Decision Making Under Uncertainty

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Lecture - 19
Newsvendor Problem: Background, Model and Analysis

Topic 3 of this course: Decision Making Under Uncertainty, is about repeated decisions.

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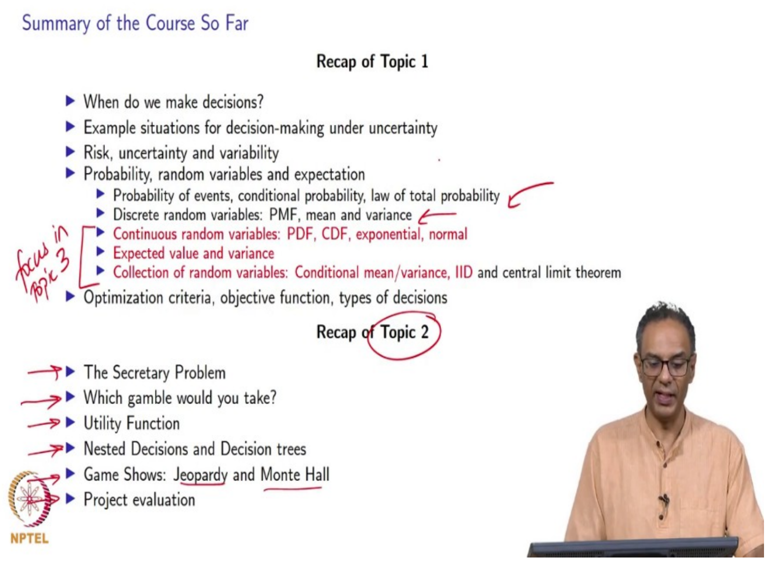

Before we say more about repeated decisions, let us do a quick recap of what we have seen so far. In topic 1, that was 2 weeks ago, we looked at making decisions, when do we do that? We also looked a little bit about some example situations, where we were looking at decision making under uncertainty and impact of uncertainty, and things like that. Then, the third topic was risk, uncertainty and variability. We spoke a little bit about what they are and how one goes about characterizing uncertainty and variability.

Finally, we moved into some mathematical items such as probability, random variables and expectations. And among those, these items in red, are the ones that we would focus on in topic 3 today. So, topic 3 would be one in which you would have to know a little bit more about continuous random variables, in particular about the PDF, CDF, the exponential distribution and the normal distribution; a lot of normal in this course especially in topic 3. We will also look at expected value and variance and also how to compute functions of a random variables expected value.

So, if this is a topic that you are not fully comfortable, I would suggest going back to topic 1, looking at those a little bit and make sure that you are very comfortable because I am expecting that level of comfort, when we look at topic 3. We will also touch upon collection of random variables a little bit; talking about IID random variables, not a whole lot, but if you can look to that. The main topic there is conditional mean and conditional variance; this is where we will use that concept that we learned in topic 1.

So, please go ahead and review that material if this is not particularly familiar; if not you are welcome to go ahead. The other topics and blocks such as probability of events and discrete random variables; we will not see a whole lot of in this topic. We also talked about optimization criteria. This one in this topic the optimization criteria would be somewhat straightforward, it would not be terribly complicated. We will look at a few of those. Then, after topic 1 and topic 2, we talked about a few things, we in particular talked about the secretary problem. This was also called the marriage problem or where we had n possible interviews and at the end of each interview, you had to decide whether or not to hire the person as a secretary.

Now, turns out that that problem is a onetime decision because you only hire a secretary at one time and after that you are not making any repeated. However, the topic for topic 3 is what is called repeated decision; you want to make decision over and over again. Now, after the secretary problem, we also looked at the gambling options; I gave you 3 options a, b and c and where the amount of money you put in was different and the amount of money you got back was also significantly different and we asked the question which one would you use.

Then, we talked about this concept of utility function that kind of normalizes this behaviour. I guess the word normal means something different in this course. It kind of puts things in one function, where if you had an exponential utility function for smaller values of the parameter r we were being a little aggressive and as r becomes larger we would become conservative and that is captured in the utility function. Then, we talked about nested decision, where we had this Island with the green cat and these we were finding ways to improve the cat population and we had to make nested decisions. What type? There were three different ways.

And we were coming up with strategies of whether to do a in vitro fertilization or whether we do cross breeding within the population or to do some type of a mainland captivity cross breeding, and seeing what is the best option. There to remember the decision was a onetime decision, although it was over 2 years, and although there were some nesting in there. The nested decisions were really if we think of the whole thing, the 2 year project, it was a onetime decision; unlike what we are going to see today. The game shows, we saw 2 game shows: Jeopardy and the Monte Hall problem. The Jeopardy problem we were trying to decide in the final Jeopardy: how much to wager? What is a good amount to wager?

And finally, in the Monte Hall problem, we looked at what would be a good way to make a decision in terms of should we pick the first door or should we go with the door that is not open? So, that was the problem, where there was a car behind one door and there were goats behind the other two and the game show host, who knows what is behind each asks you to pick 1 and then opens one of the other doors and ask do you want to switch?

It would be a good idea to go do a quick recap of that if you have not seen that already. Finally, towards the end of the topic, we looked at PERT Project Evaluation and Review Technique. There, we were looking at how to figure out if we can meet a deadline for a project or what deadlines to tell your customer. And one of the things there is, this is a onetime project; something you have never done before and how do you go about making that decision and that is the reason it was in topic 2, which was a onetime decision. In topic three, like I said in the previous slide this is about repeated decisions, we are making decisions again and again.

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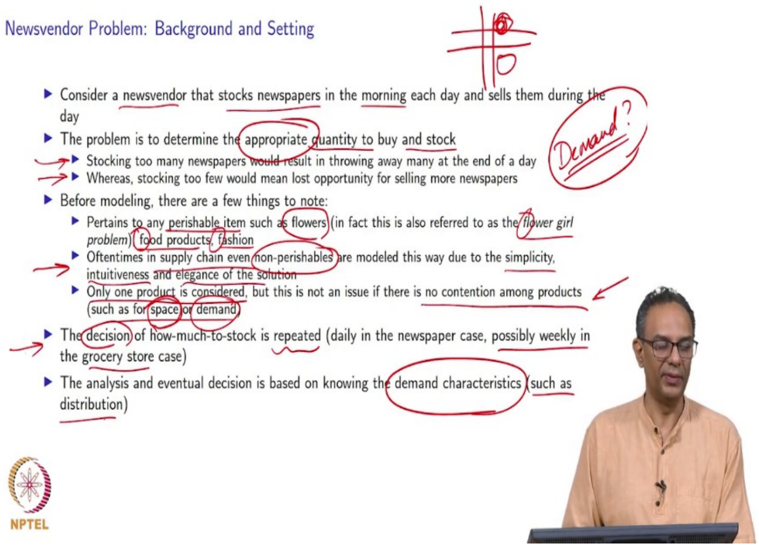

So, when you are in that type of a scenario, we will see that using an example called the newsvendor problem. In that type of a scenario, you know you are probably thinking maybe expected value is a good objective, which will be the case and all the problems we would see today. So, think of a newsvendor. We have all seen newsvendors in corner of a street and there will be a little shop, and in that little shop they would stack a bunch of newspapers. And in fact, they would have several newspapers; they would have various dailies in different languages and so on.

And this newsvendor; let us take one particular newspaper, just so that I can explain some of these things easily. So, our newsvendor stocks a bunch of newspapers early in the morning each day, then throughout the day they sell. In fact, 2 days ago I went and bought a paper at 4 PM so, that was pretty late in the day. However, you know typically the newspapers come really early in the morning, say even before 6 o'clock. And the vendor will need to sell those newspapers during the day. Then at the end of the day the newsvendor will throw away all the papers that are not used because nobody wants to buy the previous day newspaper.

So, in that way we want the newsvendor to decide: what is an appropriate quantity to buy and stock? So, how many newspapers does this person have to buy early in the morning? So, what are some of the issues? Well if this guy had way too many newspapers, then what will happen at the end of the day? Surely he is going to have a bunch that he will have to throw away.

Now, if you had too few then he is missing opportunities to sell more newspapers. So, think about this. So, if you had too few newspapers and people are coming and asking you: hey, do you have this paper and you are going- sorry sir it is all over or you have too many and this person is going- I have one too many. So, what is an appropriate amount? So, that is why we use the word appropriate. What is an appropriate amount to stock at the beginning of each day?

Now, to make that decision you are thinking- what would we need to know? Well, we need to get an idea of what is going to be the demand. So, the demand for newspapers-what is that going to be? Now, demand is a random quantity, usually you do not know exactly how many you are going to get each day. So, if you did, then it is easy. You stock exactly how many; the number that you are going to sell.

But since it is random, what is a good amount that you want to keep? Now, it turns out the demand in many situations is difficult. However, in the newspaper at the vendor in the corner of a store, it is not too bad because everybody who wants newspapers are going to ask him- do you have this particular newspaper? So, you have an idea of what your demand is. So, you could do some type of a study for a while to understand the demand distribution. So, I am assuming that one can do something like that.

Now, turns out that this is not just useful for the newspaper that we were talking about. Many other people do this. Now, there is one common feature: one of the feature is that the item is perishable. So, whenever you have something perishable like flowers. So, the newsvendor problem is used to be called the newsboy problem; it sounded too male oriented. So, they came up with something called the flower girl problem, which is exactly the same problem except instead of having newspapers.

So, this girl has flowers at the corner; another street corner and selling flowers and at the end of the day the flower is gone. Next morning she will have to buy a new set. Same situation- food products have the same perishable quality that means after a while the food products will perish. So, you have to do decide how much to have. Fashion is another thing that is perishable. I do want to warn you that just because flower girl starts with an f, food product starts with an f, fashion starts with an f; all our examples do not have to. This just so happen that way. So, fashion is another example that would perish after a while and then a new fashion would come in.

So, you will have to decide: how many do I want to keep in order to be sure that we want to maximize our expected profits. If you were; that is usually the objective. Now, turns out that in many supply chain situations, even non-perishables are modelled using what we call as newsvendor. Turns out that it is an extremely convenient way of modelling because it is simple; we will see that next. It is also intuitive which will again see and it is also elegant, but at least I think it is really elegant a solution.

So, it will give you a number to say this is a number that you need to hold provided you know the demand the distribution. If you know the demand, you just need one number. And that will be very nice and elegant and simple. Now, it is also important to know that we are only talking about one product, one particular newspaper, one particular brand of newspaper. But, as long as there is no contention among the products. If you think about a newspaper; people typically say: I want this! I want the Indian express for example. This person exactly knows what they want. So, there is no contention among the products.

So, and there is also no contention in terms of both space or demand. The person who comes in knows exactly the newspaper they want. If they want a newspaper: a regional language, they ask you for that. So, there is no contention between one newspaper and another. More importantly, there is no contention in terms of space. If the ones, the stores that I have seen; these guys typically just pile up a bunch of newspapers outside. There is not really much of an issue in terms of space. They could pile up ten times that amount if they wanted to.

However, if there is one, then we will have to think of different ways of modelling. So, we are going to just focus on one product, with the understanding that we will do the same type of analysis for all the products. Let us say you have a grocery store and you have food products in there. You want to do the same thing for all your food products. And when there is contention you will have to do some different analysis, where because there will be constraints now.

Now, let us move on to the next point, which says this decision that we make is repeated. Notice that the newsvendor is going to do this day after day; it is a daily newspaper. So, every morning this person has to decide. Or in the flower girl situation, the flowers perish. The next morning a new set of flowers and you have to decide that. Now in the grocery store case, it is probably weekly. Most of fruits and vegetables last for a whole week and therefore, you could make this decision a weekly manner.

So, I am not restricting the units, but we will pretend like you know we have some type of a pattern: daily or weekly. For example, fashion could be yearly and things like that. Now, turns out that we need to know the demand characteristics. We need to know the distribution of the demand. So, our analysis is going to depend on knowing that. So, you need some historical data or some way of knowing what is the distribution of the demand that you can expect.

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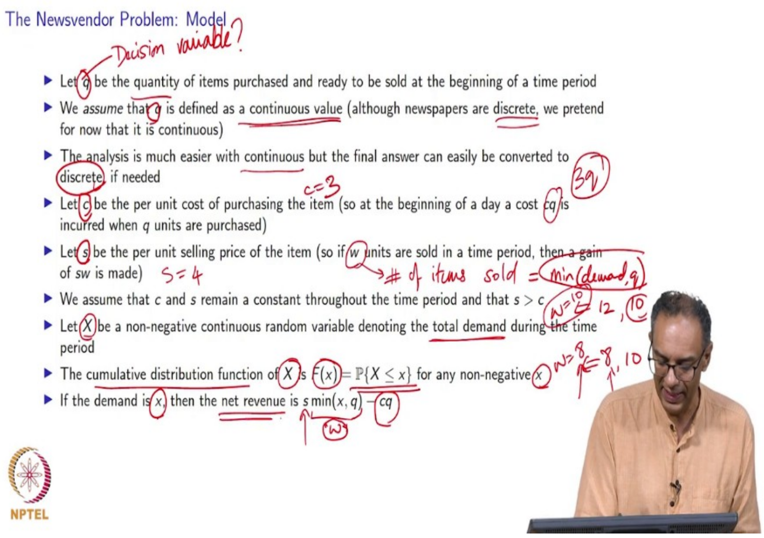

So, let us model the newsvendor problem. So, I am going to say the following: So, I am going to say let q: this is q for quantity that is why we use that letter q. So, let q be the quantity that you would stock at the beginning of the day. So, this is how you are going to keep many newspapers in your store, when the day begins. So, when you have the newspaper retailer you are going to tell them: hey, drop me off say 50 newspapers of this time.

We are going to assume that this q is a continuous quantity. I know the newspapers are clearly discrete, but we will see, we want a more general analysis. We will do an analysis pretending that this quantity is actually a continuous value because at some point we have to take the derivative and equate it to 0. And you know it really does not work very well when it is discrete. So, for that reason and others we will see, we are going to pretend like it is continuous. It really does not change a whole lot of the analysis, if it is discrete.

Now, turns out that you can get a nice cute expression if you had continuous and we are going to go with that. Now, we could always convert the final answer to a discrete quantity. I don't want to make a big deal out of it. So, let us just pretend like q is a continuous quantity. Now, we have two numbers that are important other than q; q is our decision variable. q is what we need to decide, (Refer Slide Time: 14:29) this is our decision variable.

Now, our solution is going to be in terms of 3 things: one is c, one is s, and the third is F(x). So, these 3 are important to make our decision. So, let me tell you what they are: c is the cost per item. I am going to keep it generic, but in particular we are thinking of a newspaper. So, for example, a newspaper could cost rupees 3: c = 3 rupees, for example. It is the price that the newsvendor pays for a newspaper. So, if the newsvendor buys q newspapers then it is 3q, that will be the cost that they would incur. So, for example, if the person bought 10 newspapers, then it would cost them 30 rupees to stock those newspapers.

Now, the selling price for example, let us say, s = 4. Some of the newspapers, the one that I buy, it costs 4 rupees. So, the person sells for 4 rupees and therefore, if they stock up, let us say, 10 newspapers; and they sell all 10 of them. Then, they will make 40 rupees. Therefore, (40 rupees - 30 rupees) and net profit is 10 rupees. However, there is a chance that not all 10 newspapers will be sold. However, many papers are sold, so that is w. So, this (Refer Slide Time: 16:00) is the number that are sold: number of items sold.

And we will find that that quantity is equal to essentially either: this is going to be either the minimum of the demand and q; right? If your demand is higher than q. ok. If your demand is higher than q, you are not going to be able to sell it. For example, if your demand is 12 newspapers and you only have 10 newspapers; you are only going to be able to sell 10. Right? Because even though there are 12 demand, the last 2 demands will not be satisfied because you do not have those 2 newspapers.

However, if your demand is only 8; you will only sell 8 newspaper. So, let us look at this example. Let us say q is 10 and let us say your demand is 12. Then, w is the smaller of the 2 numbers. Right? Because even though your demand was 12, the last two people who came to ask for newspapers; you are going to say: sorry! I don’t have the paper. ok. Because you only had 10 and you have sold them all.

However, in the second case, let us say you had; you had; a; I mean sorry you had 10 newspapers and your demand is only 8. Your w will then be 8 that is because you are going to only sell 8 newspapers. You are not going to sell 10 newspapers, when the demand is only 8. So, your profit in this case: in the 8 case will be, 8 times 4 is 32; minus you bought papers for 30 rupees; So, the profit is only 2 rupees.

However, when the w equals 10 case, your profit is going to be 10 rupees because you are going to get 40 rupees, and you have used up 30 rupees to buy your item. So, your profit is 10 rupees. So, now we are asking the question: So, we have this random variable X that is going to be the demand, and we are asking the question: what is the optimal value of q? So, what is the decision variable q?

So, now the random variable x we would like to characterize using the CDF; remember the cumulative distribution function from topic 1. This is the probability that the random variable x is less than or equal to little x, $P(X \leq x)$, for any non-negative x. The demand for items are usually non negative; nobody wants a negative quantity. So, you are going to pretend like it is a positive number. (Refer Slide Time: 18:28) So, the demand is x, then like I said before, this is w that we had before. This is a number you will sell; you will either sell the x itself, if x is smaller than q.

So, right. If your demand is smaller than how many you had, then you will sell that many. If your demand is larger, if x is greater than q, then you will only sell q of them because that is how many you had. I do want to repeat this because this is usually very confusing to a lot of students. So, we say the number that you sell is the smaller of the two quantities, which we stated here (Refer Slide Time: 18:54): smaller of the demand and how much you have in stock. So, if you had more demand than how much you had in stock, it is what you have in stock that you will sell.

If you had lesser demand than what you had in stock, then what you had demand is what you will sell. ok. So, you will you will sell the smaller of the two and the selling price is s rupees. So, s multiplied by the w, which is the minimum of x and q, is the amount of money you will get. But, the profit is going to be minus how much you spent in terms of the item. ok. So, if you spent c times q, then that is the total amount of money. This entire quantity is going to be what is called the net revenue. (Refer Slide Time: 19:35) It is $s\ min(x,q)-cq$. This is going to be how much revenue you are going to get. ok. Now, the question is: can we find a value of q that maximizes my net revenue?

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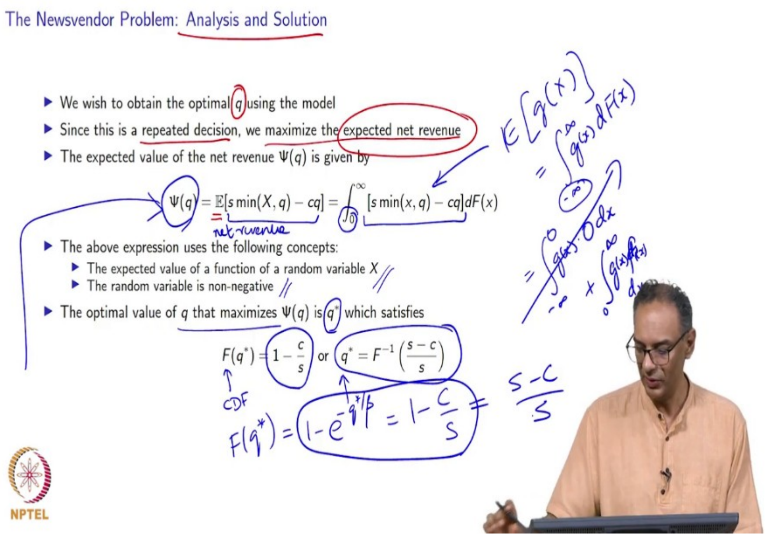

So, that is what we are going to analyze. We are going to say a little bit about how we analyze this and come up with a solution. It turns out that we want to get the optimal q, like I said using the model that we had in the previous slide. And this is a repeated decision! So, maximizing the expected net revenue is a good idea because on average you want to do well. ok.

So, maybe the first day you would get a profit, let us say for example, the demand was 8 and you will get a profit of only 2 rupees. Next day, let us say your demand is 12, you get a profit of 10 rupees. Next day let us say your demand is 11, again because you only sell 10; your profit is going to be 10 rupees. Then, the next day let us say your demand is only 4 in fact, you will go into a loss at that time and it goes up and down. So, we want to maximize the expected net revenue per day or you know per decision period. ok.

If it is in the fashion case, it is per year, and in the grocery store case it could be per week. ok. So now, turns out what we want to do is: we want to take the expected value of this random quantity: *s min(x,q)-cq*. We are saying that this is what we want to minimize! So, x is one realization of the demand; what we want to do is we want to take the expected value.

So, if you take the expected value of *E[s min (X,q) – cq]*. So, this is your expected net revenue, and you want to minimize that. How do we compute? Remember this from the expected value of a function of a random variable (say, *E[g(x)]*). So, that is equal to,

$$\int_{-\infty}^{\infty} g(x)\,dF(x).$$

This was from topic 1; we saw this. We are doing exactly the same thing. So, now our function is,

$$\int_{0}^{\infty} [s\,min(x,q)-cq]\,d\,F(x).$$

Now, the limits of the integral is only going from 0 to infinity as opposed to negative infinity.

That is because I could easily write this down as $\int_{-\infty}^{0} g(x).0\,dx$. Because the demand is always non negative! So, for all negative values, the probability density function is 0. So, that is why that goes away. I just want to show you that why we only go the limits from 0 to infinite

$\int_{0}^{\infty} g(x)f(x)d(x).$

So, this is my function (Ψ) that I would like to minimize,

$\Psi(q)=E[s\,min(X,q)-cq]=\int_{0}^{\infty} [s\,min(x,q)-cq]\,dF(x)$

And I want to say a couple of things. One is that to compute this, we did two things: we computed the expected value of a function of a random variable, and we also use the fact that the random variable is non-negative. And that is why we got that expression the way it is there. We will go over this calculation later.

But right now, I just want to tell you one thing. (Refer Slide Time: 23:17) if I were to compute the maxima of this function ($\Psi(q)$). The maxima occurs at q star. So, that is the best I could do and that q star is given by the CDF.

$F(q^{¿})=1-\frac{c}{s}$

The CDF at q^{*} equals $F(q^{¿})$.

$$q^{¿} = F^{-1}\left(\frac{s-c}{s}\right)$$

An F inverse is essentially the inverse of the CDF.

So, for example, if you had a certain distribution function (Refer Slide Time: 23:50). Let us say a particular CDF for example,

$$F\left(q^{¿}\right) = 1 - e^{\frac{-q^{¿}}{\beta}} = 1 - \frac{c}{s}$$

Then, all I am saying here is, I can solve for q star. And the way I write q star in terms of the other things is essentially writing down $q^{¿} = F^{-1}\left(\frac{s-c}{s}\right)$. So, taking the inverse of that it gives me F of q star. So, that is essentially what we going to do. I will show you how to compute that and show you an example when we get to it next. We will stop here for now!