Automation in Production Systems and Management

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Lecture - 12
Mathematical Models for Interaction between Design and Manufacturing

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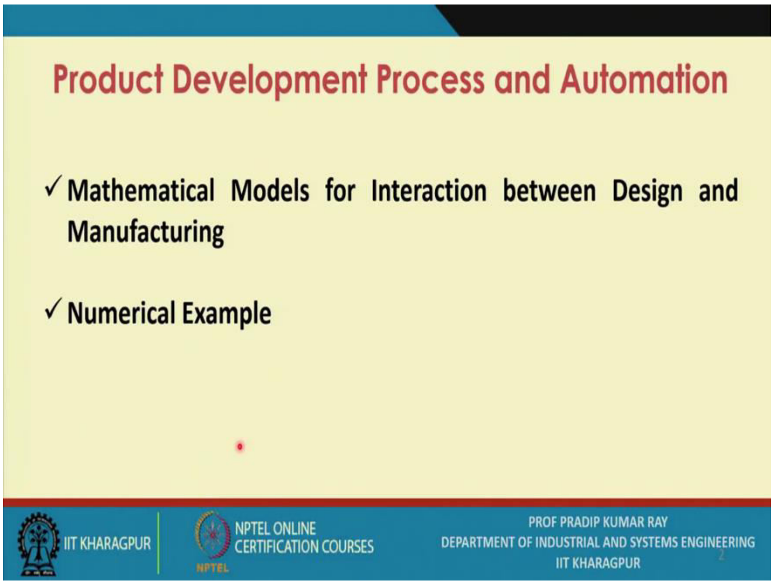

During this lecture session I am going to discuss the Mathematical Models for the Interaction between Design and Manufacturing. For product development there are two main approaches. The first one is the sequential approach or the serial engineering, and the second one is the simultaneous approach or concurrent engineering. Now, for both these approaches you must know the relationship between design and manufacturing.

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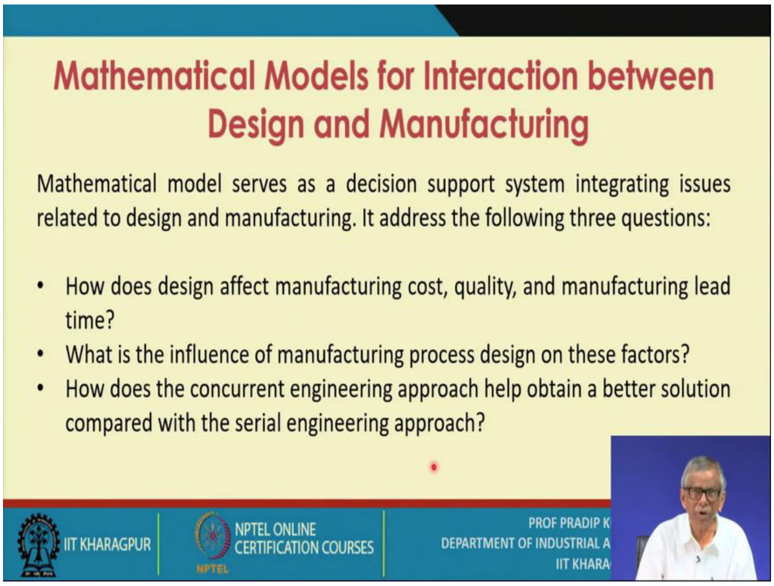

When you talk about the mathematical model, it serves as a decision support system. Many times, for automation, you need to say use decision support system or DSS.

There has to be mathematical models. Behind a decision support system mathematical model serves as a decision support system.

If you have a mathematical model, it helps integrate issues related to design and manufacturing. Product development process has three stages. First is process design, next one is the product design, followed by manufacturing. When you talk about the design, you consider both product design as well as the process design.

How these two kinds of design are related to manufacturing? At what extent the manufacturing system which you have established are influencing or affecting process design as well as the product design? It is not that you opt for a product design and that is not manufacturable or you do not have that kind of manufacturing system.

For a new product, definitely you need to add some new kinds of processes in an existing manufacturing process. But it does not mean that you opt for a particular design, and it just cannot be manufactured with the existing manufacturing system.

When you talk about this relationship between design and manufacturing and you opt for a mathematical modeling, then three questions are asked and those three questions you can definitely answer in explicit terms. How does design affect manufacturing cost, quality, and manufacturing lead time?

In order to achieve these two goals or two objectives, you run your manufacturing process in such a way that the quality is maximized, cost is minimized, and manufacturing lead time is reduced. That means, a reduced manufacturing lead time or minimum manufacturing lead time you have to ensure.

What is the influence of manufacturing process design on these factors? What are these factors? The first thing is you select a particular design, and then you select a particular process related to the design, and then for that particular design you use that process.

While you start using that process, given a particular component, given a particular part, for which the design is recommended, for which the process is being used what is your production cost, what is your quality of the product, or the component or the part which you are going to produce, and what is your manufacturing lead time or sometimes this is referred as the throughput time.

The third question is to be answered is how does the concurrent engineering approach help obtain a better solution compared with the serial engineering approach?

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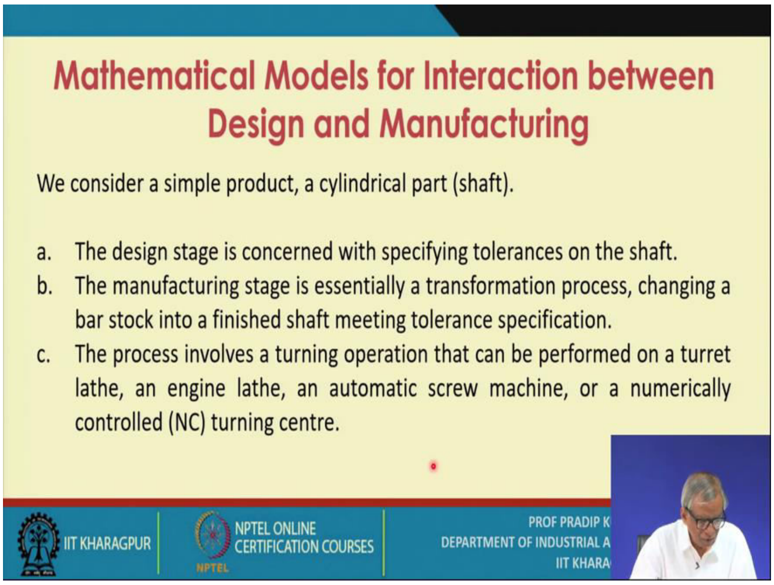

Now, we consider one example. We consider a simple product a cylindrical part like a shaft, may be solid shaft or say the hollow shaft, whatever you select. The design stage is concerned with specifying tolerances the shaft, and while you opt for a particular tolerance, a particular design approach you adopt.

This tolerance is related to a particular quality characteristic of the given or the considered part. In this case it is a shaft; maybe the outer die or the length. There are many other quality characteristics of a particular component. You should select one important quality characteristic.

The manufacturing stage is essentially a transformation process, changing a bar stock into a finished shaft meeting tolerance specification. The process involves a turning operation. The process involves a turning operation that can be performed on a turret lathe, an engine lathe, an automatic screw machine, or a numerically controlled (NC) turning centre.

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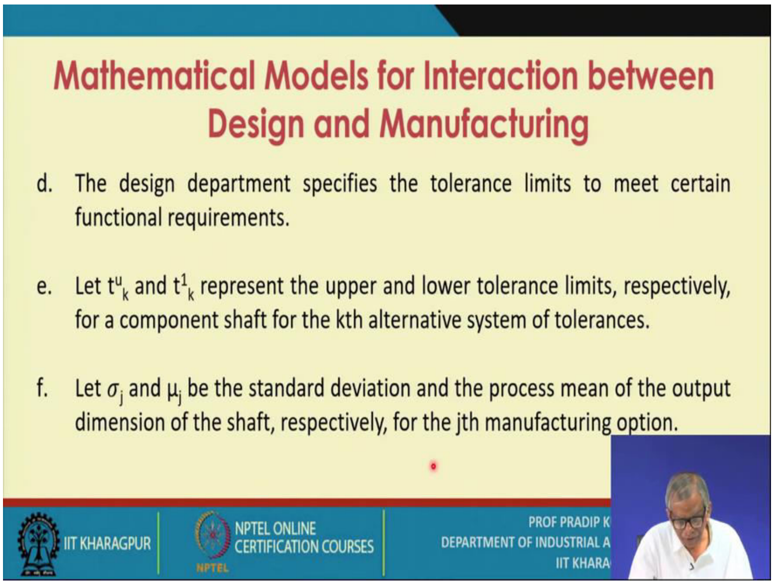

The design department specifies the tolerance limits to meet certain functional requirements. Now, we will be using certain notations. Let t^u_k and t^1_k represent the upper and lower tolerance limits, respectively, for a component shaft for the k-th alternative system of tolerances.

These were basically against a particular quality characteristic, say the diameter of the shaft, what is the upper specification limit or upper tolerance limit. These are physical dimensions. That is why the specifications are nothing but the tolerance limits.

Let σ_j and μ_j be the standard deviation and the process mean of the output dimension of the shaft, respectively, for the j-th manufacturing option. With respect of a particular process and for the given quality characteristics. Then, you can definitely estimate the value of sigma σ_j that is the standard deviation and the value of μ_j that is the mean of the process.

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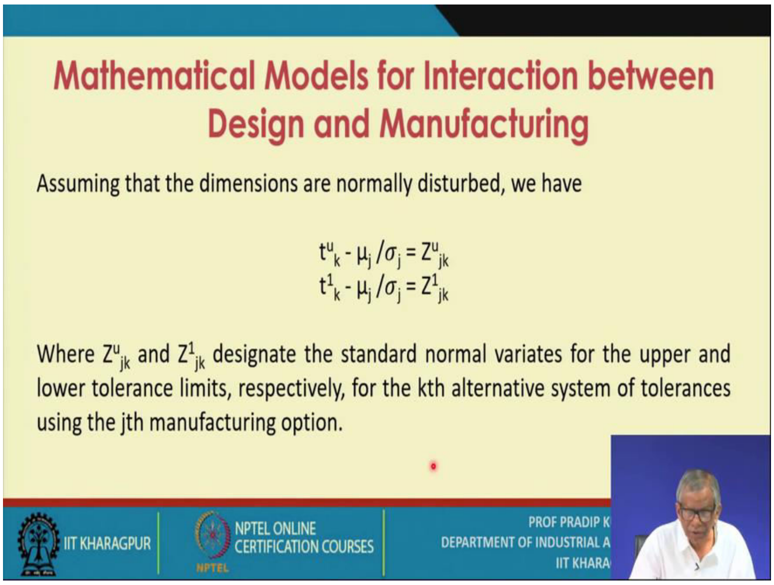

Assuming that the dimensions are normally disturbed, we have

$$t^u_k - \mu_j / \sigma_j = Z^u_{jk}$$

$$t^1_k - \mu_j / \sigma_j = Z^1_{jk}$$

Where Z^u_{jk} and Z^1_{jk} designate the standard normal variates for the upper and lower tolerance limits, respectively, for the kth alternative system of tolerances using the jth manufacturing option. This Z values are basically the standard normal variates for the upper and lower tolerance limit for the k-th alternate system of tolerances; that means, the k-th design using the j-th manufacturing alternative or the option.

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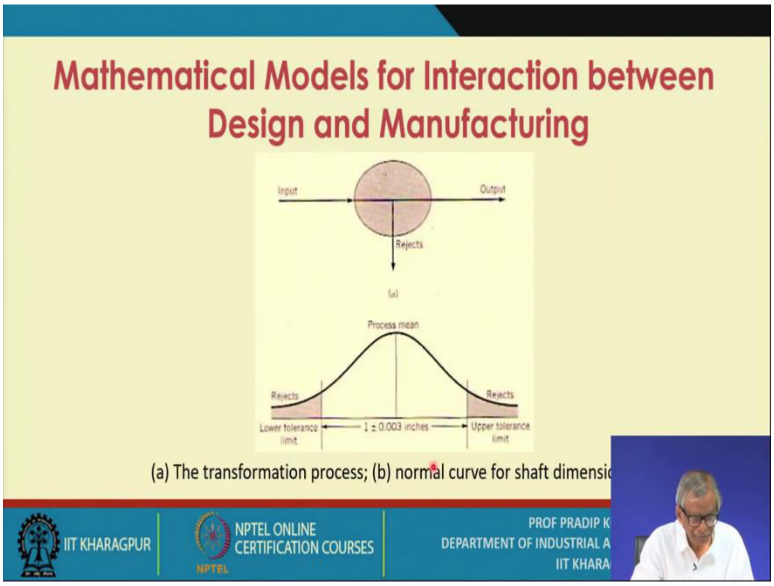

Here you have the inputs. This is the transformation process. How do you say represent the transformation process? You have the inputs, say raw materials, then you produce the output and simultaneously you also produce the rejects or the scrap.

As far as the transformation process is concerned, the input of the materials are transformed into two types of outputs, one is acceptable output and other is unacceptable output or rejects or scraps.

For the shaft dimensions you have collected data against a particular process and you find the lower tolerance limit and the upper tolerance limit. This is basically your nominal dimension and the nominal dimension is 1 and, upper tolerance limit will be 1.003, and the lower tolerance limit will be (1 - 0.003) = 0.997.

When you get this normal curve, the μ is somewhere here and if suppose the process mean is just the same as the nominal dimension, we say, yes, the process is centered.

What is centering of the process? Given a quality characteristic for which the nominal dimension is say *m*, and you are producing this quality characteristics in large number; when you get this value of μ you find that there is hardly any difference, there is no significant difference between μ and m, that is why you say that they are same and that is why you say that the process is a centered one.

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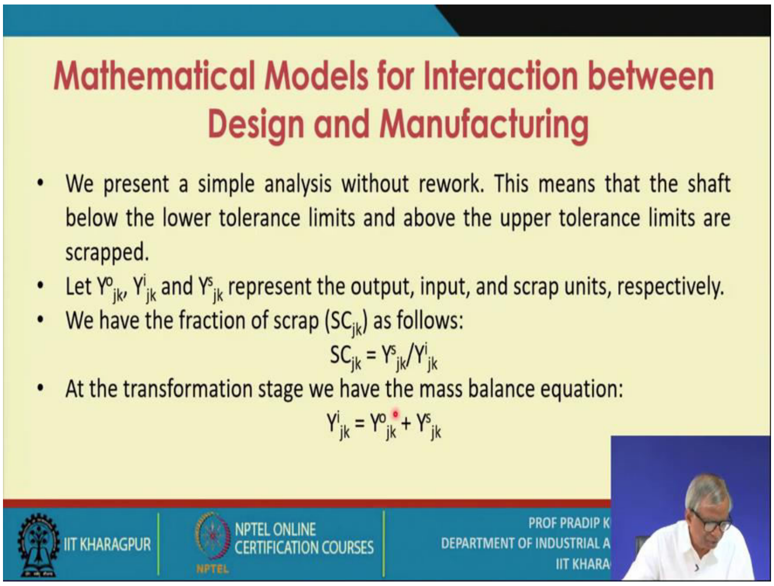

We present a simple analysis without rework. This means that the shaft below the lower tolerance limits and above the upper tolerance limits are scrapped. The shaft below the lower tolerance limits and above the upper tolerance limits are scrapped, it all depends on which portion is the scrap and which portion is rework, depending on the whether the dimension is inside dimension or outside dimension.

Suppose it is a hollow shaft, you have the outer diameter as well as the inner dia. For the outer dia, the value you get is less than lower specification limit. Obviously, it will be treated as scrap and the value which is greater than the upper specification limit may be considered as rework.

If it is inside the dimension; that means, inner dia, the rework portion will be the value which is less than lower specification limit. And the scrap will be the value for value is greater than the upper specification limit.

Let Y^o_{jk}, Y^i_{jk} and Y^s_{jk} represent the output, input, and scrap units, respectively.

We have the fraction of scrap (SC_{jk}) as follows:

$$SC_{jk} = Y^s_{jk}/Y^i_{jk}$$

At the transformation stage we have the mass balance equation:

$$Y^i_{jk} = Y^o_{jk} + Y^s_{jk}$$

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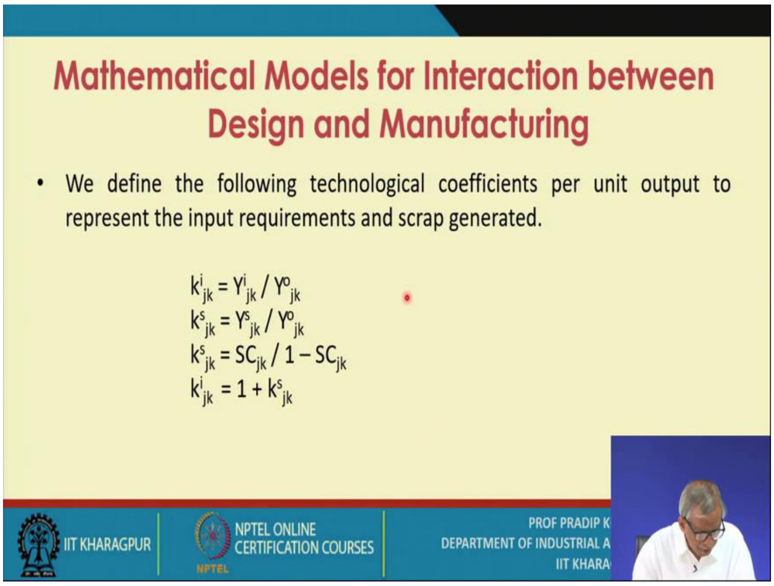

We define the technological coefficient per unit output to represent the input requirements and the scrap generated. That means, how much inputs you require to produce the same amount of output and how much will be your scrap.

It all depends on the kinds of process. And when once you say that it is a one type of process, they referring to one type of technology. The technology used for engine lathe is different from the technology you use for the CNC machining centers.

Each process essentially represents a particular technology to what extent the technology is responsible for producing the scrap. Once, one particular machine or one particular technology may produce say 200 units of scrap, but another machine just can produce just 10 units of scrap. When you consider this particular performance related measure, then you can define the technological coefficient.

The technological coefficient with respect to the inputs *k* is $k^i_{jk} = Y^i_{jk} / Y^o_{jk}$. Your output remains fixed, say 1000 units you need to produce. How much the inputs you require? It depends on the type of technology you use.

Similarly, how much the scrap you generate? It also depends on the type of technology you use or the type of process you use. Y^o_{jk} remains fixed, and to produce this output you given a particular technology, you generate this amount of scrap. This is the technological coefficient for the scrap, This is the technological coefficient for say the input.

$k^s_{jk} = Y^s_{jk} / Y^o_{jk}$

$k^s_{jk} = SC_{jk} / 1 - SC_{jk}$

$k^i_{jk} = 1 + k^s_{jk}$

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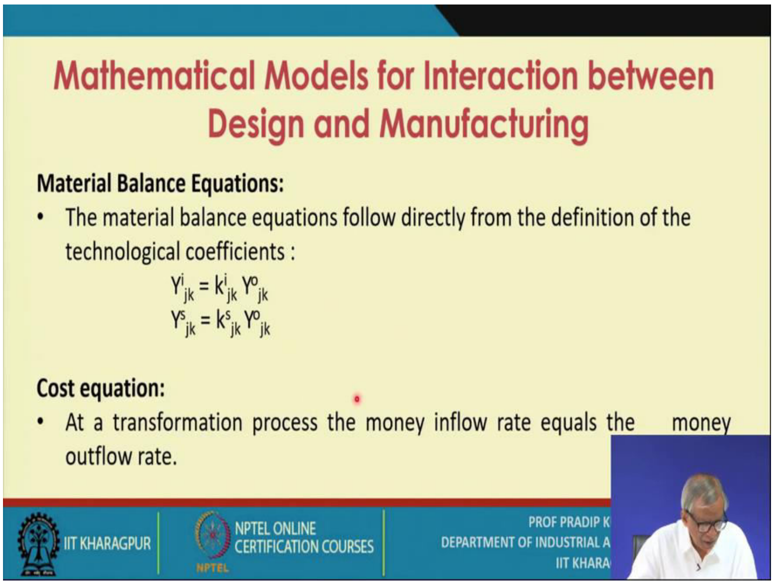

Now, we go for the material balance equation. The materials balance equations follow directly from the definition of the technological coefficients.

$Y^i_{jk} = k^i_{jk}\ Y^o_{jk}$

$Y^s_{jk} = k^s_{jk}\ Y^o_{jk}$

Next, we go for the cost equation. When we calculate that the scrap proportion, it basically a measure for quality or a pseudo measure of quality. In that sense, it is a negative approach. Instead of saying that what is your amount of yield, we are saying that what is your amount of rejects? But still, you get an idea about the quality. Essentially the quality cost equation you need to develop.

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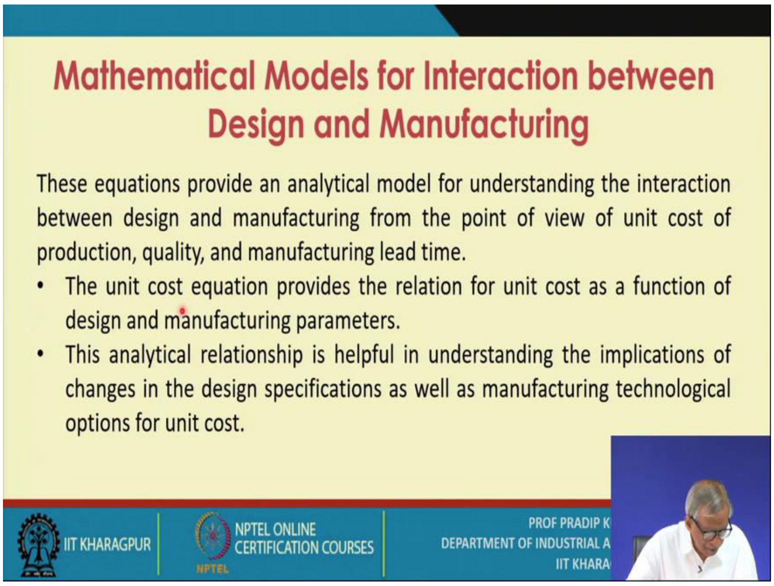

The unit cost equation provides the relations for unit cost as a function of design and manufacturing parameters. This analytical relationship is helpful in understanding the implications of changes in the design specifications as well as the manufacturing technological options for unit costs.

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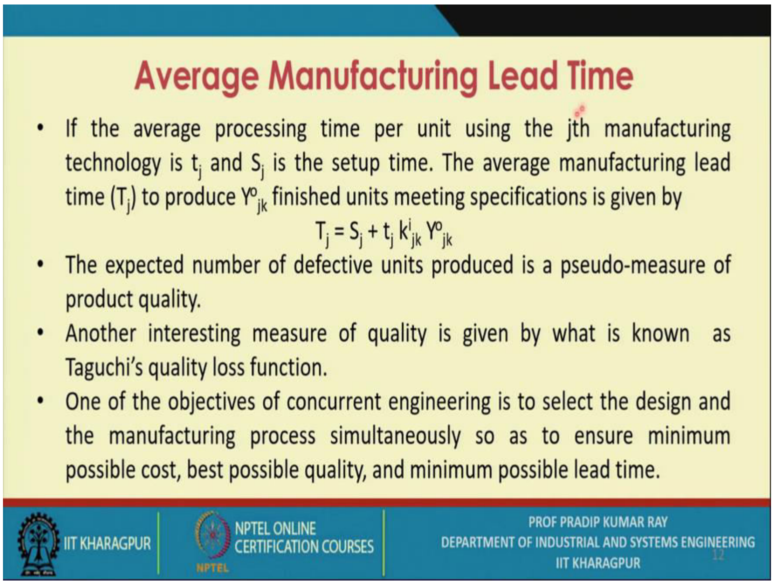

This is basically your average manufacturing lead time or sometimes this is referred to as the throughput time. This is for the j-th process and this is the setup, S_j for the j-th process, and this is basically the unit time, and this is the technological coefficient for the inputs and this is the output.

The average manufacturing lead time (T_j) to produce Y^o_{jk} finished units meeting specifications is given by $T_j = S_j + t_j\, k^i_{jk}\, Y^o_{jk}$. This way you calculate the manufacturing lead time, the cost you can estimate, you can estimate the quality and you also can estimate the manufacturing lead time for a given a particular process, output amount, and given a particular design alternative.

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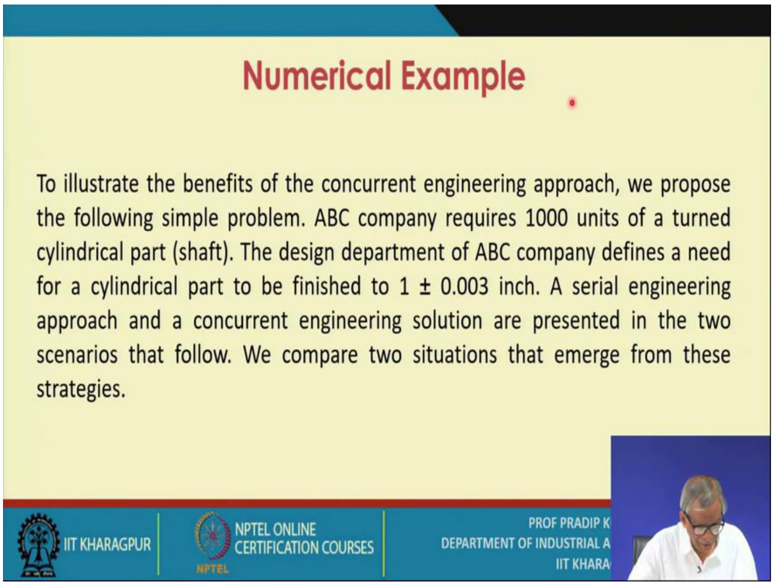

This is a numerical example. To illustrate the benefits of the concurrent engineering approach, we propose the following simple problem. ABC company requires 1000 units of a turned cylindrical part (shaft). The design department of ABC company defines a need for a cylindrical part to be finished to 1 ± 0.003 inch. A serial engineering approach and a concurrent engineering solution are presented in the two scenarios that follow. We compare two situations that emerge from these strategies.

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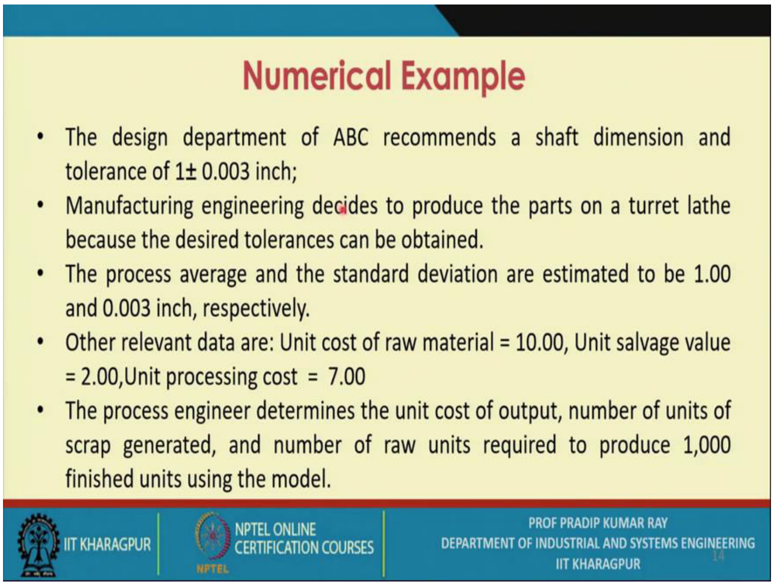

We have all these data, the design the specifications are known, then the standard deviation, the process average, and the standard deviation these are given 1 and 0.003. We are assuming that process is centered. Then, unit cost of raw material, unit salvage value, and a unit processing cost, these are all given. The process engineer determines the unit cost of output, number of units of scrap generated and the number of raw units required to produce 1000 finished units using the model.

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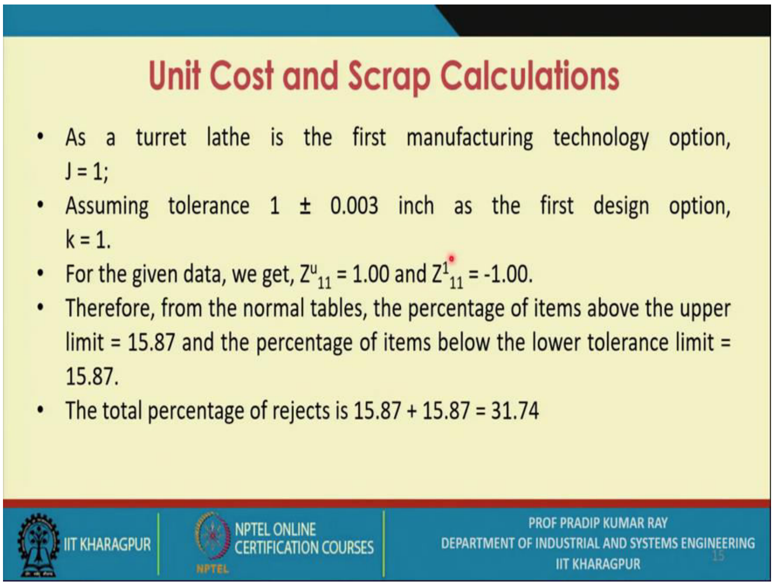

You follow the third procedure you get the value of Z, Z at lower specification limit, Z at upper specification limit, and then you refer to the standard normal distribution, And the percentage of items above upper specification limit and less and below the lower specification limit or lower tolerance limit, that is 15.87 + 15.87 = 31.74.

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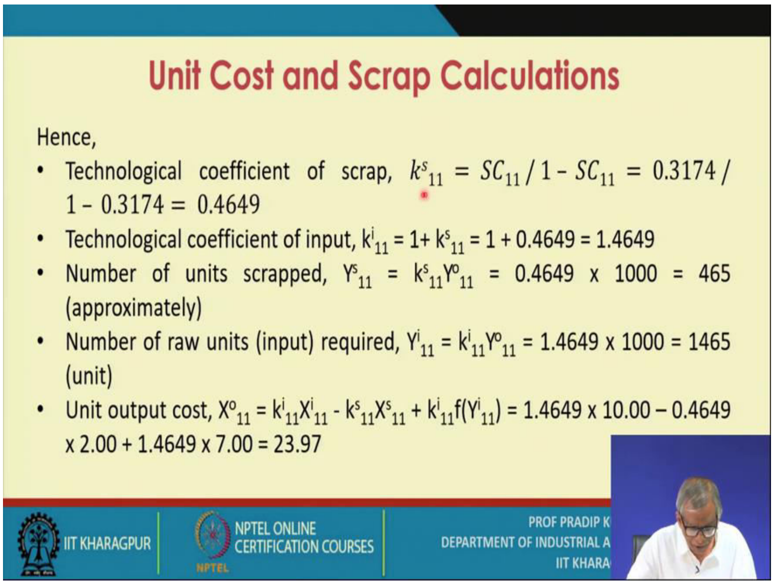

Now, you calculate the technological coefficient for the scrap. Similarly, that you calculate technological coefficient for the inputs, and then you determine the amount of scrap you generate, given a particular process and to produce say 1000 units.

And then you have the cost equation, and ultimately against this combination you get the total cost total cost as 23.97. This is the inputs one, this is the scrap, and this one is actually the processing unit. The processing X is the cost per unit. This is basically the input output cost, X^o_{11} = 1, first design alternative.

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