Decision Modelling.
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Lecture-08.
Decision Problem with Experimentation.

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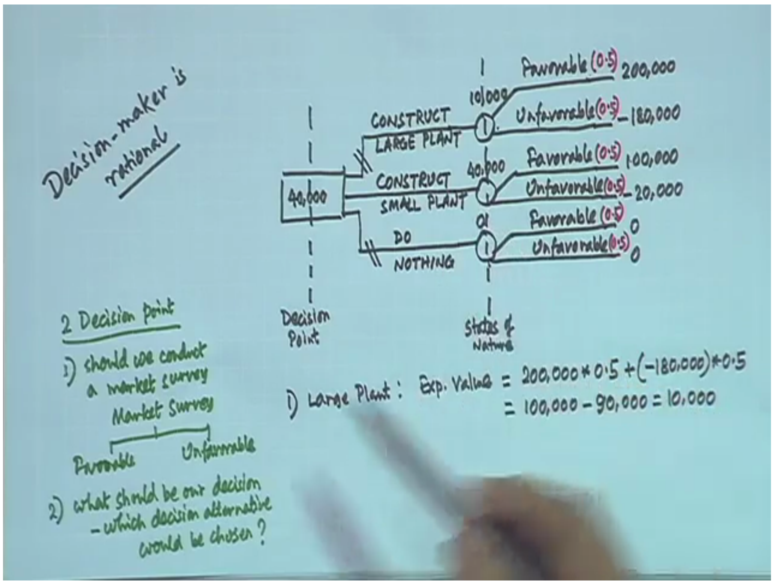

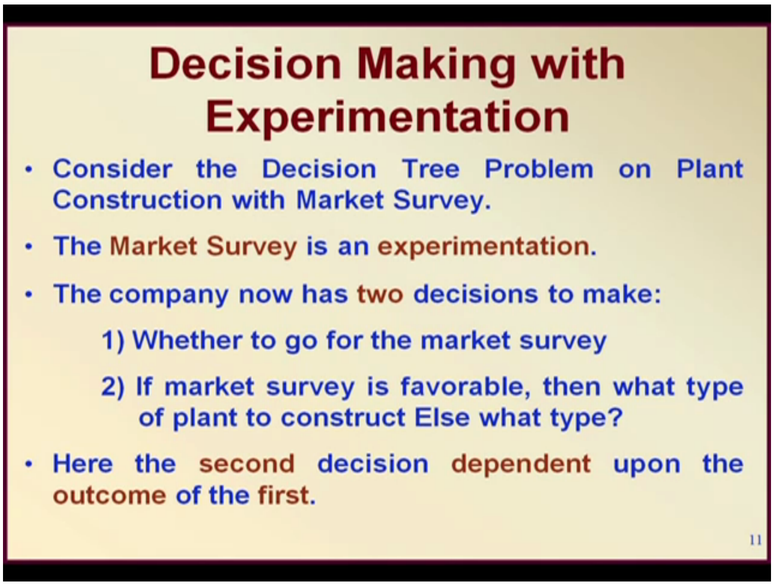

So today we are going to discuss what is known as decision making with Experimentation. Please look at this diagram here, this particular diagram we had drawn in our previous class very quickly if we look at, then there are 3 decision options before us construct a large plan, construct a small plan or do nothing and market could have been favourable or unfavourable as there are some prior probabilities and the payoffs were computed and based on which we got 40,000 and we constructed a small plant. But the question came that the large plant had 200,000 option, so is there any alternative way we can look at.

So one such solution was that doing a market survey. If we do a market survey, the result could be favourable or unfavourable, what should be our decision if the market is favourable or the market is unfavourable? Now look at the slide, so for this particular problem we can say that the, for this decision problem the plant construction with market survey, the market survey is an experimentation. The company has 2 decisions to make, whether to go for market survey and a market survey is favourable, what type of plant to construct, else what type. Here the 2nd decision is dependent upon the outcome of the 1st. Right.

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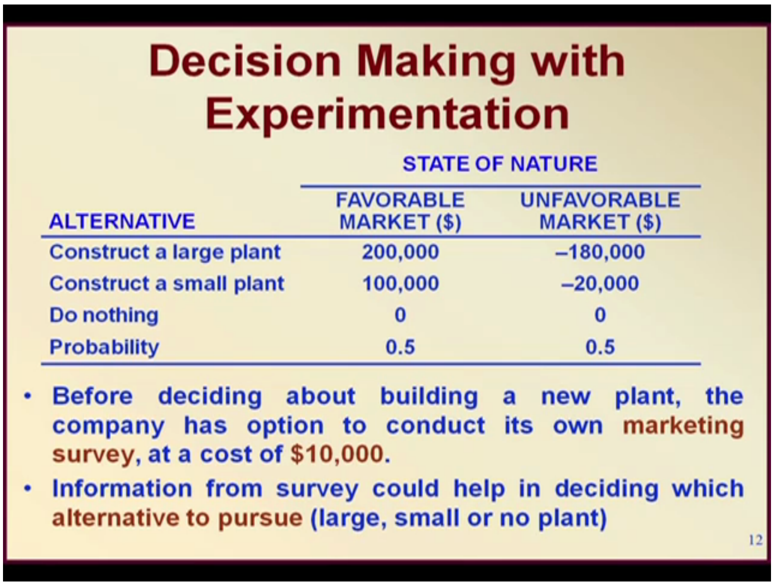

So it is a kind of situation where there are number of decisions and the 1st one is the answer of the 1st one would really look you know create the kind of 2nd decision option. So if we take up this problem, so there are some additional considerations, already we have seen the problem number of times. Before deciding about building a new plant, the company has an option to conduct its own marketing survey at a cost of 10,000 dollars. So it is not that market survey is free, the market survey also has a cost, the question is that this 10,000 additional amount we are paying, obviously with the hope that better results would be obtained.

The information from survey could help in deciding which alternative to pursue, that is either large or small or no plant. That means whenever we have a decision-making with experimentation, the experimentation itself cuts a little bit of payoffs, we give up that payoffs thinking that a better decision could be made and the outcome of the survey would decide the kind of alternative to pursue in the future. So that is the kind of situation. But there is something more to it, what is that?

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The market survey is to be conducted at a cost of 10,000, that is the point number-one, the point number 2 is that market survey will not be able to predict the state of nature with 100 percent accuracy. If it would have done with 100 percent accuracy, things would have been very simple. We simply could have you know done with if, these are our payoffs, if market survey is favourable, do this, market survey is not favourable, do this. But here since it is not 100 percent accurate, we have to do what is known as a Bayesian analysis.

The market survey can only give additional information of the probable state of nature in the form of following conditional probabilities, right. It could be survey favourable, given the market is favourable, survey unfavourable, given the market is favourable, the 3rd option survey favourable, given the market is unfavourable, survey unfavourable, given the market's unfavourable. So all these 4 conditional probabilities, they have some value. But please remember that they should occur in pairs. The 1st 2 are you know survey favourable, given favourable market, survey unfavourable, given favourable markets, they should add up to 1.

The same thing about the conditional probabilities with respect to the unfavourable market, right because when the market is favourable survey could be either favourable or unfavourable and those are going to be their probabilities. Before we embark on this problem, let us look at this at a slightly bigger respect because whenever we talk about experimentation, there are different experimental situations that can occur.

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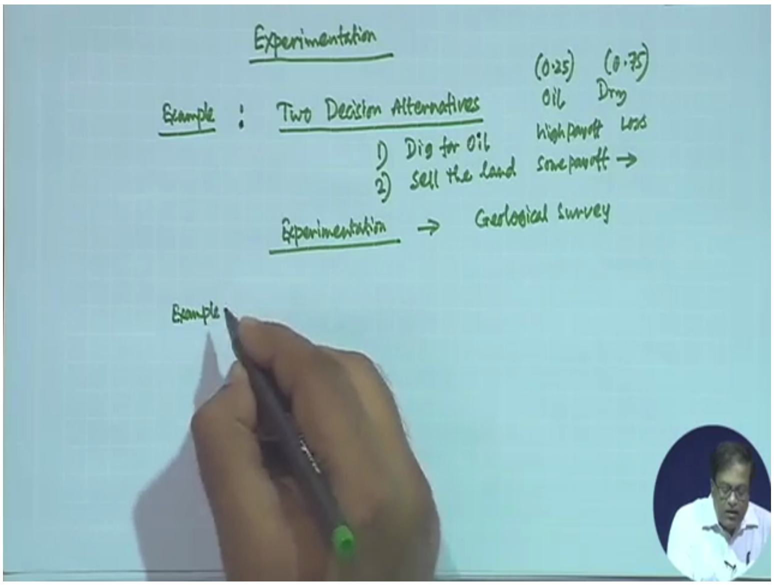

One example which you will get in what is known as the Hillier and Lieberman's book, that is about a particular situation where decision is to be made whether to you know dig for oil or do not dig for oil, simply sell the piece of land. So these are decisions one has to make. The decisions, this is an example, this example basically looks at the 2 choices, the 2 decision alternatives, one is dig for oil and 2 is sell the land. A piece of land is obtained, the 2 choices are there, either dig for oil or sell the land, because if you dig for oil and there is no oil, obviously there would be a lot of expenses and there will be loss.

Now the question is they have certain payoffs if we dig for oil, then again 2 things can happen that either it is oil is available or it is dry. Suppose oil is available, then you get high payoff and if it is dry, then loss. But if you sell the land, then some payoff you will get under both situations, right, both situations, some payoff, not much at some payoff will be obtained. The question therefore comes that should be big for oil and should the sell the land. Obviously, if you dig for oil, it is like a kind of fortune hunting because it could be high payoff or it could be dry.

And obviously the probability for the oil would be on the lower side, something like suppose this probability could be something as low as 0.25, whereas the dry probability could be as high as 75 percent. Now question is that on the face of it if you make a decision problem calculation, then you might get , best decision could be sell the land. But obviously the person who has bought the land definitely has gone for digging for oil in search of high payoff. So what it can do, it can, the company can actually go for experimentation.

Now what is that experimentation? Experimentation could be a geological survey, right, a geological survey and that geological survey use some instrument and through that instrument it can come out with a set of conditional probabilities that if you know the Ultra, you know the kind of test that it conducts. If the test becomes positive, then what is the probability that it is oil exists or what is the probability that it is dry. So when those conditional probabilities are given, then obviously based on that, those conditional probabilities the dig for oil and the sell the land will have now a, the initial decision that whether to conduct that geological survey or not.

If we conduct then should we dig for oil or should we sell the land and if it, the geological survey tells that it is dry, then whether do dig for oil or sell the land. Right. So this is a kind of example that we can think of.

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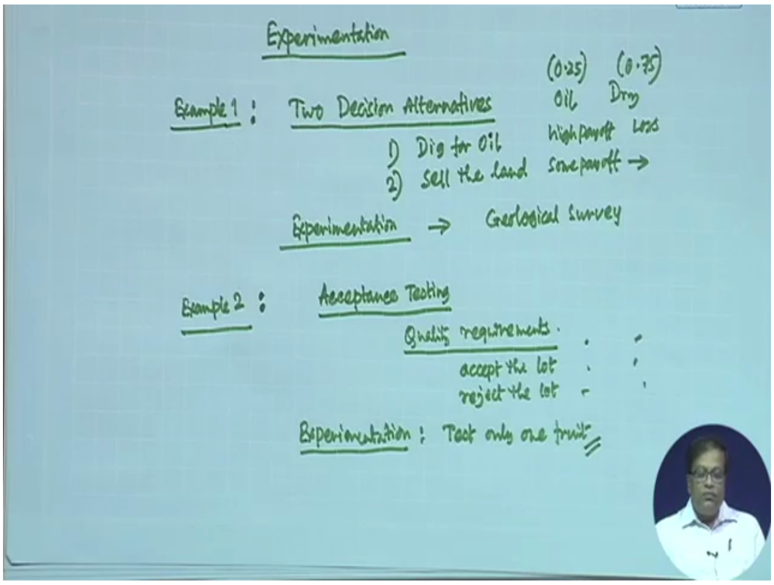

The 2nd example could be that in a particular situation, particularly let us say testing or acceptance testing, you know, we can think of an acceptance testing a situation, acceptance testing. So sometimes what happens that when you are in a particular market and you are buying let us say fruits, then what happened that you have and to alternatively, that whether to accept, supposing a lot has come and whether to accept the lot or not to accept the lot, right. So when you have to accept the lot, then there are certain quality requirements.

So there are quality requirements, quality requirements. If the fruit lot is good, then you get certain payoffs but if the fruit lot is bad, then your payoff is low. So the question is you know whether to accept the lot or reject the lot, right and obviously based on the various situations.

So here again, usually it is resorted to certain experimentation and that experimentation is that as the example shows about acceptance testing that test only let us say 1 fruit. Now the question is when we test only one fruit, is it good or it is bad.

Suppose the fruit is good, then how does it modify our you know accept and reject decision and if the fruit is bad, what should be our decision because unless we create a kind of acceptance testing norms, because it is see particularly fruit, we cannot test 100 percent, if you test 100 percent percent, then nothing remains. So we have to sample only in limited number, let us say one or 2 or 10 maximum. Now the question is this particular experimentation, when test only one fruit, how does this particular experimentation devices a particular decision.

So like that we can think of several such examples, you know you can think of an example from credit rating, we can think of examples from sharemarket or as I has given example previously, that suppose someone is having the disease and goes to a Doctor, the Doctor actually asks the patient for a test, that is, you know if the test is positive, then one kind of decision situation, that whether the person is having the given disease or not and if the test is negative, then the probabilities get modified in a different way. So all these different decision situations, they all have a similarity, basically whenever we do experimentation, there are few things to see.

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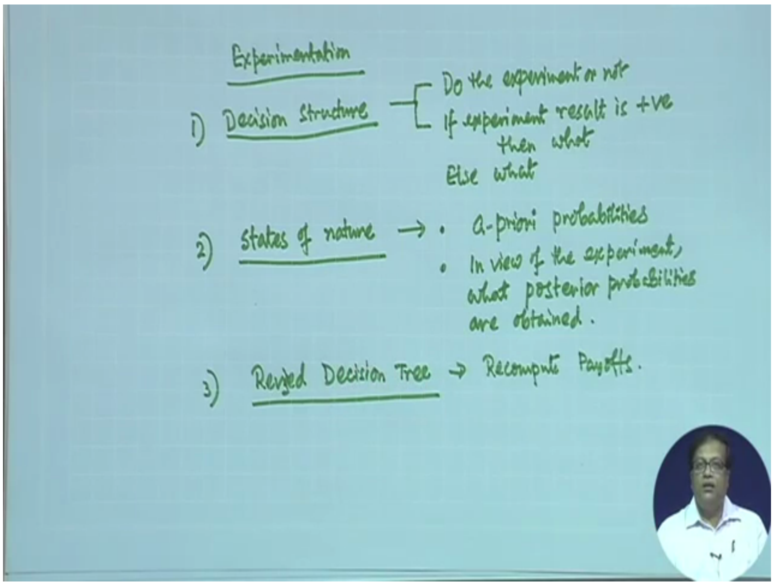

Any experimentation, a few things to see. The 1st one to see is the decision structure, means as I have discussed already that what is our 1st decision, what is our 1st decision that is do the experiment or not, that is the 1st decision. Then 2nd set is if experiment result is positive, then what, else what. So that is the decision structure, do the experiment or not, if experiment result is positive, then what, else what. Then number 2 what we need to think of is what are the states of nature. Now when we look at the states of nature, we have to understand what are, what were a priori probabilities. And next thing to see is in view of the experiments or experiment, what posterior probabilities are obtained.

Right, so 1st of all there are a priori probabilities and then in view of the experiment, what posterior probabilities are obtained because the a priori probabilities will now be replaced by the posterior probabilities. And again there are 2 different sets of posterior probabilities, one is for if the experiment result is positive, the other is if the experiment result is negative. So this has to be done and the 3rd one is revised decision tree. Right, the decision tree also has to be revised, right. Recompute payoffs. So the all the payoffs have to be recompute, recomputed on the basis of Number-one that is the decision structure and number 2 in the states of the nature.

So all of this when we are putting together, we can actually do the experimentation in a proper manner. So this is the broad structure under which we shall be able to do the experimentation.

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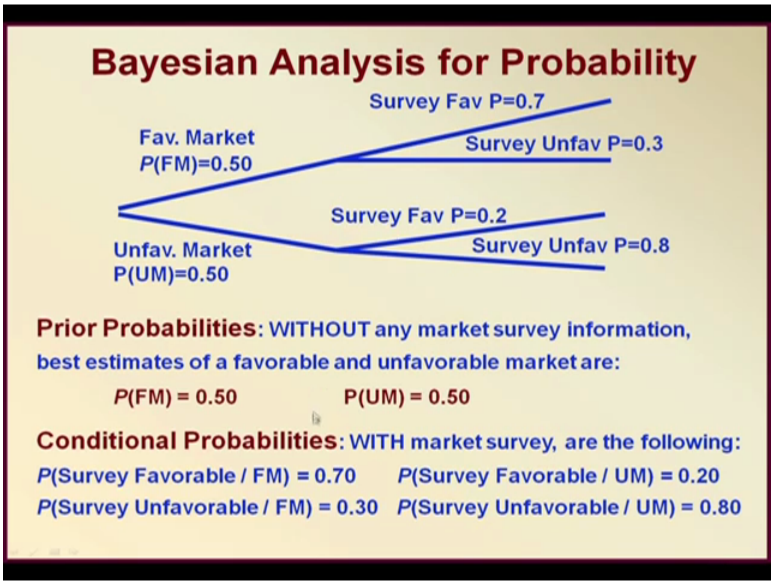

Now let us come back to the problem once again , what we are, in between that there was a market survey, the market survey was having a cause, market survey is not 100 percent accurate. And therefore there are conditional probabilities and those we have already discussed. Do you see we have to do 1st of all a Bayesian analysis. So what are those, 1st of all you see that there was a probability of favourable market, the a priori probabilities, that was without any market survey information the best estimates of the favourable and the unfavourable market, where P FM, favourable market was 0.5 and P UM, unfavourable market was 0.5 also.

And then there are these 4 options, for favourable market, the survey could be favourable for survey could be unfavourable, here is survey could be favourable, survey could be unfavourable. So these conditional probabilities are written here. So this is the Bayesian analysis diagram and these are the initial probabilities. Let us see how these computations are done. These computations we have already done for a previous problem but let us have a repeat of this. Now 1st one is what is, we have to compute what is known as the joint probabilities.

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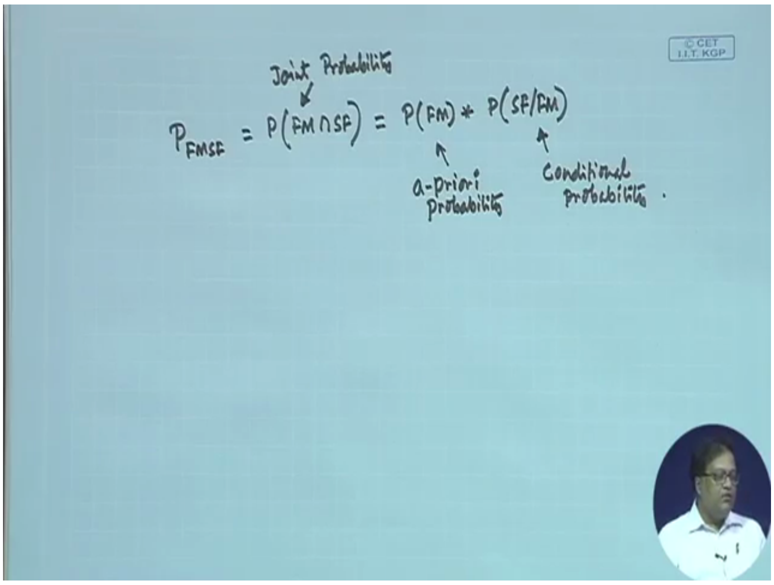

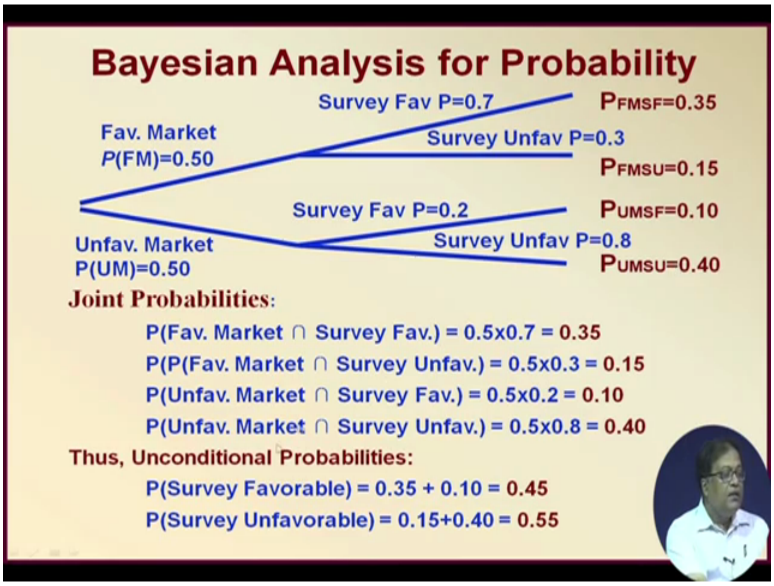

What is joint probability, a joint probability is that the P of favourable market Intersection survey favourable is basically a multiplication of the probability of the a priori probability, probability for the favourable market multiplied by the conditional probability, right. So this lets write down and understand this once again that what is, how these computations are done. That probability of P FMSF equal to probability of favourable market Intersection the survey favourable, right. So again there are 2 probabilities, one is a priori probability which is the probability of FM, the favourable market and multiplied by probability of survey favourable given favourable market.

Right, so what are these, this is, these are 1st of all, we have to understand this is a joint probability. The joint probability is a multiplication of the a priori probability and the conditional probability. Right. So basically that is how the calculations are done, the joint probability of favourable market and survey favourable is the a priori probability of favourable market multiplied by the corresponding conditional probability, probability of survey favourable given favourable market. So when you do this multiplication we get what is known as the joint probability of favourable market and survey favourable.

So let us go back there once again. So this is where the probability of favourable market and survey favourable, the interaction and you know, the intersection, the intersection would be 0.5 into 0.7 equal to 0.35. So similarly we have got all of these calculations and after these calculations, look here then we can actually calculate what is known as the unconditional probabilities. The unconditional probabilities are basically, look here, one side we have these a priori probabilities for favourable market and unfavourable market, on the common are basically there are 2 types of situations, survey favourable and survey unfavourable.

Now what we could do, if we take out the survey favourable cases, right, so there are able to survey favourable cases. One is FMSF and UMSF, if you add these 2, you know, if you add these 2, that is P FMSF + P UMSF, that is 0.35 + 0.1, that is called what is known as the unconditional probabilities, is it all right.

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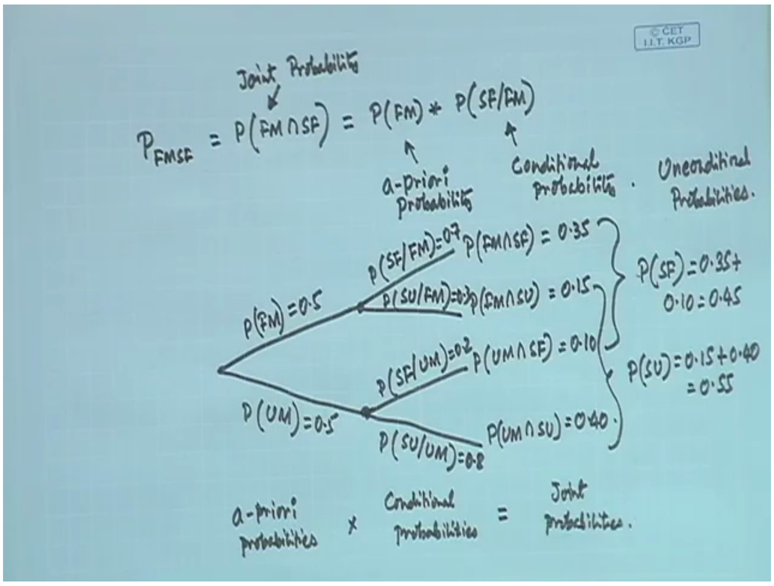

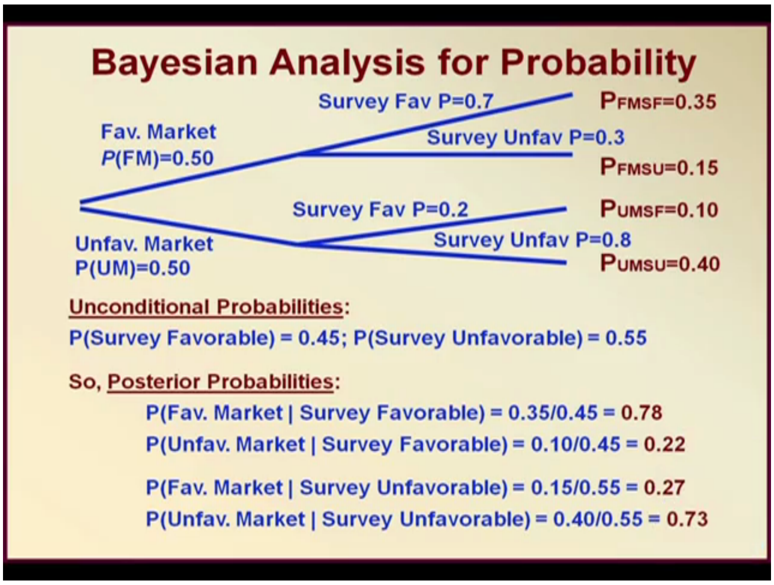

So let us look at this once again just for because I am repeating this because these are slightly difficult topic and one should be very very clear what exactly we are talking about. So this is favourable market, this is unfavourable market, this is survey favourable, given favourable market, this is survey unfavourable given favourable market. This is survey favourable given unfavourable market and this is survey unfavourable given unfavourable market. So these are some probabilities, right. So these are a priori probabilities and these are conditional probabilities.

When you multiply, you get what is known as the joint probabilities, right. So these are the joint probabilities. So a priori probabilities multiplied by conditional probabilities equal to the joint probabilities. So once again this was 0.5, this was 0.5, these values are 0.7, 0.3, 0.2 and

0.8. So when you multiply, we got equal to 0.35, we got equal to 0.15, we got equal to 0.10 and we got equal to 0.40. Now if these 2 are added, right, because in both cases we have what is known as the SF condition, right, survey favourable condition.

Then we get what is known as P SF which is equal to 0.35 + 0.10 equal to 0.45. Right., Whereas if I add these 2, then we get probability of survey unfavourable, which is 0.15 + 0.40 equal to 0.55. Okay. So this is how and these probabilities are called unconditional probabilities, sometimes they are also called total probability. Right, so anyhow, this, by multiplying a priori probabilities and conditional probabilities, we have got the joint probability and adding corresponding joint probabilities we get what is known as the unconditional probabilities.

So look at the slide, you know, we have put those figures that there are unconditional probabilities, 0.45 and 0.55. Now if we then understand, then look here, that survey favourable is 0.45, it has got 2 components, one is for favourable market, which is 0.35 and unfavourable market 0.10. So obviously if we want the posterior probabilities, that when there is survey favourable, then how much is unfavourable market, how much is for unfavourable market, that has been divided into these ratio, 0.35 is to 0.10. So what happens, we have survey favourable and for survey favourable, see we can draw what is known as a secondary diagram, right.

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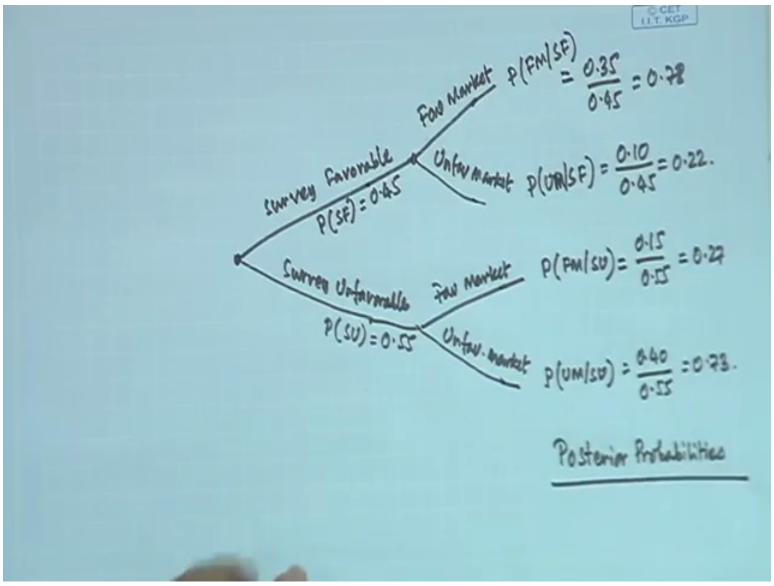

What is a secondary diagram, that is we can actually now look at this way. Suppose survey is favourable and this is survey unfavourable, right. So what we got, we have got, P of survey favourable, unconditional probability which comes to 0.45 and how much we have got survey is we have got 0.55. Now out of that what are the 2 components, the 2 components is again, it could be 2 things, one is favourable market and unfavourable market, right. And this side also, favourable market and unfavourable market. Okay. And those are since you know this one, probability of favourable market given survey favourable and probability of unfavourable market given survey favourable.

So this one, since the ratio has divided into 0.35 is to 0.45, so this one would be 0.35 by 0.45 equal to 0.78. And this one will be 0.10 by 0.45 equal to 0.22, right. So look at the slide once again, that what we have, that probability of the favourable market to survey favourable is 0.78 and the other one is 0.22. Similarly on the other side, the probability of favourable market given survey this thing and probability of unfavourable market given survey unfavourable, that has divided by you know 0.15 and 0.40. So this one 0.15 by 0.55 and this one is 0.40 by 0.55 equal to 0.27 and equal to 0.73.

So these are called what is known as posterior probabilities. Right, so this is how the posterior probabilities are calculated, and I will stop here, we will continue the similar discussion in the next class.