

Decision Modelling.
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Lecture-09.

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Decision Making with Experimentation

ALTERNATIVE	STATE OF NATURE	
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0
Probability	0.5	0.5

- Before deciding about building a new plant, the company has option to conduct its own **marketing survey**, at a cost of **\$10,000**.
- Information from survey could help in deciding which **alternative to pursue** (large, small or no plant)

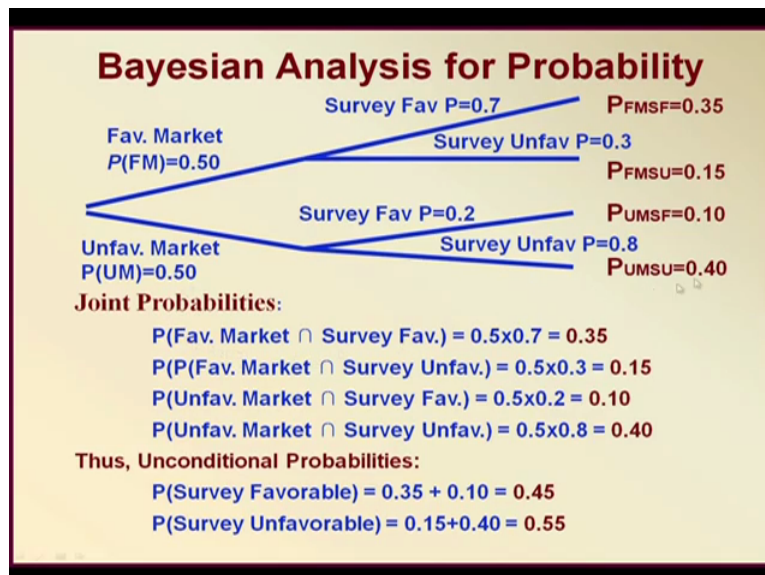
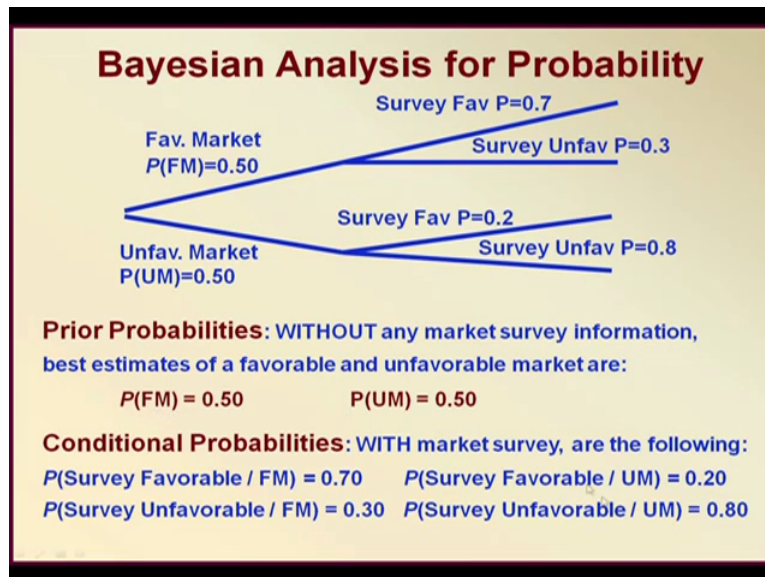
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The Experimentation

- The market survey is the experimentation conducted at a cost of \$10,000.
- However, the market survey will not be able to predict the state of nature with 100% accuracy.
- The market survey can only give additional information on the **probable states of nature** in the form of the following conditional probabilities:

P(Survey Favorable / Favorable Market) = 0.70
P(Survey Unfavorable / Favorable Market) = 0.30
P(Survey Favorable / Unfavorable Market) = 0.20
P(Survey Unfavorable / Unfavorable Market) = 0.80

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So we were in the middle of this decision-making with Experimentation, a very quick review of what we have done so far. Basically there are 3 alternatives construct a large plant, a small plant and do nothing and there are 2 states of nature, favourable market and unfavourable market and there are these payoffs and there are some a priori probabilities of 0.5 and 0.5. Now we can decide a new plant based on there is a market survey option we can also have. So when we have market survey, it has a cost of 10,000 and the market survey actually gives a set of conditional probabilities for survey favourable given favourable market or unfavourable market and similar thing for survey unfavourable. So these are the conditional probabilities.

Then what we did is the Bayesian analysis and in that Bayesian analysis we calculated the prior probability is already known, the conditional probabilities are given, based on these

prior probabilities and the conditional probabilities we calculated what is known as the joint probabilities and from the joint probability is by taking the similar cases of the survey results that is survey favourable and survey unfavourable we could obtain what is known as the unconditional probabilities.

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Bayesian Analysis for Probability

	Survey Fav P=0.7	P _{FMSF} =0.35
Fav. Market P(FM)=0.50	Survey Unfav P=0.3	P _{FMSU} =0.15
Unfav. Market P(UM)=0.50	Survey Fav P=0.2	P _{UMSF} =0.10
	Survey Unfav P=0.8	P _{UMSU} =0.40

Unconditional Probabilities:
P(Survey Favorable) = 0.45; P(Survey Unfavorable) = 0.55

So, Posterior Probabilities:

P(Fav. Market | Survey Favorable) = 0.35/0.45 = 0.78
P(Unfav. Market | Survey Favorable) = 0.10/0.45 = 0.22

P(Fav. Market | Survey Unfavorable) = 0.15/0.55 = 0.27
P(Unfav. Market | Survey Unfavorable) = 0.40/0.55 = 0.73

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Joint Probability

$$P_{FMSF} = P(FM \cap SF) = P(FM) \times P(SF/FM)$$

↑ a-priori probabilities
↑ Conditional probabilities
↑ Unconditional Probabilities.

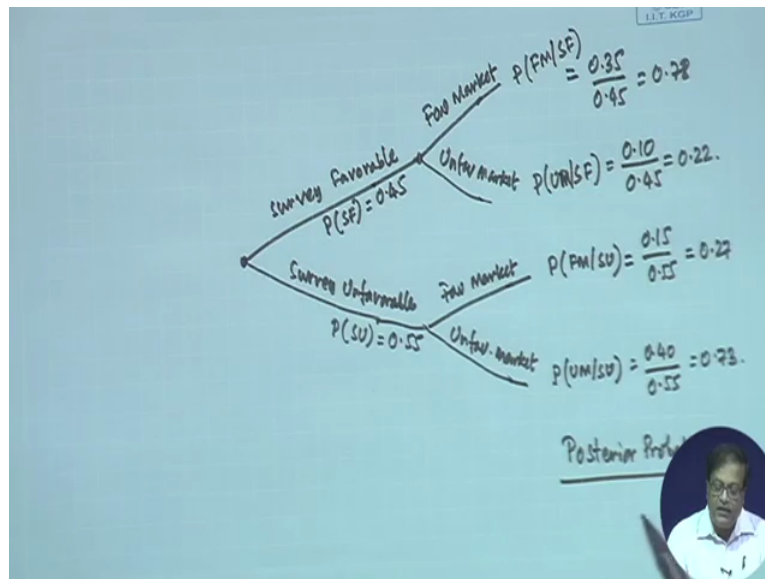
Bayesian Analysis

$P(FM) = 0.5$ $P(SF/FM) = 0.7$ $P(FM \cap SF) = 0.35$
 $P(SU/FM) = 0.3$ $P(FM \cap SU) = 0.15$

$P(UM) = 0.5$ $P(SF/UM) = 0.2$ $P(UM \cap SF) = 0.10$
 $P(SU/UM) = 0.8$ $P(UM \cap SU) = 0.40$

$P(SF) = 0.35 + 0.10 = 0.45$
 $P(SU) = 0.15 + 0.40 = 0.55$

a-priori probabilities × Conditional probabilities = Joint probabilities.



And after obtaining unconditional probabilities we could actually obtain what are known as the posterior probabilities and those posterior probabilities are you know for 2 cases, favourable market given survey favourable, unfavourable market given survey favourable, they are 0.78 and 0.22 respectively. And similarly for survey unfavourable the values came to 0.27 and 0.73. But look here, there is something more to it, look at this particular diagram, in this diagram what we have done, we had done what is known as Bayesian analysis.

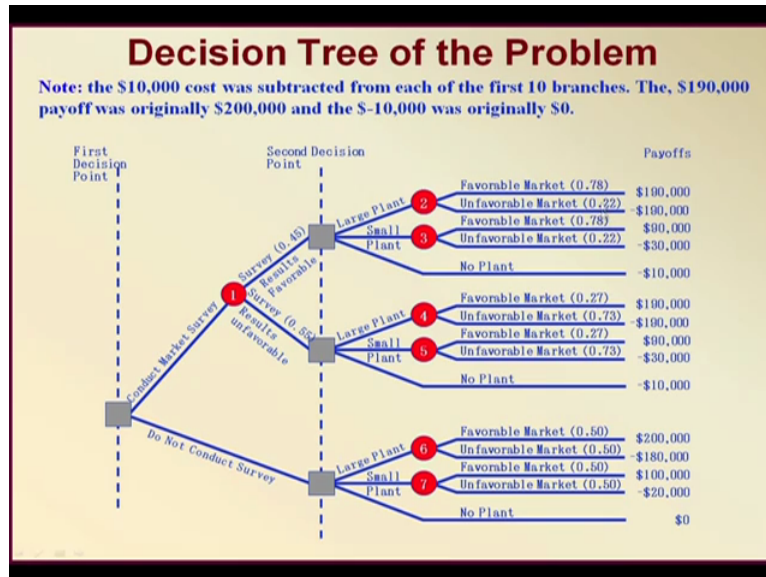
And in this Bayesian analysis, you know what is basically important here is that, you know one side we have, these are the a priori probabilities, that is the favourable market and the unfavourable market. And then we did the survey and then we did the you know Bayesian analysis, we opted the conditional probabilities and obtained joint probabilities, etc. But after the a, the posterior probabilities are obtained, then look here, this diagram is not changed, right. Now we could draw actually the decision tree in name different manner.

What is the different manner, we could put the decision 1st, right. What was the decision, decision was whether to go for survey or not going for survey. Now the result could be 2, it could be either survey is favourable or survey is unfavourable. If the survey is favourable, what was the unconditional probabilities, the PSF was 0.45, what was the unconditional probability for survey unfavourable, it was 0.55, so we put them here. Now you know there could be 2 situations, it could be favourable market, it could be unfavourable market, right.

Basically the survey says it is favourable but in spite of that actually the, because of uncertainties, the market can actually be favourable or unfavourable with posterior probabilities of 0.78 and 0.22. Similarly if the survey comes out to be unfavourable, it could

be favourable market for 27 percent times and it could be unfavourable market for 73 percent times. So this is the net result out of our Bayesian analysis and you know we can then put these results into our decision tree.

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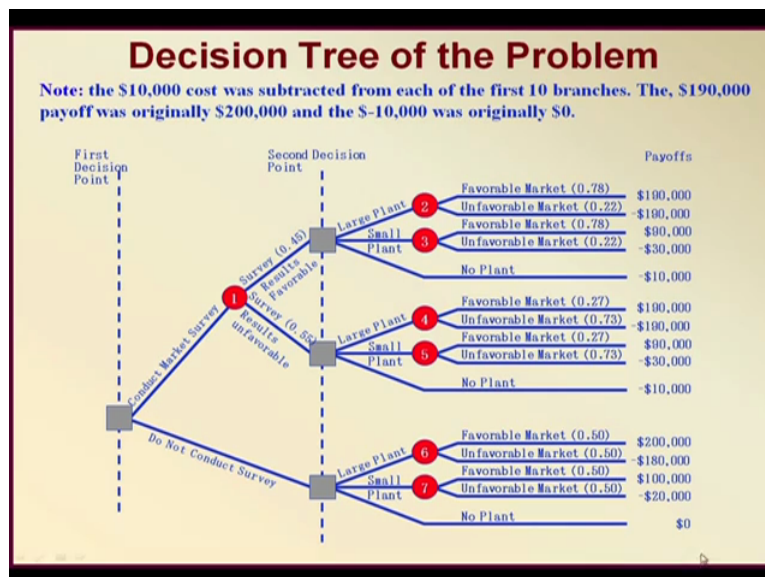
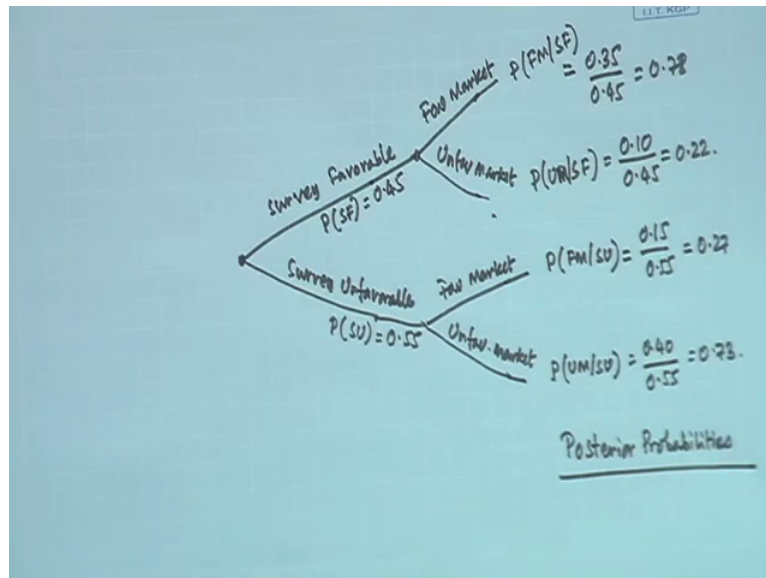
Now let us look how we can do that, you know these are our calculations, so after these calculations, look at how the decision tree of the problem can actually look at. So look how this particular decision problem actually is looking after. So what we have done, we have now reconstructed the whole thing. That how we have to reconsider the whole thing, let us look here. Now at the very 1st decision point there are 2 decisions for the decision maker, that is the company. What is that, either it can conduct a market survey or it may not conduct a market survey.

If it does not conduct the market survey, then this part of the problem is nothing but the original decision problem that we have taken up in the class. Right, we have already can have this problem which is the decision tree problem without experimentation. That means we have not conducted survey. And what are these situations, that we have 3 options, large plant, small plant and no plant. If we go for a large plant there are 2 states of nature, favourable market or unfavourable market with probabilities of 50-50 each. And similarly for small plants and these are the payoffs.

Now on the other hand suppose we actually conduct the market survey, then when we conduct the market survey then we found that unconditional probabilities are the survey results favourable, that is 0.45 and survey results unfavourable 0.55, these are the

unconditional probabilities. And what are the your posterior probabilities, these are the posterior probabilities, right. Supposing if the survey results are favourable, then favourable market was 78 percent and unfavourable market was 22 percent. But if the survey results are unfavourable, then favourable market was 27 percent and unfavourable market was 73 percent. So those are the things.

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And that could be constructed for the different, the decision points. Now look at these you know thing once again. What we have done, what we have done is that this is where we gave the unconditional probably these and for each case we are getting 2 situations where we put the posterior probabilities the favourable market, unfavourable market, favourable market, unfavourable market and these are our posterior probabilities. Now look at the slide once

again, the same 0.78 and 0.22, and same 0.27 and 0.73, they are actually put here. Now what is then the experiment has actually done?

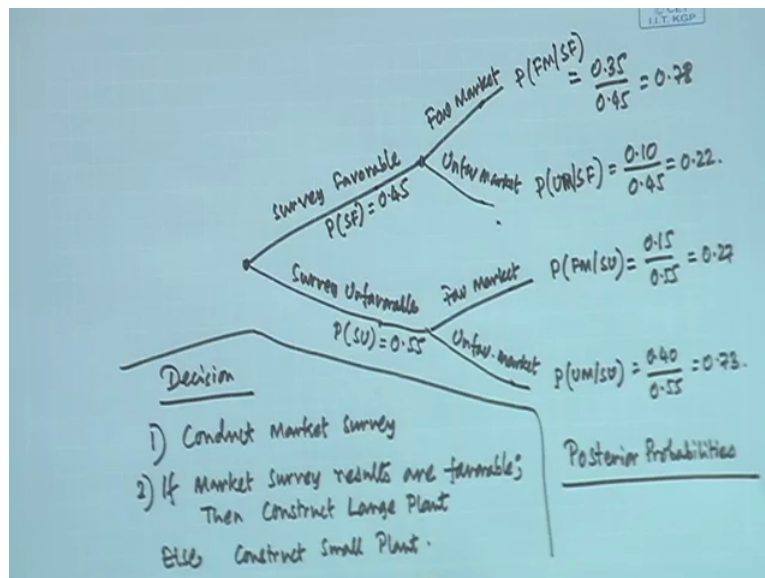
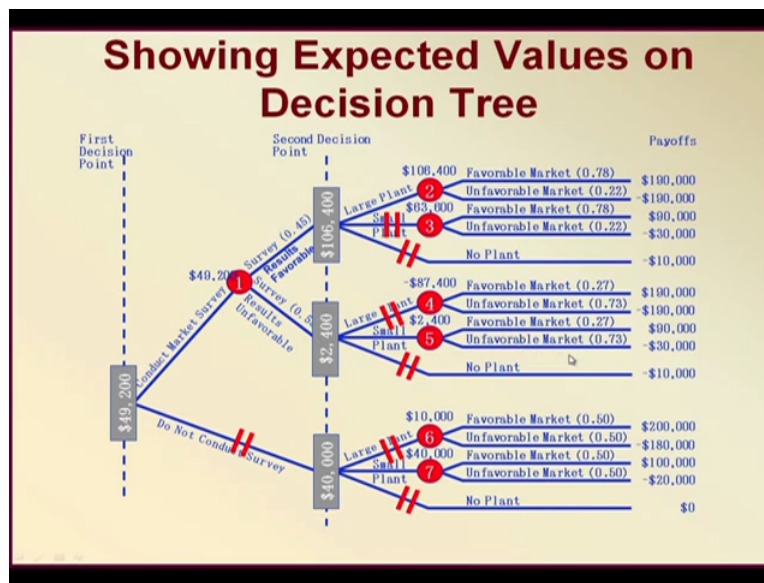
Look here, when you do not conduct the survey, we do not know exactly the ground situation and therefore there is a 50-50 chance that there would be favourable market, unfavourable market. Why, because we do not know, our knowledge is limited but when you have conducted market survey, if the result comes out to be favourable, then we believe that favourable market may come in 78 percent because that is what survey has said. The survey has said that the survey results are favourable.

But if the survey results are unfavourable, then favourable market falls to 27 percent, right. So this is exactly what you know we have to do really when we do experiment, we have to bifurcate the decision point, we have to obtain the unconditional probabilities as the states of nature when we conduct market survey and at the 2nd decision point when those original decisions are taken, again and the corresponding states of nature which are favourable market unfavourable market, we have to write what is known as the posterior probabilities. Right. So when we do that, putting posterior probabilities here and putting the unconditional probably these here which are the results of the market survey, we have been able to construct the decision tree. Right.

If you look at this decision tree carefully, then this particular decision tree has got 7 different points where we have to calculate what is known as the payoffs. Right. 1, 2, 3, 4, 5, 6, 7, right, those payoffs are to be calculated. And look at the payoffs they have written there also, if we do not conduct survey, then the payoffs are coming out to be 200,000, -1 80,000, 100,000 and -20,000. And the survey itself you know has a certain cost and that cost is 10,000, so that 10,000 is now deducted from each of the payoffs. So therefore this becomes 190,000, - 190,000, 90,000, 30,000 and no plant -10,000, why, because survey has taken up 10,000, right.

And similar payoffs figures here. Sometimes people do not believe in the play-offs and they calculate what is known as another term called Utility. Right, sometimes the payoff may be high but Utility maybe a little lower than that. Sometimes the payoffs may be low but Utility may be still some value. So basically utility is a function of payoff, so instead of really using payoff, some people actually use utility. Anyhow we are not going to use utility here and we are going to use the payoffs as they are.

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So you know these are the some calculations, we will show the calculations later, 1st let us see how the decision trees are actually obtained. Right, so you can see here that these calculations, if favourable market is 78 percent, unfavourable market is 22 percent and the payoffs are 1 90,000 and -1 90,000, then the actual payoffs that will be obtained is 0.78 into 1 90,000 + 0.22 into - 190,000 and it comes to 106,400, right. On the other hand if we go for a small plant, then because of 90,000 in - 30,000 payoff, it comes to 63,600 as the payoff, for no plant it will be -10,000.

So now again these are the different payoffs, 106,400, 63,600 and -10,000. When you combine them all, we get 106,400, that is the payoff that we get and it comes from going for a large plant, right. Go for a large plant and we will get what is known as 106,400 as payoff.

On the other side if we go for the survey results are unfavourable, there the payoff calculations are showing - 87,400 and 2400, right. And with those calculations, the 2400 and this is -10,000, this is the only positive term, rest are negative, so we actually go for the 2400. And these calculations were already done, that is 40,000 as the net calculation that was with our going for the small plant that we have already seen.

So do not conduct survey you know, so if you look at the ultimate decision because there was 45 percent and 55 percent weight, so if we put those weights, then the net payoff comes as 106,400 into 0.45 and 2400 into 0.55, that is 49,200, that is our payoff, 49,200, that is payoff for if we conduct market survey and if we do not conduct market survey, it comes to 40,000. So since 49,200 is higher, we go for 49,200, so at the 1st decision point, finally the evaluation is the net payoff is 49,200 and it is better to conduct market survey because payoff is higher, which is higher than 40,000.

So go for market survey. So after going for market survey, 2 states of nature, survey favourable, survey unfavourable. If survey is favourable, then 106,400 is the 2nd decision point and it is better to go for large plant. But if the survey results are unfavourable, then go for small plant. Right. So this is the situation that actually can happen and if you do not conduct survey, the decision was small plant. Right. So this is the decision, let us write down the final decision here, the decision number 1, conduct the market survey.

If, number 2, if market survey results are favourable, then construct large plant, else, that is market survey results are not favourable, construct small plant, right. So this is the final decision, that is we construct, conduct the market survey because that gives better payoff and then if market survey results are favourable, then construct large plant and else construct small plant. So you see the advantage here is that based on market survey results we can actually change our decision points. So that is how to construct really, we will take up another example later on, so that the understanding becomes clearer.

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Company's Dilemma

Company is not sure how much value to place on market survey. They want to determine the monetary worth of the survey. Company is also interested in how sensitive the decision is to changes in the market survey results.

- Expected Value of Sample Information (here, market survey)
- Sensitivity Analysis

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Now before moving on, there are 2 important decisions that are also to be decided by the company, that 1st of all the companies not sure how much value to place on market survey. Because in this case we have given 10,000 but one should know also that okay we are going to have market survey, how much value should be put. So they want to determine the monetary worth of the survey, company is also interested in how sensitive the decision is to changes in the market survey results. So sometimes that sensitivity analysis is also required. So 2 things are important, what is expected value of sample information, here market survey and 2nd is the sensitivity analysis.

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Expected Value of Sample Information (EVSI)

- Company want to know the actual value of doing the survey
- Defined as increase in expected value resulting from the sample information

$$\text{EVSI} = \left(\begin{array}{c} \text{Expected value} \\ \text{with sample} \\ \text{information, along with} \\ \text{the cost to gather it} \end{array} \right) - \left(\begin{array}{c} \text{Expected value} \\ \text{of best decision} \\ \text{without sample} \\ \text{information} \end{array} \right)$$

= (EV with sample information + cost) – (EV without sample information)

EVSI = (\$49,200 + \$10,000) – \$40,000 = \$19,200

The company could have paid up to \$19,200 for a market survey but the survey actually costs \$10,000.

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So the EVSI really tells how much money to place on the market survey and sensitivity analysis tells that you know what should be the changes in the decision based on the market survey results. So let us look at the 1st one, that is expected value of the Sample information, it is called EVSI. So EVSI is defined as increased and expected value resulting from the Sample information, right, resulting from the Sample information. In this case that expected value of Sample information is the expected value with Sample information - EV without Sample information, right.

So in this case 49,200 is the actually we got but then 10,000 be paid also, so if we add that, than 59,200 would have been EVSI. So one another way of looking at it is, you know what would have been the expected value of Sample information, you know without considering the cost, that is also, so that would have been 59. If suppose there was a 0 cost for that, then what would have been our expected value. So it would come to 59,200 and this 59,200 and 40,000 is if Sample information is not available.

So net difference is 19,200, so that means the company could have paid up to 19,200 for a market survey but the survey actually costs 10,000. Let us look at the diagram once again. So we have got 49,200, 10,000 be paid for the survey, that means another 10,000 we could have got. So this 49,200 would have been 59,200 if the market survey would have been obtained at 0 cost. But if we do not conduct survey, then our payoff is only 40,000, right. So that means this 59,200 - 40,000, 19,200, that is our EVSI, expected value of Sample information, right.

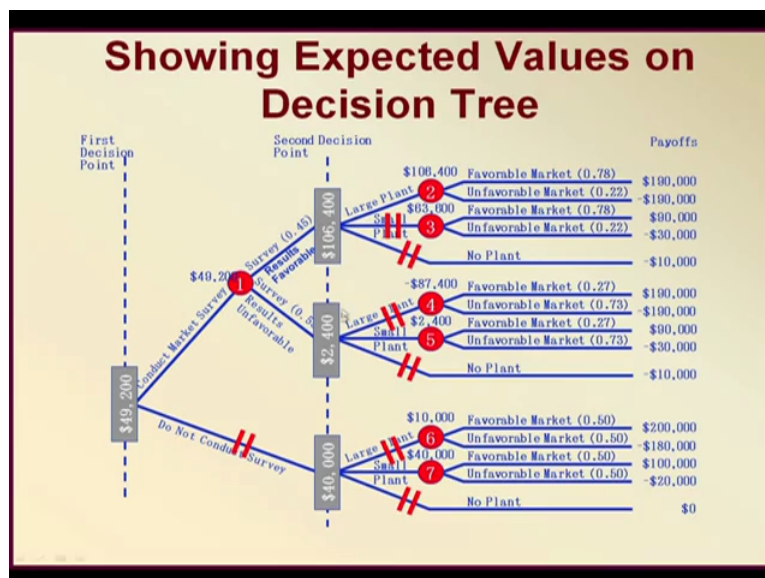
So expected value of Sample information once again is the expected value with Sample information + the cost that is given and that is the expected value that we have obtained - the expected value without Sample information. So that means the company could have paid up to 19,200 for a market survey but the survey actually costs 10,000. So therefore it is worth going for the survey, right, that means survey is meaningful.

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Sensitivity Analysis

How sensitive are the decisions to changes in the probabilities?

- How sensitive is our decision to the probability of a favorable survey result?
- That is, if the probability of a favorable result ($p = 0.45$) were to change, would we make the same decision?
- How much could it change before we would make a different decision?



The 2nd important thing is what is known as the sensitivity analysis. So the question here is how sensitive are the decisions to change in the probabilities. How sensitive is our decision to the probability of a favourable survey result? That is if the probability of a favourable result P equal to 0.45 were to change, would we make the same decision? How much could it change before we would make a different decision? So what was that 45, let us look at this diagram once again.

Here we see that survey favourable 45 percent and survey unfavourable 55 percent, here sensitivity means that suppose this survey favourable result is not 0.45 but 0.40, right and this survey unfavourable result is not 0.55 but 0.60. So when this 0.40 and 0.60, they changes, let us say that posterior probabilities still remain constant because of the way calculations are

done. Suppose, this is only an assumption, suppose we make this assumption, as long as this probability remains same, this payoff values would remain same also.

Right, so what happens, but if it is less favourable towards this and more favourable towards the other side, then this 49,200 would have reduced, right. Now obviously how much margin we have, the margin is 40,000 only. The moment it goes below 40,000, then the situation becomes, the sensitivity result shows that you know conducting market survey is meaningless. So therefore, there is a certain probability of here the total probability or unconditional probability of the survey result becoming favourable, beyond which it is not worth really going for the market survey.

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Sensitivity Analysis


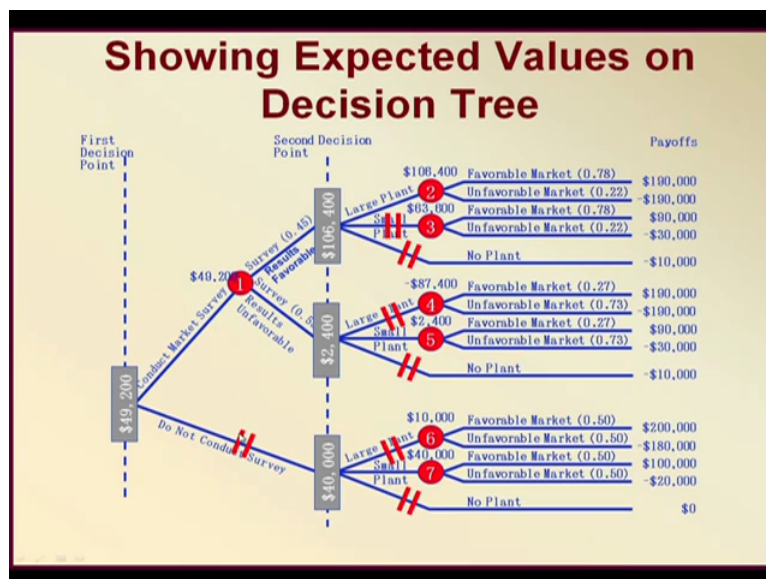
p = probability of a favorable survey result
 $(1 - p)$ = probability of a unfavorable survey result

$$\begin{aligned} \text{EMV}(\text{node 1}) &= (\$106,400)p + (\$2,400)(1 - p) \\ &= \$104,000p + \$2,400 \end{aligned}$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, \$40,000

$$\begin{aligned} \$104,000p + \$2,400 &= \$40,000 \\ \$104,000p &= \$37,600 \\ p &= \$37,600 / \$104,000 = 0.36 \end{aligned}$$

$p > 0.36$, the decision will stay the same
 $p < 0.36$, do not conduct survey

So let us look at this once again, about the sensitivity analysis that if we do a calculation, then we find that if the probability of a favourable survey result is P and $1 - P$ is the probability of a unfavourable survey result. So the expected monetary value would be 106,400 because that was with the survey favourable and 2400 with the survey unfavourable, obviously these are obtained with there is no change in the posterior probabilities. If the posterior probability values themselves change, then obviously these values, that means the payoff values will change also.

So there is an inherent assumption that those values do not change. So if we multiply, then we get 106,400 into $P + 2400$ into $1 - P$, so total expected monetary value is $104,000 + 2400$. So we are indifferent when the expected monetary value of node one is the same as the expected monetary value of not conducting the survey, that means 40,000. So if we equate the 2, if we equate the 2, then we get $104,000 P + 2400$ equal to 40,000, in In other words $104,000 P$ equal to 37,600. So what would be the value of P , the value of P comes out to be 37,600 by 104,000 equal to 0.36, or in other words 36 percent.

So when we have a value of 36 percent, p greater than 36, 0.36, the decision will stay the same. But if the probability becomes below 0.36, then the result comes out to be do not conduct survey. So what is the essential meaning of the sensitivity analysis, let us go back to this diagram once again, that is we have a value of 36 percent, that means if the survey favourable becomes 0.36 and survey unfavourable becomes 0.64, then this payoff would become 40,000. And this 40,000 would be equal to this 40,000, then that is basically we are indifferent to conducting market survey or not conducting market survey.

In other words we may not conduct market survey, right. So that means that if the survey favourable probability falls below 36 percent, then there is not worth conducting market survey. Right. So this is exactly what we have learnt, that 1st of all whenever there is a decision point and we have to do an experiment, we put that experiment, conduct the experiment or do not conduct the experiment as a decision point and when we conduct the market survey or in this case the experiment, then the results that can come, they will be the states of nature and the kind of probabilities, the posterior probabilities that we have got, that would be should be used for making our computations and calculate the payoffs and make the decision.

And finally calculate the expected value of Sample information and also do a sensitivity analysis if possible, right. So with that I stop here, in our next class we shall again take another example to really understand in more detail, thank you very much.