Project Management: Planning, Execution, Evaluation and Control

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Welcome to the course Project Management Planning, I am Professor Sanjeev Choudhury from Indian Institute of Technology, Kharagpur. Today, we will be continuing the module 6, Developing Project Plan and this will be the last lecture on that. Here we will be covering probability of completions of project on time. Let us move to the concepts that will be covered. All these we have shown before, these we have covered in our previous 3 lectures. Today, we will be discussing about uncertainty in activities, then introducing of 3-time estimates, then probability of completions of project on time.

So, to start with to give you the certain information that what we have covered in the last 3 lectures, we have taken a deterministic time that is one time estimates which are generally used in critical path method or the CPM. But in part program evolutions and review technique, we use probabilistic time because you the time estimates are not accurate and you do not have the past experience to get it. So, here we introduce 3-time estimates, those 3 times estimates we have discussed are optimistic time, probabilistic time, pessimistic time, pessimistic time and most likely time. Generally, it is found statistical distributions of these is called beta distribution.

We use statistical beta distribution to find an estimate the likely time, time estimates for this type of activities. So, the expected durations, what is the expected durations of this timing? The expected durations are for this for this beta distribution, it comes say t e that is the expected time duration is a plus 4 m plus b divided by 6. What is a? A is the optimistic times and b is the pessimistic time and m is the most likely time. So, and you also find out variance or the standard deviations, we call it variance is will be b minus a divided by 6 square and standard deviations is b minus a by 6. This we will be using for the 3-time computing, the 3 time estimates this, this will be very useful to you.

Let us now see the these 3, these also let us discuss before going. So, what we will be doing it now as you have seen, we have discussed 3-time estimate, then average activity time that

we have estimated, standard deviations for each activity we have we have shown it in the previous slide, but the standard deviations for the project is different. You know this is the this is the distributions for the activity that follows a beta distribution. We have found out the standard deviations for that average activity time also in that in the last in this slide, but the standard deviations of the project are different thing. Its standard deviations of the project follow a normal distribution.

More on these why it follows a normal distribution, we will discuss more on these after a one or two slides. So, we will be in the next slide, we will be talking about standard deviations for the project. These will give you the probability of, but you remember the SD standard deviations of the project is a normal distribution. So, now we will be finding out what is the probability of completions of project on time. For that as a decision maker or as a project manager, sometimes you want to know this question what is the probability of completions of a project on a specified date or specified time period.

So, how do you will you find out, how will you answer this for that we will be requiring some assumptions and these assumptions are based on scientific basis. So, this all of us know that this assumption is first you make it that the we have seen that independent random variables, if there are an independent random variable these are so, the project activities the activities we are talking about those work packages. Activity time is independent random variables because any activities time are not dependent on the other activities times and all it is independent random variables. So, and another is that what is a critical path? Critical path is a summation of the summations of the time of the critical activities. This critical activity are independent random variables, then the summations of these random independent random variables will also be a random variable.

So, project completion times will also a random variable. Now we also know this probability that central limit theorem of probability says that an independent random variable, when the summations of these independent random variables tends to infinity that is n tends to infinity, then these they tend to approximate as a normal distribution. So, a large project having lot many critical activities, when n tends to infinity or n tends to a large number. So, it will be it is the project completion time will also follow an approximated normal distribution. This is assumption is based on central limit theorem of probability.

So, now look at the normal distribution a project completion time follows a normal distribution. A normal distribution is characterized by a mean that is mu and standard deviation that is dispersion sigma. We all know a normal distribution has mu mean and plus sigma minus sigma it and the and the areas under this curve represent the probability that is say we will come to that z value later, but we know the areas covered under mu plus minus

sigma is 62.2 percent and the areas covered mu plus minus 2 sigmas' are 95.44 percent and mu plus minus 3 sigma covers 99.

74 percentage of the areas. So, what we will be doing we must need to find out the z value, how do you get the z values. Now, suppose this will be covered will be clearer when we solve a problem a numerical will be giving and solving it. Say how do you find the completions of the project on time for that you will be requiring what we will be requiring we will be requiring a specified time units T s that is time unit that is time is specified durations specified time specified time that is given. Then T e that is expected time estimated time for completions estimated time for completion time for completion time for completion project completion.

Then we wanted to find the z value what is z? z value this z value is the, but it is nothing, but number of standard deviations standard deviations from the mean from the number of standard deviations from the mean. So, what happens what we get z equal to time specified time minus expected or estimated time for completions divided by root over sigma t e i square that is that is the z value because why do you take the sigma square because sigma is non-additive sum total you know if you plus sum total of error becomes 0. So, we find out root point square error no similarly here the variance is additive. So, you take variance square sigma is you cannot add. So, now to find out probability of z probability of z is less than equal to t specified minus what is t e? t e is your critical path duration critical path duration.

So, what is its critical path duration is also is your here equal to mu. So, we can write it t s minus mu by sigma ok p z probability of z you have to find out this z value probability of z is time specified to you minus critical path duration that is mu divided by sigma of what sigma is the variations of critical activities those variations you take variance of critical activities critical activities not the non-critical activities this you must keep in mind you can find it out. Now, we will be solving a problem that will this problem we have to solve now this will clear all of your doubts. So, go through it the activity times of a project the activity times of a project is given below like 1, 2, 2, 3, 2, 4, a is the optimistic time, m is the most likely time, b is the pessimistic time. So, compute the expected time and variance of activities you have to compute it how do you compute it we have got the formula beforehand know we have derived the formula this was the formula we have derived t expected time a plus 4 m plus b by 6 and variance is b minus a by 6.

So, we have the formula. So, based on this we will be so, what is the 1, 2 duration this duration will be 7 ok 17 this is 17 plus 4 into 29 plus 47 divided by 6 know. So, what is how much it comes it comes I think 30 30 and what is the variance comes variance is you write it

as sigma square variance comes is sigma square is will be coming how much 47 minus 17 divided by 6 squares how much it comes 47 minus 30 30 by 6 is 5 it is 25 oks. Similarly, you do for 2 3 2 4 and all what will be the this is for this then this will be how much say 6 plus 4 into 12 plus 24 divided by divided by 6 how much does it come 2 3 it comes how much 13 hm 13 and sigma square comes how much here sigma square will come 24 equal to 24 minus 6 divided by 6 square equal be 18 by 6 3 3 this will be 9 ok. So, similar way you can find out each and every activity then you have to so, have to compute the expected project durations determine critical path showing early start early finish then late start late finish and slack finish for each activity we will be doing it ok.

So, this we have part a is done here we have we have done this say 30 30 25 13 these we have the expected time and these are the variance we have found out. Now, next what we will be doing we have to compute the compute the critical path and find out the things how do we compute the critical path we have 1 2 this is activity on arrow this is not activity on node this is activity on arrow. So, control activity on arrow. So, how do you do that we do say compute it 1 2 2 3 4 4 3 4 then 5 1 2 2 3 2 4 4 5 then 5 6. So, you have drawn the network now this is activity on arrow.

So, 1 2 this where a what is the we have found out we have to find out estimated time estimated time for 1 2 was 3 and variance was 25. Similarly, 2 3 we found out 13 and variance was 9 similarly if you compute we have found that we have computed it 20 and variance is 4 3 5 is 16 and variance is 1 then it is 6 4 5 6 and variance is 4 5 6 it is total time estimate is 5 and variance is 1. So, now the problem says we have to find out what problem says compute the expected project duration and determine critical path showing early start early finish and all those. So, we have to find out the early start early finish of this 1 2 early start is 0 early finish will be 30 for 2 3 early start is 30 early finish 30 plus 13 43 then this 2 4 early start is 30 then 20 50 50 is the early finish then this 3 5 is early start is the 43 early finish 43 plus 16 is 59 and this 4 5 early start is the 50 and early finish is 50 plus 6 56 then 5 6 ah 5 6 early start is 59 plus 5 days is 64 ah. Now you go for the backward pass backward pass it is 64 minus 5 59 slack is 0 now 59 is the early late finish late start the late finish and minus 16 is 43 late start is 43.

So, this becomes 0 now here what is the late finish of this is late start of 5 6 late start is 59. So, what should be the late start of 50 will be late start of 4 5 will be the minus 4 it is how much 50 no it is minus 6 59 minus 6 is how much 53 59 minus 6 is 53. So, what is the slack is 3 59 minus 56 or 53 minus 3. Now coming to this one this is 43 is the late start of this is 43. So, late finish is 43 minus 13 30.

So, what is the slack is 0. So, similarly what is the late finish of this is 53 because this now minus 20 it becomes 33. So, what is the slack? Slack is 3 here now this 30 and 33. So, this is a burst activity 1 2 is a burst activity 2 3 and 2 4. So, what should be the late finish of this will be the minimum of late start 30 and 33 minimum of 30 and 33 is 30.

So, then it is duration is 30. So, it is 0. So, what is the slack is 0 now which one is the critical activity is having 0 slack. So, critical activities are 1 2 this is 2 3 2 3 then 3 5 is also a 0 slack 3 5 0 slack and 5 6 these are 0 slack and this is having slack 3 and 5.

So, slack. So, critical activity critical path is what 1 2 2 3 3 5 5 6 and duration is duration is how much duration is 64 days or time units and this is non-critical. Now, the what are the now this is the we found out you're that now the we will go to the part c these are non-critical activities. So, part c if we go. So, we have computed we have computed part 1 and part 2 both part 1 we have completed part 2 also we have done for slack for each activity and now we will be doing the important one what is the probability of completing the project by 67 days and within 60 days.

So, we have to calculate this. So, probability of completing project by 67 days 67 days is the specified time given to you and 60 days specified time. So, what how will you do that we have we have calculated it know that t specified minus t estimated time of completion you get z value. So, we have derived this. So, let us go this and we can solve this ok. So, what is the z value we have to find out we have to find out z value probability of we have seen probability of z value is less than equal to t specified minus t estimated divided by divided by root over sigma t i square this now we have we have already known t s is the specified time unit specified time of completion time of completion that is that is given as in this problem 67 days to e is the estimated project completed estimated time of project completion estimated time of project completion.

What is it? It is the critical path duration critical path duration how much we got it we got it is 64 days here that is the equal to mu that is mu and sigma square that is the variance you will take only which are on the critical path which are on the critical path equal to sigma this is this is sigma this is this is standard deviation sigma square t e t i t i square. So, only that critical activities it is a critical activity you take sigma square of critical activities. So, what is the critical activities we have found critical activities are 1 2 2 3 3 6 5 6 this is variance at 25 plus 9 plus 1 plus 1. So, sigma square of critical activities is variance of 25 plus 9 plus 1 plus 1. So, equal to how much 36 roots over 36 equals to 6 we have found it out now let us go to here.

So, what is the z value? z value is, T is 67, T is given 67, so 67 days minus 3, is 64 days that is mu, that is we got it from the critical path. Now sigma is sigma square that it is we have T we have got it root over this is 3 by root over 25 plus 9 plus 1 plus 1 last slide we did it equal to 3 by 6 equal to this is equal to half that means, point 0.5. Now it has z value is 0.5 what does it mean? It means that it is it is 0.

5 standard deviation from it moves from the mean 0.5 is somewhere here from the standard deviation it moves from the number of standard deviations from the mean. So, we have to find out the 0.5 here and the areas under the curve areas left under the curve left to that points left to that z value will be the probability.

Suppose z value was 1.20 this is here above the mean. So, left areas left to that at the tail is the z value now here our z value is 0.5. So, we have to look at the what we have to look at the that table z value is 0.

5. So, look at the table for the for the z value table. So, how do you look at it? Here is the say standard normal distribution table value to the left of the z score here it is from minus 5 9 3 9 this is 0.01 0 2 0 7 it is given. So, you come to the z value for 0.

5 this is the positive one. Here z value 0.500 is 0.69146 we have seen that this is the 0.69146 z value is the is coming 0.69146. So, your we will go to the here we this is probability of z probability of z is 69.

14 percent. The probability that the project will be completed within 67 days is 69.14 percent. Now another is given that z value with probability of completing in 60 days. So, what you will become z equal to 60 days minus 64 divided by sigma square that is 6 equals to minus 4 by 6 equal to minus 0.

67. Now what is the probability of z 0 7? We have to find out from the from the normal distribution table. So, normal distribution table if we go now look at its normal distribution table.

So, these are 0.1. So, this is minus 0.67. So, you go to the minus 0.67 ok 7 is 0.6 will be there 0 7 is this third row from the left yeah points minus 0.

67 is this one 0.25143 0.25143. So, we go to the let us go to here. So, how much it is? So, we can we can tell probability of minus 0.067 is 20 how much was it? 25 is 25.14 percent. So, probability that it will be completed in in 60 days is only 25.

14 percent it is poor. So, this way we had now completed this this this part what is the probability of completing the project by 67 days and within 67 days. So, we have solved this problem 2 ok. So, these are the these are the we have completed these also you can find out 0.5 the normal distribution from the and minus 0.67 also find out from the from the excel sheet excel table say I will just show you this how do you do it in excel say in excel suppose the probability of z no you are doing probability of this.

So, what probability p probability ok capital p probability of z probability of z how will you find out here equal to normal distributions you go no norms distribution ok into and give the z value z value was 0.5 hm 0.5 say you got 0.69142 we have got that and another p z was another p z was point equal to norm norms distribution another was hm sorry minus 0.

67 no. So, how much it is it is 25.1429. So, you can also calculate it from the excel sheet. Now coming to this we are we will be ending it we have completed the problems now to summarize what we have learnt today is further to the last lecture this chapter illustrates the uncertainty in activities such as new product development R and D work etcetera. It introduces three time estimates namely optimistic pessimistic and most likely times which are generally used in program evolution and review technique. It further determines the probability of project completions on time to assist project managers and senior management to take decision and plan accordingly. These are some of the reference book you can consult to and enhance your knowledge further.

So, thank you. So, these are the entire reference books. So, thank you very much for attending today's lecture.