Decision Modeling
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Lecture 05
Probability Concepts

Today we are going to discuss the probability concepts in connection with the decision making situations.

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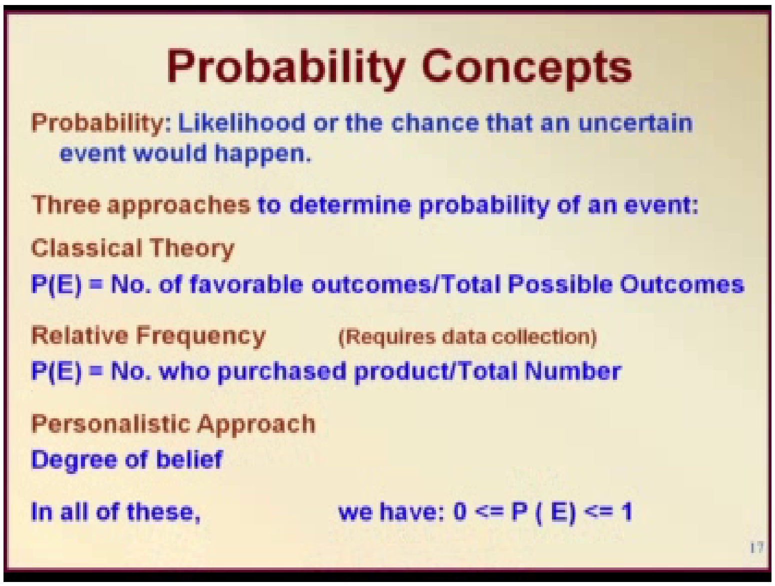

Now probability can be defined as the likelihood or the chance that an uncertain event would happen right

And usually when but very fast before when we do any kind of probability analysis we have to put or establish probability values in a decision situation, now how do we do that, it can be done basically there are three approaches, what are the three approaches?

The very first is the classical theory, what happens in classical theory, in classical theory you can actually do an experiment, how do we do experiment; suppose we have a coin and we need to know is it a biased coin or it's a coin where exactly equal number of heads and tale would come out that means it's an unbiased coin, what we can do we can actually do experiment right, simple thing flip the coin, as you flip the coin you will get head or tail.

But if you do only once you will get either head or tail, and suppose you get head then you tail this coin is hundred percent head, it's not correct, we have to do very large number of experiment, even not fifty hundred maybe in millions and billions right, so if you do it let's say a million times right, you are suppose to get head or tail, let us say fifty percent time but you will not exactly get, there will be still some differences right.

What is to be done then, you have to repeat those experiments, in simulation terms they are called replications, so if you do many replications you will not only get an expected value of the number of heads or expected value of number of tail but you will also get what is known as a distribution right, so moment you get that distribution then you can do a better estimation of the probability and you can have a confidence interval.

On which you can tell that this value could be this, this is classical theory based on certain kind of experimentation, the second method is what is known as the relative frequency right, to do a relative frequency we need to calculate or based on the data collected right, so here is an example number who purchased a product by the total number, so you know it could be collect lot of data about people who purchased a given kind of product.

Out of all the customers who visited the market, so suppose five hundred people visited the market and hundred people purchased a given product the probability of buying this particular product could be hundred by five hundred, but again the experiment has to be repeated and statistical estimate has to be obtained right, many a situation where really we do not have much information, not much of theory can be applied.

The relative frequency method is a very important method through which probabilities can be established, the third method which is known as the personalistic approach, it depends on a degree of belief, what exactly we do we ask experts right, supposing you would like to know that people who comes to the bank right, how many times they really come for withdrawing money, so out of hundred customers who visit bank let us say in the first one hour in the bank.

How many are coming who are for withdrawing money right, and how many are other customers, so obviously you can really approach a relative frequency method you can do a survey, you can ask each customer or you can look at the bank data and you can establish this fax, but if you want a quick estimate you can always ask the bank manager right, the bank manager has an idea because the bank manager is going through all the records.

All the data on a day to day basis because if you collect data on a given day, that day maybe special, that maybe the salary day right, so may not be a correct data could be obtained, so here the degree of belief asking an expert could be a good method to establish and Brock probability value, whatever method you may follow the probability value of an event should be between zero and Y.

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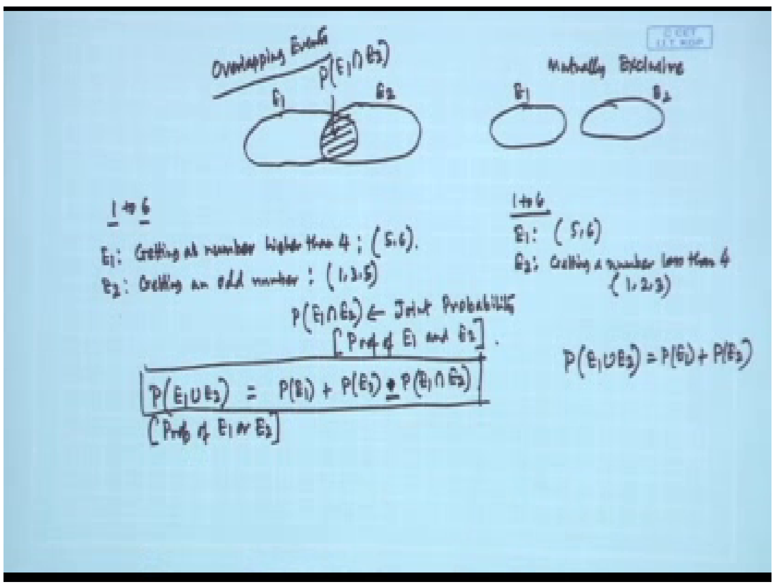

Now let us say there are two events, E one and E two right. So these two events E one and E two you can look here that two things can happen, say this is one situation where this E one and E two they are overlapping, this is another situation where E one and E two they are mutually exclusive right,

So in one situation the events E one and E two they are overlapping, in another situation they could be mutually exclusive, suppose we have a dye and we cast the dye, the dye can really come up with one to six right.

Suppose event one is getting a number higher than four right, and E two is getting an odd number, so basically E one consists of five and six and E two consists of one three five right, so as you rule the particular thing you might get anything that maybe coming up maybe anything between one to six and if we take two events, the first event we may take getting a number higher than four which could be five and six.

And E two could be getting an odd number which could be one three or five and you see they have an overlap, that is the overlapping portion isn't it, this is the overlapping portion, so in this case that is five right, but then supposing in this case one to six suppose E one remains same that is getting a number higher than four which is five, six and suppose here E two is getting a number less than four, so this really will be point two three is it alright.

So you see they are mutually exclusive, so the events could be overlapping, events could be mutually exclusive, here you see when there are overlapping we have to come up with a new definition which is known as probability of E one and E two overlapping right, so this is the intersecting point so this portion is really probability of E one and E two, this kind of probabilities are called joint probability, so this is basically E one and E two.

What is it, this is probability of E one and E two, but then the entire portion is called the probability of E one union E two can be also defined as probability of E one or E two and that is equal to probability of E one plus probability of E two plus probability of E one and E two right, so basically the probability of E one union E two is the probability of E one plus probability of E two plus probability of E one intersection E two right.

So this is one relation we know and this relation is true for overlapping events right, so whenever we have overlapping events we have a relation probability of E one union E two is equal to probability of E one plus probability of E two plus probability of E one and E two or probability E one intersection E two which is called a joint probability, but in case of mutual exclusive scenario the events are mutually exclusive.

So the relationship is also very simple probability of E one union E two is simply probability of E one plus probability of E two right, so this is the basic difference between the two different situations that is intersection events because there is no joint kind of thing, sorry this one is minus, just one small mistake, please correct it that probability of E one intersection E two is the E one plus probability of E two minus probability of E one intersection E two right.

Once you got this now let us look at how this overlapping events and mutually exclusive events really they can be put together the concepts and we can try to get the probability in a given situation, so let us do it with the help of an example.

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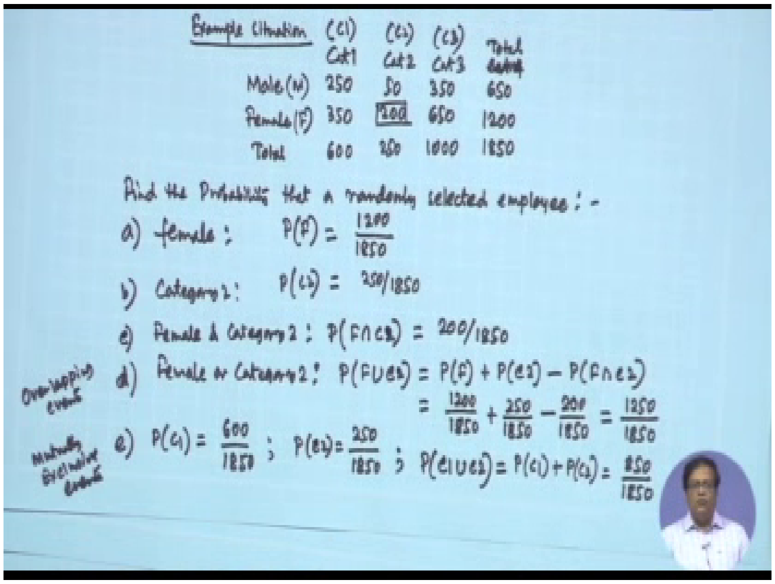

Suppose we have an example situation that is number of male and female employees in a company and there are different categories, four categories and these are the different number of male and female employees.

Two fifty, three fifty, fifty, two hundred, right, so if we make the whole total, ya this is not category four, this is total and we can also make category wise totals, these are six hundred, two fifty, one thousand and eighteen fifty right, so in this example situation suppose we have to really find the following, find the probability or chance that a randomly selected employee will be A female, so what is that probability.

So you see this is very straight forward because the probability of here we can make some abbreviation male and female f and category one, two and three, so what is the probability of a randomly selected employee will be female, this we can straight way obtain by looking at the total figures because total number of females are twelve hundred, so P F will be twelve hundred by eighteen fifty then question B that is again a simple one is category two.

What is the probability that a randomly selected employee will be of category two, so here we can make out that it should be that category total number of category two employees are two fifty out of eighteen fifty, so this will be two fifty by eighteen fifty is it alright, now the question comes, suppose this is another question we ask ourselves what is the probability of female and category two right, so this probability female.

And category two is can be written as probability of female intersection C two right, how many are there, so if you look at this then out of the two fifty category to employees two hundred are female right, so since only two hundred people are female and category two that is this figure right, we can write this is equal to two hundred by eighteen fifty, so we have got this, now suppose we are asked to find out.

What is the probability of an randomly selected employee is female or category two right, so basically we have to see first of all that it is an mutually exclusive event or overlapping event, the probability of female randomly selected employee becoming female and a randomly selected employee becoming category two, they are actually overlapping because some of the females are also category two, so if that happens then we can directly use our formula.

The probability of female or category two is equal to probability of female plus probability of category two employees minus the intersecting point that C two right, what is it, the probability of female is twelve hundred by eighteen fifty, the C two is two fifty by eighteen fifty and the intersection portion is what is known as the fifty, two hundred by eighteen fifty, right so if we have this then we can actually calculate that this would be probability of female.

Or category two is twelve hundred plus two fifty minus two hundred so fourteen fifty minus two hundred that is twelve fifty by eighteen fifty right, so this is exactly how we can really workout the kind of things because if we really look at the total number of category two employees at two fifty, total number of female employees at twelve hundred, so this should be twelve hundred plus two fifty that is fourteen fifty but the common portion two hundred.

So that if you deduct then you get twelve fifty, so this is how we can actually establish what is known as the probability in many different situations, but suppose we have to find out the probability of, what is the probability let us say number E probability of category one employee is, you see probability of category one employee is will be six hundred by eighteen fifty and what is the probability of category two employees, that is two fifty by eighteen fifty.

Now if you combine that two then you will get that probability of category one and category two employees what it should be, it should be, is there any intersecting portion between category one and category two no, so what we get here that while this case was overlapping events, in this case we have what is known as mutually exclusive, the events are mutually exclusive events right, so we have what is known as the mutually exclusive events.

Because we have mutually exclusive events therefore it should be simply probability of C one plus probability of C two and it should be nothing but eight fifty by eighteen fifty right, so this is how you can actually calculate the different probability concepts right,

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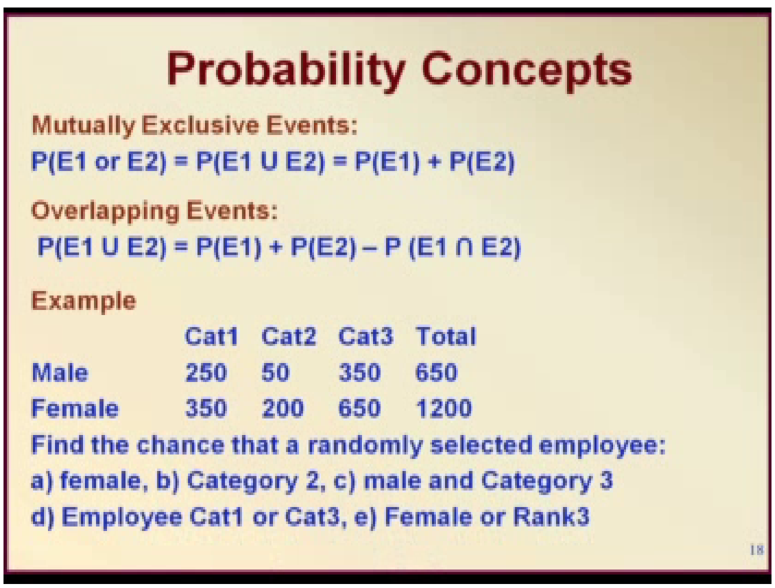

So what we really found so far is that there could be mutually exclusive events and the overlapping events and based on which we can actually calculate all the different probabilities. And try to put them in the appropriate context.

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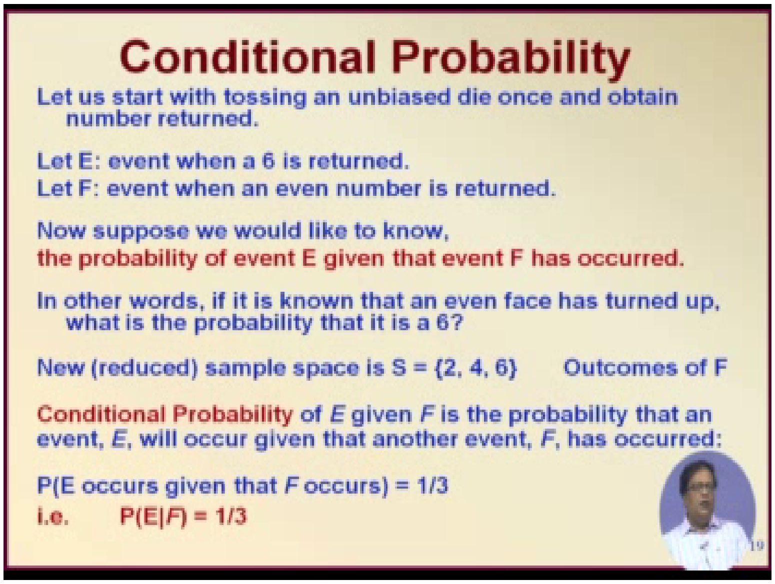

Now let us look at another very very important concept which is known as the conditional probability conditional probability let us start with tossing and unbiased die once and obtain the numbers returned right, let us say that we get E as an event which means a six is returned, and F is another event when an even number is returned, now suppose we would like to know the probability of event E given that event F has occurred.

In other words, what is the probability of getting a six knowing that an even number has returned, look here what is the probability of getting a six, out of six possibilities, it is one by six but if you know that an even number has come out, what is that, even number could be two, four, or six right, and we know that even number has come out so that means now the sample space is reduced to three from the original six.

Because earlier the probability was between one, two three, four five, six and one possible event is six right, so it was one by six but under condition, the condition is that it’s an even number, the sample space is now reduced to only two, four and six so it is only three, so the conditional probability of E given F is the probability that an event E will occur given that another event F has occurred, so P E occurs given that F occurs equal to one by three right.

So that is what we write probability of E given F equal to one by three right, and that is what is our conditional probability,

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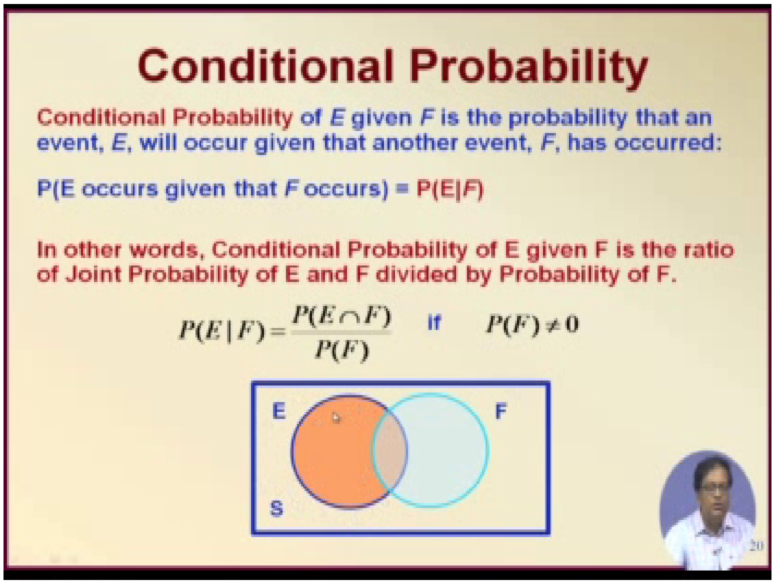

So we know that the conditional probabilities are calculated in this manner and here if you look at this particular diagram you will find that there is a sample space, there is an event E, there is an event F and there is an intersection of the event E and F, so if look at this diagram, if this is E this entire portion is E, this entire portion is F.

This is the intersection E and F right, if you now look at the F domain see earlier we had the entire sample space, but now we have only the cut of sample space of the reduced sample space of F right, in the reduced sample space of F this is the portion where the E and F has intersection, so having known that F has occurred right, that is given that F has occurred, so once F has occurred then this annular portion of E has no meaning, because it cannot happen.

Just cannot happen right, so if E has any portion which is, suppose you have told that you have cast an die and you will be getting declared a winner, if you get something above four right, so if you'll be declared winner if you get above four but suppose you have told that the number that has come is an even number right, so if it is coming even number then possibility is only two, four and six right, so but you will be winning only.

If even number is above four which is five and six right, so you will be winning if you have five and six but you have been told an even number has come that is two, four and six that means five has not come isn't it.

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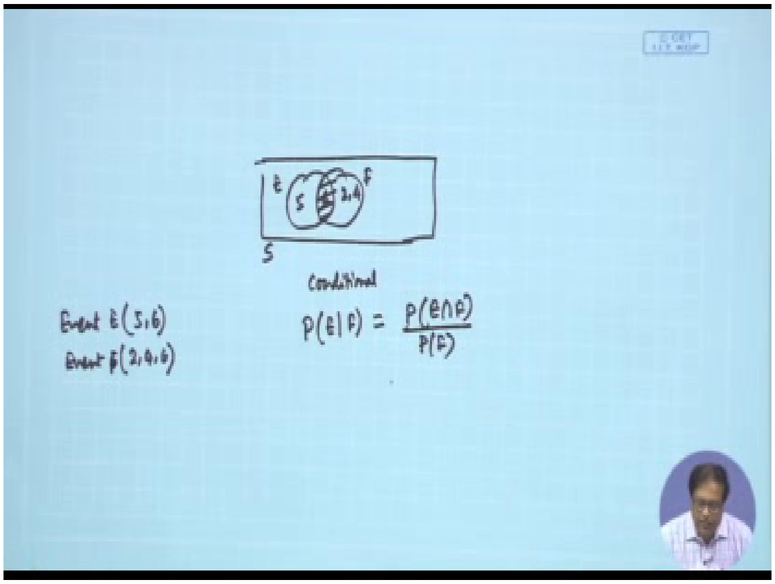

So if I draw this diagram once more here then, this is my sample space S, this portion is E which is the winning, so event E really contains five and six and you were told that an even number has come so that is event F which has got two, four, and six.

So this is event F then six is here because six is out of all, so two four is also here and five is here, so look here this is the common portion, so that means the probability, what is the probability suppose two, four, six they are all equally likely, then the probability will be oen by three right, so this is the essential learning from here that the conditional probability, probability of E given F is probability of E intersection F divided by probability of F alright.

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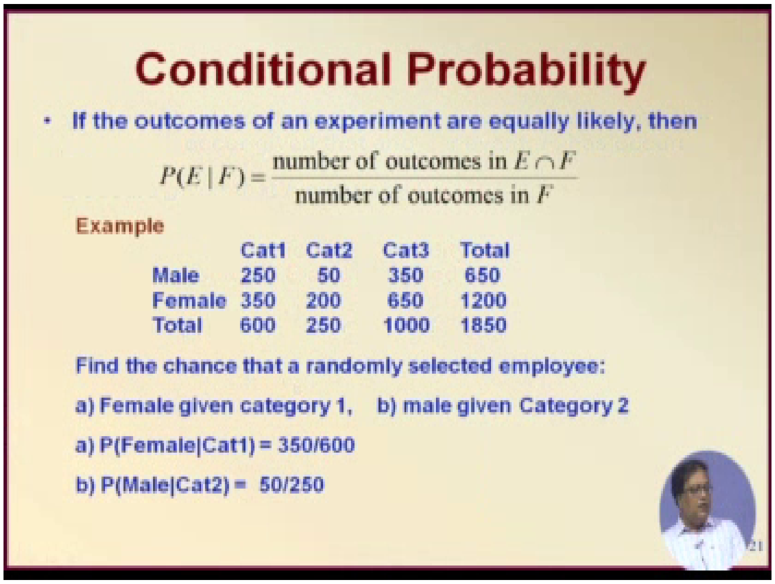

So this is very important result, this example once again that if the outcomes, same example look at it once again find the chance that a randomly selected employee is a female given category one right, so we know what is the probability of female and category one right, so let us look at that example once again.

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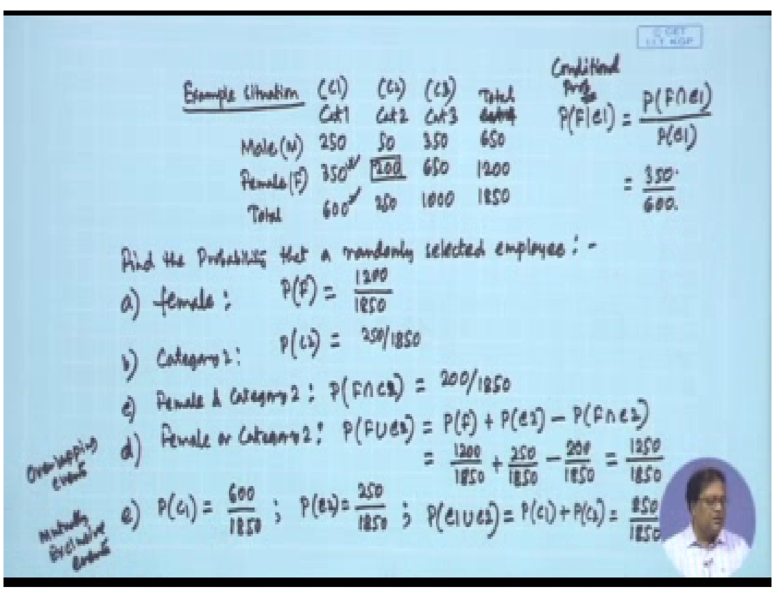

So this is the example that we are doing it is the same example, so if we know that probability of, we have to find probability of female given category one right. So it will be equal to probability of female intersection category one divided by probability of category one isn't alright,

So what is the female intersection category one is three fifty and how many category one employees are there six hundred isn't alright, so we see here this is three fifty which is the female intersection category one and total category one employees are six hundred,

So this is the intersection three fifty, this is the total category. So this is probability conditional probability of female given category one will be given by three fifty by six hundred,

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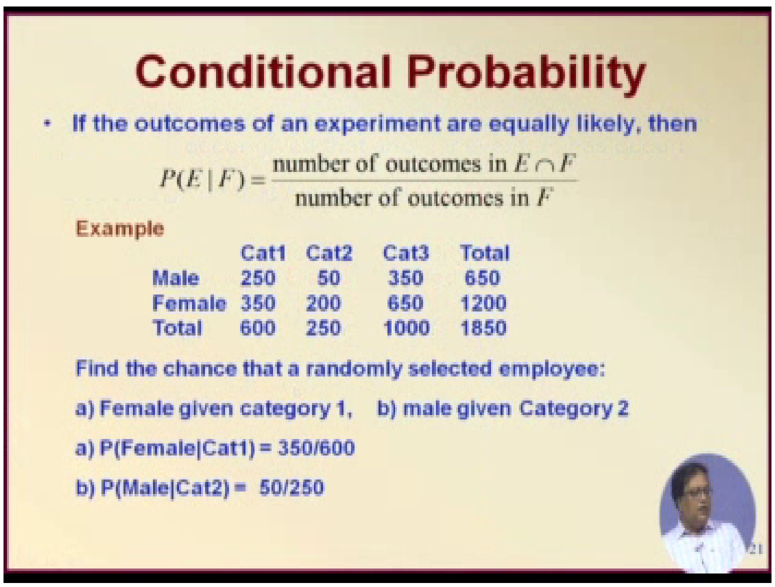

So you can look at the slide once again that you can now workout probability of male given category two, why it is fifty by two fifty, think about it you can easily find out what is the intersection male category two fifty, what is the total number of category two, two fifty, so conditional probability will be fifty by two fifty right. So this is about this conditional probability things

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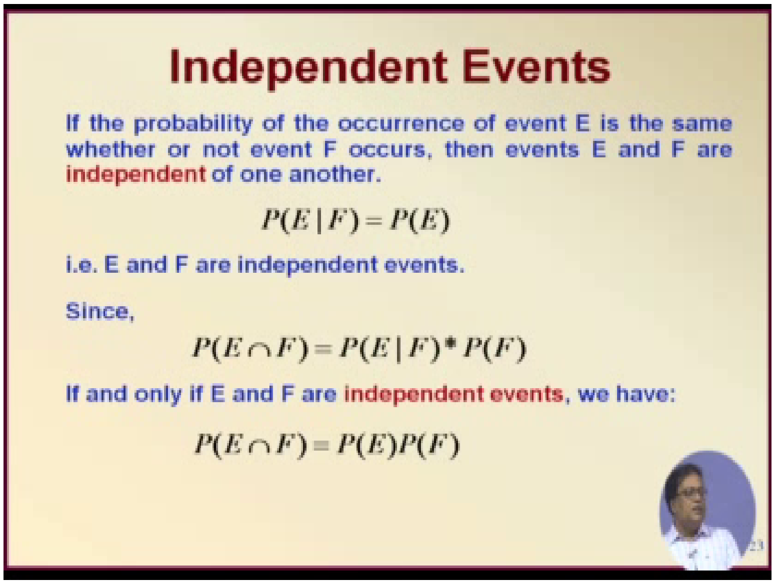

And in case of independent events if the probability of the occurrence of event E is the same whether or not event F occurs, then they are called independent so if they are independent then probability of E given F is nothing but probability of E right, and since that happens and since we can also right this because look at this previous equation,

Probability of E given F Equal to is probability of E intersection F by PF. If you put PF on the other side then you get that equation that probability of E intersection F is probability of E given F star PF but if conditional probability is nothing but probability E in case of independent events then the intersection between E and F that probability will be P E multiplied by P F Right?

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So for exclusive events what happens, the exclusive events the independent events verses mutually exclusive events or disjoint events.

If two events are independent then probability E given F equal to P E and probability E intersection F, the joint probability is PE into PF

But if two events are mutually exclusive they do not share common outcomes right, so in this particular section, we've looked into the mutually exclusive events how the probabilities are obtained, the overlapped events how the probabilities are computed, and also the basic concepts of conditional probability.

This concept is very important because in our subsequent section when we talk about the Bayesian theorem the idea of conditional probability will be very useful right, so thank you very much.