Commodity Derivatives and Risk Management
Prof. Prabina Rajib
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur
Week-05
Lecture 25
Pricing of Commodity Options

Welcome to the 25th lecture on Commodity Derivatives and Risk Management. And today we are going to discuss more on Pricing of Commodity Options. So, how anybody would go about deciding what is the long call, short call, long put, short put premium to be paid and received. Just to recapitulate which we had discussed in the previous session, please note that the call or put premium is a function of 5 under 5 factors. These are the spot price of the underlying asset, strike price, time to maturity, underlying asset volatility and the interest rate. In fact, in the previous session we discussed in detail how each of these parameters will be influencing the call premium and put premium.

Now, with this understanding let us go and understand a little more about the Black-Scholes option pricing model. This model is used to calculate the call and put premium for European options. Please note that in the year 1973, 3 professors, Professor Fisher Black, Myron Scholes and Robert Merton came up with the option pricing model and this particular model is popularly known as a Black-Scholes model. And they came up with a research paper titled pricing options and corporate liability in the journal of political economy which was published in May 1973. And in this particular paper, they gave a closed ended formula for calculation of option premium and that is known as your Black-Scholes option pricing model. And as you can see, this is such a path breaking and such an important moment for anybody who is interested in learning about derivatives that in the year 1997, these 3 professors were awarded the Nobel Prize. Of course, by that time Professor Fisher Black was no more, hence the Nobel Prize was awarded to Professor Myron Scholes and Robert Merton. And this is the particular newspaper article which was published in Wall Street Journal on 15th October 1997, and which mentions about the Nobel Prize and as you can see, the part of the Black-Scholes option pricing model is mentioned here in this particular newspaper article. And this newspaper article is available at this link, if you want to read more about this particular article it will be available in this in this link. Now coming to the Black-Scholes option pricing model as you can see this particular closed ended Black-Scholes option pricing model C stands for your call premium, P stands for your put premium, S 0 stands for your spot price, X stands for your exercise price, R is your risk free rate of rate, T is your time to maturity or exercise duration and sigma is your underlying asset volatility. As you recall, just now we discussed there are 5 factors which influence the price of call option or a put option. So, these 5 factors are the spot price, exercise price, risk free rate of interest, time to maturity

and underlying asset volatility. So, what is this C? C is your [$S_0N(d1)-Xe^{-rT}*N(d2)$]. As you can see this N(d1) and N(d2) and N(-d1) and N(-d2) are nothing, but the probability values from the standard normal cumulative distribution. We will not be going into the exact derivation of this particular formula. Again, those who are more interested to go and understand from the first principle onwards the derivation related to this particular formula you can go and read the respective chapters from the John Hull book which is futures options and other derivatives. Let us price a European option using this particular formula this exercise I have done using a excel calculator. Let us see our discussion related to the you know branded atta manufacturer, if you recall, we were discussing this particular company which wants to enter into a long call option on wheat and it had agreed to pay a premium of 2 rupees. In fact, in the previous example I just gave the 2 rupees of a premium without discussing anything about how exactly this premium has arrived. One may think that I have just pulled the number from thin air it is correct I just pulled the number from thin air and gave for the sake of giving an example, but in real life both long call and short call and long put and short put option holder will be using the black source option pricing to arrive at the arrive at the call premium as well as the put premium. So, how exactly this will be done let us take a numerical example. So, the day BAM limited wants to enter into a long call option that is spot price of wheat is let us say 28 rupees, exercise price it wants to enter into long call at 23 rupees as an exercise price, days to expiry is 36 days and wheat return volatility, please note we have to calculate the wheat return volatility which is 21.70 percent. How exactly we will calculate the wheat return volatility, I will be showing it in this excel calculation and for the calculation of the European option we also require the risk-free rate, and this risk-free rate has to be converted to continuous compounding risk free rate. Please recall in the initial part of the lectures in one of the sessions we have done extensive discussion and done some numerical exercises related to converting a discrete compounding rate to a continuous compounding rate. So, the applicable continuous compounding rate in this case is 7.30 percent per gram. Again, in real life this data has to be actually arrived from some kind of a you know some kind of data available here this 7.30 percent I have just taken for the sake of example. And let me repeat here, this is the formula for the black Scholes option formula for the European options just now we discussed the meaning of these notations C and P stands for call premium and put premium and rest of the other 5 factor just now I have discussed. Now, let us go to the excel model to see how the same thing can be calculated. Please note that what is available to us without doing any other calculation is your spot price. What is negotiated or agreed upon by both parties is this exercise price. Days to expiry, also both parties know when they would like to exercise the option. So, normally it depends upon the business requirement. Let us say BAM limited wants to buy within 36 days 35-36 days later hence the option expiry is going to be 36 days, but what is not available to us is the volatility associated with the underlying asset. So, we need to calculate that particular volatility normally the same thing is calculated from the

historical data. So, wheat price data will be used to arrive at the volatility associated with the wheat return. So, let us go to the excel calculation. As you can see, I have just taken some 10-12 days of data and I have taken some random price of the wheat. So, wheat per kg in real life when you will be doing this exercise you will be needing to have access to the actual wheat price from some Monday or some authentic sources. Let us say the wheat price is given and from the wheat price we will be calculating the daily return daily return will be calculating based on the continuous compounding return that is your ln pt by pt minus 1. This aspect also we have discussed in earlier sessions as you can see based on this, we will be able to the formula for a calculation of standard deviation volatility is nothing, but the standard deviation of the return series. So, volatility we have to calculate on an annualized basis. So, whatever the standard deviation we are getting we are multiplying with the square root of 252 to get them to get the wheat return volatility. So, in this case we are getting 21.7 percent. Please note that without even changing the initial price and the last price I have changed some in intermittent price just random change I have made, and you can see that the wheat volatility has gone up significantly by 74.2 to 74.8 percent. So, let us for the model building we will be taking this volatility as 21.7 percent. So, other factors as we have discussed the spot price of wheat is 28 rupees, exercise price is 26 rupees, exercise price is 23 rupees, exercise date in is 36 days and we need to annualize it and exercise date in years is going to be 36 divided by 365 or which is your risk-free rate continuously compounded risk-free rate is 7.30 percent. The sigma which is your underlying asset volatility is 21.7 percent with this rest of them these details which is mentioned here are basically your calculated values as you can see for this particular underlying asset call premium should be 5 to 5.16 rupees and put premiums would be 0.0007 rupees. And as you can see why call premium is higher than your put premium, please see that the spot price is much higher than the exercise price. In fact, if I change this one let me change you know to let us say I am making it to 24, if I make it to 24 as you can see the call premium is going down and put premium is increasing. In fact, I can change this numbers to let us say I am making it to 72 days as you can see that currently call premium is 1.38 rupees and put premium is 0.21 rupees and let me make it to the exercise date as 72 rupees as you can see the call premium is also increasing and put premium is increasing. Similarly, if I change one of these figures let us say I am changing this figure to let us say I am making it 29. So, when I am making this figure to 29 you can see significantly the volatility is increasing to 89.3 percent and as you can see when the volatility increases both call premium and put premium has it is going up. So, this particular excel file will be available to you to play along and change different parameters to see how these factors are influencing the call premium and put premium. Let us spend a minute on this risk-free rate of return if you recall we had discussed that if the risk-free rate of return increases call premium will be going up and put premium is going to go down. Let us increase this one to please to 8 percent please remember that this is 4.37 and 3.04 let us make this one to a significantly higher let I am

making this one as let us say I am making it as a 9 percent. So, if I am making this one as 9 percent. So, as you can see it is call has gone up 4.40 and put premium has gone down. So, with this we will be ending the discussion related to the calculation of call premium and put premium using a black souls option pricing model as I mentioned this particular excel file will be available to you for you know practice and change these numbers, but let me also highlight that in real life when somebody will be actually wanting to know the call premium and put premium for a traded asset this underlying asset volatility has to be calculated based on the actual figure not the you know not the theoretical or random numbers which I have put it just for the sake of calculation. So, from here as you can see, we use this particular formula to arrive at the C and P and this call premium and put premium is going to be the theoretical premium and actually actual buyers and sellers of call and put will be basing their expectation related to the call and put premium based on this particular black Scholes option pricing model. Please note that this black Scholes option pricing model will be able to evaluate or help us in getting the call premium or put premium for options which are European in nature, but if the options are American in nature where the long call and long put position holder will be able to exercise the option much before the maturity, those options cannot be valued using your black souls option pricing model that those options will be valued using something called a binomial options. Again, as I have mentioned earlier the scope of evaluating this particular binomial option is beyond the discussion related to this particular subject. If any one of you is interested to again learn more on how binomial options are used to calculate the American option long call and long put premium or call premium and put premium, I would again advise or encourage each of you to read the book on John C. Hull futures options and derivatives. Now please note that there are lot of option calculator free options calculator available on the internet and you can use that to arrive at the option a premium and this is again out of many of these option calculators I have just randomly taken this option calculator the details are available in this particular link. So, I have just taken the European option the price some random price of spot price is 1225 strike price is 1200 and other parameters like days to expiration 90 days volatility underlying asset volatility is 25 percent interest rate is 6 percent and please note that this is a calculator which requires us to give the dividend amount which is applicable to equity with the dividend as underlying asset. When I filled in this detail as you can see the call premium is 80.2108 and the put premium is 42.4693 and it exactly the same data as you can see the call premium for American option is 80.2716 and put premium is 43.7582 and as you can see in case of the American option the call premium is higher than the European option. Please note that even the calculator looks like that they have used the same method no American option can be valued using a Black-Scholes option pricing model. So, in this particular this particular website must have used the binomial option pricing model as the background as the back end to arrive at this American option call premium and put premium. And there are also as you can see something called delta, gamma, theta, vega,

rho, these are called your option Greeks. Again we will not be discussing this option Greeks and the relevance of this option Greeks for option premium and option pricing these are very important, but this is not the scope of this particular subject if any one of you again are interested to know what is the interpretation and what is the relevance of these option Greeks you can again go through the textbook on John C. Hull. Now with this, we come to an end with our discussion related to all things on options. Now coming to the most important aspect, which is your commodity options, please note that the commodity options do not have commodities spot as the underlying, but commodity futures as underlying. So, the Black-Scholes option pricing model or the binomial option pricing model what we just discussed is related to the option pricing when the underlying asset is a spot asset, but in case of a commodity options the underlying are not commodity spot, but the underlying are commodity futures. So, now let us first understand the contract specification related to commodity options and then we will proceed with the use of detailed different method of method to value these commodity options. Black-Scholes option pricing model is changed to certain extent to arrive at the call premium and put premium associated with the commodity options because commodity options do not have spot commodities as the underlying, but they have commodity futures as underlying. Now to proceed further let us analyze the contract specification of a commodity option contract as given by the multi commodity exchange. So, this is a silver option contract where the underlying is silver futures contract. Please note that the underlying is not silver, but the underlying is silver futures contract. As you can see, this is an option on silver future and what is the option type in the exchange we have at multi commodity exchange we have European call and put options gets traded and what is the trading unit, one MCX silver futures contract and rest of the things which we have discussed also in the earlier in case of a futures contract those remain same, but what is interesting or what is different here is the number of strikes. Please note that at any given point in time multi commodity exchange allows 25 in the money 25 out of money and one near the money contract. So, what do we mean by this in the money, out of money and at near money, we have discussed in the last session but let us understand what the meaning is of this 25 in the money, 25 out of money and one near the money contract with respect to the strikes. Please see that here also the exchange mentioned that the minimum price interval is going to be 250. So, let us see this in the money and out of money will be always calculated based on the daily settlement price for the futures. Let us say we are standing on 28th May 2023. So, as a trader, which exercise prices will be available to me. So, this will be based on the daily settlement price of the futures contract which is prevailing on the previous day. Let us say on 27th May 2023, the underlying futures contract the contract which is you know underlying this particular option contract that particular contract closed at 70,235. So, the daily settlement price is 70,235. So, this is going to be the price based on which other exercises will be available. So, what do you mean by that when our settlement price is 70,235 the near the money options strike price

is going to be 70,250. One may ask why the 70,000 contract will not be considered as near the money, as you can see 70,250 is going to be nearest to the 70,235 as compared to 70,000. So, on this date near money option strike price is going to be 70,250 and what is going to be the upper strike price range. The upper strike price range is going to be 25 contracts. So, 70,250+25 *250 is going to give us 76,500 and the lower price range is going to be 70,250-25*250 which is going to be 64,000. So, on 28th may what are the strike price available for somebody to trade this is going to be 25 contracts in a multiple of 250 over and above 70,250 and 25 contracts in multiple of 250 below 70,250. Please note that why this is being done is because this these option contracts are exchange traded contracts. In the case of a bilateral OTC both parties are free to decide on any exercise price, but because these contracts are exchange traded contracts, there is standardization in terms of the expiry as well as standardization in terms of the strike price. So, this is how the standardization comes into picture and the exchange informs at any given point in time how many in the money option will be there and how many out of money option will be there. So, with this let us proceed to the next part of the contract specification. And this particular table also shows how options get traded on a given day. So, as you can see, the trade date is 26 May 2023, the type as you can see the type is OPTFUT that means, options on future. So, the trade date is 26 May 2023 and the option expiry this particular option is expiring on 21 June 2023, and this is an example of call option. And what is the exercise? Exercise is 70,000. So, we see that exercise will always be in the multiple of 250. So, exercise cannot be anything other than the multiples of 250 as just now we discussed. So, this is an example of call European CE stands for call European, PE stands for put European. Had it been American option it would have been written as CA or PA. As we mentioned that in India, we do not have any options organized exchange traded options which are American in nature and all these traded options are European in nature, hence the denomination is related to CE and PE. So, the exercise price is 70,000. I have 70,000 different exercise prices and this is a snapshot which I have taken from the multi commodity exchange website. And what I want to draw your attention is that these are the option premium. Please note that what buyers and sellers trade on the exchange platform is the option premium. So, what is the meaning of open, high, low, close? Open price is the price at which the first long call, short call option position to have been executed on 21 June 2020 on 26 May 2023 for the option which is expiring on 21 June 2023. Similarly, high is throughout the day what is the highest price at which both long call and short call position has been exchanged. And low and close exactly the same logic of close is not the last traded price close is the last half an hour average price of all call option prices, all call and long call and short call option prices to have been traded on that day. And volume indicates how many contracts have traded, open interest is on that day OI stands for your open interest and open interest is 142. Again, this is just a snapshot of the data to show how exactly exchanges report or inform the option of traded at their platform and please note that these options have the futures as

underlying. So, as you can see OPTFUT indicates options on futures. Now, the second block shows the traded details at CME Chicago Mercantile Exchange. As you can see, this product name is options and so have been meal futures. The way we also have at multi commodity exchange or NCDEX options have underlying futures also in CME you have commodity options have an underlying contract as futures and as you can see, they have the exercise style is American. In case of India we have European style, in case of you know CME they have a combination of American option and European option. For certain futures they have European options, for certain commodity or commodity futures they have European options and for certain commodities they have American option. We will come back to this margin which is what is equity here and also as you can see the underlying product name in case of a soyabean options and soyabean meal future the underlying is a soyabean meal future. So, this part is self-explanatory. Now, coming to the margin style please note that in case of an option, only the short call and short put option holder pay the margin. In case of a futures please note that daily mark to market margin both long futures as well as short futures can pay margin or receive margin, but in case of an options the long call and long put position holders have no risk only the short call and short put position holders have risk. So, in that case the whatever they are supposed to pay long call and long put pays the full premium upfront to the exchange. So, when they pay the full premium upfront to the exchange, they do not have any risk. So, they pay the full premium and future style margining indicates like daily mark to market style of margining and this is not applicable in Indian case, if some of you are more interested to understand what do you mean by future style margining that you have you can go to the Chicago Mercantile Exchange CME website and you can understand more on this future style margining. And let me summarize here what we discussed in India we have only European options gets traded, in CME you have American option as well as European options trade and when we are talking about margin style margin, style equity means the long call and long put position holder pay the premium upfront to the exchange and beyond that they do not have to pay any mark to market margin. But in case of the short call and short put position they have a tremendous amount of risk. So, they will be paying daily mark to market margin to the to the exchange, but only the long call and long put position holder whatever they are supposed to pay the premium they will pay that amount upfront to the exchange and beyond that they do not have to pay anything as part of this option trading. Now, come to the most interesting part of the commodity option which is the commodity option exercising mechanism. Please note that this first block shows the snapshot which I have taken from the multi commodity exchange website. So, I have restated the same thing here, as you can see on expiry of the option contract all ITM option positions are devolved into underlying futures position. Please note that the options have commodity options have futures as underlying and if you recall when we were talking about in the money, out of money and at the money option context. So, all in the money options will be exercised and when these in the

money options are exercised, the long call, short call, long put and short put position holder will be receiving the futures position they will be taking futures position they will not be taking an underlying spot position. So, long call positions are devolved into long futures position. So, somebody whoever has a long call ITM position that particular ITM option will be exercised, and that particular party will be getting a long futures position. Similarly, long put position holder will be devolving into the short futures position, the short call position will be devolving into the short futures position and the short put position will be devolving into the long futures position. And please note that all such devolved futures positions shall be opened at the strike price of the exercised option. So, one may you know get confused with this combination. So, let us take a numerical example to understand what we mean by long call position devolving into long futures position, long put position devolving into short futures position etcetera. Now, so, this particular table or this particular diagram explains this concept in detail, please note that let us say on day 0 a trader takes an option position. The trader can take a long call, can take a short call can take long put or short put, trader is free to take any of the option. So, this is on day 0 and let us say on day t, please see the denominator or please take note of the you know notation t is your small t is the expiry date of the option and on this expiry date of the option all in the money options will be exercised. So, trader takes an option position, these option positions have futures contract and underlying and as you can see this option expiry, please note that option expiry will always precede the underlying futures expiry. In this case the underlying futures expires on day t and the option is expiring on small t which is before the futures expiry. So, let us say on this day on the day t the option which is coming to an end all ITM options will be exercised and all ITM options will be taking either the long position or a short futures position on the they will be taking either the long futures position or short futures position which is going to be expiring on day t. Please note that on day 0 the long call and long put position holder will be paying the premium upfront, but from day 0 till day t the margin will be paid by short call or short put position holder. The long call and long put will not pay any margin because whatever they are supposed to pay to the counterparty in terms of the premium they have already deposited with the exchange. Now, on day t when the all in the money options are exercised traders will be taking long or short futures position and from this date till capital t date, they will be answerable to day mark to market margin the way we had discussed earlier that long and short futures position will be either paying or receiving the mark to market margin. So, with this let us go to a real-life example to understand more on this. As you can see all long call ITM option holders will take a long futures position and whenever a long call takes a long future position, the counterparty to the long call which will be a short call option holder will take a short futures position. Similarly, all long put in the money option holder will take a short futures position and the counterparty to a long put who is happens to be holding a short put position that particular party will be taking a long futures position. Let us say on the option expiry date

on that small t date, let us say the future price is future settlement price is 72,500. So, on the option expiry date which is the small t the futures are trading please note that the future is the underlying and future is trading and, on that day, the daily settlement price for the future is 72,500. Now, this 72,500 will be compared with all option holder positions. Let us say somebody is having a long call option position at a strike price of 71,750. The first thing we have to understand is whether this is an in the money option or out of money option. So, underlying is 72,500 and exercise is 71750 as that a call option will be in the money option if the underlying is greater than the exercise. So, in this case the underlying is greater than the exercise price it is going to be a in the money option. So, if it is in the money option so, long call holder please note that the long call in the money option holder will take a long futures position. So, what will be as part of the exercise what will be done as part of the exercise it will receive 750 from the short call position holder. Please note that for every long call there will be a short call. So, the moment this long call position holder is holding a in the money option holder there will be a short call option holder who will also be holding the holding the option at a exercise price of 71750. So, as part of this exercise both parties will be paying or receiving money in that sense the short call will pay 750 rupees to the long call position holder and the long call position holder will be taking a futures position will take a long futures position at 72500. So, where exactly this 750 is coming from please see that 72500 -71500 comes to your 750 rupees. So, when the underlying asset price is 72500 and the exercise price is 71750 long call is benefiting by 750, short call is losing by 750. So, short call will pay 750 rupees to the long call position holder and long call will take a long futures position at 72500 and the counterparty will take a short futures position at 72500. Now, coming back to another exercise price, let us say somebody is holding a long call at 73000, will it be in the money the answer is no it will not be a in the money it will be an out of money option how f t is 72500 if f t would have been greater than x then only it would have been a in the money because it is a call option. So, it is an out of money option. So, nothing will happen, the option will expiry no exercise will happen similarly let us go to a put option position. Let us say somebody is having a long-put position at 71750. So, the long-put option holder is holding an exercise at 71750, our underlying asset price is 72500. So, will it be a in the money option no because it is a put option put option will be in the money when x is greater than f t, x is not greater than f t, hence it will be an out of money option the moment it is out of money option nothing will happen the option will expire. Let us go to the next you know contract where you have a long-put option holder, but he is holding an asset or holding the option having an exercise price of 73000. So, is it an in the money option, yes. Why it is in the money option because x is greater than your f t. So, x is greater than f t by 700, x is greater than f t by 500 rupees. So, in that case it receives 500 from the short put option holder and it takes a short futures position at 72500. And exactly the counterparty payoff is going to be the opposite the short put position holder will be paying 500 to the counterparty and will take the long futures

position. So, as you can see from this particular table on the option expiry date you will have some party who is taking a long call at 72500 another party who is taking a sorry on the option expiry date somebody who is taking a long futures at 72500 somebody who is taking a short futures at 72500. So, these two parties are becoming counterparty to each other as part of a futures contract. Similarly, if you see that this party is taking a short futures contract and this particular party is taking a long futures contract. So, these two parties are becoming counterparty to each other. So, from this day which is your small t small t to the future expiry which is your capital T date both parties will be these parties will be trading with each other and will be acting as counterparty to each other. With this we will come to our end on discussion related to commodity options. Commodity options have commodity futures as underlying and when the commodity options expire depending on whether the commodity option is in the money out of money or at the money option for the traders will be receiving the futures position, they will either take long futures position or short futures position. In fact, I would urge each of you to spend a considerable amount of time to understand this particular detail because it is not a very easy thing to apprehend at the first time. So, I would appreciate if you wrote down in a piece of pen and paper what is the meaning of in the money and out of money and at the money option in context of commodity option and when a commodity option expiry happens how that you know in the money options are getting devolved into long futures position and the short futures position. This is one of the most critical or very interesting yet very confusing aspects of commodity options. Of course, it will come to one can understand it in a it is in a better manner if one spends some amount of time understanding the nitty-gritty of it. With this, we come to end of our discussion and today's session the remaining part of pricing and valuation of commodity futures and a commodity options aspect I will be discussing in the next class. Again, I eagerly look forward to interacting with all of you in the next session. Thank you all of you.