

**Investment Management**  
**Prof. Abhijeet Chandra**  
**Vinod Gupta School of Management**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 09**  
**Pricing and Valuation of Bonds (Contd.)**

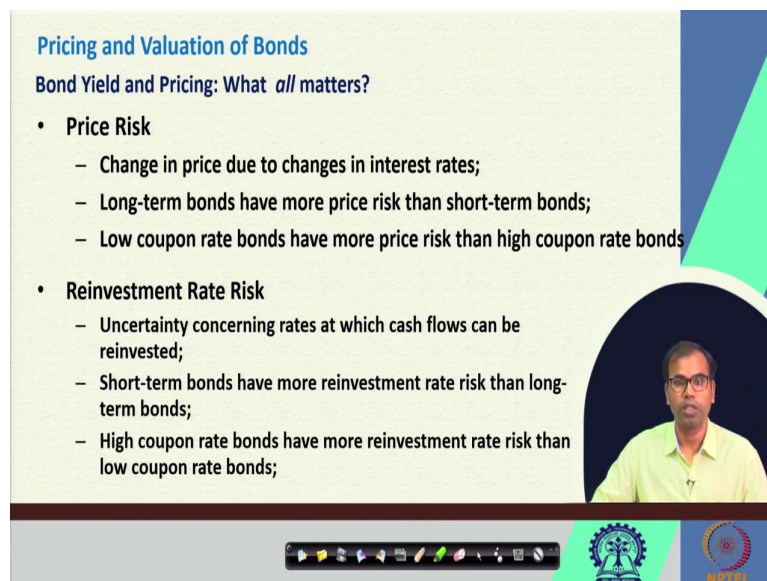
Hi there. So, continuing with our discussion on Bonds as an Investment. We have been discussing about different factors or to inputs that we use for finding the value of a bond. And in this session, we will continue with our understanding and discussion on Pricing and Valuation of Bond. And we will try to elaborate on the relationship between bond yield and the price or the value of the bond.

(Refer Slide Time: 00:56)



So, essentially, this session focuses on understanding how bonds yield and price are related. And we will also see how yields can be calculated with the help of information that are available with respect to any bond as a financial instrument. So, before we go to the relationship between price and yield of a bond, we need to understand what are other factors that might be of interest when it comes to understanding the characteristics of a bond.

(Refer Slide Time: 01:32)



**Pricing and Valuation of Bonds**  
**Bond Yield and Pricing: What all matters?**

- **Price Risk**
  - Change in price due to changes in interest rates;
  - Long-term bonds have more price risk than short-term bonds;
  - Low coupon rate bonds have more price risk than high coupon rate bonds
- **Reinvestment Rate Risk**
  - Uncertainty concerning rates at which cash flows can be reinvested;
  - Short-term bonds have more reinvestment rate risk than long-term bonds;
  - High coupon rate bonds have more reinvestment rate risk than low coupon rate bonds;

The slide features a video inset of a presenter in a light green shirt. At the bottom, there is a navigation bar with icons and logos for IIT Bombay and NPTEL.

So, we all know that bonds can be of different characteristics. For example, if there are bonds issued by same issuer, but at different points of time, one might have more time to maturity, whereas, the other bond might have less time to maturity. Because one bond is senior, another bond is junior.

There would be cases where one bond might have higher coupon rate because of this its inherent risk. And on the other side another bond might have lower coupon rate because of

lesser risk carried in the project that is funded by the funds or the money raised through this bond.

So, when we talk about the relationship between bonds yield and with pricing, what matters here is basically the interest rate risk. Earlier, we have touched upon the idea of interest rate risk being one of the important risk aspect. Here, we will just touch upon two aspect, price risk and much a reinvestment rate risk.

So, if we look at price risk, we know that the price of a bond can change because of the change in underlying interest rates. As we have shown earlier, if the inter prevailing interest rate in the market changes, then the rate at which coupon is paid for a particular from a particular bond, might not look as attractive and that is why price can change.

Similarly, a bond which have with higher longer maturity, let us say long term bonds will have a higher price risk compared to a short term bonds. Because for long term bonds, the uncertainties related to the change in interest rate will be higher and that is why the investor might perceive it as a more risky riskier instrument compared to a bond with shorter maturity.

In similar lines, a bond with lower coupon rate can have a more price risk than a bond with higher coupon rate. So, these are some factors which might change the price risk associated with the interest rate in case of a bond. So, we can look at these characteristics such as what kind of maturity the bond carries, what is the coupon rate whether it is lower or higher compared to the other bond that we are evaluating or assessing for investment.

And then, we can understand about the price risk associated with a particular bond as an instrument for investment. In the same context, reinvestment rate risk is also arising because of interest rate changes. So, as we understand reinvestment rate risk is associated with the return that we might be generating on the fund that are reinvested. Suppose, we have invested 1000 dollars today and 1 year later, we get 7 percent return, so we get 1070 dollars of total money.

Now, from next year onward, let us say rate of interest changes from 7 percent to 6 percent. Now, the amount of return that I will be generating on 1070 dollar will be relatively lesser than the amount of return that I have generated on 1000 dollar for the first year. So, reinvestment rate risk actually relates with the change in interest rate and uncertainty is associated with that.

So, uncertainty concerning interest rates at which cash flow can be reinvested is basically indicating the reinvestment rate risk. So, for example, a short-term bonds can have more reinvestment risk than long-term bonds and higher coupon rate bonds may have more reinvestment risk than the lower coupon bond lower coupon rate bond, because of the same argument as we have explained earlier.

So, interest rate is basically one of the major factors or inputs that we need to look at while evaluating or comparing bonds of different maturity or different coupons for the purpose of investment.

(Refer Slide Time: 06:17)

**Pricing and Valuation of Bonds**

**Bond Yield and Pricing: What all matters?**

Suppose a bond with a **10% coupon rate** and semiannual coupons, has a face value of **\$1000**, 20 years to maturity and is selling for **\$1197.93**.

- Is the **YTM** more or less than 10%?
- What is the semiannual coupon payment?
- How many periods are there?

Handwritten notes on the slide:

- FV = \$ 1000
- Coupon = 10%, semi-annually =
- Tenor = 20 yrs,  $\rightarrow t = 40$  (20 x 2)
- CMP = \$ 1197.93


Timeline diagram showing cash flows:

Timeline:  $t_0$  to  $t_1$  to  $t_2$  to  $t_3$  to  $t_4$  to ... to  $t_{40}$

Cash flows:  $t_0$ : CMP = 1197.93;  $t_1$  to  $t_4$ : 50;  $t_{40}$ : 50 + 1000

Equation:  $1197.93 = \frac{50}{(1+\frac{y}{2})} + \frac{50}{(1+\frac{y}{2})^2} + \frac{50}{(1+\frac{y}{2})^3} + \dots + \frac{50}{(1+\frac{y}{2})^{40}} + \frac{1000}{(1+\frac{y}{2})^{40}}$

Handwritten solution:  $y = ?$ , YTM = 4%



Now, let us take a look at a simple example here. Suppose, there is a bond with 10 percent coupon rate and semi-annual coupons this bond has a face value of 1000 dollars, a maturity of 20 years and current market prices 1197.93. So, the question that is arising here is the YTM or yield to maturity for this particular bond is more or less than 10 percent?

So, 10 percent is the coupon rate; if we just highlight the key information that we are going to need semi-annual coupon, face value of 1000 dollar, 20 year is maturity and current market price is 1197.93. So, we know that face value is 1000 dollar. Coupon rate is 10 percent, tenor is 20 year semi-annually. So, semi-annually and current market price is 1197.93 dollars.

Now, if we apply the same argument that we have been discussing so far, we know that on a timeline like this where we have  $t_0$ ,  $t_1$ ,  $t_2$  which is indicating the semi-annual period and  $t$

40, because remember this is 20 year which means  $t$  will be 40 that is 20 into 2 semi-annually, right. So, because of that we will have 40 periods.

And 10 percent coupon is paid. So, when we pay semi-annually it becomes 5 percent semi-annually. So, we have 5 percent on 1000, so 50 dollar, 50 dollar, 50 dollar. So, every time we pay a coupon of 50 dollar and at the end we pay 50 dollar and 1000 dollar. So, if we use the same argument where we have to find or equate the current market price of 1197.93. So, equate with all the value of coupons discounted with the semi-annual coupon rate sorry yield rate.

And  $1 + y/2$  to the power 2, 50 divided by  $1 + y/2$  to the power 3 and so on till 40th period where we have coupon  $1 + y/2$  to the power 2 to  $y/2$  to the power 40 plus 1000 divided by  $1 + y/2$  to the power 40 and that is how we can find this value. Now, if we equate this expression, we know that  $y$  will be certain percent.

Now, if we look at these numbers intuitively, we can assume that or we can sort of infer that whenever the price of the bond is higher than the face value typically yield is lesser than the coupon rate. That is the argument that we have already explained and discussed in earlier sessions, where yield is supposed to be yield is expected to be lesser than the coupon rate when the price of a bond is higher than the face value.

So, if we do this calculation, apparently, we get to know that yield is 4 percent where at discounted at 4 percent of  $y$ , value of  $y$ . We can find the value of current market price of this particular bond to be 1197.93. So, we know the semi-annual coupon payment is 50. The period is going to be 40 which we have considered here. And YTM is if we calculate this is going to be 4 percent. So, this is how we can find the missing information related to a particular bond.

(Refer Slide Time: 11:50)

**Pricing and Valuation of Bonds**

**Bond Yield and Pricing: What all matters?**

Suppose a bond with a 10% coupon rate and semiannual coupons, has a face value of \$1000, 20 years to maturity and is selling for \$1197.93.

- Is the YTM more or less than 10%?
- What is the semiannual coupon payment?
- How many periods are there?

$$PV = \frac{C}{y} \left( 1 + \frac{1}{(1+y)^t} \right) + \frac{FV}{(1+y)^t}$$

And if we want to apply a sorted route, probably you would refer to any standard corporate finance text or the formula that we learn in corporate finance where we know that the present value of a particular constant cash flow constant annuity cash flow which is the case here.

So, the constant annuity cash flow present value is calculated as  $C$  by  $y$  into  $1$  plus  $1$  by  $1$  plus  $y$  to the power  $t$  and then we have face value divided by  $1$  plus  $y$  to the power  $t$ . So, essentially, we will try to find the present value of all these coupons that are due or that are paid. Particularly, the in case of semi-annual it is calculated, accordingly and period is multiplied with  $2$  because it is semi-annual

(Refer Slide Time: 12:49)

**Pricing and Valuation of Bonds**  
Bond Yield and Pricing: What *all* matters?

Suppose a bond with a 10% coupon rate and semiannual coupons, has a face value of \$1000, 20 years to maturity and is selling for \$1197.93.

- Is the YTM more or less than 10%? *Coupon > YTM, FL CMP*
- What is the semiannual coupon payment? *= 5% semi-annually*
- How many periods are there? *= 40 periods*

The slide includes a video feed of a man in a light green shirt in the bottom right corner. At the bottom of the slide, there is a taskbar with various application icons and logos for institutions like IITM and IITB.

So, 40 periods semi-annual coupon is 5 percent semi-annually. And we will also see why when coupon is more than YTM, the face value is typically less than the current market price, that we will see subsequently.

So, with this example, probably we get more clarity about the linkages between different information provided along with a bond certificate or a bond as an investment instrument. So, we know that given the given certain information such as coupon rate, face value, time to maturity, current market price, we can find the yield or vice versa.



(Refer Slide Time: 14:00)

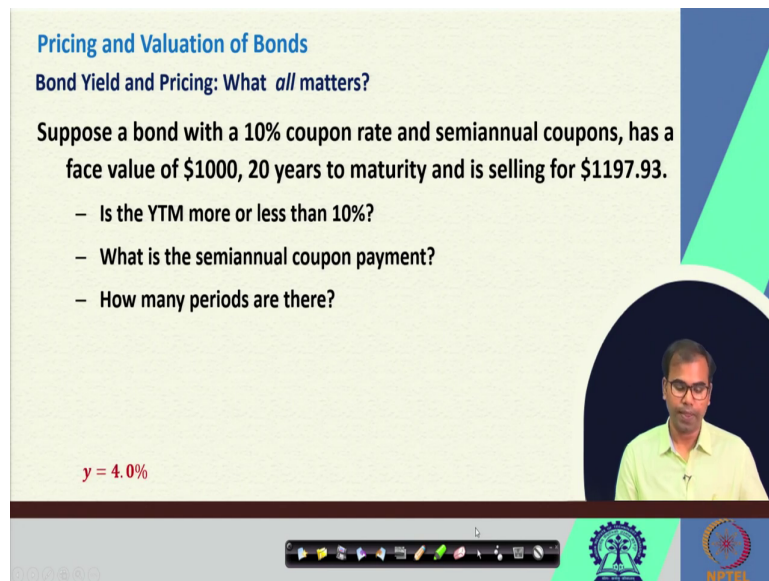
**Pricing and Valuation of Bonds**

**Bond Yield and Pricing: What all matters?**

Suppose a bond with a 10% coupon rate and semiannual coupons, has a face value of \$1000, 20 years to maturity and is selling for \$1197.93.

- Is the YTM more or less than 10%?
- What is the semiannual coupon payment?
- How many periods are there?

$y = 4.0\%$



So, let us try to see more examples here.

(Refer Slide Time: 14:01)

**Bonds as an Investment**  
Relationship between Bond Yield and Coupon

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000 (Face Value)
- Maturity = 5 years
- YTM = 11%

$$PV = \frac{C_1}{(1+YTM)^1} + \frac{C_2}{(1+YTM)^2} + \dots + \frac{(C_5 + FV)}{(1+YTM)^5}$$

Handwritten notes: 'PV' is circled, and 'CMP' is written below it with an arrow pointing to the PV term in the formula.

The slide also features a video inset of a presenter and logos for IIT Bombay and NPTEL at the bottom.

We can we can take this example. Let us say again I use the same argument. So, here coupon rate is given 10 percent and this is annually paid. So, we have a face value of 1000 dollar, coupon rate is 10 percent annually, maturity is 5 year, YTM is 11. So, we can find the present value of a cash flow which is coming in the form of coupon.

So, annual coupon first coupon 1 plus YTM to the power n or t C 2 1 plus YTM to the power 2 and here C 5 that is coupon 5 plus face value divided by 1 plus YTM to the power 5. So, this particular expression will give us the present value which we can say as current market price or current value of the bond. So, if you try to calculate this using simple financial

calculator or spread sheet or for that matter numerical calculation, we can find the value of this particular bond.

(Refer Slide Time: 15:36)

**Bonds as an Investment**  
**Relationship between Bond Yield and Coupon**

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000
- Maturity = 5 years
- YTM = 11%

Coupon	10%					
FV	1000					
Maturity	5					
YTM	11%					

Year	1	2	3	4	5
Interest	\$100.00	\$100.00	\$100.00	\$100.00	\$1,100.00
Discount with YTM					
Discounted Value	\$90.09	\$81.16	\$73.12	\$65.87	\$652.80
Current Price	\$963.04				

$$= \frac{100}{(1.11)^1} + \frac{100}{(1.11)^2} + \frac{100}{(1.11)^3} + \frac{100}{(1.11)^4} + \frac{1100}{(1.11)^5}$$

$$= \$963.04 \leftarrow \text{CMP}$$

So, we are given coupon rate, we are given face value, we are given time to maturity and YTM. So, if we use individual cash flows, so 10 percent coupon rate means 100 dollars of coupon is accruing every period which is 5 years and in 5th year 1000 or 100 dollars of coupon and 1000 dollars of face value is accruing say 1100.

So, if we discount it with YTM, if we discount this value with YTM which is 11 percent, we can find the present value of this cash flow to 100 dollars cash flow at the end of 1 year is 90.09. This 100 dollar which is accruing at the end of second year, the present value is 81.16. Similarly, for third year cash flow we get 73.12 and fourth year 65.87. And fifth year is since 1100, we get 652.80.

So, if we just put plot, it on a timeline, we can know that this is  $t_0$ ,  $t_1$ ,  $t_2$ , this is 3,  $t_4$  and  $t_5$ . So, we have 100 dollars of coupon 1.11 that is 1 plus YTM to the power 1, 100 dollars of cash flow, 1.11 to the power 2, 100 dollars of cash flow third time, 1.11 dollar to the power 3, 100 dollars of cash flow for fourth year, 1.11 to the power 4.

And in case of in fifth year we have 1100, so we have 1100 divided by 1.11 to the power 5. And this gives us 963.04 dollars that is the current value of the bond that we are talking about. Fair enough. So, we have YTM as the discounting rate here for the cash flow that we are expecting to generate from this investment in bond.

Now, this is very simple. We all have understood the approach of discounting future cash flows back to the present time by using a discount rate which in this case is YTM. So, we can look at the change in YTM and its impact on the market price. So, remember, let us take a note here that current market price for this particular bond at YTM being 11 percent is 963.04.

(Refer Slide Time: 18:46)

**Bonds as an Investment**  
**Relationship between Bond Yield and Coupon**

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000
- Maturity = 5 years
- YTM = 11%

Coupon	10%				
FV	1000				
Maturity	5				
YTM	11%				
Year	1	2	3	4	5
Interest	\$100.00	\$100.00	\$100.00	\$100.00	\$1,100.00
Discounted Value	\$90.09	\$81.16	\$73.12	\$65.87	\$652.80
Current Price	\$963.04				

Price < Face Value

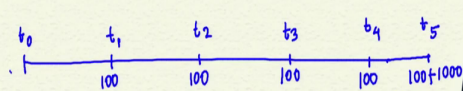
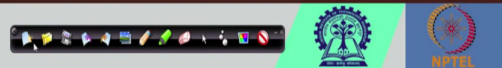

When YTM > Coupon rate → Price < Par = "Discount Bond"

So, if we change this YTM, what happens to the price of this particular bond? So, this is the takeaway. So, we have current market price to be 963.04 at YTM being 11 percent, which means if YTM is greater than coupon rate, coupon rate is 10 percent right. So, if YTM is greater than coupon rate, price will be less than par value. So, price is expected to be less than face value of the bond and we call it discount bond, ok. So, this is the case when YTM is less than the coupon rate.

(Refer Slide Time: 19:38)

**Bonds as an Investment**  
Relationship between Bond Yield and Coupon

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000 (Face Value)
- Maturity = 5 years
- YTM = 8% ← Risk country Rate


$$PV = \frac{100}{(1+8\%)^1} + \frac{100}{(1+8\%)^2} + \frac{100}{(1+8\%)^3} + \frac{100}{(1+8\%)^4} + \frac{1100}{(1+8\%)^5}$$


Now, let us take a different scenario where YTM is greater than we have seen the case where YTM is greater than coupon rate and now we will see YTM being less than the coupon rate as well. So, here we will follow the same approach. Earlier we have seen a YTM being greater than the coupon rate. So, the price of the bond is typically a less than the face value of the bond.

So, if we have YTM being less than the coupon rate, then the value of the bond should ideally be more, right. So, we will apply the same approach. We have different time period. So, other factors are same for this bond. We have 10 percent coupon annually accruing. So, on 1000 dollars of face value, so we have 100 dollars, 100 dollars, 100 dollars, 100 dollars in 4th year and 100 plus 1000 dollars in 5th year. So, what we are going to do is to find the present value, we will use 8 percent as the discounting rate.

So, we will have 100 divided by 1 plus 8 percent to the power 1, 100 plus 100 divided by 1 plus 8 percent to the power 2. Then, for third year 100 divided by 1 plus 8 percent to the power 3 then, for fourth year 100 divided by 1 plus 8 percent to the power 4 and in fifth year we have 1100. So, we have 1100 divided by 1 plus 8 percent to the power 5. And whatever value we get will be the current market price.

(Refer Slide Time: 22:00)

**Bonds as an Investment**  
**Relationship between Bond Yield and Coupon**

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000
- Maturity = 5 years
- YTM = 8%

*A discounting Rate*

Coupon	10%				
FV	1000				
Maturity	5				
YTM	8%				
Year	1	2	3	4	5
Interest	\$100.00	\$100.00	\$100.00	\$100.00	\$1,100.00
Discounted Value	\$92.59	\$85.73	\$79.38	\$73.50	\$748.64
Current Price	\$1,079.85				

*CMP (1079.85) > face value (1000)*

When YTM < Coupon rate → Price > Par = "Premium Bond"

So, if you look at the numbers here and we do the calculation as we did in case of previous example, here since YTM or discounting rate is less than the coupon rate that is 8 percent here compared to the coupon rate of 10 percent. Then, current market price in this example is higher than the current market price being 1079 dollar 85 cents is greater than the face value of 1000 dollars. Earlier, it was less, now it is more.

So, the conclusion is if we have YTM being less than coupon rate which is the case here, the price of the bond will be greater than the face value. And that is where we call such a bond as premium bond, which means a bond that is being sold at a premium. So, if you remember the example in previous session where we look at the dashboard of bonds traded in the capital market, where most of the bonds last traded price was higher than the face value.

So, we can presume here that those bonds are trading at a premium and for those for such bonds being traded at premium, the YTM that is yield to maturity is less than the coupon rate and that is why price of the bond is greater than the par value or face value of such bonds. So, this is basically to indicate the relationship between YTM that is yield, coupon rate price and face value.

(Refer Slide Time: 24:06)

**Bonds as an Investment**  
**Relationship between Bond Yield and Coupon**

- Coupon rate = 10%
- Annual coupons
- Par = \$1,000
- Maturity = 5 years
- YTM = 8%

Coupon	10%				
FV	1000				
Maturity	5				
YTM 11%	8%				
Year	1	2	3	4	5
Interest	\$100.00	\$100.00	\$100.00	\$100.00	\$1,100.00
Discounted Value	\$92.59	\$85.73	\$79.38	\$73.50	\$748.64
Current Price	\$1,079.85				

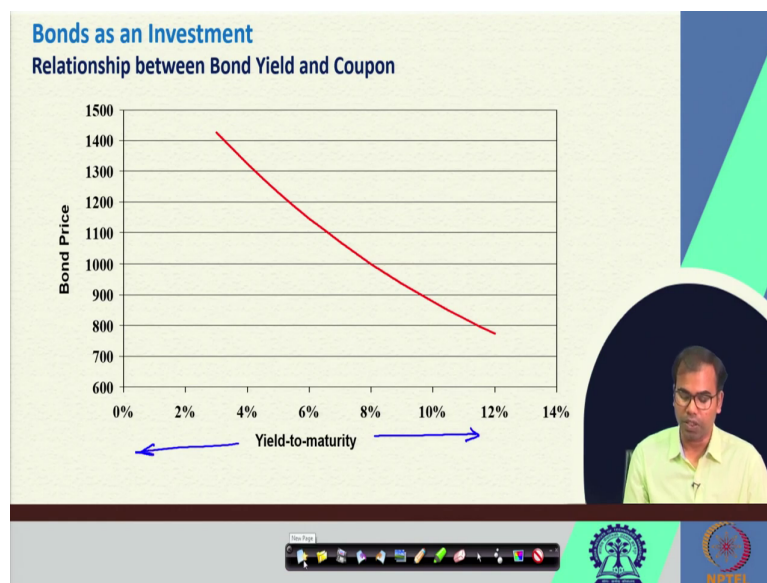
$YTM > C$	$P_0 < FV$
$YTM < C$	$P_0 > FV$

When  $YTM < \text{Coupon rate} \rightarrow \text{Price} > \text{Par} = \text{"Premium Bond"}$



So, we have understood two scenarios, YTM being greater than coupon rate. So, price today is going to be less than the face value. And if YTM is less than coupon rate then price today is going to be greater than the face value. So, we with this takeaway we can replicate this calculation, this example for multiple scenarios or multiple combinations of yield and coupon, and we can see how it translates or how it reflects in terms of the final price or the value of the bond.

(Refer Slide Time: 24:55)



So, just to summarize this is the way we can interpret the relationship between YTM and bond price. So, as YTM increases, given other things same, we know that the bond price will accordingly subside and yield will as yield will reduce or yield will decrease, the price of the bond is supposed to increase and vice versa.

(Refer Slide Time: 25:35)

**Bonds as an Investment**  
**Relationship between Bond Yield and Coupon**

	Bond A	Bond B	Bond C	Bond D	Bond E	Bond F
Face Value	₹1,000.00	₹1,000.00	₹1,000.00	₹1,000.00	₹1,000.00	₹1,000.00
Semi-annual Coupon Rate	5.50%	7.50%	5.00%	9.00%	7.00%	6.00%
Annual Coupon/Interest	₹27.50	₹37.50	₹25.00	₹45.00	₹35.00	₹30.00
Yield (Market Interest Rate)	5.75%	6.50%	7.00%	7.25%	7.25%	6.00%
Issue Date	01/12/93	15/12/03	01/04/00	01/12/15	15/08/20	16/12/20
Settlement Date (Purchase Date)	01/12/22	15/12/22	01/12/22	01/12/22	01/12/22	15/12/22
Maturity Date	01/12/23	15/12/25	01/04/30	01/12/35	15/08/50	15/12/30
Time-to-maturity (years)	1.00	3.00	7.34	13.01	27.72	8.01
Bond price (% of par value)	99.76	102.69	88.67	114.58	97.03	100.00
Bond Value	₹997.60	₹1,026.89	₹886.75	₹1,145.80	₹970.31	₹1,000.00

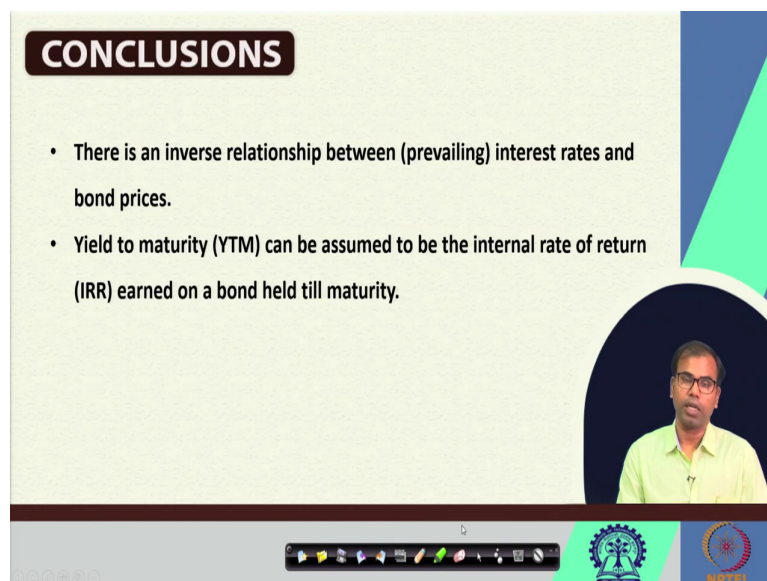
So, just to complete the understanding of the relationship between bond yield and coupon, here is one scenario analysis where we can see different bonds. So, here we have 6 bonds or bonds with different information. In all these cases the face value of the bond is presumed to be same 100 1000 rupees, but we have changed the coupon rate. So, what we have done is we have assumed that coupon rate in these bonds are going to be different.

For bond A, it is 5.5 percent, bond B 7.5 percent, bond C it is 5 percent, bond D 9 percent, bond A 7 percent and bond F 6 percent. So, accordingly we can calculate the coupon. So, coupon will be calculated on the basis of face value and semi-annual coupon rate converted to annual coupon rate coupon value. And then, we bring in the yield or YTM which is different in all these cases. So, here in case of bond A, you can see that yield is greater than the coupon rate.

Now, the moment yield is greater than coupon rate, we can see that the value of the bond is less than the face value. And in every instance where yield is less than the coupon rate, we can see that the bond is a bond valuation is changing. And accordingly, the relationship between bond yield and coupon is reflected here in different examples. When it comes to the yield being the same to be coupon rate, then the bond is traded at face value itself, the price of the bond is going to be the same.

And with such scenario analysis we can understand, the value of the bond depends on yield and coupon rate. So, coupon rate are typically predefined yield might change depending on the tenure or the time to maturity and other characteristics of the bond. Particularly, yield is predefined for most of the cases because yield represents the underlying risk and the other economic characteristics of the bond.

(Refer Slide Time: 28:19)



**CONCLUSIONS**

- There is an inverse relationship between (prevailing) interest rates and bond prices.
- Yield to maturity (YTM) can be assumed to be the internal rate of return (IRR) earned on a bond held till maturity.

The slide includes a video inset of a man in a light green shirt speaking, a navigation bar with icons, and logos for IIT Bombay and NPTEL.

In case of bond, we know that there is an inverse relationship between the interest rate and bond prices. And for better understanding of valuation of a bond, we can assume that YTM or yield to maturity is the most commonly used indicator as it represents the internal rate of return earned on a bond if it is held till maturity. And with this, I wind up this session.

Thank you very much.