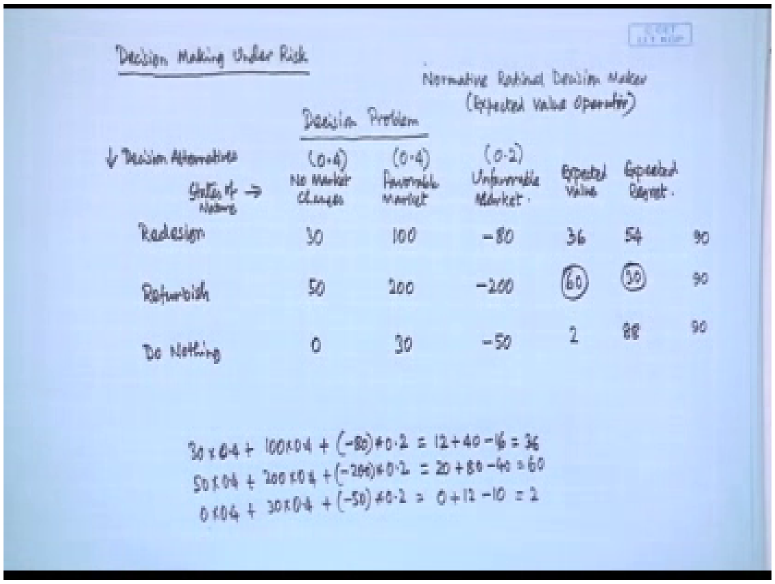

We have seen in our previous this thing that the under decision making under risk we have an expected value and we also have an expected regret and the expected value and expected regret.

When you add we actually get a constant value. So this brings us to a concept of the expected value of perfect information right,

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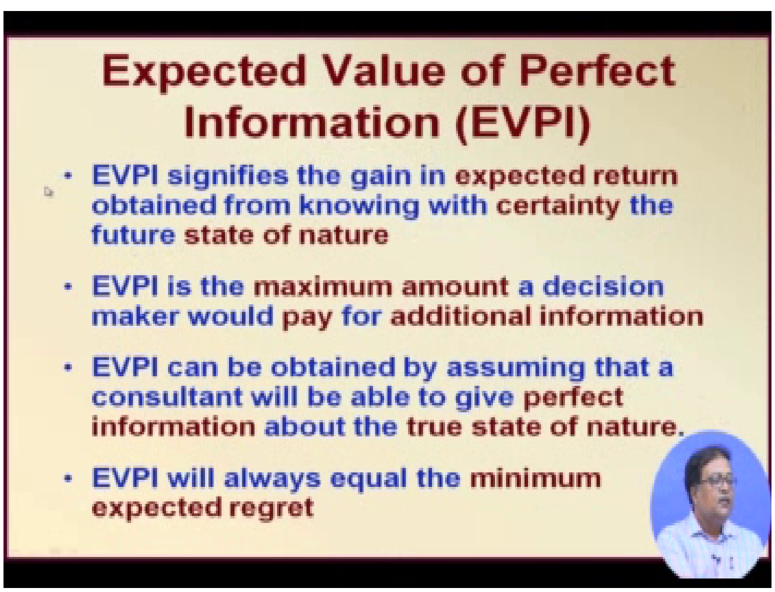

So EVPI signifies the gain in expected return obtained from knowing with certainty the future state of nature right, so EVPI is the maximum amount, a decision maker would pay for additional information, so that we can call as the expected value of perfect information, expected value of perfect information can be obtained by assuming.

That a consultant will be able to give perfect information about that true state of nature and this EVPI will always equal to the minimum expected regret, so we can if you look at the calculations once again we see,

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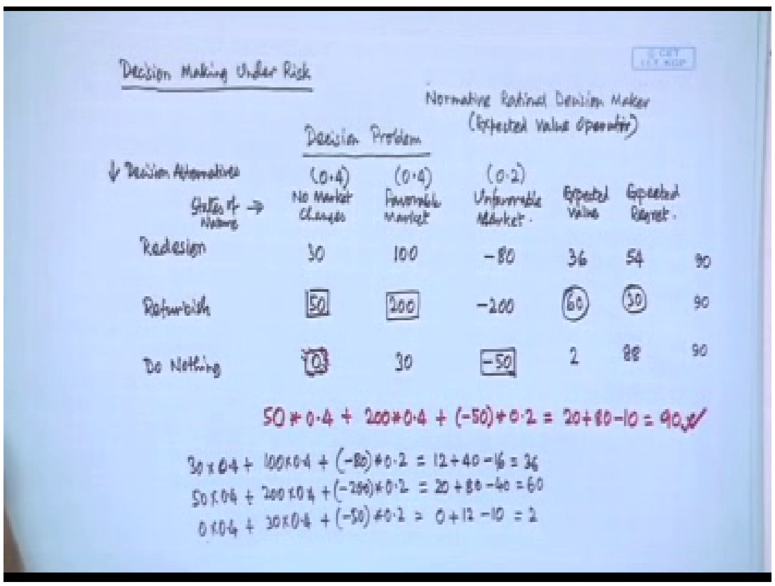

This particular expected regret value which was fifty four for redesign the refurbish for thirty and do nothing for eighty eight, so actually if you look at it from another point of view that assuming perfect information was available.

When perfect information is available then the person would have really gone for a decision like do nothing, suppose it would have gone for do nothing it would have gone, if there are no market changes, and the person would have gone for refurbish for a favorable market situation right, and the person again would have got an unfavorable market situation this do nothing for unfavorable market situation, in other words if perfect information is available.

The decision maker could have maximize their probabilities by appropriately taking a particular decision alternative based on what is the payoff at that particular situation, so what could be our payoff under that situation, the payoff would have been let us calculate, let me repeat once again that under the perfect information a person would have gone for refurbish because that is what it gets a maximum payoff of fifty, then would have gone for refurbish.

Once again in case of favorable market and would have gone for do nothing in case of the unfavorable market situation, so in that case it would have got fifty into point four, two hundred into point four and minus fifty into point two that is a total payoff of ninety and this ninety is actually called what is known as the total amount that it would have got maybe you can call the payoff with perfect information, so payoff when perfect information is available.

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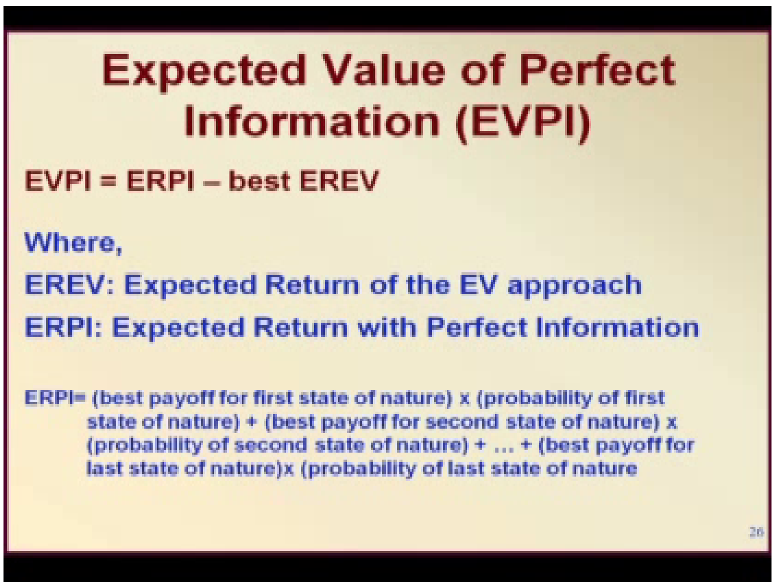

So that actually gives us another term that is known as the expected value of perfect information is coming to two things. One is the expected return of the expected value approach and expected return with perfect information right,

So the ERPI minus the bets expected return that we have got in the EV approach, so that would have been the expected value of perfect information, so coming back, so under this situation you see let’s look once again.

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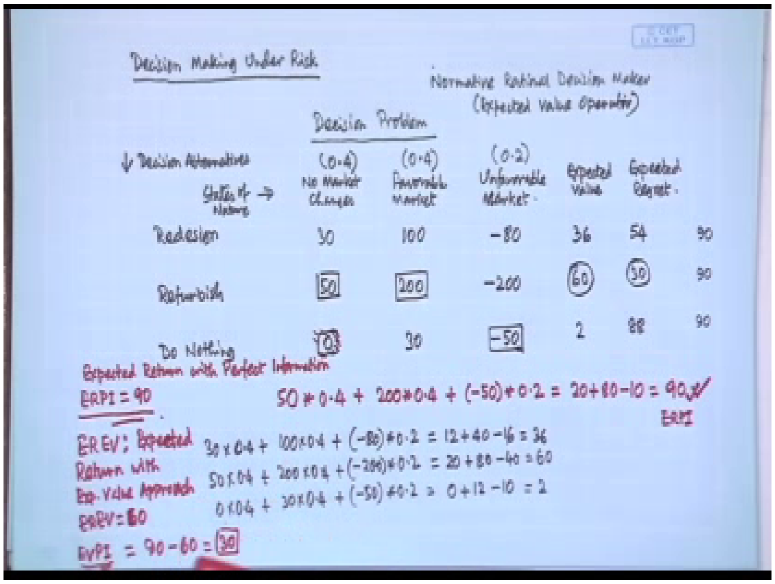

So in this case the ERPI so this is our ERPI or expected return with perfect information right, so let me sum it up that ERPI is ninety right, why that is the expected return with perfect information is ninety and the second one is the EREV, what is EREV expected return with EV or expected value approach, so what is the best possible expected return with the expected value approach so in this case we have got the EREV equal to sixty.

So sixty is the expected value that we have got with the best possible decision that means we have got an expected return is sixty, but we could have got up to ninety right, so sixty is what we have got, ninety is what we could have got, so EVPI or expected value with perfect information, if perfect information is available then we would have got a return of ninety minus sixty that is thirty, so an additional thirty we could have got if we would have got.

What is known as expected value with perfect information, now how perfect information can make a difference, let us take the problem in fact let us look at another problem and try to compute this from the first principle right, so that would have really got give us another exercise and we could have do the entire calculation once again,

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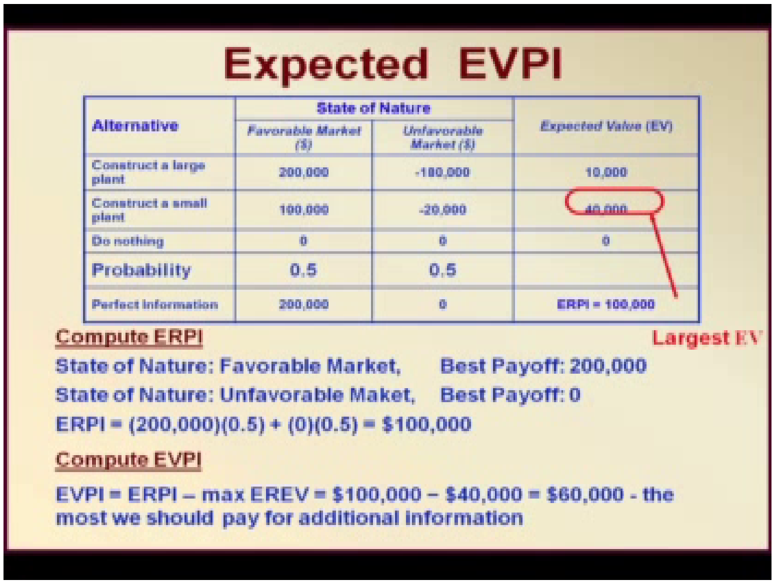

Let us look at this particular problem where we have these states of nature and the different alternatives. And how these calculations are coming together, so let's take up this problem once again and look at it from that point of view,

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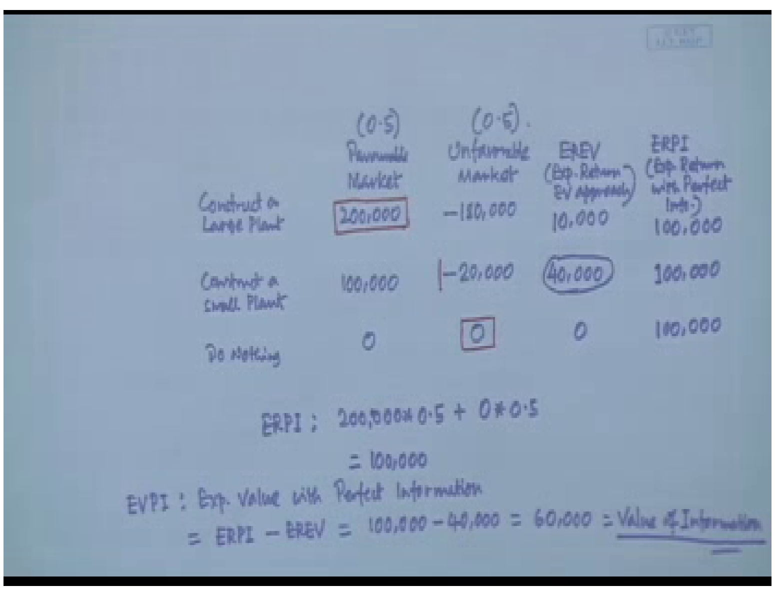

So first of all we have construct a large plant, Construct a small plant and then do nothing, and we have the two states of nature favorable market and unfavorable market, the probabilities are point five and point five and let us take the two hundred thousand is the payoff here, hundred thousand is the payoff here.

Zero minus one eighty thousand is the payoff here minus twenty thousand is the payoff here and zero is the payoff here, so what would be the EREV, that is expected return with the expected value approach, for the expected return with the expected value approach, what is the return that we are expecting, how much we can get is basically two hundred thousand into point five plus minus one eighty thousand into point five and it comes to total around ten thousand.

So ten thousand will be our EREV, but if we would have constructed a small plant then we would have got hundred thousand into point five minus twenty thousand into point five and it would have been forty thousand, so this would have been our expected return with the expected value approach, so this would been an obviously we would have constructed a small plant because that is giving us a better expected value.

But the question comes that if we would have had perfect information then if it would have been a favorable market we would have constructed what is known as the large plant because that would have given us better expected value and if it would have been a unfavorable market, we would have constructed as we would have done do nothing, because that is giving us the better or the best possible payoff.

So under that situation we can actually calculate what is known as the ERPI that is expected return with perfect information, what is the expected return with perfect information right, expected return with perfect information that would have been something which will be dependent on both these values and that would have been the two hundred thousand, so ERPI would have been two hundred thousand multiplied by point five plus zero.

Multiplied by point five, why because two hundred thousand is the best possible payoff when we construct a large plant, if favorable market comes up, if we would have had perfect information we would have gone for constructing a large plant then would have got a payoff of two hundred thousand, similarly had it been an unfavorable market, we would have done nothing because perfect information is available and we would have got a payoff of zero.

So it comes out to be the ERPI value which would be around one hundred thousand and this ERPI value of one hundred thousand is independent of whether we make a perfect this decision or the other decision and therefore that is what would have been our ERPI hundred thousand right, so we have an ERPI of hundred thousand and we have an EREV of expected return of the EV approach is say ten thousand, forty thousand and zero.

And our decision would have been forty thousand because that is the highest, so these actually gives us the calculation of what is known as EVPI that is expected value with perfect information and that would be our ERPI that is expected return with perfect information minus expected return with expected value that means it would be hundred thousand minus forty thousand because that is the return that we have got and that would be sixty thousand.

This sixty thousand is what is known as the expected value with perfect information and this is essentially what is known as can be called as value of information, you see that is the focus of discussion of today's class that the value of information really is sixty thousand, that means if someone would have given us the perfect information we could have paid that person maximum of sixty thousand, why maximum because beyond that if we pay then we would have made loss.

Because forty thousand is the payoff we have already got, that we can reach up to hundred thousand with perfect information and if perfect information is available we could have gone all the way up to sixty thousand another additional thing, but question is different, question is that is it really we can give this sixty thousand, answer would have been very difficult really why, because you see what is going to happen is although we can theoretically give.

Up to sixty thousand because that is the value of information but how much can we really depend on the exact information given by the person the expert and how much we can depend on that, that is where and very interesting thing really comes up that is when an expert gives an information obviously again there is a doubt, the doubt in the sense that information may not be hundred percent correct.

So that kind of analysis we require what is known as a Bayesian analysis, we had to therefore construct a probability matrix and really think that how much of that information we can really depend upon right, and based on which we can recalculate and we can come up with a different set of values, in fact in the decision making literature that is really called what is known as an experimentation, so if we go for an experimentation.

How with that experimentation we can get a different sort of result and how we can modify our decision structures that is what we shall discuss in our consequent classes right, so thank you very much and we will continue this in our next class.