Investment Management
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Lecture - 10
Prices, Yields, and Duration of Bonds

Hello there. Welcome back to the course Investment Management. And from our discussion so far, we have learned that yield related to a bond is one of the most important factors that an investor should be concerned about. In today's session, we will discuss about the relationship between Price, Yields and Duration of the Bond.

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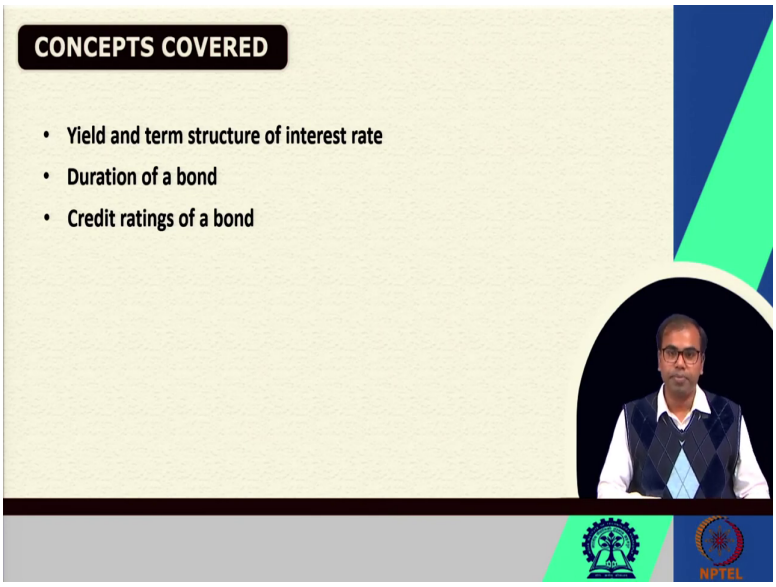

So, basically, we are going to talk about the relationship between yield and in interest rate, which we know as term structure of interest rate with respect to the duration and the interest rate.

We will also talk about duration of the bond particularly in the context of a bond with long term a maturity for which an investor has purchased the bond after the issuance and wanting to hold the bond till maturity to know about the interest rate or the interest that he or she is going to get. We will also touch upon finally, the credit ratings as an important input for decision making with respect to bond investment.

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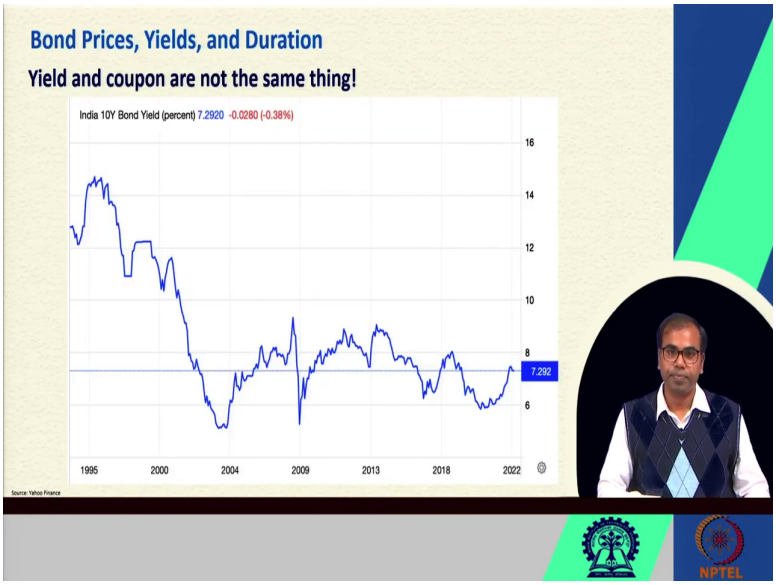

So, to start with, we know that yields are the most important things when it comes to the numerical characteristics with respect to a bond and we have also seen that yield and coupon are not the same thing. As far as we understand, we know that coupon are the rate of interest

that bond issuer has promised to pay every period and an investor when subscribes a bond is entitled to get a coupon that is in terms of rate of interest upon the face value.

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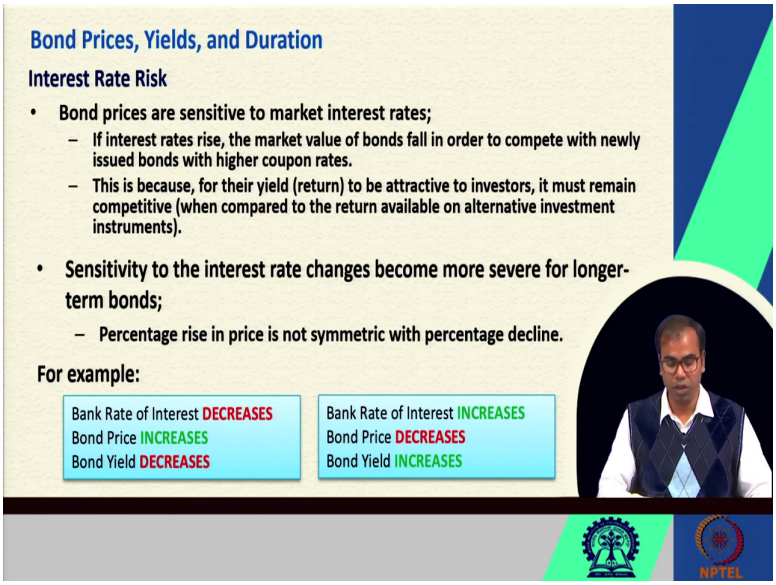

But when it comes to yield, we agree that yield is the rate of interest that an investor is supposed to get if the bond is held till maturity. So, it is all about the dynamics between different rate interest rates. We have also seen earlier that bonds prices are very much sensitive to the prevailing interest rates. If interest rate in the market rises, then market value of bonds fall because investor would not find it as attractive compared to the prevailing interest rate in the market.

So, bond prices fall in order to compete with newly issued bonds with higher coupon rate. And this is why, for every yield to be attractive with respect to any bond, it must retain its competitiveness because when it is compared with the return available on alternative

investment instrument in the market. The rate of return on the bond with smaller yield is no longer attractive as attractive and that is why the value of the bond falls in order to compete with the newly issued bonds with higher coupon rate.

We should understand that the sensitivity to the interest rate changes become more severe for bonds with longer maturity because the uncertainty is more in that case and that is why percentage rise in prices is not symmetric with percentage decline. And we will see how this translates into the duration of the bond.

So far to understand the relationship between coupon and yield or rather interest rate and yield, we know that when bank rate of interest decreases, the price of a bond increases and yield on such a bond would decrease as well. And when it becomes opposite situation where bank rate of interest increases, the price of a bond decreases and the yield on such a bond increases which means effectively.

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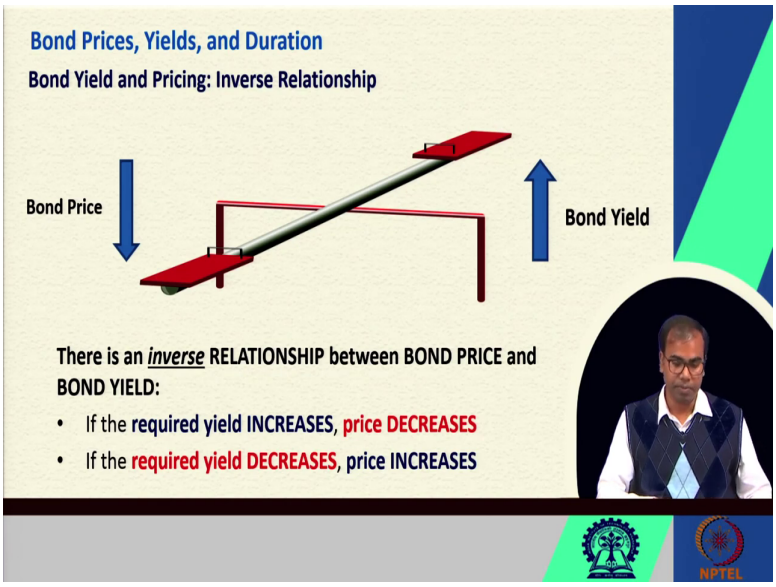

There is an inverse relationship between the yield on a bond and the price of such a bond. So far wherever we have talked about yield, we intend to indicate the yield to maturity which means the rate of return on a bond if the bond is held till maturity. So, when we say that there is an inverse relationship between bond price and bond yield, essentially it is bond price and bond yield to maturity.

So, whenever there is a bond price increases, as a result of bond decrease in bond yield and vice versa. So, if the required yield or expected yield on a bond increases, the value of such a bond decreases and if the required yield decreases, then value or price of a such a bond increases.

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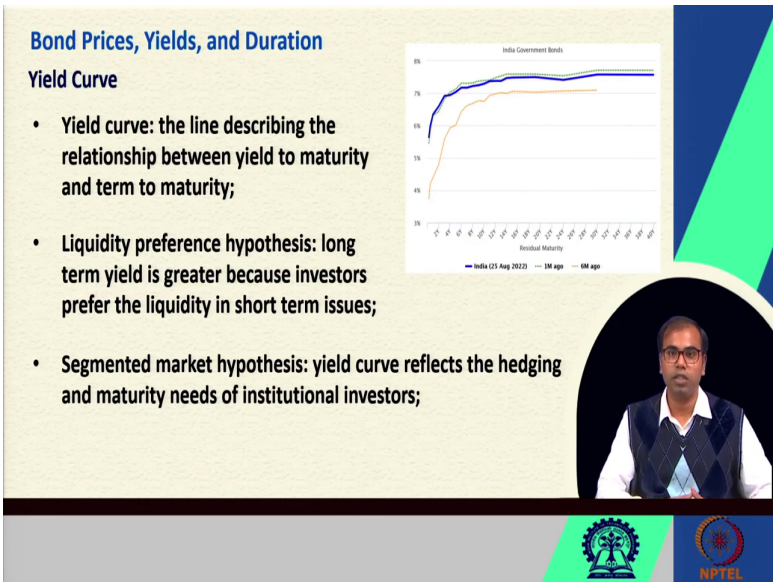

This sort of relationship between the maturity of the bond and the yield is essentially reflected in terms of the graphical representation of what we know as yield curve. So, yield curve basically is the relationship or the line indicating the relationship between yield to maturity and time to maturity.

Which means if the yield to maturity is plotted against the time to maturity with respect to a bond, the graph that we obtain is known as yield curve. And here one important aspect we need to understand that the liquidity preference hypothesis comes in the picture, which states that long term yield is greater because investors prefer the liquidity in short term issues and vice versa.

In another supplementary hypothesis here is the segmented market hypothesis which states, yield curve reflects the hedging and maturity needs of institutional investors and these two

hypothesis combined together explain the behavior of yield curve, which is nothing but the relationship between yield to maturity and the time to maturity.

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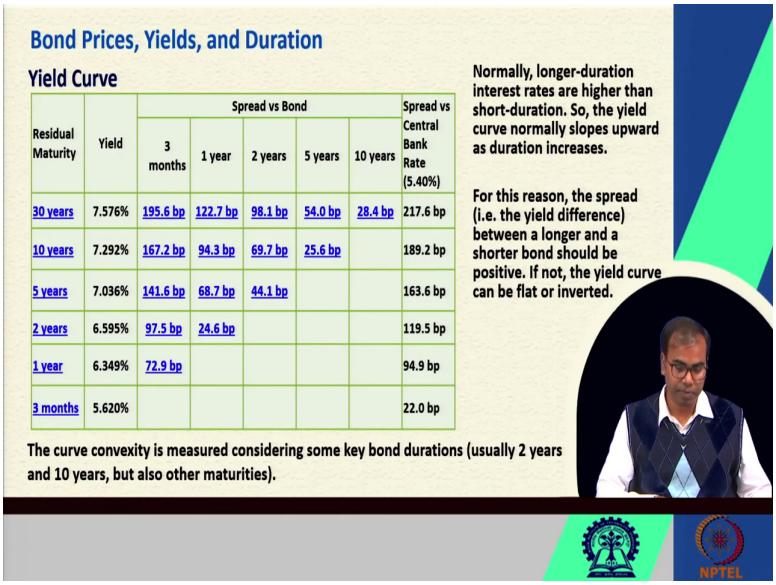

Here is one example which can showcase the relationship between the yield to maturity and time to maturity from the India market. So, basically if you look at this table, this table indicates the yield curve for different bonds or bonds with different maturity and time to maturity and in this table for benchmarking we have also indicated spread versus central bank rate which we obtain by finding the yield and the prevailing spread.

So, normally as stated earlier, longer duration interest rates are higher than shorter duration interest rate. So, the yield or the yield curve normally slope towards upward direction and it happens as duration increases which essentially shows the graph that we have seen earlier.

And for this region the spread that is the difference between the yield and the prevailing interest rate between a longer and a shorter bond should be positive and if it is not the case then the yield curve becomes flat or in some cases inverted as well.

So, if you look at the numbers in the table, we see that the prevailing bank rate is assumed to be 5.40 percent and the yield for that particular bond for which residual maturity is 30 years is 7.576 percent and for different duration the spread versus bond duration is given here for 3 months, 1 year, 2 year, 5 year and 10 years.

And we can see the difference between the yield and the central bank rate which is the prevailing interest rate that is 5.40 percent is indicated in terms of basis point. Which means for a bond with residual maturity of 30 years and the prevailing bank rate being 5.40 percent, the yield is 7.576 percent and the spread with respect to central bank rate is 217.6 basis point which is essentially 2.176 percentage.

Similarly, for a bond maturing with residual maturity of 10 years, the yield is 7.292 percent whereas, the bank rate is 5.40 percent and the spread can be seen as 189.2 basis point which is 1.892 percentage. In case the bond with the residual maturity of 5 years, we see that the yield is 7.0636 percent and the bank rate is 5.40 percent. So, the spread is163.6 basis point which is effectively 1.636 a percentage point.

And as we see further a bond with residual maturity of 2 years 1 year and 3 month the yields are 6.595 percent, 6.349 percent and 5.620 percent respectively for which we can see the spread to be 119.5 basis point, 94.9 basis point and 22 basis point respectively. So, when we try to plot such a relationship between yield and time to maturity or residual maturity.

We get yield curve and sometimes yield curve convexity or the shape of the yield curve actually is measured considering some key bond duration and particularly in the context of a market such as India, the usually the bond duration which is considered usually is 2 years and 10 years, but also it can be considered for other maturity.

And when we obtains this relationship the shape of the relationship essentially shows as the convex curve and that is also referred to in the fixed income literature as convexity of a bond and to continue with the discussion on yield and duration.

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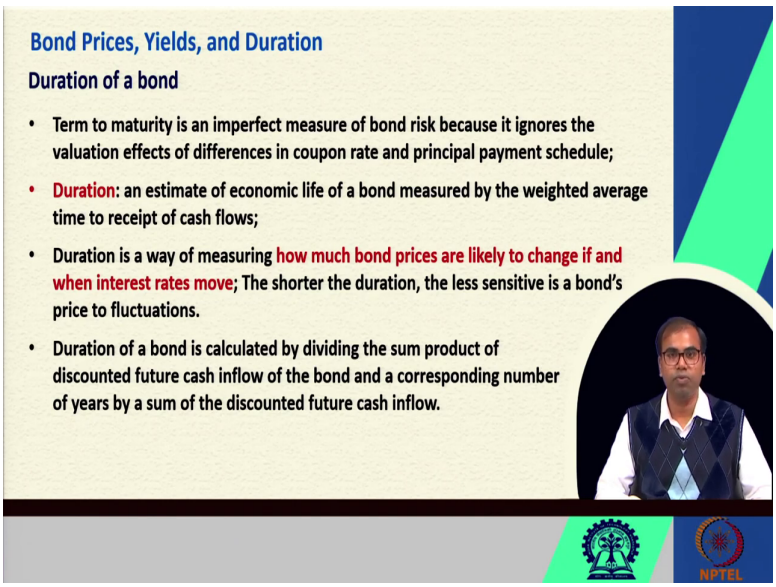

Let us quickly touch upon the duration of a bond as pointed out earlier. Duration of the bond essentially is the related to term to maturity or time to maturity which is basically an imperfect measure of bond risk because it ignores the valuation effects of differences in coupon rate and principal payment schedule.

So, when we look at the timeline of a bond, suppose an investor invades a bond in year 1 in the very beginning of the year and expects to receive the first coupon at the end of first year,

subsequently is a if it is annually paid then at the end of second year, the second coupon will be paid and it will keep on being paid at the end of every year.

Now, since the investor has not invested in the bond in the very beginning when the bond were issued for the first time, then the time to maturity will be an imperfect measure for the risk associated with the bond and that is why we need to understand a more refined measure of risk particularly with respect to the time to maturity or term to maturity with respect to a bond. And for that we have a measure called duration.

So, duration is defined as an estimate of economic life of a bond and it is measured in terms of weighted average time to receipt of cash flows. Which means whatever cash flows is expected out of a bond, which is in terms of coupons or interest to be paid to the holder to the bond holder. So, we have to see the duration of a bond in terms of weighted average time of time to receipt of a all the cash flows associated with that particular bond.

So, if we try to measure duration or quantify the duration of a bond basically, we will measure it in terms of how much bond prices are likely to change, if and when interest rate moves. Because whenever we try to find their value in terms of present value, then the prevailing interest rate will be impacting the valuation and whenever there is a change in interest rate then the price of the bond is likely to change.

Earlier we have seen that the interest rate and bond prices are inversely related, which means if interest rate increases prices are likely to decrease and vice versa. How much this decrease or this change is supposed to happen is indicated in terms of duration of the bond, when it is translated into number of periods or number of years.

So, effectively we can say that, the shorter the duration of a bond the less sensitive is a bond's price to fluctuations in the interest rate. And when it comes to the calculation of duration essentially duration of a bond is calculated by dividing the sum product of discounted future cash inflows coming out of a bond investment and a corresponding number of years by a sum of discounted future cash inflows.

Now, if you look at these two components, there are two components of the formula or the calculation approach with respect to duration. One component says the sum product of discounted cash inflows related to a bond and a corresponding number of years and divided by the sum of discounted future cash inflows. So, if you look at the sum of discounted future cash inflows component the second component, it is nothing but the price of a bond.

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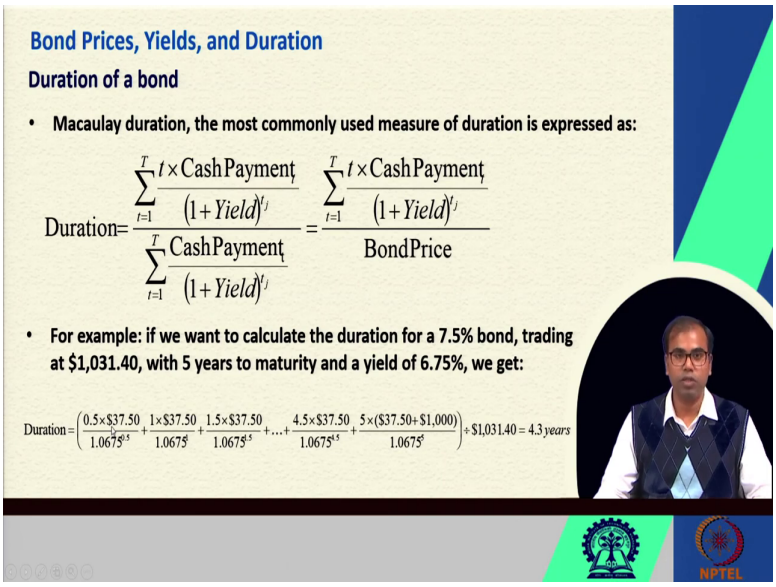

So, when we try to indicate duration in terms of number of years or number of periods by using this formula, we use a very popularly used mode method called Macaulay duration, which is the most commonly used measure of duration in bond literature and we can express the formula of Macaulay duration as following.

So, if you look at this formula, you see that the first component that is numerator is essentially the expected cash payment and divided by 1 plus yield to the power t which is

nothing but the time period multiplied with t. So, and then we sum it all. So, we have the present value of all future cash payment, present value being calculated with respect to the yield. So, yield is acting as the discount rate here.

So, we first discount all the cash flow that we are expecting to generate from an investment in bond. We discount it to the present time; we find the present value by using the yield as a discount rate. Multiply it with number of years for corresponding period and then sum it all that gets us the first component or the numerator part of the formula.

And then for denominator part we discount all the future cash payment which is basically the coupons or interest that is coming to this bond holder as well as the lump sum value at the end of the maturity, with respect with the yield being the discount rate and then sum it all. So, when we look at the denominator, we see it is nothing but the price of the bond.

Remember the price of the bond can be expressed in terms of the present value of all future cash flows that include coupons as well as the face value repayment at the end of maturity. So, the formula effectively is expressed as the sum total of time weighted, future cash flows discounted with the yield being the discount rate divided by the bond price.

So, let us take an example here, in this example we have a bond with pays 7.5 percent annual coupon and it has 5 year's time to maturity and prevailing yield is 6.75 percent and this particular bond is currently trading in the market at a price of 1031.40 dollars. So, if this is the case and if we implement this example this these numbers into the formula of Macaulay duration, we can see that for every coupon we find the present value by discounting it with the yield which is 6.75 percent.

So, since this is semi-annual coupon, we use this semi-annual discounting rate for every coupon that is to be paid multiplied with the corresponding number of period which is half year, one year, one and a half year and so on and so forth. And then divide it with the current price of the bond which is 1031 dollar 40 cents.

So, if we implement this formula, we get a value of 4.3 years and this shows the duration of the bond. So, in a very Layman terms, we can say that duration of the bond is the period within which an investor is expected to recover all the investment in current values or in present values terms. So, this is how we can calculate the duration.

Now, the implication of duration for an investor is the argument that we have already put forth. It says that the shorter the duration of a bond is the better it is for the investor because it will be less impacted by the future fluctuations. So, whenever as an investor we are looking for a bond and we are able to calculate the duration for a such a bond or let us say a set of bonds which we are considering for investment, then we should always look for a bond with shorter duration.

Which means, in a typical sense for a risk averse investor, the shorter duration the shorter the duration of a bond is the better it is because the investor's investment will be less impacted by the fluctuations in interest rates in future. So, shorter duration bonds are preferable for a typical risk covers investor who is looking for an investment in bonds.

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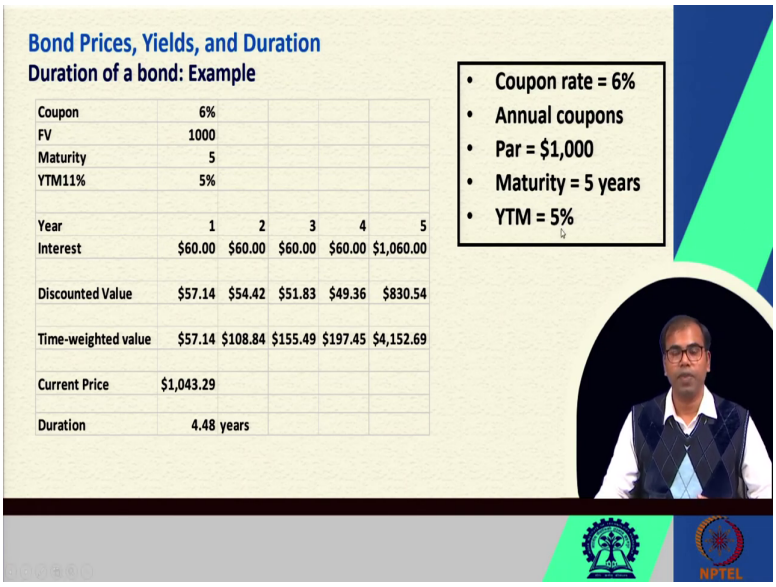

To continue with some example so, here is another example which helps us understand the calculation of a duration of a bond. So, in this case we have a bond which pays annual coupon of 6 percent and the face value of this bond is 1000 and maturity of this bond is 5 years, YTM which is yield to maturity is 5 percent.

So, if you try to calculate this the duration of this bond, we know that duration of the bond is calculated by finding the present value of the coupons which are to be received by the investor by the bond holder in present terms. So, first we calculate the present value of each of the coupon, in this case 60 dollar. So, 5 coupons of 60 dollars and 1000 of par value payment at the end of maturity.

So, in fifth year we have a cash flow of 1060 dollars. We find the present value by discounting it with yield to maturity that is 5 percent. So, we find the present value

discounted value of all these cash flows for 5 years, then we time we find the time weighted value which is basically nothing but multiplying this discounted value with year.

So, for first year we multiply 57.14 dollars with 1, in second year we multiply 54.42 dollars with 2 and so on and so forth. So, we find time weighted value of bond. And then we sum it all. So, sum of all these time weighted value of coupon that are coming to the bond holder and at the same time we have calculated the present value of all the future cash, which is basically the sum of discounted value and that gives us the current market price or current value of the bond in the absence of any other information.

And when we divide it the sum total of time weighted value of these cash flows with the current price of 1043.29 dollars, we get a value of 4.48 which is interpreted in terms of years and this is the duration of the bond. So, this particular bond has a duration of 4.48 years. Now, there may be some variation when we change one or the other factor.

As we see in this example, we have multiple inputs including coupon rates, par value, maturity and yield to maturity. In some cases, we might as well have the current market price given; here we have calculated assuming that there is no further information. So, we use current market price in terms of the sum total of present value of all future cash flows. So, if we change one or one of these factors, how will it impact the duration?

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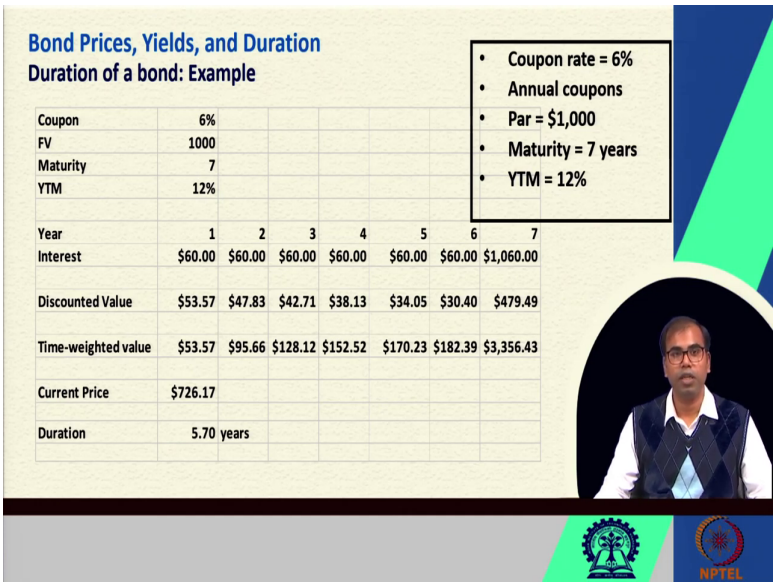

Let us look at another example and find out. So, in this case, in this example we have coupon rate as previous case 6 percent which is annually paid, par value of 1000 dollars, maturity has been changed from 5 years to 7 years and YTM is also changed from 5 percent to 12 percent.

Now, in previous example we had seen that coupon rate is 6 percent and YTM was 5 percent whereas, here coupon rate is remains 6 percent, but YTM changes to 12 percent which means, YTM or yield is higher than coupon rate. Now, let us see how it changes the duration. So, in this case let us first take a look at the numbers. The number says that coupons are coming to the investor for 7 years, 60 dollars every year plus 1000 of lump sum payment at the end of maturity, SO 1060 dollars in 7th year.

We find the discounted value by discounting it with YTM that is yield to maturity or yield, which is discount rate. We find the present value of these sum total of discounted value as the

current market price, so current market price is 726.17 dollars and we find the time weighted value of all these present value of coupons.

So, this row shows the time weighted value which is discounted value multiplied with year. So, 53.357 dollars into 1 is in the first year, 47.83 dollars in multiplied with 2 in second year and so on. So, we get the total value of time weighted value of future cash flows, and we divide sum total of time weighted value of future cash flows by current market price or current value that is 726.17 dollars and we get the value of 5.70 years, which is 5.7 years as the duration of this bond.

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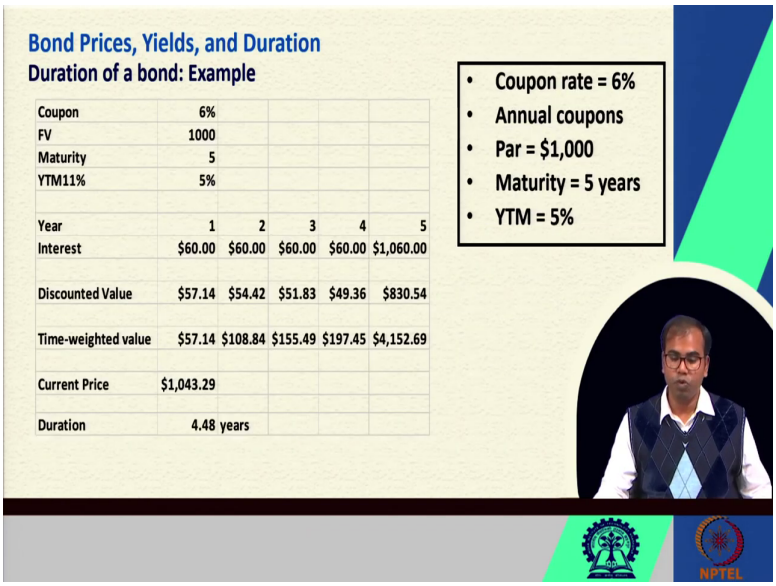

So, can we say that if coupon rate is greater than yield to maturity, the duration is 4.48 year here and if we change the coupon rate rather if we change the yield to maturity that is greater than coupon rate, we find a case where we have the current market price found as the sum

total of present value of all the coupons and final payment and we get the duration as 5.7 years.

So, this way we can see duration for different bonds and particularly if I am an investor and I am looking for a bond to invest my money in, then I can consider probably the duration as one of the key inputs for decision making.

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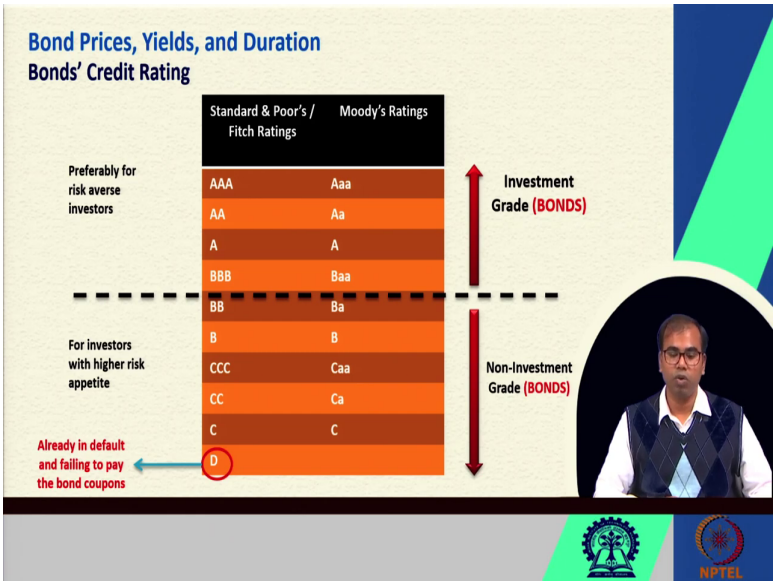

One last bit before we wind up the discussion on bonds is the credit rating. As we know that every financial instrument is supposed to be rated or evaluated for its credit worthiness and same goes with bonds. So, there are multiple credit rating agencies which provide credit rating for different bonds and when as an investor I am looking for an investment a bond for investment.

I look at these ratings and I can interpret in a very basic sense that a bond with rating with such as triple A or double A or A, I can consider those bonds as rather safe investment and when it comes to a double B, B or triple C, C, double C, C and D these are bonds which are not recommended for those people who are risk averse.

So, essentially there is some sort of threshold above which is considered as investment grade bonds and which are preferred by the risk covers investors and below this threshold these are considered non-investment grade bonds, which is or which are basically more suitable or more preferable by investors who are seeking higher risk who have higher risk appetite.

And of course, it goes without saying that a bond with a rating of D is already in default and it is going to it is going in the process of failing to pay the coupons. So, it is not recommended for most of the typical investor's, maybe bond rating is something that we should look at and consider this as an important input in our investment decision making.

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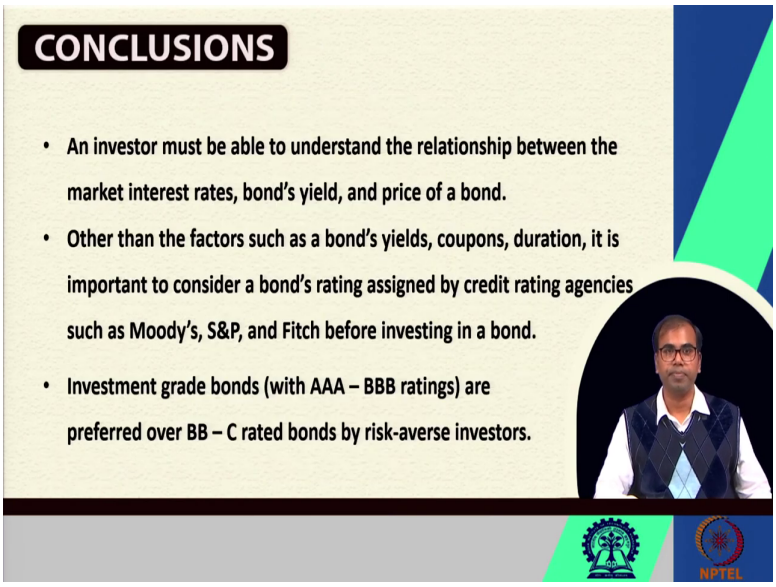

So, to conclude this session, we have already discussed about different characteristics of bonds as an investment tool and as an investor, we must be able to understand the relationship between those characteristics particularly market interest rates, bonds yield and price of a bond.

And other than the factors such as bonds yield, coupon, duration it is important to consider a bonds rating as well that is assigned by in different agencies such as Moody's, Standards and Poor's and Fitch before investing in a bond.

And we have just seen that bonds with investment grade rating such as, triple A to triple B ratings are preferred over double B to C rated bonds by risk averse investors and preferably a

typical investor would not like to invest in bonds, such with a rating of D because it is already in default. With this I conclude this session.

Thank you very much.