## Management of Commercial Banking Professor Jitendra Mahakud Department of Humanities and Social Sciences Indian Institute of Technology, Kharagpur Lecture 15 Valuation of Fixed Assets-3

In the previous class we discussed about the different concepts, different relationship between the different inputs which are required for the valuation of the bond and as well as a we try to derive certain implications from that particular relationship. Today we will be discussing about a concept that how the total return from the particular bond can be calculated. If the bond will not be hold upto the maturity.

(Refer Slide Time: 00:47)



So, if you, if you see what exactly it means for example if somebody is holding your bond and the he has not hold the bond upto the maturity he wants to redeem that bond and sell that particular bond between that then how much return he can generate from that particular investment that is basically our focus of discussion.

Other concept what will be discussed in today that is a concept called duration what will be going to use this concept in the future sessions as a strategy in the different context to minimize the different type of risk in the banking system. Then how the duration is used to measure the price volatility of the bond that is another thing. And in general if somebody wants to know the practical sense that in the research sense that what are those factors which affect the price of the bond return. We can also find out certain kind of probable factors which determine the value of

the bond return in the particular market. So, these are the issues or these are the concepts what basically we are going to discuss in this particular session.

(Refer Slide Time: 01:59)



Already I told you the total returns means what. For example somebody is holding a bond and bonds maturity period is 10 years. But after 8 years he has sold the bond but if after 8 years he has sold the bond then how much return he has realized, how much return he has realized from that particular investment.

But whenever we are talking about the total return, the total return means what? There is a already we know that bond gives you coupon, right. The coupon is the cash flow on that basis the price of the bond is based upon. Number 2, what we get? We get a face value. But another thing whenever every period wise we get the coupon there is a possibility that the coupon also can be reinvested in the market. After 1 year you received that coupon, the coupon if the bonds maturity period is 10 years then, that after 1 year whatever coupon money you got that money can be reinvested in the market for another 9 years.

So, like that whenever you get the coupon after 2 years some amount of money then that money can be invested for another 8 years. Like that when you are receiving that money than the rest of the period the particular money can be reinvested in the market.

So, if that will be reinvested in the market then, obviously we can get some interest on interest from this. For example if 10 percent is the coupon, 1000 is the, par value then after 1 year you

got 100 rupees. After 1 year whenever you got the 100 rupees then obviously that can be invested for how many years? For another 9 years. Then from that particular 9 years you get some interest out of this particular coupon interest. Coupon itself is an interest then out of that coupon interest you get another type of another income, interest income from this that is called the interest on interest.

So, whenever we calculate the total return from the particular bond or the realized yield from that particular bond we basically assume that the cash flows has to be, have to be reinvested to the investors horizon at an assumed reinvestment rate and the horizon of the bond is sold at an assumed rate given the horizon is not at the maturity.

Because if you are holding upto the maturity then there is no risk involved the return what you are expecting from the beginning that you are going to get. But if you are going to sell the bond or you are holding the bond upto the before the maturity period then only the concept of total return is more relevant for the discussion.

So, therefore assuming this things the concept of total return can be determined by estimating the horizon value, the total monetary return and bond price at the horizon. And given the current price or the value of the horizon value if you can solve for that particular rate then we can find out the total return from this. So you have ending value, you have a beginning value. So, how much money you got whenever you have sold the bond and how much money you have paid whenever you have started investing in that bond.

So, if these 2 data we are getting then simply whatever way we calculate the return from any type of investment upto the horizon period this particular rate of return from that particular investment or particular bond can be calculated. So, that is basically called the realized yield.

(Refer Slide Time: 06:01)



How basically it is done? Let us see one example. Suppose, one investor buys a 4 year 9 percent coupon bond paying the coupon annually, the coupons are paid annually. And the maturity period of the bond is 4 years and coupon rate is 9 percent. The coupon rate is 9 percent, maturity period is 4 years and there is a annual coupon.

Market interest rate 10 percent means the discount rate is 10 percent. The par value of the bond is 1000. Assume the investor needs cash at the end of the year 3. That means its horizon period is 3 years. Here we considered, we name it the horizon period. The horizon period is 3 years even if the bonds maturity is 4 years the investor wants to sell that bond after 3 years.

If after 3 years the investor wants to sell that particular bond then what happens that, we have to calculate that how much return we are going to get or we are going to realize from this. So in this context what basically what we are trying to do that we have to find that exactly how much money you will get considering the coupons are reinvested in the market.

If the coupons are reinvested in the market and as well as he is getting the coupon periodically and also we are getting the value of the bond by selling the bond on that particular day. We are combining this 3 then finally the total value can be calculated, if the total value can be calculated then the beginning value already how much money you have invested in that particular bond then the total return can be calculated from this. So, here we are assuming there is a flat yield curve. So what do you mean by a flat yield curve? The yield curve is basically nothing but we shows the relationship between the term to maturity and as well as the yield. So, here the assumption is flat yield curve means this discount rate is not changing the market interest rate is not changing in this particular system.

If the market rate is not changing that means across the period the interest rate is fixed. So, then how basically we are going to calculate this total data. Then how we can calculate? For that there are certain steps we have to follow.

The first step is we have to find the HD value. What is the horizon value? And the horizon value is nothing but what the investor basically gets or obtains from selling that particular bond at the horizon rate and the value of the coupon at the horizon rate. So, first is we have to calculate the horizon value and the beginning price of the bond and as well as the total value of the bond at the time of horizon. Then we can calculate the return. Let us see, how it is basically worked out.

(Refer Slide Time: 09:15)





First of all we have to find out the purchase price. What is the purchase price of the bond? The purchase price of the bond is already we know we have consider the coupon is 9 percent, the discount rate is 10 percent, maturity period is 4 years, maturity is 4 years and face value is equal to 1000.

So, if we want to calculate the price of the bond at they, whenever the investor really started investing in that bond, how much money he has paid? How we can get that, that means you have to find out the market price of the bond then how you can get the market price of the bond? Then obviously the same way then your P 0, B if you denote it this is a nothing but your 90 divided by 1.1, plus 90 divided by 1.1 square, plus 90 divided by 1.1 to the power 3, plus 90 divided by 1. It is not 90 it is 1090 divide by 1.1 to the power 4. After 4 years so, if you calculate the present value of this particular cash flow then that will be the price whatever the investor has paid whenever he started investing in that particular bond.

So that will be coming here 968.30 this one so if you solve this you will be getting 968.30 because after 1 year he got 90 rupees then after 2 years he got another 90, after third year he got another 90 and after fourth year he got 1090, the 1000 is the principle or the face value and 90 rupees is the coupon.

Then you got the total value means whenever he started investing he got 968.3. He has sold the bond after 3 year. So after 3 years, whenever he has sold the bond already how much year is

remaining for that particular bond? The remaining period is only 1 year if the remaining period is only 1 year then only 1 coupon is left out.

Then your the bonds maturity is now become 1 year then coupon is basically only 90 rupees and face value of the bond is 1000. That is not going to be changed. So whenever he has sold the bond after 3 years he will be getting this 90 rupees plus 1000 divided by 1.1 to the power 1 because only 1 year left out.

So, then that price become 990.9091. So whenever he has sold the bond he will be getting 990.9091 and that is one thing what it is said number 1 component. What is the number 2 component he has received the coupon he has received the coupon of 90 rupees after first year. And he has reinvested that coupon in the market if he has reinvested the coupon in the market then how money he got? 90 into and we are assuming that market interest rate has not changed the 90 into 1.1 to the power 2 he has got 108.9 rupees.

Then after 2 years he got another 90 rupees again he has reinvested it for another 1 year because after 2 years he has received that, after 3 years he has sold that because of that only 1 year is left out then the 90 rupees has been again reinvested in the market then that has become 99 rupees. And after third year he got 90 rupees. And that 90 rupees is not reinvested, that 90 rupees is not reinvested the reason is, he has no time to reinvest that at that time he has sold the bond.

Immediately he receive the coupon after that he has sold the bond. Then now how much money he got? He got 108, he got 90, he got 99 and as well as 90. So if you add out this three he will be getting basically 108 plus 99 plus 90 and as well as he has got 990.9091 after the selling the bond. Then if you add out this three you will be getting 1088.81. The total value of the bond after 3 years has become 1088.81. So, that basically consist the bond value at 990.91 here, coupon 270 rupees and 27.8 rupees is the interest on the coupon. Interest on interest. So, now how we can get the total data?

Actually he has paid 968.30 whenever he has invested in that particular bond. Whenever he has bought the bond. And now the bond value has become 10 sorry 1288.81. So now if you want to go for calculating the return from this then this is way return can be calculated. The HD value, the horizon value divided by the purchase price of the bond which was there in the beginning.

The purchase price of bond was 968.3 and your ending value was 1288.81 and he has hold it for 3 years only then to the power 1 by n that is 1 by or 1 by HD whatever we can say. 1 by 3 minus 1 that will give you 9.89 percent. So, effectively the total return of that particular bond is 9.89 percent.

If the investor has sold the bond after 3 years even if the maturity period is 4 years. So that concept is called the total return or the realized return from that particular investment. So this what basically always we consider whenever anybody wants to sell the bond before the maturity.

(Refer Slide Time: 15:50)

Duration · A bond's duration (D) can be defined as the weighted average of the bond's time periods, with the weights being each time period's relative present value of its cash flow It is the weighted average on a present value basis of the time to full recovery of the principal and interest payments on a bond. It measures the weighted average maturity of a bond's cash flows on a present value basis. \*\*\*\*\*\*\*

Then we have another concept called the duration. Then what exactly the duration is, the duration is nothing but it is basically a measure it is a weighted measure for the time. And the concept of duration is mostly used as a strategy to minimize the interest rate risk in the market that we will discuss later whenever we talk about the investments of management of the investments of the commercial banks.

But now the concept of duration is basically is explained in this way, the duration is nothing but it is a weighted average of bonds time period and the weights are basically being each time periods relative present value of the cash flows. So, this is the way the duration is calculated. So, it is the weighted average on the present value basis on the time to full recovery of the principal and interest payments on a bond and it measures the weighted average maturity of the bonds cash flow on a present value basis. So, that means you have to find out the present value of the bond and as well as the present value of the cash flows then that proportion once basically will calculate that will be multiplied with respect to t and take the summation of that, then that will give you the duration. So, this the way basically the duration can be calculated if you see this example it will be more clear.

(Refer Slide Time: 17:17)



So, before that what basically the duration what it measures? It basically measures the effective maturity that incorporates the timing and size of the security's cash flow. It captures the combined impact of the market rate, size of the interim payments and maturity of security's price volatility.

Conceptually if you, if you observe duration is nothing but a measure of interest elasticity in determining the security's market value. If the interest rate is going to change by a 1 percent how the market value of that particular security or particular bond is going to be changed that is basically measured through the duration.

If a security's duration is known then, an investor can readily estimate the size of a change in the value or the price of the bond for different interest rate changes. So, duration is basically a concept which is mostly used for the investment strategy in the market.

(Refer Slide Time: 18:24)



So see in this example how the duration can be calculated. So, this a bond whose maturity period is 4 years, the maturity period is 4 years, coupon is 9 percent, par value is 1000 then 10 percent is the discount rate. So, now as per our definition if you see then what is the periodic cash flow we are getting for year 1 we are getting 90, for year 2 we are getting 90, for year 1 we are getting 90, year 2 90, year 3 90 and year 4 we are getting 1090 because in the end of the fourth year we will getting here principal amount 1000 and 90 rupees is the coupon.

Now we have to find out the present value of the cash flows whatever we have receiving to find out the value of the bond on that particular day. So if you discount it with respect to 10 percent then you are getting 81.81, 74.38, 67.61, 744.485 then finally the value of the bond has become 968.30.

The 968.30 is equal to the value of the bond and now what is happening the present value we are saying that we are trying to find out the proportion that what is the present value with respect to the total present value. Present value of the cash flow with respect to the total present value. Then 81.818 divided by 968.30 that will give you this one.

This one divided by this one that will give you this one, this one divided by this one that will give you this one and this one by this one that will giving you this one. Then what will happen this multiplied by 1, that will give you this then this multiplied by 2 that will give you this and

this multiplied by 3 that will give you this and this multiplied by 4 that will give you this. Then the duration become 3.52.

So, for a term to maturity of 4 years of 9 percent coupon and 10 percent discount rate the duration is 3.52 of that particular bond. So now there are certain kind of properties, certain kind of observations what we can draw from this. That we will see.

(Refer Slide Time: 20:56)



But if you have many bonds with you or you are holding many bonds then how the duration basically of that particular portfolio of the bonds whatever you are carrying that can be calculated. If the duration of a bond portfolio is nothing but the weighted average of each of the bonds duration with the weights being given to the different bonds on the basis of the investment funds allocated to each bond.

What does it mean? It means that for example your total investment is 1000 crore, total investment is 1000 crore and you have 4 bonds available or 5 bonds available. Then how much money you have invested on bond 1, how much is the 2, how much is the 3, how much is the 4 and how much is the 5.

So that proportion you have to find out. And that proportion is basically what let you have invested 200 crore here, 50 crore here, 150 crore here, 250 crore here then it will be 400, 650 then 350 crore here then you find out 200 by 1000 then 50 by 1000 then 150 by 1000 so on then

350 by 1000. Then finally what basically you can do? It is 350 by 1000 this different proportion that is basically your w.

Then for each bond you have the duration which is available then obviously the total duration of that particular portfolio is wi by summation wi into Di. That is basically the duration of the bond portfolio can be calculated and then, then the weights are basically given on the basis of the proportion of investment funds allocated to each bond. This is the way the duration of the bond portfolio is calculated in the particular market.

(Refer Slide Time: 23:13)



The duration can be considered as a price sensitivity measure because duration is defined as the weighted average of the bonds time period. It is also important measure of the volatility. Already I told you for unit change of interest rate how the price of the bond is going to be changed that is basically measured as the duration. So as a measure of the volatility duration is defined as the percentage of change in the bond price given a small change in the discount rate, that is the y.

So, mathematically if you want to derive the duration then you can take the first order derivative of the equation of the price of the bond. With respect to the yield. Then you divide that bond price and expressing the resulting equation in the absolute value. Then the duration can be calculated from this.

(Refer Slide Time: 24:11)



But one thing you remember there are 2 types of duration we have one is Macaulay's duration and another one is the modified duration. In the previous example, numerical example whatever we have explained from 4 years term to maturity we got 3.52 that is called the Macaulay's duration.

And if you basically trying to calculate from Macaulay's duration to modified duration then you have to consider that how many times basically the cash flow you are receiving. But whenever we are calculating the duration with respect to the price volatility or change in the due to the change in the interest rate or change in the yield we generally calculate the modified duration and from the modified duration we calculate the Macaulay's duration. So, the here of you see dP by P, by dy is equal to nothing but 1 by 1 plus y into summation t is equal to 1 to N, t Pv into CFt divided by P0B.

So, the bracketed expression is nothing but the weighted average of the time period whatever we have defined in the last section. But formally, the weighted average of the time period is called the Macaulay's duration that already I told you. And the equation which defines the percentage change in the bond price for a small change in the yield in the absolute value is called the modified duration.

So, your Macaulay's duration divided by 1 plus y that will give you the modified duration. Then y basically nothing but the discount rate. But one thing you remember, that whenever we are

going by that numerical calculation this is the Macaulay's duration but if you are finding it mathematically by using this derivatives then basically we can calculate the modified duration. And from the modified duration you can find out the Macaulay's duration. That is the basic difference in terms of the calculation of the duration either from the numerical way using any kind of numerical data or you can go by the mathematical way.

(Refer Slide Time: 26:14)

Duration as a Price Sensitivity Measure Cont... · Note that the price of a bond that pays coupons each period and its principal at maturity is Taking the first derivative of this equation, dividing through by P, and expressing the resulting equation in absolute value provides a measure of duration for a bond paying principal at maturity: 1  $(1 + y)^{2}$ Modified Duration P.t \*\*\*\*\*\*\*\*\*\*\*\*\*

So, now if the price of the bond you already know that P is equal to C into 1 minus 1 by 1 plus y to the power n or 1 plus y to the power N divided by y plus F by 1 plus y to the power N that already we know this is the formula for the bond valuation. So, if you take the first derivative of the equation with respect to the y, then or divide the expression by P then we can get your C by y square, 1 minus 1 by 1 plus y to the power n, plus n into F minus C by y divided by 1 plus y to the power N plus 1, divided by P.

So, that is basically your modified duration. So, if you got the already you know that your modified duration, modified duration is equal to the Macaulay's duration divided by 1 plus y. Then if you got modified duration already then you can get this Macaulay's duration Md into 1 plus y. So this is the way the relationship between Macaulay's duration and modified duration can be established.

(Refer Slide Time: 27:25)



So, the above measure of the duration is basically defined as in terms of the length of the period between the payments. So, if the cash flow is distributed annually then the duration reflects basically years. If the cash flow is semiannually then the duration reflects half years. But generally convention on is, the has to express the duration as an annual measure.

So, annualized duration if you want to find out then you divide this duration by the number of payments per year. So, annualized duration is equal to duration for the bond with n payment year divided by n. That will give you the annualized duration.

(Refer Slide Time: 28:07)



Then properties lower the coupon, greater will be the duration. Longer the term to maturity, greater the duration because lower the coupon the value will be increasing obviously the cash flow divided by price that will be less. If that will be less than automatically your duration will be less.

But the longer the maturity obviously the greater the duration. Because that we are multiplying with respect to the proportion of cash flow to the total present value. For the 0 coupons bonds the Macaulay's duration is equal to the bonds term to maturity and the modified duration already we know that the term to maturity divided by 1 plus y the discount rate.

Higher the yield to maturity lower the duration. Higher yield to maturity means again the price will be if the interest rate is basically is increasing then the price of the bond will declined then obviously what will happen that the price will be declining and then if the price will be declining then we can observe that the that proportion also will be declining. So, if the proportion will be declining then automatically your duration is also declining.

So these are the properties of the duration with respect to yield to maturity, with respect to coupon and as well as the term to maturity. So, these are about the properties and the for the 0 coupon bond the duration is equal to the term to maturity particularly the Macaulay's duration.

(Refer Slide Time: 29:41)



So, these are the different factors in the practical sense which affect the bond return like credit rating of that particular bond, the maturity of the bond, size of the company or size of the bank, liquidity, downside risk means we are basically referring towards the what is the loss, probability of the loss, what loss which is happening to that particular kind of business, quality of the credit of that particular bond issuer then the market risk. Mostly the market risk is measured through the beta. So these are the common factors which may affects the returns from the bond market. So, whenever practically we see the investments or we try to take the position in the bond investments we consider these factors for this.

(Refer Slide Time: 30:32)



And the conclusion is, the total return basically consist of both coupons, interest on coupons and horizon value. Bonds duration is the weighted average of the bonds time period and the weights are being each time periods relative present value of the cash flow.

Then the percentage change in the bond price for small change in the yield in the absolute value is called the modified duration and there is a relationship between the modified duration and the Macaulay's duration. And the market risk, liquidity, size, maturity, credit rating etcetera are the major determinants of the bond returns in the market. That is basically on the practical sense we observe in the market.

So this is about the different kind of approaches we use whenever we deal the fixed income securities in either of the organizations for there it is commercial bank or any other organization but it is also applicable for the banks investments in the fixed income securities. In the next session we will be talking about the different type of risk what the bank faces and how we can measure there is type of risk in a statistical way or the quantified way because that is the major concern for the commercial banks and what are those risk management process the commercial banks follow.

(Refer Slide Time: 31:50)



So these are the references what you can go through for this particular session. In to find out the detail analysis on this. Thank you.