Commodity Derivatives and Risk Management
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Lecture 16
Pricing of Futures(contd.)

Welcome to the 16th lecture on Commodity Derivatives and Risk Management. In the previous session we were discussing the pricing of futures contract, and we will be continuing with the pricing of futures contract in today's session also. And let us recollect what we had discussed in the previous session. As you remember in the previous session, we had discussed that the commodity underlying as well as financial underlying futures contract for commodity underlying, and financial underlying can be calculated using the cost of carry model and we had taken some examples of financial contracts as well as some commodity contract. Now, let us revisit the same concept with respect to two commodities, which are silver and coriander.

And this data as I have said I have taken from the MCX website and the spot price for silver is 71,785 rupees per kg prevailing on 19th May 2023 and the near month contract which is maturing on 30th June 2023 has a contract maturity for 42 days. And storage charges per day is 1 rupee 15 paisa as mentioned in the you know website as mentioned in the warehouse details given by the multi-commodity exchange. As you can see, the storage charges on daily basis for 1 kg of silver is 1 rupee 15 paisa up to 90 days and it is 1 rupee 50 paisa 91 day and above. So, factoring into 1.15 rupees of storage charges per day and that leads to the total storage charges of 48.3 and using your cost of carry based model we are able to find out what should be the theoretical price. As you can see, the theoretical price that is cost of carry based model price when I am meaning theoretical future price means this price is derived from cost of carry model. So, the near month price ah cost of carry based model price is 72,384, 73,315 and 74,680. Similarly, for coriander the spot price is 6,688 days to maturity is 31 because the contract matures on 19th June both these contracts have the same spot price, but the coriander contract the near month contract is maturing on after 31 days and the storage charges again is mentioned in the warehouse detail of the exchange. So, it is 85 paisa per day. So, factoring in all this you know detail we have been able to calculate the theoretical future prices of 6,752 and 6,817. Of course, the assumption here is that continuous compounding rate per annum has been assumed to be 6.639 percent and number of days in a year has been assumed as 365 days. So, with this information, we have been able to calculate the theoretical future price.

Now coming to the most important question, does the theoretical price hold true in real life? The model helps us to calculate or derive a price. In reality, does the actual price is the same or closer to it or significantly different than it? As you can see, these are your theoretical you know future price and these prices are different than the theoretical price. And this leads to an interesting discussion related to the ah cost of carry model and if the actual price is different than the theoretical future price one would be able to make some arbitrages benefit out of it and that arbitrage benefit we will be discussing in the next slide. Now let us go to what do we mean by arbitrage benefit. Arbitrage is a concept where we ah arbitrage is a concept which indicates one can make a risk less profit and this can happen when somebody buys low and sells high.

Let me repeat one can make arbitrage benefit which is a risk less profit by buying something at a low price and selling at a high price. Now going back to our understanding that the actual price is different than the cost of carry based model price, that gives us an opportunity to make some arbitrage benefit. So, let us go to the left column of this table, let us say $F_{(actual)}$ that is the future price prevailing in the market which is the actual price is greater than $F_{(theoretical)}$ or cost of carry based model price. If this happens then we will be able to undertake an arbitrage opportunity which is known as your cash and carry arbitrage. Let me repeat if the actual future price is greater than the theoretical or model best price one would be able to undertake cash and carry arbitrage.

So, what do we mean by cash and carry arbitrage? This cash and carry arbitrage will also be following the same principle of buying low and selling high. So, that means, where one will be buying and where one will be selling. Please note that the actual future price is greater than the theoretical price. So, if this situation happens somebody will be able to undertake arbitrage benefit and to get the arbitrage benefit somebody will be selling the futures contract. Please note that the future price actual future price is higher.

So, that party will be entering into a futures contract or sale futures contract. Simultaneously the same party will be borrowing the spot price and buying the underlying. Basically, borrow spot price buy underlying and sell the futures contract. This particular strategy is known as cash and carry arbitrage and as you can see this is a English meaning cash and carry indicates borrow money buy the underlying buy the cash asset carry it till maturity and simultaneously enter into the sale futures contract. And this is going to be the strategy on day 0. What is going to be the activity related to the arbitrage opportunity on the futures contract maturity date? Because the party has sold the futures contract that party will be delivering the underlying, receive the $F_{(actual),}$ the future price and return the S_0 with interest. So, this is going to be the cost of carry arbitrage. There could be the reverse situation where the actual market price will be less than the model-based cost of carry model-based price. That will give rise to an arbitrage opportunity which is known as reverse cash and carry arbitrage. So, in reverse cash and carry arbitrage what a trader will do? Traders will borrow and sell the underlying asset at

S_0. It will borrow the underlying commodity from somebody and sell that same commodity in the spot market and will receive S_0. And lend or invest S_0 for a period equal to the maturity. Of the future contract and simultaneously buy the futures contract. So, this is going to be the strategy on day 0. Simultaneously on the contract maturity date because this person has invested S_0, he will get S_0 with interest.

And he has entered the buy futures contract. So, he will get the delivery of the underlying you know underlying asset and pay the future price pay the actual future price. And simultaneously he will return the underlying asset with the fee. Please note that he had borrowed the underlying asset from somebody and had agreed to return the underlying asset with the fee. So, this part he will be returning the underlying asset with a fee.

And this will result in a financial benefit this will also result in a financial benefit. Now, let us take some numerical examples to understand the cash and carry arbitrage and reverse cash and carry arbitrage in real life situation. Let me revisit this table as you can see the actual price for silver price is much higher than the theoretical future price in all three cases. That means, there exists a cash and carry arbitrage for the silver. However, when we are coming to the coriander, please see the actual price at NCDEX is much lesser compared to the theoretical price.

Hence this kind of a situation is I mean able to reverse cash and carry arbitrage. Now, let us go to the you know actual ah situation how a particular person will be able to do a cash and carry for silver and reverse cash and carry for coriander and these two-arbitrage strategy is given in these two blocks. As you can see the spot date is 19th May 2023, we are able to find out that the actual price for silver is actual price for silver for a contract which is maturing on 30th June 2023 is 73,285, but the theoretical future price we are calculating as 72,384. As you can see, this is a significant difference, and this gives an indication to us that we will be able to make arbitrage benefit and the nature of arbitrage is going to be the cash and carry arbitrage. So, in cash and carry arbitrage what we do it on the day 0 which is the spot date or 19th May 2023.

We will borrow 71,833 which is nothing but your spot price plus the storage charges, buy 1 kg of silver and store it for ah for 42 days and simultaneously enter into a short futures contract for 30th June 2023 expiry. Now, on the expiry date which is your 30th June 2023 we will deliver 1 kg of silver as part of the short futures. We will receive 73,285, which is the actual futures price market future price. We will return 72,384, which is the actual futures price market future price. We will return 72384 for the amount which we borrowed and the interest cost and as you can see, it will give us a benefit of 901 rupees per kg.

So, this is an example of how if the actual price is mispriced as compared to the model-based price somebody will be able to make cash and carry arbitrage. Without

going into the actual nitty gritty of ah the reverse cash and carry I would like to draw your attention that you know benefit for the coriander ah reverse cash and carry is going to be 116 rupees per quintile and this table is self-explanatory I am not going to let us not ah spend a lot of time ah going through this detail ah at this point in time at your spare time you can go ah and exactly see how the reverse cash and carry arbitrage has been undertaken. Now, the question is, is this cash and carry arbitrage and reverse cash and carry arbitrage always possible to be exploited? The answer to this question is only if the price difference between the theoretical future price which is model-based price and actual ah future price is significantly different then only we will be able to get an arbitrage benefit. If the benefit is small, we may not be able to get the arbitrage benefit. Now, let us understand what we mean by a significant difference in the model-based price and the actual price.

This diagram shows an upper limit and lower limit for no arbitrage bound as you can see, the a is the upper limit and b is the lower limit and if actual future price remains this range, then we will not be able to make some ah we will not be able to do any arbitrage activity. Let us take an example here let us say in the spot market the market price is spot price is 100 and future price is 103. So, just by merely looking at these two price points somebody may be able to think that there is an arbitrage possible, but whenever a trader is buying the spot because arbitrage relates to buy low and sell high as the spot price is lower than the future price let us say trader buys the spot. And when the trader is buying the spot, so trader is not only paying 100 rupees the trader is also paying certain transaction charges. Hence the total payment of the trader is 103.

Similarly, when the trader is entering into sale futures contract, he will be selling the futures contract at 103, but simultaneously he will be paying let us say 3 rupees as transaction charges. So, the total receipt is going to be 103 he is receiving 103, but he is paying the transaction fee of 3 rupees. So, as you can see, the total payment was 103 and the total receipt was 100. So, that nullifies any arbitrage benefit. So, this table indicates that the arbitrage benefit must be the price difference between the theoretical future price and the actual future price should be significantly higher.

And please note that the rate at which different traders will be borrowing money and lending money, that rate is also different. Hence the no arbitrage price is not an individual or a single price no arbitrage price follows a range. So, this a is your upper bound related to the no arbitrage price which is governed by your $S_0 * e^{r(borrow)}$continuous compounded rate for borrowing rate plus the transaction charges. So, this gives the upper limit and $S_0 * e^{r(lent)}$, lent means lending rate into contract maturity period minus the transaction cost is going to give us the lower bound. And if the actual price is above this no upper bound then only, we will be able to make a cash and carry arbitrage.

Similarly, if the actual price is less than the lower bound then we one will be able to undertake the reverse cash and carry arbitrage. Hence what I am deriving it is that actual future price may be different than the ah theoretical price or cost of carry model based future price, but the price difference must be significantly different enough for traders to undertake arbitrage. And if the price difference is significantly different arbitrage will be entering into the market and bring the price to this range or a future price will be trading in such a range that the arbitrage range that there is no arbitrage possible. Now, how this will happen? Let us go back to please note that if somebody wants to do a cash and carry arbitrage on day 0 that trader will be borrowing money and buying the underlying and selling the futures contract. Please note that if everybody is private to this information that there exists arbitrage opportunity everybody will borrow money everybody will buy and store the silver and the moment when everybody starts buying or many people starts buying silver the silver price is going to go up.

And when people entering into a short futures contract so, many people will be entering into a short futures contract and in that case the future price will go down. So, when such kind of an arbitrage opportunity significant arbitrage opportunity is there, it will not be available for long there will be arbitrages who will be entering into different kind of contracts such that the arbitrage benefit is taken away very quickly. With this let us understand of the third category of investor. Please note that a couple of sessions ago we spent a considerable amount of time trying to understand who a hedger is and who is a speculator. Here with the cash and carry arbitrage and reverse cash and carry arbitrage we are able to know that there is another category of investors who operate in the spot and futures market, and they are known as arbitrage.

And please note that arbitrages take position in both spot and futures market to make risk less profit. Even though they take spot market position they are not commodity producer consumers or value chain partners, or they are not hedgers. They take position in the futures market, they take position in the spot market, but they are not hedgers. So, this is a very important understanding one must have that in case of commodity derivatives market you have three categories of investors, one is your hedgers, one will be speculators and the other categories will be the arbitrages who will be taking position in both spot as well as futures market. And arbitrages ensure that the cost of carry model holds true for pricing forward or futures contract.

And this cost of carry model holds true for financial asset, this also holds true for investment commodities like gold and silver, it also holds true for consumption commodities, but without any supply constraint. If any consumption commodity does not have any supply related constraint or freely available, then the cost of carry model will be holding true. The future price will be closer to the actual price will be closer to the cost of carry based model price. But the cost of carry based model will not be holding true for any supply constraint, it will hold true when the underlying is going to be consumption

commodities but facing significant supply constraint. Let us understand why cost of carry model will not hold true for consumption commodity are facing supply constraint.

And please recall in case of a coriander the theoretical price is very high, it is very much smaller as compared to the actual price sorry the theoretical future price is much higher compared to the actual price. Now, why this will happen or when this happens as we know a spot price will be greater than futures price, we call this one as a backwardation market. And when backwardation market happens there is a clear caught indication that there is arbitrage opportunity somebody will be able to undertake a reverse cash and carry arbitrage. And what do we mean by reverse cash and carry arbitrage? In case of a reverse cash and carry arbitrage, somebody must borrow and sell coriander in the spot market and simultaneously take long futures position on day 0. This is a clear-cut case for reverse cash and carry arbitrage as you can see the theoretical future price is significantly higher than the actual future price or actual future price is significantly lower than the theoretical future price.

In fact, the actual future price is much smaller than the spot price. Please recall in normal circumstances the actual future price should be more than the spot price because the spot price is positive, interest rate is positive and time to maturity is positive. So, with all three positives underlying the future price the actual future price should be more than the spot price. But in this case, as you can see the actual future price is less than the spot price. Now why this happens and why people are not able to arbitrage away this benefit.

This is not possible, or one is not able to take arbitrage because the coriander may be in short supply. And as we have discussed taking arbitrage or to get the arbitrage benefit, one has to borrow and sell coriander. So, today standing you know on 19th May 2023 somebody may be able to borrow and sell coriander in the spot market take long futures to get the arbitrage benefit. But when he or she is wanting to borrow coriander from somebody there is nobody who is willing to lend coriander. Why because the inventory is low and when inventory is low the demand for commodity outstrips the supply.

And commercial users who are holding the commodity prefer holding the physical commodity rather than lending the commodity to arbitrage to generate some financial benefit. And this benefit which a commercial user of the commodity is benefiting by holding the inventory of coriander that is known as your convenience yield. So, a manufacturing company which uses the commodity as a raw material would prefer to hold the physical commodity as physical inventory smoothens the production process helps it maintaining the market share and these intangible benefits are coined as convenience yield. So, what is this convenience yield indicates that even if somebody is wanting to borrow coriander and sell coriander in the spot and takes long futures to get

arbitrage benefit there is nobody who is willing to lend coriander. Whoever is holding coriander they are holding the coriander because they have some use for it, they are maybe you know ground spice manufacturing company.

If they are going to lend coriander to some arbitrage yield, then their business is going to be negatively affected and they are going to lose market share. So, even if they have coriander, they do not have surplus enough coriander to give it to somebody just. So, that somebody else will be able to get some arbitrage benefit and they will be able to get some lease rental. So, people who are holding the coriander are benefiting by holding the coriander and that benefit is major that benefit is known as convenience yield. With this, we will come to today's lecture session on the pricing of futures contract and if any miss price happens how cash and carry and reverse cash and carry is used by arbitrageurs to minimize the price difference between the cash and carry based minimize the price difference between cost of carry based model price and the actual price.

The cash and carry and reverse cash and carry model holds true for financial assets holds true for consumption assets or consumption commodities holds true for investment commodities holds true for consumption commodity without having supply constraint. But this arbitrage opportunity cannot be taken away when the underlying consumption commodities are facing some supply constraint. With this we will come to the end of this discussion the remaining part of the pricing of futures contract we will be discussing in the next session.